

Research Article

Bouncing Cosmology in $f(G, T)$ Gravity with Logarithmic Trace Term

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This study is devoted to explore bouncing cosmology in the context of $f(\mathcal{G}, T)$ theory of gravity. For this purpose, a Gauss–Bonnet cosmological model with logarithmic trace term is considered. In particular, the possibility of obtaining bouncing solutions by considering two equations of state parameters is investigated. A graphical analysis is provided for analyzing the obtained bouncing solutions. The energy conditions are discussed in detail. It is interesting to notice that null and strong energy conditions are violated near the neighborhood of bouncing points justifying the accelerating universe in the light of the recent observational data. The behavior of the scale factor, red shift function, deceleration parameter, and Hubble parameter is also debated. An important feature of the current study is the discussion of conservation equation in $f(\mathcal{G}, T)$ gravity. The possibility of some suitable constraint equations which recover the standard conservation equation is discussed, and all the free parameters are assumed accordingly. All the results in this study suggest that the proposed $f(\mathcal{G}, T)$ gravity model provides good bouncing solutions with the chosen EoS parameters.

1. Introduction

Bouncing universes are possible alternatives to standard big bang cosmology. Unfortunately, some of the older studies of bouncing universes raise the issues such as cusp and angular-momentum barrier, thus leaving this turnaround of nature quite ambiguous [1]. Interest in bouncing universes was declined after the onset of first cosmological singularity theorem [2, 3]. In particular, some new types of isotropic cosmological models were proposed without singularity [4]. However, a standard big bang cosmological model has also some shortcomings such as horizon problem, flatness issues, transplanckian problem, and entropy problem. Therefore, in recent era, the investigation about bouncing cosmological solutions is an interesting and attractive topic for researchers. These solutions involve the cosmological models that replace the big bang cosmological singularity with a “big bounce” scenario, a smooth transition from contraction to the expansion phase [5, 6]. In this situation, the contraction phase of the universe is dominated by matter, and also, a nonsingular bounce is occurred. Moreover, the density

perturbations having spectrum consistent with the observational data can be produced [7]. The so-called BKL instability can be witnessed after the contracting phase making the universe anisotropic [8]. The possible ways of avoiding BKL instability and issues of the bounce in the ekpyrotic scenario have been studied [9–12]. It has been explicitly confirmed that spatially flat nonsingular bouncing cosmologies corresponds to effective theories of gravity [13]. Libanov et al. [14] studied flat bouncing cosmological models with the early time Genesis epoch in the context of generalized Galileon theories. They found that the bouncing models either possessed these instabilities or had some singularities. The presence of a nonsingular bounce in a flat universe may imply violation of the null energy condition (NEC), which can be obtained via a ghost condensation phase. Kobayashi [15] argued that the NEC could be violated in the generalized Galileon theories supporting the possibility of nonsingular cosmology. However, it was argued that on many occasions, cosmological solutions were plagued with instabilities. Furthermore, within the framework of the effective field theory and beyond Horndeski theory, it has

been shown that stable bouncing cosmologies can be constructed [16–20]. The investigation of some cosmological issues in extended gravity seems interesting because the big bang singularity could be replaced by a big bounce using modified gravity models [21–25].

Barragan et al. [26] studied oscillating cosmology in $f(R)$ Palatini formalism, and it was shown that the big bang singularities could be replaced by a big bounce without violating energy conditions. In fact, the bounce is possible even for pressureless dust in $f(R)$ gravity background. The metric version of $f(R)$ gravity has also been used to find the behavior of bouncing cosmology, and it was concluded that this behavior could solve the singularity problem in standard big bang cosmology [27]. Singh and his collaborators [28] investigated a cosmological model in a flat homogeneous and isotropic background in the context of $f(R, T)$ gravity. They proposed the Hubble parameter in a functional form such that it fulfilled the successful bouncing criteria to investigate the solution of the gravitational field equations without any initial singularity. Oikonomou [29] showed how a cosmological bounce could be evolved by a vacuum $f(\mathcal{E})$ gravity model, and the stability of the obtained solutions was addressed by analyzing a dynamical system of equations of motion. Analytic bounce in nonlocal Einstein–Gauss–Bonnet cosmology has been discussed, and the bouncing solutions are found to be stable during the bounce phase [30]. Escofet and Elizalde [31] investigated some Gauss–Bonnet (GB) extended gravity models exhibiting the bouncing behavior. They argued that how the addition of a GB term to a viable gravity model could influence some properties and even the physical nature of the obtained cosmological solutions. In particular, some new dark energy models can be proposed in which the equation of the state (EoS) parameter leads either to a big rip singularity or to a bouncing solution evolving into a de Sitter spacetime. A new theory by coupling GB term and trace of energy momentum tensor has been proposed and named as $f(\mathcal{E}, T)$ gravity [32], and it was proved that due to the presence of extraforce, the massive test particles could follow nongeodesic lines of geometry. Though $f(R, T)$ theories of gravity are the simplest modifications with matter coupling, the construction of some viable $f(R, T)$ gravity models is not an easy task. The main reason behind this is that Ricci modified gravity may produce a strong coupling between dark energy and a nonrelativistic matter in the Einstein frame [33]. While, some modified GB models may be consistent with solar system barriers under certain constraints [34]. Nojiri et al. [35] explored some $f(\mathcal{E})$ gravity nonminimally coupled models with matter Lagrangian and concluded that theories with such coupling may unify the inflationary era with current cosmic expansion. Thus, modified GB gravity with matter couplings can be more fascinating to study the universe in comparison with $f(R, T)$ gravity.

Motivated from the above discussions, this study focussed to examine bouncing cosmology in the context of $f(\mathcal{E}, T)$ theory of gravity. For this purpose, a GB cosmological model with logarithmic trace term is considered. The analysis is based upon two important EoS parameters. A

brief structure of the study is as follows: in Section 2, some basic important preliminaries to develop a required background for the analysis are provided. Section 3 is devoted to provide cosmological solutions and a detailed graphical analysis. Last section provides a brief summary of the work and conclusive remarks.

2. Some Important Preliminaries

The modified GB gravity can be described using the action [32]

$$\mathcal{A} = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} [R + f(\mathcal{E}, T)] + \int d^4x \sqrt{-g} \mathcal{L}_M, \quad (1)$$

where \mathcal{E} and T represent the GB term and trace of the stress-energy tensor, respectively. \mathcal{L}_M stands for the standard matter Lagrangian, R being the Ricci scalar, κ is a coupling parameter, and g is the determinant of metric tensor. The modified field equations are obtained by metric variation of action above [32].

$$\begin{aligned} R_{\eta\zeta} - \frac{1}{2}g_{\eta\zeta}R + [2Rg_{\eta\zeta}\nabla^2 - 2R\nabla_\eta\nabla_\zeta - 4g_{\eta\zeta}R^{\mu\nu}\nabla_\mu\nabla_\nu \\ - 4R_{\eta\zeta}\nabla^2 + 4R_\eta^\mu\nabla_\zeta\nabla_\mu + 4R_\zeta^\mu\nabla_\eta\nabla_\mu + 4R_{\eta\mu\zeta\nu}\nabla^\mu\nabla^\nu] f_{\mathcal{E}}(\mathcal{E}, T) \\ - \frac{1}{2}g_{\eta\zeta}f(\mathcal{E}, T) + [T_{\eta\zeta} + \Theta_{\eta\zeta}] f_T(\mathcal{E}, T) + [2RR_{\eta\zeta} - 4R_\eta^\mu R_{\mu\zeta} \\ - 4R_{\eta\mu\zeta\nu}R^{\mu\nu} + 2R_\eta^{\mu\nu\delta}R_{\zeta\mu\nu\delta}] f_{\mathcal{E}}(\mathcal{E}, T) = -\kappa^2 T_{\eta\zeta}, \end{aligned} \quad (2)$$

where all the symbols involved have their usual meanings. Moreover, the subscript \mathcal{E} and T appearing in the functions are to denote the partial derivatives and $\Theta_{\zeta\eta} = g^{\mu\nu}(\delta T_{\mu\nu}/\delta g_{\zeta\eta})$. The trace of equation (2) turns out to be

$$\begin{aligned} R + \kappa^2 T - (T + \Theta) f_T(\mathcal{E}, T) + 2f(\mathcal{E}, T) + 2\mathcal{E} f_{\mathcal{E}}(\mathcal{E}, T) \\ - 2R\nabla^2 f_{\mathcal{E}}(\mathcal{E}, T) + 4R^{\zeta\eta}\nabla_\zeta\nabla_\eta f_{\mathcal{E}}(\mathcal{E}, T) = 0. \end{aligned} \quad (3)$$

It is worth noticing that if $f(\mathcal{E}, T) = 0$ is replaced in equation (3), GR is recovered as

$$R + \kappa^2 T = 0. \quad (4)$$

The important aspect of equation (3) is that it relates R , \mathcal{E} , and T differentially. However, as evident by the corresponding GR version in equation (4), R and \mathcal{E} are manipulated algebraically. This clearly suggests that the modified field equations will have more solutions than usual GR. The covariant divergence of equation (2) is given by

$$\begin{aligned} \nabla^\zeta T_{\zeta\eta} = \frac{f_T(\mathcal{E}, T)}{\kappa^2 - f_T(\mathcal{E}, T)} [(T_{\zeta\eta} + \Theta_{\zeta\eta}) \nabla^\zeta (\ln f_T(\mathcal{E}, T)) \\ + \nabla^\zeta \Theta_{\zeta\eta} - \frac{g_{\zeta\eta}}{2} \nabla^\zeta T]. \end{aligned} \quad (5)$$

It can be seen that the conservation equation is not covariantly divergent here as in the case of GR. It is due to the involvement of higher order derivatives of the matter components in the modified field equations due to the matter coupling. Thus, a little drawback is that this theory might be suffered by divergences at cosmological scales. This is an issue with other theories as well that includes higher order terms of energy momentum tensor, such as the $f(R, T)$ theory of gravity. However, to deal with the issue, some constraints are put to equation (5) to recover the standard conservation equation. In present work, cosmology in this modified theory by considering the flat Friedmann–Lemaître–Robertson–Walker (FLRW) spacetime is investigated:

$$ds^2 = dt^2 - a^2(t)[dx^2 + dy^2 + dz^2], \quad (6)$$

and assumed that the universe is filled with perfect fluid:

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - pg_{\mu\nu}, \quad (7)$$

where a , ρ , and p represent the cosmic scale factors of universe, energy density, and pressure of the fluid, respectively. The study of energy conditions involving these important parameters (energy density and pressure) has many significant applications in cosmology. For example, one can easily investigate the validity of the second law of black hole thermodynamics and Hawking–Penrose singularity theorems by using energy conditions [36]. In relativistic cosmology, many interesting constraints have been described by the use of energy conditions [37–46]. Mainly five different types of energy bounds are found in the literature:

- Trace energy condition (TEC), now abandoned
- Null energy condition (NEC)
- Weak energy condition (WEC)
- Strong energy condition (SEC)
- Dominant energy condition (DEC)

The TEC suggests that the trace of the energy-momentum tensor should always be negative (or positive depending on metric conventions). This condition was popular among the researchers during the decade of 1960. However, once it was shown that stiff EoS, such as those which are appropriate for neutron stars, violates the TEC [47, 48]. Thus, the study of this energy condition was not further encouraged, and it is now completely abandoned, in fact no longer cited in the literature. However, the remaining four energy constraints are known as a necessary feature for the cosmological discussions. For an acceptable cosmological model, these constraints should be validated. For this purpose, the most important requirement is the positivity of energy density. However, the negative pressure may indicate the presence of exotic matter. In fact, the violation of energy conditions may lead to some fascinating cosmological features. The violation of these conditions may yield some instabilities and ghost pathologies in the presence of a canonical scalar field. The SEC is currently the most heated subject of discussions. It has been argued that the SEC should be violated in the inflationary era [48]. Most

importantly, the SEC is violated on cosmological scales in the light of the recent observational data regarding the accelerating universe [37]. Also, the minimal condition for a cosmological bounce rather than a big bang singularity requires the violation of SEC [12]. Moreover, violation of DEC is typically associated with either a large negative cosmological constant or superluminal acoustic modes [49].

2.1. $f(\mathcal{G}, T)$ Gravity Model with Logarithmic Trace Term. Now, for FLRW spacetime (6) with perfect fluid, the field equation (2) takes the form

$$6\frac{\dot{a}^2}{a^2} - 24\frac{\dot{a}^3}{a^3}\dot{f}_{\mathcal{G}} + \mathcal{G}f_{\mathcal{G}} - f - 2(\rho + p)f_T = 2\kappa^2\rho, \quad (8)$$

$$-2\left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right) + 16\frac{\dot{a}\ddot{a}}{a^2}\dot{f}_{\mathcal{G}} + 8\frac{\dot{a}^2}{a^2}\ddot{f}_{\mathcal{G}} - \mathcal{G}f_{\mathcal{G}} + f = 2\kappa^2 p, \quad (9)$$

where $f \equiv f(\mathcal{G}, T)$ and $f_{\mathcal{G}} \equiv f_{\mathcal{G}}(\mathcal{G}, T)$ are considered for the sake of simplicity. Due to complicated and highly nonlinear nature of field equations, sometimes it becomes very difficult to choose a particular $f(\mathcal{G}, T)$ model which could provide some viable results both analytical and numerical. The simplest choice is to consider a linear combination [51]:

$$f(\mathcal{G}, T) = f_1(\mathcal{G}) + f_2(T). \quad (10)$$

In present study, $f_1(\mathcal{G}) = \mathcal{G} + \lambda\mathcal{G}^2$, with λ being a real constant is proposed. This choice is important as the similar power law $f(\mathcal{G})$ gravity model has been studied with some interesting results [50]. Most importantly, following the work of Elizalde et al. [21], consider $f_2(T) = 2\beta\text{Log}(T)$, where β is an arbitrary model parameter. Thus, the proposed $f(\mathcal{G}, T)$ model takes the form

$$f(\mathcal{G}, T) = \mathcal{G} + \lambda\mathcal{G}^2 + 2\beta\text{Log}(T). \quad (11)$$

To the best of our knowledge, this is the first such attempt to consider logarithmic trace term in the study of $f(\mathcal{G}, T)$ cosmology. Thus, in this case, TEC must be satisfied to obtain realistic results from the logarithmic term.

2.2. Bouncing Cosmology and Equation of State. In recent years, there has been an increasing interest of the researchers in cosmological models that replace the big bang cosmological singularity with a “big bounce,” a smooth transition from contraction to the expansion phase. In order to resolve the fundamental problems in cosmology, the study of cosmological dynamics in modified gravity seems interesting because the big bang singularity could be avoided by a big bounce using modified gravity models [21–25]. The behavior of the bouncing universe can be judged by the evolution of the scale factor and Hubble parameter. One of the indications is that the size of the scale factor gets contracted to some finite volume not necessarily zero and then shows an increasing trend. Another possibility to indicate a bounce is when the Hubble parameter becomes zero and then blows

up. Mathematically, there must exist some finite points of time at which the size of the universe attains a minimum value. Another indication is the violation of NEC for some period of time near the neighborhood of the bounce point in the context of FLRW spacetime. Moreover, the EoS parameter goes in the negative range, especially a bouncing cosmology with $\omega \approx -1$ justifies the current cosmic expansion [52–54]. The most interesting aspect of studying bouncing cosmology is that the cosmic singularity problem can be avoided, geodesically complete evolution can be enabled, chaotic mixmaster behavior can be eliminated, the horizon problem can be resolved, the smoothness and flatness issues can be tackled, and the small entropy at the onset of the expanding phase can be naturally explained [55].

EoS parameter is an important constituent to study cosmological dynamics, in particular in the context of modified gravity. In this analysis, two interesting proposals were discussed [21]. First, the possibility of obtaining a bouncing solution in $f(\mathcal{G}, T)$ gravity described by the following EoS parameter is considered:

$$\omega_1(t) = -\frac{k \text{Log}(t + \varepsilon)}{t} - 1, \quad (12)$$

where k is any arbitrary constant, and ε is the very small real parameter. It is interesting to notice from equation (12) that ω varies from negative infinity as $t \rightarrow 0$ to $\omega_1 = -1$ (cosmic expansion phase) when $t = 1 - \varepsilon$. Second, the bouncing solution of $f(\mathcal{G}, T)$ gravity models by considering the following EoS is investigated:

$$\omega_2(t) = \frac{r}{\text{Log } t} - s, \quad (13)$$

where r is a negative parameter, while s is a positive parameter. In this case, witness that ω_2 varies from negative infinity as $t \rightarrow 1$ to the cosmic expansion era at $t = e^{(r/s-1)}$ and moves on, eventually coming back to again the same phase as t approaches positive infinity and $s = 1$. Elizalde et al. [21] obtained viable cosmological solutions in the framework of $f(R, T)$ theory of gravity using these two interesting choices of the EoS parameter. In this study, their work in the context of $f(\mathcal{G}, T)$ gravity is extended.

3. Cosmological Solutions and Numerical Analysis

In this section, the main focus is to discuss the evolution of energy density and pressure profile by using some suitable choice of the scale factor. For this purpose, the possible choice of the Hubble parameter which could provide viable bouncing cosmology is first discussed. A well-known functional form of the Hubble parameter as described in the following equation is considered [21].

$$H(t) = \alpha h(t) \text{Sin}(\xi t), \quad (14)$$

where $h(t)$ is an arbitrary smooth function, and α and ξ are the real constants. Mathematically, it is an interesting form of the Hubble parameter as the trigonometric sine function vanishes at some periodic values of t . Furthermore, such an

analytic form of $h(t)$ which could provide the nonzero value at these points can be considered. Thus, a specific form of $h(t)$ is given by

$$h(t) = e^{\zeta t}, \quad (15)$$

where ζ is any arbitrary real parameter. Thus, the complete parameterized form of the Hubble parameter turns out to be

$$H(t) = \alpha e^{\zeta t} \text{Sin}(\xi t). \quad (16)$$

This form of the Hubble parameter is important as it allows to obtain the behavior of the cosmic scale factor in the later stages of the evolution of the universe. This form of the Hubble parameter provides the scale factor:

$$a(t) = \kappa \exp\left(\frac{\alpha e^{\zeta t} (\zeta \text{Sin}(\xi t) - \xi \text{Cos}(\xi t))}{\zeta^2 + \xi^2}\right), \quad (17)$$

where κ is an arbitrary integration constant. Now, investigate bouncing cosmology in the below subsections for the abovementioned two different EoS cases.

3.1. $f(\mathcal{G}, T)$ Cosmology: $\omega_1(t) = -(k \text{Log}(t + \varepsilon)/t) - 1$. Here, the universe is assumed to be dominated by the matter with the EoS given by (12). The field equations (8) and (9) are simplified using equation (12), for details see Appendix. The evolution of energy density and pressure of the universe using the scale factor (17) are shown in Figure 1. The left plot indicates that the energy density is negative within the neighborhood of the bouncing point $t = 0$. Though, it is not physical and one can get a better result (positive in the immediate neighborhood) by manipulating the parameters involved. Since this negative trend is for a very small duration and positive energy density is witnessed very soon, leave as it is. The right plot indicates that the pressure is negative, which might be an indication of accelerated expansion of the universe. The left plot of Figure 2 depicts that NEC is violated near the neighborhood of the bouncing point.

This justifies the indication of bouncing universe with violation of NEC for some period of time near the neighborhood of the bounce point in the context of FLRW spacetime. Similarly, WEC and SEC are also violated as evident from left plots of Figures 1 and 2. This also ensures the fact that the SEC is violated on cosmological scales in the light of the recent observational data regarding the accelerating universe [37]. Figure 3 describes that DEC and TEC are satisfied with the chosen values of parameters. This provides the justification that the chosen cosmological model is well behaved. In particular, the choice of parameters strictly depends upon the evolution of energy density and TEC. It is worthwhile to mention here that TEC sometimes gets violated; however, in our case, it must be satisfied due to the involvement of the logarithmic function in the $f(\mathcal{G}, T)$ gravity model. The well behaved behavior of the scale factor and red shift function is shown in Figure 4.

According to a successful bouncing model, the Hubble parameter passes through zero from $H < 0$ when the

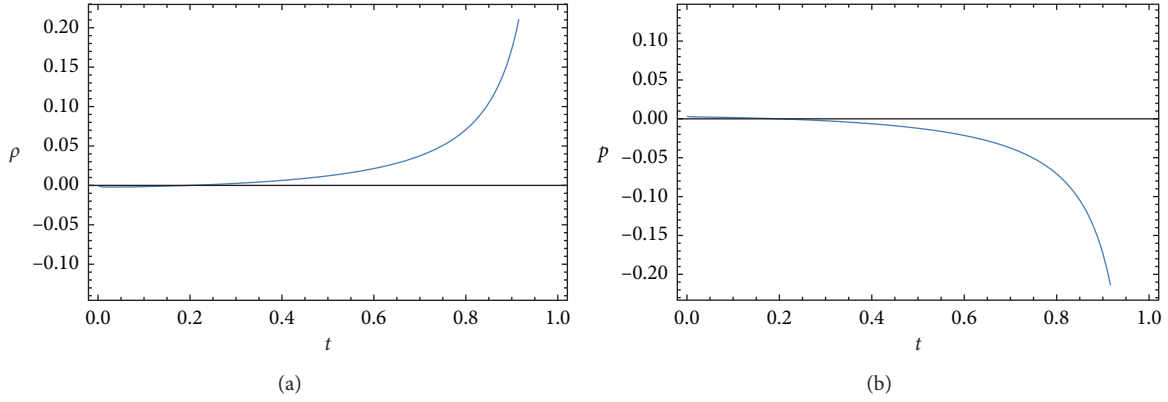


FIGURE 1: Energy density and pressure profiles for $\omega_1(t) = -(k\text{Log}(t + \varepsilon)/t) - 1$ with $\alpha = 0.001$; $\kappa = 0.5$; $\varepsilon = 0.001$; $\zeta = 0.01$; $\xi = 0.01$; $k = -0.001$; $\beta = 0.005$; $\lambda = 0.005$.

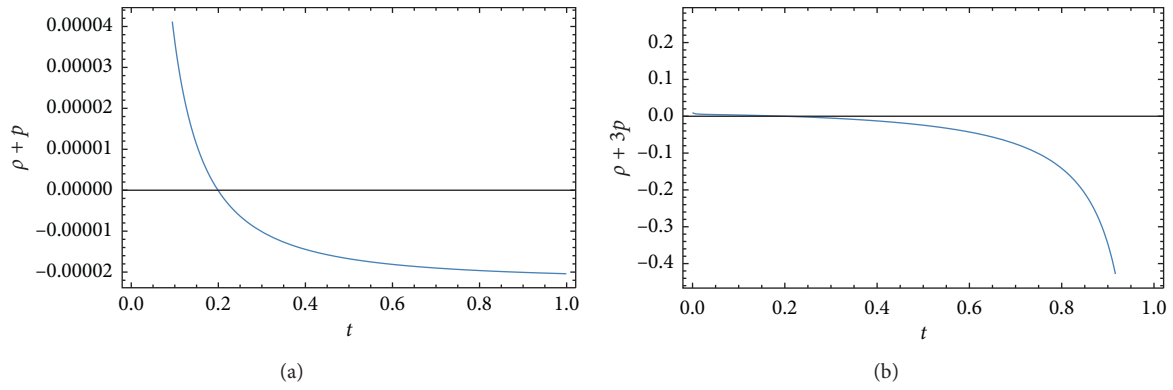


FIGURE 2: Evolution of NEC and SEC for $\omega_1(t) = -(k\text{Log}(t + \varepsilon)/t) - 1$ with $\alpha = 0.001$; $\kappa = 0.5$; $\varepsilon = 0.001$; $\zeta = 0.01$; $\xi = 0.01$; $k = -0.001$; $\beta = 0.005$; $\lambda = 0.005$.

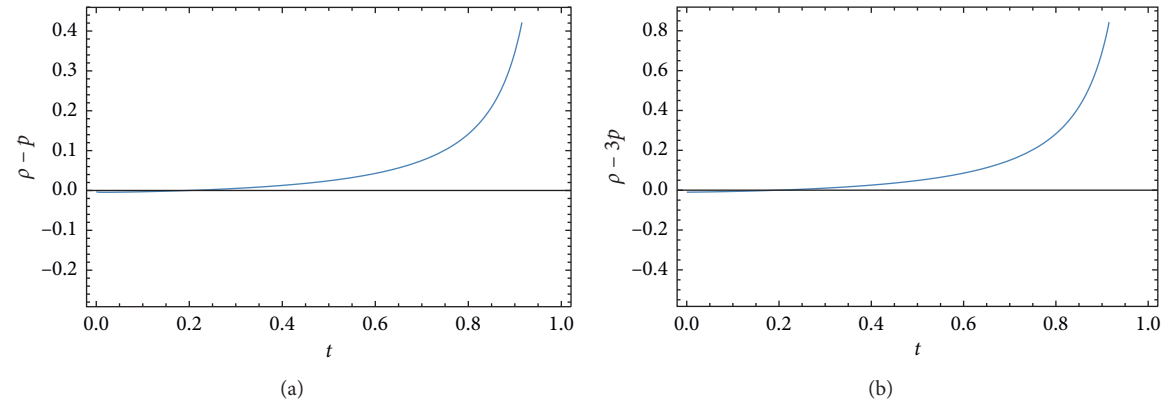


FIGURE 3: Evolution of DEC and TEC for $\omega_1(t) = -(k\text{Log}(t + \varepsilon)/t) - 1$ with $\alpha = 0.001$; $\kappa = 0.5$; $\varepsilon = 0.001$; $\zeta = 0.01$; $\xi = 0.01$; $k = -0.001$; $\beta = 0.005$; $\lambda = 0.005$.

universe contracts to $H > 0$ when the universe expands and $H = 0$ when the bouncing point occurs. This feature of a bouncing model in our case is evident from the left plot of Figure 5. The deceleration parameter q in cosmology is defined as

$$q = -\frac{\ddot{a}a}{\dot{a}^2}. \quad (18)$$

The negative trend of the deceleration parameter is shown in the right plot of Figure 5. The behavior of the EoS

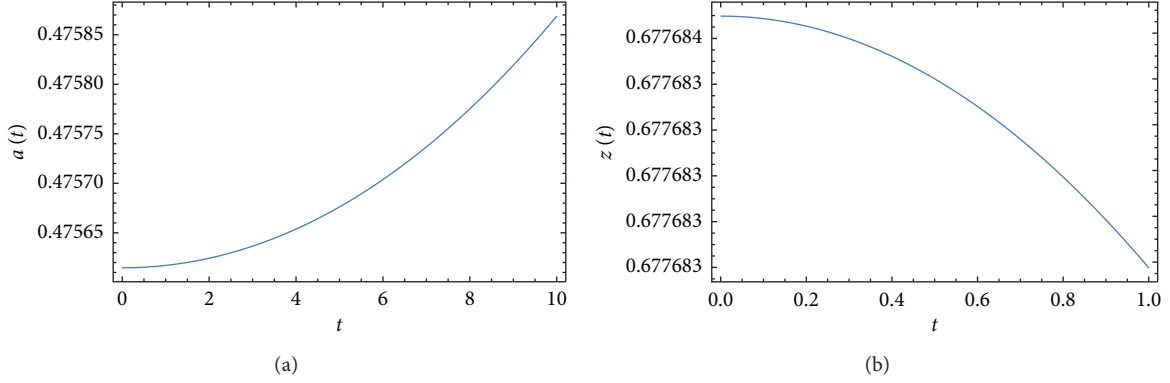


FIGURE 4: Evolution of the scale factor and red shift function for $\omega_1(t) = -(k\text{Log}(t + \epsilon)/t) - 1$ with $\alpha = 0.001$; $\kappa = 0.5$; $\epsilon = 0.001$; $\zeta = 0.01$; $\xi = 0.01$; $k = -0.001$; $\beta = 0.005$; $\lambda = 0.005$.

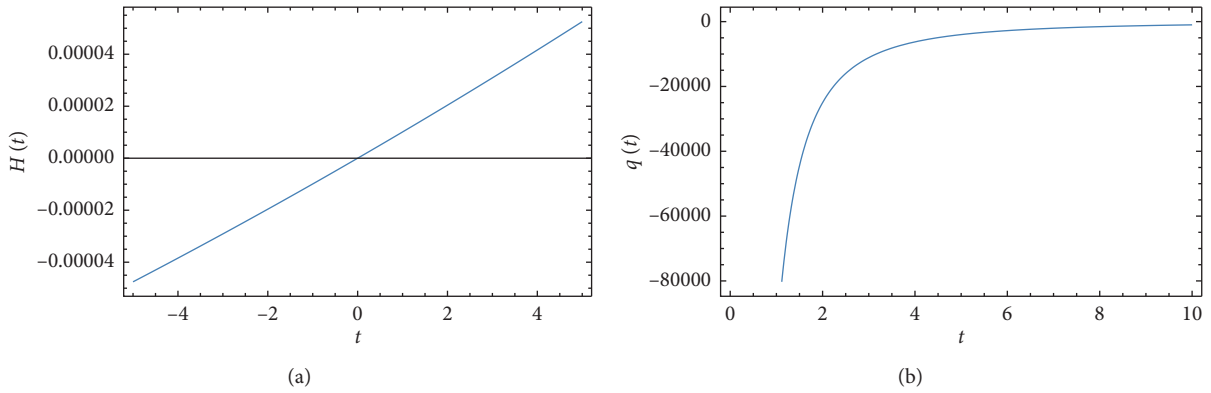


FIGURE 5: Evolution of Hubble and deceleration parameters for $\omega_1(t) = -(k\text{Log}(t + \epsilon)/t) - 1$ with $\alpha = 0.001$; $\kappa = 0.5$; $\epsilon = 0.001$; $\zeta = 0.01$; $\xi = 0.01$; $k = -0.001$; $\beta = 0.005$; $\lambda = 0.005$.

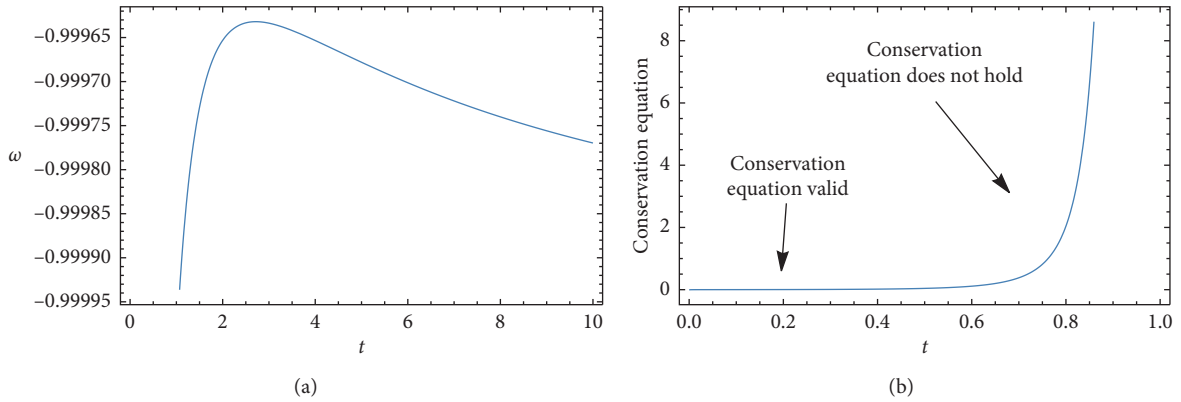


FIGURE 6: Evolution of EoS parameter $\omega_1(t) = -(k\text{Log}(t + \epsilon)/t) - 1$ and behavior of conservation equation with $\alpha = 0.001$; $\kappa = 0.5$; $\epsilon = 0.001$; $\zeta = 0.01$; $\xi = 0.01$; $k = -0.001$; $\beta = 0.005$; $\lambda = 0.005$.

parameter is depicted in the left plot of Figure 6. It is shown that the EoS parameter remains negative after the bouncing point, in particular $\omega_1 \approx -1$, which justifies the current cosmic expansion [52–54]. In particular, bouncing universe in this case supports the lambda cold dark matter (Λ -CDM) model; since when the cold dark matter dominates the

evolution of the universe, the value of EoS parameter is negative and closer to zero (as evident in our case).

One of the important aspects of this study is the discussion of conservation equation. Modified theories of gravity which involve matter curvature coupling do not act as per usual conservation of GR. In case of $f(\mathcal{L}, T)$ gravity,

this issue is evident from equation (5). Perhaps, there is a little drawback that the theory might be plagued by divergences at astrophysical scales. However, one can put the following constraint on equation (5) to deal with the issue and standard conservation equation can be recovered.

$$(T_{\zeta\eta} + \Theta_{\zeta\eta})\nabla^{\zeta}(\ln f_T(\mathcal{E}, T)) + \nabla^{\zeta}\Theta_{\zeta\eta} - \frac{g_{\zeta\eta}}{2}\nabla^{\zeta}T = 0. \quad (19)$$

The exact solution of this equation is very difficult to obtain due to highly nonlinear terms. However, in the current study, the parameters are in such a way that this equation is partially satisfied. It can be seen in the right plot of Figure 6 that conservation equation is satisfied in the neighborhood of the bouncing point but deviates as the time passes. All the above discussions and graphical analyses suggest that the proposed $f(\mathcal{E}, T)$ gravity model provides good bouncing solutions with the chosen EoS parameters.

3.2. $f(\mathcal{E}, T)$ Cosmology: $\omega_2(t) = (r/\text{Log } t) - s$. Now, the universe is considered as dominated by the matter with the EoS given in (13). Here, also the field equations (8) and (9) are simplified using equation (13) as in the previous case, for details see Appendix. The evolution of energy density and pressure of the universe is shown in Figure 7. Left plot indicates that the energy density is positive within the neighborhood of the bouncing point $t = 0$. However, the right plot indicates that the pressure is negative, which might be an indication of accelerated expansion of the universe. Left plot of Figure 8 depicts that NEC is violated near the neighborhood of the bouncing point. In this case, the indication of bouncing universe with violation of NEC for some period of time near the neighborhood of the bounce point in the context of FLRW spacetime is also justified. Here, WEC and SEC are also violated as evident from the left plot of Figures 7 and 8. Figure 9 describes that DEC and TEC are satisfied with the chosen values of parameters. This provides the justification that the chosen cosmological model is well behaved. In particular, the choice of parameters strictly depends upon the evolution of energy density and TEC. It is worthwhile to mention here that TEC sometimes gets violated; however, in our case, it must be satisfied due to the involvement of logarithmic function in the $f(\mathcal{E}, T)$ gravity model. The well behaved behavior of the scale factor and red shift function is shown in Figure 10.

The negative trend of deceleration parameter is shown in the right plot of Figure 11, while the behavior of the EoS parameter is shown in the left plot of Figure 11. Here, the EoS parameter also remains negative after the bouncing point. From Figure 12, it is evident that for $t < 0$, $H < 0$, while for $t > 0$, $H > 0$, so that around $t = 0$, i.e., in the early universe, the bouncing behavior of the universe can be justified. Two bouncing points are shown in Figure 12 (a magnified view is also inserted in the figure for better understanding), one around $t \approx -7$ and the other around $t \approx 6$. These results

are similar to already obtained bouncing solutions in the context of modified GB gravity without matter coupling [23].

4. Outlook

In present study, bouncing cosmology in the context of $f(\mathcal{E}, T)$ theory of gravity is examined. For this purpose, a GB cosmological model with logarithmic trace term, i.e., $f(\mathcal{E}, T) = \mathcal{E} + \lambda\mathcal{E}^2 + 2\beta\text{Log}(T)$, is considered. In this study, the possibility of obtaining bouncing solutions by considering two EoS parameters is investigated. A detailed graphical analysis is provided for discussing the obtained bouncing solutions. To best of our knowledge, this is the first such attempt in the frame-work of $f(\mathcal{E}, T)$ gravity. The main results of present study are itemized as follows.

The analysis is based upon two EoS parameters, i.e., $\omega_1(t) = -(k\text{Log}(t + \varepsilon)/t) - 1$ and $\omega_2(t) = (r/\text{Log } t) - s$. The evolution of energy density and pressure profiles of the universe for both these cases are shown in Figures 1 and 7. The energy density is positive within the neighborhood of bouncing points while the pressure profiles are negative, which might be an indication of accelerated expansion of the universe.

As evident from Figures 2 and 8, NEC is violated near the neighborhood of bouncing points. This justifies the indication of bouncing universe with violation of NEC for some period of time near the neighborhood of bounce point in the context of FLRW spacetime. Similarly, WEC and SEC are also violated for both these cases. This also ensures the fact that the SEC is violated on cosmological scales in the light of the recent observational data regarding the accelerating universe [37]. Figures 3 and 9 describe that DEC and TEC are satisfied with the chosen values of parameters. This provides the justification that our chosen cosmological model is well behaved. In particular, the choice of parameters strictly depends upon the evolution of energy density and TEC. It is worthwhile to mention here that TEC sometimes gets violated; however, in our case, it must be satisfied due to the involvement of logarithmic function in the $f(\mathcal{E}, T)$ gravity model.

The well behaved behavior of the scale factor and red shift function is shown in Figures 4 and 10. From Figures 5 and 12, it is evident that for $t < 0$, $H < 0$, while for $t > 0$, $H > 0$, so that around $t = 0$, i.e., in the early universe, the bouncing behavior of the universe can be justified. Two bouncing points are shown in Figure 12 (a magnified view is also inserted in the figure for better understanding), one around $t \approx -7$ and the other around $t \approx 6$. These results are similar to already obtained bouncing solutions in the context of modified GB gravity without matter coupling [23].

The deceleration parameter q in cosmology is the measure of the cosmic acceleration of the universe

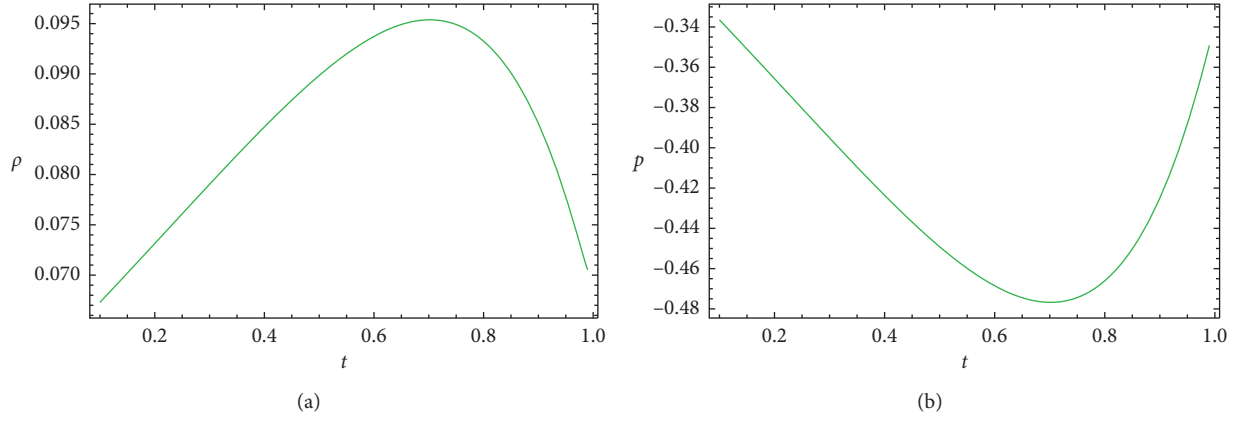


FIGURE 7: Energy density and pressure profiles for $\omega_2(t) = (r/\text{Log}t) - s$ with $\alpha = 0.2222$; $\kappa = 0.0256$; $\zeta = 0.5$; $\xi = 0.5$; $\beta = -0.04858$; $\lambda = 0.5$; $r = -0.0005$; $s = 5$.

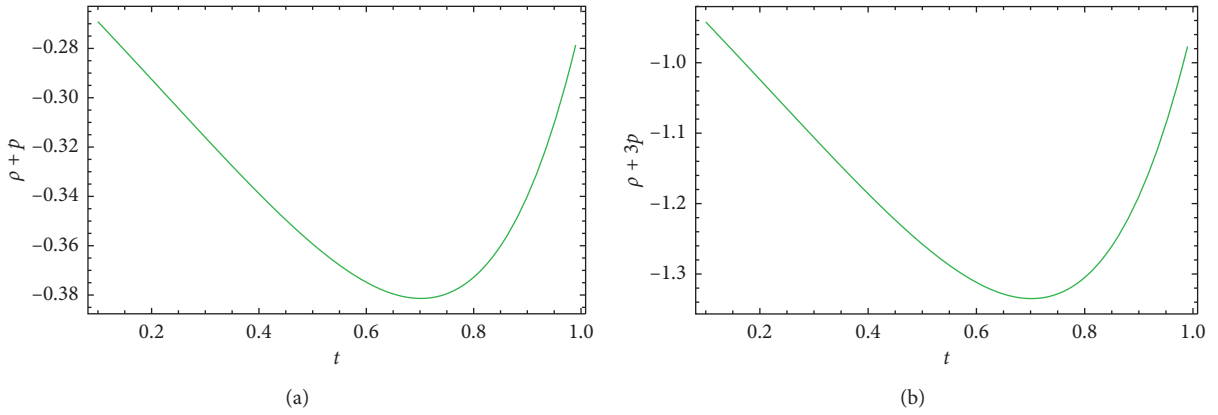


FIGURE 8: Evolution of NEC and SEC for $\omega_2(t) = (r/\text{Log}t) - s$ with $\alpha = 0.2222$; $\kappa = 0.0256$; $\zeta = 0.5$; $\xi = 0.5$; $\beta = -0.04858$; $\lambda = 0.5$; $r = -0.0005$; $s = 5$.

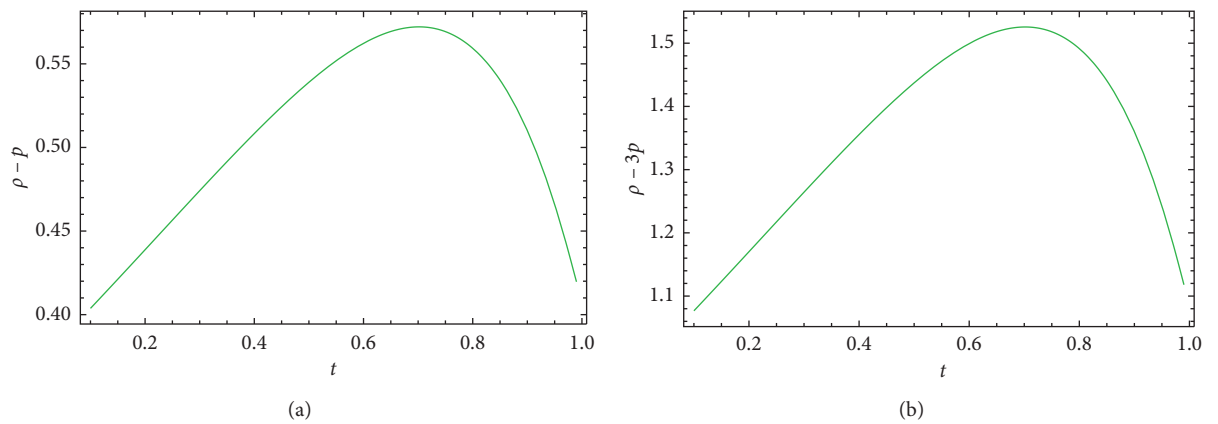


FIGURE 9: Evolution of DEC and TEC for $\omega_2(t) = (r/\text{Log}t) - s$ with $\alpha = 0.2222$; $\kappa = 0.0256$; $\zeta = 0.5$; $\xi = 0.5$; $\beta = -0.04858$; $\lambda = 0.5$; $r = -0.0005$; $s = 5$.

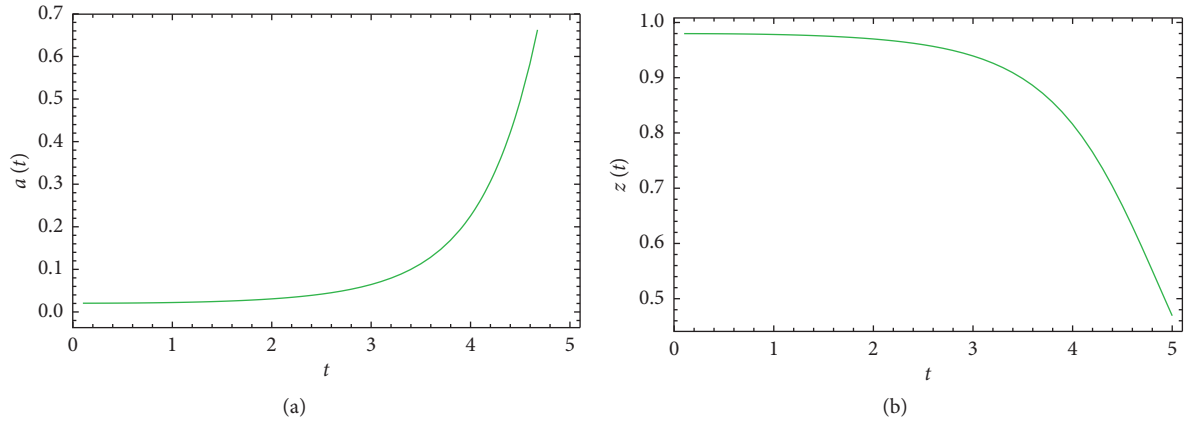


FIGURE 10: Evolution of the scale factor and red shift function for $\omega_2(t) = (r/\text{Log}t) - s$ with $\alpha = 0.2222; \kappa = 0.0256; \zeta = 0.5; \xi = 0.5; \beta = -0.04858; \lambda = 0.5; r = -0.0005; s = 5$.

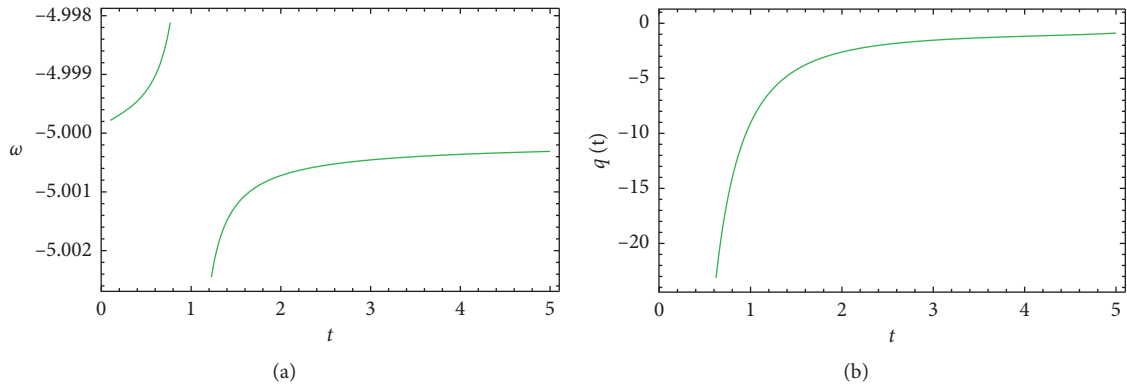


FIGURE 11: Evolution of EoS and deceleration parameters with $\alpha = 0.2222; \kappa = 0.0256; \zeta = 0.5; \xi = 0.5; \beta = -0.04858; \lambda = 0.5; r = -0.0005; s = 5$.

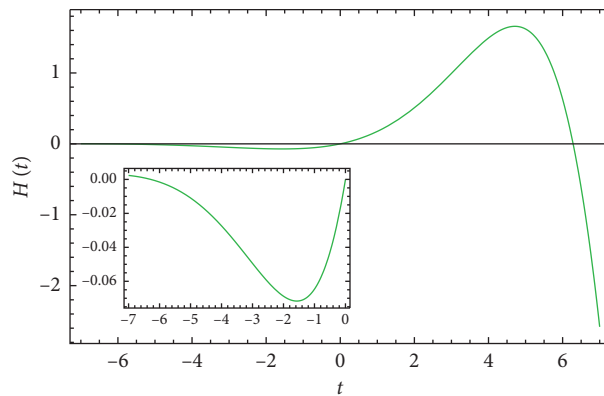


FIGURE 12: Evolution of the Hubble parameter for $\omega_2(t) = (r/\text{Log}t) - s$ with $\alpha = 0.2222; \kappa = 0.0256; \zeta = 0.5; \xi = 0.5; \beta = -0.04858; \lambda = 0.5; r = -0.0005; s = 5$.

expansion. The positive deceleration parameter corresponds to a decelerating model, while the negative value provides cosmic expansion. The negative trend of the deceleration parameter can be seen in the right plots of Figures 5 and 11. The behavior of EoS parameters is depicted in the left plots of Figures 6 and 11. It is shown that EoS parameter remains negative after the bouncing point, in particular $\omega_1 \approx -1$, which justifies the current cosmic expansion [52–54]. Moreover, the bouncing universe in the case of $\omega_1(t) = -(k\text{Log}(t + \varepsilon)/t) - 1$ supports the Λ -CDM model; since when the cold dark matter dominates the evolution of the universe, the value of EoS parameter is negative and closer to zero (as evident in our case).

One of the important features of the current study is the discussion of conservation equation. Modified theories of gravity which involve matter curvature coupling do not act as per usual conservation of GR. In case of $f(\mathcal{G}, T)$ gravity, this issue is evident from equation (5). Perhaps, there is a little drawback that the theory might

be plagued by divergences at astrophysical scales. However, by putting some suitable constraints on equation (5), the standard conservation equation has been tried to be recovered. The exact solution of the constraint equation is very difficult to obtain due to highly nonlinear terms. However, in the current study, the parameters have been set in such a way that this equation is partially satisfied. It can be seen in the right plot of Figure 6 that conservation equation is satisfied in the neighborhood of the bouncing point but deviates as the time passes.

All the above itemized discussions suggest that the proposed $f(\mathcal{G}, T)$ gravity model provides good bouncing solutions with the chosen EoS parameters.

Appendix

Simplified field equations for the case $\omega_1(t) = -(k\text{Log}(t + \varepsilon)/t) - 1$:

$$\begin{aligned}
 \rho = & \frac{2t}{ka^7 \text{Log}(t + \varepsilon)(3k\text{Log}(t + \varepsilon) + 4t)} \times (3ka^6 \ddot{a} \text{Log}(t + \varepsilon) + 4ta^6 \ddot{a} - 3ka^5 \dot{a}^2 \text{Log}(t + \varepsilon) - 4ta^5 \dot{a}^2 \\
 & + 864k\lambda \dot{a} a^5 \text{Log}(t + \varepsilon) + 1152\lambda t \dot{a} a^5 - 1152k\lambda a^2 \dot{a}^2 \ddot{a}^3 \text{Log}(t + \varepsilon) + 3456k\lambda a^4 \dot{a}^2 \text{Log}(t + \varepsilon) \\
 & - 2592k\lambda \dot{a}^6 \ddot{a} \text{Log}(t + \varepsilon) - 1536\lambda t a^2 \dot{a}^2 \ddot{a}^3 + 4608\lambda t a^4 \dot{a}^2 - 3456\lambda t \dot{a}^6 \ddot{a} - 576k\lambda \dot{a} a^2 \dot{a}^3 \ddot{a} \text{Log}(t + \varepsilon) \\
 & - 768\lambda t \dot{a} a^2 \dot{a}^3 \ddot{a} + \beta(-k)a^7 \text{Log}(t + \varepsilon)), \\
 p = & \frac{2(k\text{Log}(t + \varepsilon) + t)}{ka^7 \text{Log}(t + \varepsilon)(3k\text{Log}(t + \varepsilon) + 4t)} \times (3ka^6 \ddot{a} \text{Log}(t + \varepsilon) + 4ta^6 \ddot{a} - 3ka^5 \dot{a}^2 \text{Log}(t + \varepsilon) \\
 & - 4ta^5 \dot{a}^2 + 864k\lambda \dot{a} a^5 \text{Log}(t + \varepsilon) + 1152\lambda t \dot{a} a^5 - 1152k\lambda a^2 \dot{a}^2 \ddot{a}^3 \text{Log}(t + \varepsilon) + 3456k\lambda a^4 \dot{a}^2 \text{Log}(t + \varepsilon) \\
 & - 2592k\lambda \dot{a}^6 \ddot{a} \text{Log}(t + \varepsilon) - 1536\lambda t a^2 \dot{a}^2 \ddot{a}^3 + 4608\lambda t a^4 \dot{a}^2 - 3456\lambda t \dot{a}^6 \ddot{a} - 576k\lambda \dot{a} a^2 \dot{a}^3 \ddot{a} \text{Log}(t + \varepsilon) \\
 & - 768\lambda t \dot{a} a^2 \dot{a}^3 \ddot{a} + \beta(-k)a^7 \text{Log}(t + \varepsilon)).
 \end{aligned} \tag{A.1}$$

Simplified field equations for the case
 $\omega_1(t) = (r/\text{Log } t) - s$:

$$\rho = \frac{2\text{Log}(t)}{\left(a[t]^7(3r^2 + 2r\text{Log}[t] - 6rs\text{Log}[t] - \text{Log}[t]^2 - 2s\text{Log}[t]^2 + 3s^2\text{Log}[t]^2)\right)}$$

$$\times \left(3ra^6\ddot{a} - 3sa^6\text{Log}(t)\ddot{a} - a^6\text{Log}(t)\ddot{a} - 3ra^5\dot{a}^2 + 3sa^5\text{Log}(t)\dot{a}^2 + a^5\text{Log}(t)\dot{a}^2\right.$$

$$+ 864\lambda r\dot{a}\ddot{a}^5 - 864\lambda s\dot{a}\text{Log}(t)\dot{a}^5 - 288\lambda\dot{a}\text{Log}(t)\dot{a}^5 - 1152\lambda r\dot{a}^2\dot{a}^3 + 3456\lambda r\dot{a}\dot{a}^4\dot{a}^2$$

$$- 2592\lambda r\dot{a}^6\dot{a} + 1152\lambda s\dot{a}^2\text{Log}(t)\dot{a}^2\dot{a}^3 - 3456\lambda s\dot{a}\text{Log}(t)\dot{a}^4\dot{a}^2 + 2592\lambda s\text{Log}(t)\dot{a}^6\dot{a}$$

$$+ 384\lambda a^2\text{Log}(t)\dot{a}^2\dot{a}^3 - 1152\lambda a\text{Log}(t)\dot{a}^4\dot{a}^2 + 864\lambda\text{Log}(t)\dot{a}^6\dot{a} - 576\lambda r\dot{a}\dot{a}^2\dot{a}^3\ddot{a}$$

$$+ 576\lambda s\dot{a}\dot{a}^2\text{Log}(t)\dot{a}^3\ddot{a} + 192\lambda\dot{a}\dot{a}^2\text{Log}(t)\dot{a}^3\ddot{a} + \beta(-r)a^7 + \beta sa^7\text{Log}(t) - \beta a^7\text{Log}(t)\Big),$$

$$P = \frac{2(r - s\text{Log}(t))}{a^7(3r^2 - 6rs\text{Log}(t) + 2r\text{Log}(t) + 3s^2\text{Log}^2(t) - 2s\text{Log}^2(t) - \text{Log}^2(t))}$$

$$\times \left(-3ra^6\ddot{a} + 3sa^6\text{Log}(t)\ddot{a} + a^6\text{Log}(t)\ddot{a} + 3ra^5\dot{a}^2 - 3sa^5\text{Log}(t)\dot{a}^2 - a^5\text{Log}(t)\dot{a}^2 - 864\lambda r\dot{a}\dot{a}^5\right.$$

$$+ 864\lambda s\dot{a}\text{Log}(t)\dot{a}^5 + 288\lambda\dot{a}\text{Log}(t)\dot{a}^5 + 1152\lambda r\dot{a}^2\dot{a}^2\dot{a}^3$$

$$- 3456\lambda r\dot{a}\dot{a}^4\dot{a}^2 + 2592\lambda r\dot{a}^6\dot{a} - 1152\lambda s\dot{a}^2\text{Log}(t)\dot{a}^2\dot{a}^3 + 3456\lambda s\dot{a}\text{Log}(t)\dot{a}^4\dot{a}^2 - 2592\lambda s\text{Log}(t)\dot{a}^6\dot{a}$$

$$- 384\lambda a^2\text{Log}(t)\dot{a}^2\dot{a}^3 + 1152\lambda a\text{Log}(t)\dot{a}^4\dot{a}^2 - 864\lambda\text{Log}(t)\dot{a}^6\dot{a} + 576\lambda r\dot{a}\dot{a}^2\dot{a}^3\ddot{a} - 576\lambda s\dot{a}\dot{a}^2\text{Log}(t)\dot{a}^3\ddot{a}$$

$$\left. - 192\lambda\dot{a}\dot{a}^2\text{Log}(t)\dot{a}^3\ddot{a} + \beta ra^7 - \beta sa^7\text{Log}(t) + \beta a^7\text{Log}(t)\right).$$

(A.2)

Data Availability

No data were used to support this study.

Conflicts of Interest

The author declares that there are no conflicts of interest.

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