Research Article
Market Power, NAIRU, and the Phillips Curve

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We explore the relationship between unemployment and inflation in the United States (1949-2019) through both Bayesian and spectral lenses. We employ Bayesian vector autoregression ("BVAR") to expose empirical interrelationships between unemployment, inflation, and interest rates. Generally, we do find short-run behavior consistent with the Phillips curve, though it tends to break down over the longer term. Emphasis is also placed on Phelps' and Friedman's NAIRU theory using both a simplistic functional form and BVAR. We find weak evidence supporting the NAIRU theory from the simplistic model, but stronger evidence using BVAR. A wavelet analysis reveals that the short-run NAIRU theory and Phillips curve relationships may be time-dependent, while the long-run relationships are essentially vertical, suggesting instead that each relationship is primarily observed over the medium-term (2-10 years), though the economically significant medium-term region has narrowed in recent decades to roughly 4-7 years. We pay homage to Phillips' original work, using his functional form to compare potential differences in labor bargaining power attributable to labor scarcity, partitioned by skill level (as defined by educational attainment). We find evidence that the wage Phillips curve is more stable for individuals with higher skill and that higher skilled labor may enjoy a lower natural rate of unemployment.

1. Introduction

Dating back to at least 1926 (Fisher [1]), economists have pondered the relationship between inflation and unemployment. Irving Fisher, citing early statistical work, theorized inflation has a lagged, distributed impact on unemployment due to revenue flexibility and expense rigidity.

In 1958, A. W. Phillips popularized this inflation-employment association in his famous paper *The Relationship between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom, 1861-1957*, detailing the response of wage growth to unemployment rates. Rather than relying on the sort of the Cantillon effect Fisher described, Phillips theorized that employers tend to bid wage rates up rapidly during periods of low unemployment. Conversely, he believed workers are hesitant to reduce wage demands when unemployment is high. The rate of change of wage rates is therefore convex to the unemployment rate. The simplicity of the model combined with the strength of his empirical results generated significant interest in the subject.

Phillips' theoretical model was quickly generalized by numerous authors to relate movements in the broad price level (i.e., the inflation rate) to the unemployment rate, consistent with Fisher's approach. Samuelson and Solow [2] considered the possibility of exploiting such a relationship and the policy implications that follow, which is thought to have had significant influence on policy-setting over the subsequent 20 years (this argument is made in Hart and Hall [3]). Such a tradeoff was thought to imply a metaphorical "menu" from which politicians choose depending on the economic environment they inherit.

In 1968, Milton Friedman etched his own contributions in stone while addressing the American Economic Association (Friedman [4]). The Phillips curve tradeoff may indeed exist, but only by misleading the general public who set their own expectations regarding wage and price inflation. A policy maker may surprise the public in the short-run, increasing inflation to reduce unemployment through the effect on real wages, but eventually the veil of "money illusion" would dissipate. Inflation expectations would rise in response to observed inflation. Unemployment would return to its "natural" level, but with equilibrium inflation higher than before. Further attempts to drive unemployment down will then require accelerating inflation hikes. In the long-run, Friedman insisted,

there is only a single rate of unemployment consistent with a steady rate of inflation. This is the nonaccelerating inflation rate of unemployment ("NAIRU," or sometimes referred to as the natural rate of unemployment). (See Figure 6 in Appendix A for a graphical representation of this theory.)

In the time since, much empirical work has been performed to sort out this theoretical controversy. Emphasis is often placed on the generalized form of the Phillips curve, i.e., the relationship between broad inflation and broad unemployment. Attention has also been paid to the market power implications of Phillips' analysis (for a few examples, see Eagly [5], Aquilante et al. [6], or Dennery [7]). Some researchers have commented on the role of market power in relation to observed flattening of the Phillips curve, but few have examined the interindustry differences in market power that Phillips alluded to in his original paper (in Phillips [8] pp. 292, Phillips notes "…wage rates rose more slowly than usual in the upswing of business activity from 1893 to 1896 and then returned to their normal pattern of change; but with a temporary increase in unemployment during 1897. This suggests that there may have been exceptional resistance by employers to wage increases from 1894 to 1896, culminating in industrial strife in 1897. A glance at industrial history confirms this suspicion. During the 1890's there was a rapid growth of employers' federations and from 1895 to 1897 there was resistance by the employers' federations to trade union demands for the introduction of an eight-hour working day, which would have involved a rise in hourly wage rates. This resulted in a strike by the Amalgamated Society of Engineers, countered by the Employers' Federation with a lock-out which lasted until January 1898." This excerpt, along with others, highlights Phillips' consideration of bargaining power between employee and employer as a function of the level of unemployment and a determinant of wage growth.).

In this paper, we seek to explore potential differences in bargaining power stemming from labor scarcity/abundance by skill level as defined by educational attainment. We accomplish this using Phillips' simple hyperbolic functional form to determine the degree of convexity in the wage growth–unemployment relationship and the point of wage stability. We then turn to testing the relationship between inflation acceleration and the unemployment gap as described by NAIRU. We employ BVAR to understand the interrelationships between inflation, unemployment, and interest rates. Lastly, wavelet analysis is performed to test for time dependency in the estimated Phillips curve and NAIRU theory relationships.

The paper is structured as follows. In "Literature Review," we review the literature relevant to the topics discussed. In "Methodological Overview," we give a brief overview of the methodology employed (see Appendix F for more detailed discussion). "Data" explains the data we used and the cleaning applied. "Results and Interpretation" summarizes and interprets our findings, and "Conclusion" concludes.

2. Literature Review

Phillips' analysis in 1958 regressed nominal wage growth ($\dot{y}$) on the unemployment rate (x). He expressed the relationship as

$$\dot{y} = a + bx^c, \qquad (1)$$

or

$$\log{(\dot{y} - a)} = \log{(b)} + c\log{(x)}. \qquad (2)$$

Phillips specified $\dot{y}$ to be $[y_{t+1} - y_{t-1}/2y_t]$ where y is the wage rate. The constants a, b, and c were estimated by ordinary least squares ("OLS"). The constant a was selected by trial and error to reposition the curve. There were two other factors Phillips considered important for the wage–employment relationship. These included the rate of change of unemployment (dx/dt) and retail/import prices. To accommodate the former, Phillips considers a functional relationship of the following form:

$$y + a = bx^c + k\left[\frac{1}{x_m}\frac{dx}{dt}\right], \qquad (3)$$

where the constant k is measured by OLS and x_m is the unemployment rate at the beginning of time t. Phillips decides against this form, noting that x is trend-free, meaning $(1/x_m)(dx/dt)$ should be uncorrelated with x. This decision produced Phillips' famous "loops," where the data appeared to form neat cycles around the fitted line presumably reflecting the rate of change of the unemployment rate. (See Figure 12 in Appendix B for an example using the 1861–1868 data.)

Phillips' functional form could not be fitted to all observations due to points where $\dot{y}$ is less than the constant a since the logarithm of a negative number is undefined. To adjust for this, Phillips selected intervals based on the unemployment rate and grouped his observations, taking the mean value of $\dot{y}$ and x. (Gallegati et al. [9] note that the procedure Phillips used to group observations is akin to a primitive form of wavelet analysis. The transformation stressed low-frequency information, leading these authors to argue that Phillips' theory is not meant to convey a contemporaneous relationship.) Thus in Lipsey [10], Lipsey altered Phillips' curve using the quadratic form:

$$\dot{y} = a + \frac{b}{x} + \frac{c}{x^2}. \qquad (4)$$

This form allows all observations to be fitted without any data transformations. While Lipsey maintains that an interesting relationship between wage growth and unemployment remains in the period under scrutiny, this form gives an awkward and ill-fitting curve when applied to the U.S. from 1934 to 1958 (Hoover [11]) (which is the time period [2] consider).

In Phelps [12], Phelps applies the Phillips curve to broad price inflation rather than wage growth, incorporating inflation expectations. He describes behavior as adaptive, suggesting that the change in expected inflation ($\dot{y}_e$) is proportional to the inflation gap, i.e., $(d/dt)\dot{y}_e \propto [\dot{y} - \dot{y}_e]$. According to Phelps' and Friedman's NAIRU theory, the short-run supply function of the Phillips curve is

$$Y = Y^* + \alpha(P - P^*), \qquad (5)$$

where Y is log output, Y^* is log potential output, α is a positive constant, P is the log price level, and P^* is the log

expected price level. Rearranging gives

$$P = P^* + \frac{Y - Y^*}{\alpha} + \varepsilon, \tag{6}$$

where ε is an exogenous shocks from world supply. We may again define $P - P_{-1} \equiv \dot{y}$ as the inflation rate and $P^* - P^*_{-1} \equiv \dot{y}_e$ as the expected inflation rate, where the subscript -1 indicates the prior value.

Okun's law (displayed this way in Dritsaki and Dritsaki [13]) predicts a negative relationship between output and unemployment such that

$$\frac{Y - Y^*}{\alpha} = -\beta(x - x^*), \tag{7}$$

where x^* is NAIRU and β is a positive constant. One may think of $x - x^*$ as an unemployment gap. Utilizing these identities, we may assert the following

$$P - P_{-1} = (P^* - P^*_{-1}) - \beta(x - x^*) + \varepsilon, \tag{8}$$

or

$$\dot{y} = \dot{y}_e - \beta(x - x^*) + \varepsilon, \tag{9}$$

to be the short-run Phillips curve according to the NAIRU theory. Note that this theory suggests a negative relationship between the inflation rate and unemployment gap, as indicated by the subtraction of $\beta(x - x^*)$. Assuming inflation expectations are set based on observed inflation from the prior period, $\dot{y}_e$ can be approximated by $(P_{-1} - P_{-2} \equiv \dot{y}_{-1})$.

While adaptive expectations imply the need for accelerating inflation to maintain unemployment below the NAIRU, other expectation types may be even more pessimistic for policy-setting. Rational expectations play this role, doing away with short-run expectational errors made by individuals. Rational expectations are a modeling framework in which expectations are driven by the model itself, conditional on the information agents are assumed to have.

Under rational expectations, inflation expectations are a function of the monetary policy rule, assuming this rule is known by all agents. If inflation exceeds expected inflation, adaptive expectations suggest an increase in inflation expectations in response to the surprise excess inflation. However, if monetary policy is known to counteract inflation, rational expectations may suggest a decline in inflation expectations. Lucas [14] and Sargent and Wallace [15] provided early theoretical development for the application of rational expectations to the Phillips curve.

In the context of the NAIRU theory, the expectation at time $t - 1$ of the price level at time t can be represented as $E_{t-1}P_t$. Under adaptive expectations, monetary policy can affect output through unanticipated inflation, as described in Equation (5). Under rational expectations where the monetary policy rule is known, the predicted effects of monetary policy are already embedded in $E_{t-1}P_t$. Monetary policy can then only be effective by deviating from the rule or by with-

holding information from agents. Otherwise, monetary policy is ineffective for changing output and unemployment.

Fischer [16], Taylor [17], and Calvo [18] provide theoretical models in which rational expectations fail to sterilize monetary policy. The most influential observation stemming from this work involves the use of staggered, overlapping, multiperiod wage, and price contracts. Output is shown to be affected by current and lagged disturbances. Current disturbances cannot be offset by monetary policy, and lagged disturbances from before contract initiation were already known and taken into account when the contract was written. Conversely, lagged disturbances that occur midcontract can be offset by monetary policy.

This set of theoretical models revived the importance of the Phillips curve for short-run policy. It also provided theoretical grounding for wage and price rigidities observed at high levels of unemployment.

Stock and Watson [19] generalize the conventional Phillips curve for inflation forecasting purposes using measures of real aggregate activity other than unemployment. They set the dependent variable to be the change in inflation over periods longer than the sampling frequency and exclude supply shock measures. Their conventional Phillips curve specification is

$$\pi^h_{t+h} - \pi_t = \phi + \beta(L)u_t + \gamma(L)\Delta\pi_t + e_{t+h}, \tag{10}$$

where $\pi^h_t = (1200/h) \ln (P_t/P_{t-h})$ is the h-period inflation in the price level P_t, u_t is the unemployment rate, and $\beta(L)$ and $\gamma(L)$ are polynomials in the lag operator L. Inflation is restricted to be integrated of order one and NAIRU is assumed constant (hence its exclusion from Equation (10)). They then consider the possibility of omitted variables, replacing u_t with one or several factors including housing starts, capacity utilization, and manufacturing and trade sales growth. They find that indices of aggregate activity produce inflation forecasting improvements, implying the conventional Phillips curve may be an overly simplistic relationship and setting the stage for use of other macro variables.

Gali and Gertler [20] build on the staggered-contract work to develop a New Keynesian Phillips curve model. This model supplies microfoundations for the use of price and wage contracts by forward-looking firms. It also replaces the output gap with real marginal costs (i.e., unit labor costs) as the relevant indicator of economic activity due to the unobservable nature of potential output and because movements in real marginal cost tend to lag changes in output.

The New Keynesian Phillips curve also adjusts the expectational form, representing the expected inflation as the expectation at time t of inflation at time $t + 1$ such that $E_t \dot{y}_{t+1} \equiv \dot{y}_e$. This contrasts with the New Classical expectational form utilized by the NAIRU theory, in which expected inflation is the expectation at time $t - 1$ of inflation at time t (hence the use of $\dot{y}_{-1} \equiv \dot{y}_e$ in Equation (9) above). The authors find that real marginal costs are statistically and economically significant determinants of inflation and that the output gap is of suspect use in the NAIRU theory.

Egger et al. [21] explore the Phillips curve using a vector autoregression ("VAR") approach (see Zarnowitz and Braun [22] for a summary of early uses of VAR and BVAR for forecasting macroeconomic variables). The inflation rate, unemployment rate, and nominal interest rates (the prime rate) are all treated endogenously. This model has the form

$$X_t = c + \phi_1 X_{t-1} + \phi_2 X_{t-2} + \cdots + \phi_p X_{t-p} + \varepsilon_t, \qquad (11)$$

where X_t is a 3×1 vector consisting inflation rates, unemployment rates, and prime rates, c is a 3×1 vector of constants, and ϕ_i are 3×3 matrices of coefficients for $i = \{1, \cdots, p\}$. Specifying the system of equations in this way allows the authors to parse the feedback effects theoretically present in the Phillips curve relationship. The Schwartz information criterion suggested an appropriate lag length (p) of two. The authors also determine that unemployment and inflation are not cointegrated using the Johansen test, although the results of this test conflict with those of the Augmented Dickey-Fuller test regarding the stationarity of the underlying variables. Equation (11) is adopted in "Simultaneous Equations" under a similar methodology.

Dritsaki and Dritsaki [13] construct a vector error correction model with functional form from Equation (9). The Akaike information criterion suggested an appropriate lag length (p) of three. Using the Johansen test, they find that the inflation and unemployment rates are both integrated of order one. They conclude that these variables are in fact cointegrated (in contrast to [21]) and that inflation Granger causes unemployment in the long-run. These contrasting findings regarding the properties of the Phillips curve components highlight the difficulty generalizing such an analysis through a frequentist lens.

Karlsson and Osterholm [23] avoid the problem of cointegration by specifying a BVAR model in which the unemployment rate and PCE inflation rate are endogenous variables and there are no exogenous variables (i.e., only lags of the endogenous variables are included). For lag coefficients ($\theta = \beta_1, \beta_2, \cdots, \beta_p$), they use a diffuse normal prior for the mean and an inverse-gamma prior for the variance. While Karlsson and Osterholm conclude that the Phillips curve was relatively flat during the 2005 to 2013 period compared to the prior decade, they believe the relationship between the unemployment rate and inflation rate is fundamentally similar over time.

Gallegati et al. [9], focusing on wage inflation, liken Phillips' original analysis to a primitive version of wavelet analysis. Modernizing the approach, they apply wavelet analysis to explore the Phillips curve relationship at various frequencies. Such an approach does not require data to be stationary, and it provides estimates that are localized to time and frequency, making the approach quite flexible. Working with quarterly U.S. data from 1948 to 2009, they find that, while the wage Phillips curve relationship is unstable over the short-run and largely vertical in the long-run, it is statistically and economically significant over the medium-run (2-8 years) until the mid-1990's. Fratianni et al. [24] and Mutascu [25] come to similar conclusions (note, however, that Mutascu [25] uses general

inflation rather than wage inflation) after applying comparable methodology to different countries and time periods.

Aguiar-Conraria et al. [26] apply wavelet analysis to the New Keynesian Phillips curve using the quarterly U.S. data from 1978 to 2016. They similarly find that there is no long-run Phillips tradeoff and that short-run tradeoffs are time-dependent. They find less evidence for a medium-run Phillips tradeoff, instead concluding that medium-run inflation is explained by inflation expectations (they relied on survey methods to estimate inflation expectations, which they argue is an improvement upon backward- and forward-looking expectations) and energy prices (as a proxy for supply shocks). They do not find evidence of nonlinearities or structural breaks in the U.S. New Keynesian Phillips curve, in contrast to references therein showing a structural break in the early to mid-1990's.

3. Methodological Overview

For a more detailed explanation of our methodology, see Appendix F.

3.1. Market Power Analysis. In comparing the relationship between wage growth and unemployment rate by skill level, we use Phillips' original functional form shown in Equation (2). The following regression equation is applied to the three different datasets, each representing a different skill level.

$$\log\left(\dot{y}_{ij} - a\right) = \log\left(\alpha_j\right) + \beta_j \log\left(x_{ij}\right) + \varepsilon_{ij}, \qquad (12)$$

where $j \epsilon J = \{1, 2, 3\} = \{\text{Less Than High School, High School, Bachelors}\}$ and $i \epsilon \{1, 2, \cdots, n\}$ for $\forall j \epsilon J$. In other words, there are n wage growth unemployment rate observation pairs for each skill level. Coincidentally, we find that -0.9 is still the most appropriate value for a, although we use a to avoid undefined values for the left-hand side of Equation (12), rather than for positioning of the curve.

3.2. NAIRU Theory Analysis. In testing the NAIRU theory, we utilize the functional form from Equation (9). The unemployment gap here is treated as exogenous. The resulting regression equation is

$$\dot{y}_t - \dot{y}_{t-1} = d\dot{y}_t = \beta(x_t - x_t^*) + \varepsilon_t. \qquad (13)$$

3.3. Simultaneous Equations. To understand the interrelationships between the unemployment rate, inflation rate, and interest rate, we specify a BVAR model with the structural form from Egger et al. [21] using methods detailed in Koop and Korobilis [27]. After that, we apply the same methodology to retest the NAIRU theory. Generally, a structural BVAR model has the form

$$
\begin{aligned}
b_{11}^0 \dot{y}_t + b_{12}^0 x_t + b_{13}^0 z_t = {}& \alpha_{01} + b_{11}^1 \dot{y}_{t-1} + b_{12}^1 x_{t-1} + b_{13}^1 z_{t-1} \cdots \\
& + b_{11}^p \dot{y}_{t-p} + b_{12}^p x_{t-p} + b_{13}^p z_{t-p} + \varepsilon_{1t},
\end{aligned}
$$

$$
\begin{aligned}
b_{21}^0 \dot{y}_t + b_{22}^0 x_t + b_{23}^0 z_t = {}& \alpha_{02} + b_{21}^1 \dot{y}_{t-1} + b_{22}^1 x_{t-1} + b_{23}^1 z_{t-1} \cdots \\
& + b_{21}^p \dot{y}_{t-p} + b_{22}^p x_{t-p} + b_{23}^p z_{t-p} + \varepsilon_{2t},
\end{aligned}
$$

$$b_{31}^0 \dot{y}_t + b_{32}^0 x_t + b_{33}^0 z_t = \alpha_{03} + b_{31}^1 \dot{y}_{t-1} + b_{32}^1 x_{t-1} + b_{33}^1 z_{t-1} \cdots$$
$$+ b_{31}^p \dot{y}_{t-p} + b_{32}^p x_{t-p} + b_{33}^p z_{t-p} + \varepsilon_{3t}, \tag{14}$$

or

$$B_0 y_t = \alpha + B_1 y_{t-1} + \cdots + B_p y_{t-p} + \varepsilon_t, \tag{15}$$

where $y_t = (\dot{y}_t \ x_t \ z_t)' = ($ inflation rate unemployment rate prime rate $)'$, α is a 3×1 intercept vector, B_0 is a 3×3 contemporaneous coefficient matrix, $B_1, \cdots, B_p$ are 3×3 lag coefficient matrices, and ε_t is a 3×1 vector of structural errors (shocks). Premultiplying by B_0^{-1} gives the reduced-form

$$y_t = B_0^{-1}\alpha + B_0^{-1}B_1 y_{t-1} + \cdots + B_0^{-1}B_p y_{t-p} + B_0^{-1}\varepsilon_t, \tag{16}$$

or

$$y_t = A_0 + A_1 y_{t-1} + \cdots + A_p y_{t-p} + e_t, \tag{17}$$

where $A_0 = B_0^{-1}\alpha$, $A_i = B_0^{-1}B_i$ for $i = \{1, \cdots, p\}$ such that the elements are $a_{11}^1 = b_{11}^{0 \ -1}b_{11}^1$, etc., and $e_t = B_0^{-1}\varepsilon_t$ such that each reduced-form error is a linear combination of the structural shocks. Consistent with Egger et al. [21], we use $p = 2$, or two lags for each variable, although a robustness analysis regarding number of lags is briefly discussed in "Results and Interpretation." Then, Equation (17) becomes

$$y_t = A_0 + A_1 y_{t-1} + A_2 y_{t-2} + e_t = AX_t + e_t, \tag{18}$$

where $A = [A_0 \ A_1 \ A_2]'$ and $X_t = [I \ y_{t-1} \ y_{t-2}]$ where I is the identity matrix.

Stacking for all t gives

$$Y = AX + E, \tag{19}$$

where Y is a $T \times 3$ matrix whose t^{th} row is y_t', X is a $T \times 3$ matrix whose t^{th} row is X_t', and $E \sim MN(0_{2T}, \Sigma \otimes I_T)$ is a $T \times 3$ matrix whose t^{th} row is e_t'. T represents the total number of observations.

This approach is then repeated using $y_t = (\dot{y}_t \ x_t \ z_t)'$ $= ($ change in inflation rate unemployment gap change in prime rate $)' = ($ Inf Chg Unemp Gap Rate Chg $)'$ to assess the validity of the NAIRU theory more rigorously.

3.4. Wavelet Analysis. Wavelet analysis allows us to decompose time series data and reexamine interrelationships in the time-frequency space. We rely on the continuous wavelet transform, cross-wavelet transform, and wavelet coherence to do this. These measures can be thought of as providing the localized variance, covariance, and correlation, respectively, for two time series in the time-frequency space.

We examine the power spectrum for periods (scales) of statistically and economically significant coherence for both the

conventional Phillips curve and NAIRU theory relationships. Additionally, we analyze phase differences, helping us to uncover the nature of the coherence and the direction of the effect.

Appendix F includes a deeper dive into wavelets, wavelet analysis, and the measures described above.

4. Data

The U.S. wage index and unemployment rate data partitioned by educational attainment is sourced from Federal Reserve Economic Data (https://fred.stlouisfed.org). Raw unemployment rate data is monthly. Raw wage index data for "High School" and "Less Than High School" groups are quarterly, while for the "Bachelors" group, data is annual. For all data frequencies, the January 1st value is selected to represent the year immediately prior (i.e., the data point at 1/1/2019 represents the year 2018). Annual data for the period 2000 to 2018 was relied upon for the market power analysis.

For the NAIRU theory, simultaneous equations, and wavelet analyses, data for the aggregate U.S. unemployment rate, CPI index, and natural rate of unemployment are sourced from the Federal Reserve Economic Data. All data is quarterly. Quarterly CPI index data is transformed to provide annualized quarterly growth in CPI. The data period begins 1948Q4 and ends 2019Q4. For the NAIRU analysis using simultaneous equations and wavelets, the first data point is 1949Q1 rather than 1948Q4.

5. Results and Interpretation

5.1. Market Power Analysis. Fitting the three data sets to Equation (12) and using the mean of each parameter's posterior distribution results in the following fits (Figure 1):

Less Than High School (LHS): $\ln(\dot{y}_1 + 0.9) = 7.385 - 3.088 \ln(x_1)$

High School (HS): $\ln(\dot{y}_2 + 0.9) = 2.395 - 0.8126 \ln(x_2)$

Bachelors (B): $\ln(\dot{y}_2 + 0.9) = 2.149 - 0.9948 \ln(x_3)$.

The 95% highest marginal posterior density intervals ("HPDI") for each of the parameters are in Table 1.

Initial inspection of the results may reveal sample size difficulties. Unemployment for the Bachelors group is clustered below 5%. Additionally, the shape of the curve for the "Less Than High School" group is heavily influenced by a single observation of -0.89% wage growth corresponding to a 14.3% unemployment rate from 2010. This point significantly enhances the convexity of the Less Than High School curve, as defined by the second derivative of the fitted lines after reversing the logarithm, relative to the other two. It also reduces the error precision and widens the HPDI for this group, making the position and curvature of the Less Than High School curve highly uncertain. (See Appendix C for further discussion on this curve's sensitivity to individual data points.)

The convexity of the High School and Bachelors groups is quite similar, although the 95% HPDI is significantly tighter for the Bachelors group. In general, as skill level declines, the HPDI becomes increasingly wide for all parameters. This may suggest more stability (and predictability) of wage growth as skill level increases. (See Appendix D for graphical representation of the HPDI for each group.)
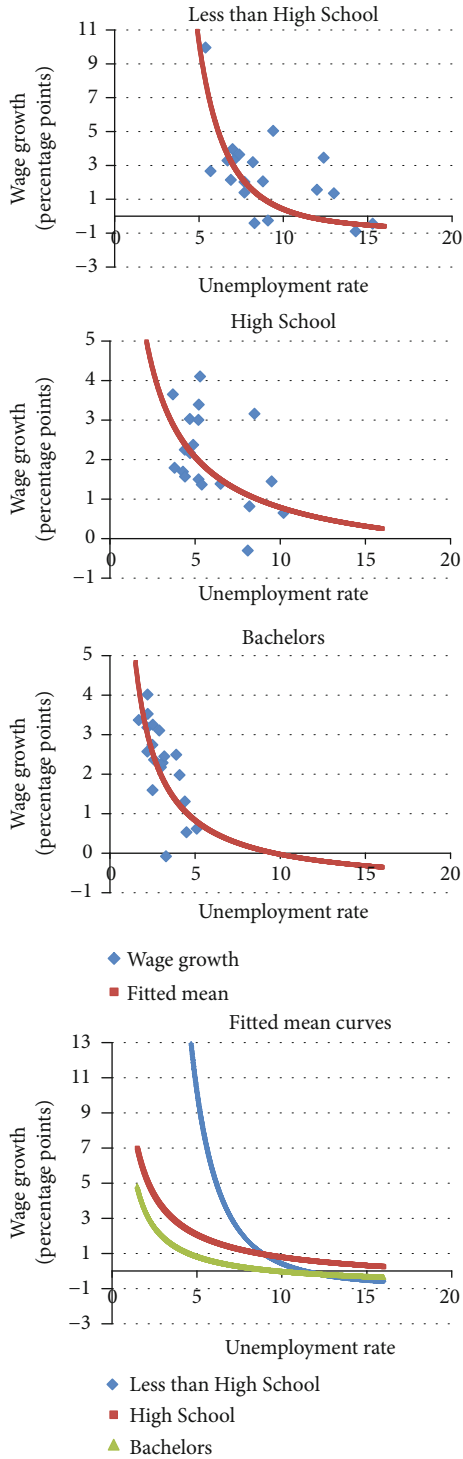
FIGURE 1: Visualization of fits of the three data sets to Equation (12) using the mean of each parameter's posterior distribution.

The intersection of the mean of the fitted curve with the x-axis is heavily dependent on numerous factors including the shape forced by Phillips' functional form, the time period, and the sample size. Nonetheless, each group intersects the x-axis at a different point. This may reflect labor market segmentation by skill level, with each segment having a unique natural rate of unemployment.

TABLE 1: 95% HPDIs.

Parameters	2.50%	Mean	97.50%
$\text{Log}(\alpha)$ (LHS)	3.028	7.385	12.000
$\text{Log}(\alpha)$ (HS)	1.215	2.395	3.647
$\text{Log}(\alpha)$ (B)	1.474	2.149	2.804
β (LHS)	-5.205	-3.088	-1.081
β (HS)	-1.523	-0.813	-0.139
β (B)	-1.581	-0.995	-0.391
σ^2 (LHS)	0.908	1.830	3.627
σ^2 (HS)	0.104	0.210	0.414
σ^2 (B)	0.071	0.144	0.286

TABLE 2: 95% HPDIs.

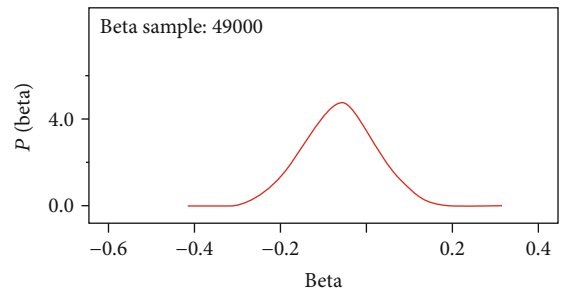| Parameters	2.50%	Mean	97.50%
β	-0.235	-0.064	0.106
σ^2	4.656	5.494	6.478

FIGURE 2: Marginal posterior distribution of β.

The Bachelors group has the lowest natural rate of unemployment of the three groups, suggesting increased skill may reduce the natural rate of unemployment. The awkward shape of the Less Than High School curve relative to the other two may signify a different degree of wage rigidity at the lowest end of the skill spectrum and possibly the need for a skill-specific functional form.

5.2. NAIRU Theory Analysis. To support the assertions of the NAIRU theory, specifically that there is a short-run tradeoff between the change in inflation and the unemployment gap, the value for β in Equation (13) (sourced originally from Equation (9)) should be negative.

The results of the analysis weakly support the NAIRU theory. The mean of the marginal posterior distribution for β is negative (-0.064); however, the results detailed in Table 2 and Figure 2 indicate a nonnegligible portion of the marginal posterior lies in positive territory.

Support for the NAIRU theory is even weaker using one-period lagged values for the unemployment gap. These results are similar to those from Dritsaki and Dritsaki [13], who conclude little or no response of inflationary changes to the unemployment gap in the short-run. However, as
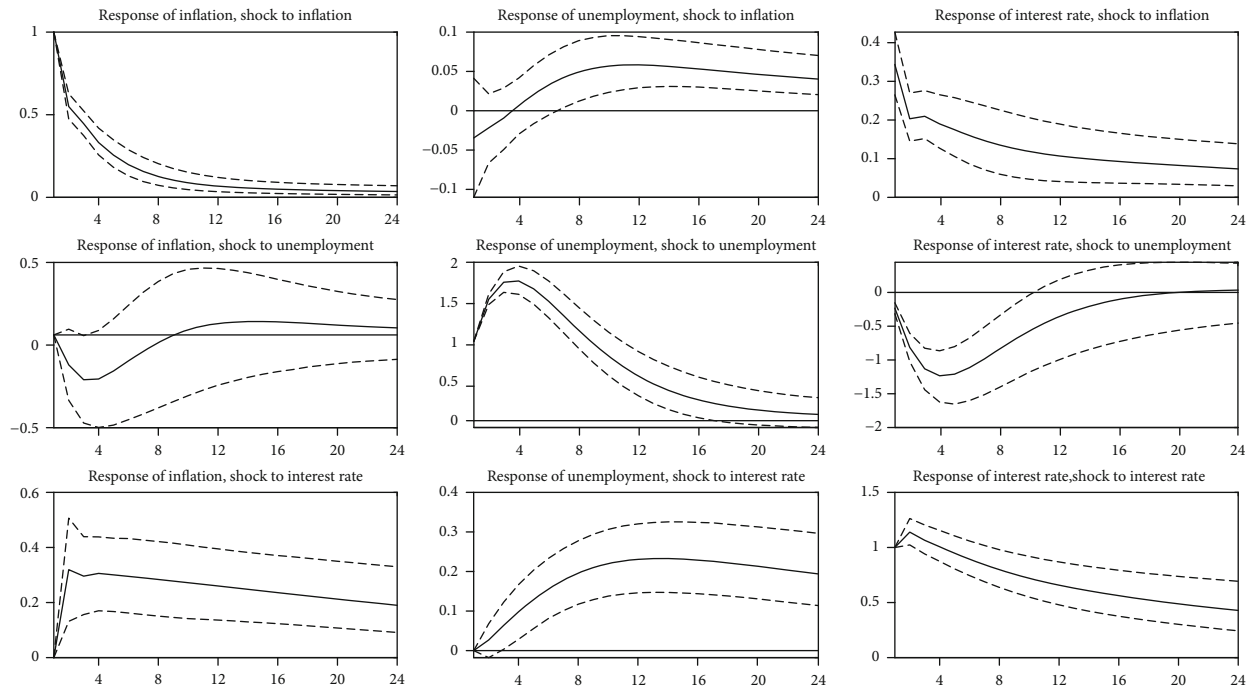
FIGURE 3: Impulse response functions.

discussed below, a BVAR framework provides more support for the NAIRU theory than the simplistic form specified here.

5.3. Simultaneous Equations.

We turn to the impulse response functions from the BVAR to interpret results. Impulse response functions allow visualization of the effect of a one-time shock to both current and future values of the endogenous variables. For the first BVAR specification involving the inflation rate, unemployment rate, and prime rate, the impulse response functions are in Figure 2.

These results generally support those of Egger et al. [21] and Koop and Korobilis [27]. (See Figures 12 and 13 in Appendix E for corresponding impulse responses for comparison.)

The diagonal of Figure 3 represents the reaction of each variable to shocks to itself. The impact of these shocks is predictably meaningful in the short-run but decaying in importance thereafter.

The response of inflation to shocks in unemployment (Figure 3, column 1, row 2) represents Phillips' curve, broadly defined. The results of this analysis are consistent with theory, in that inflation declines concavely for approximately eight periods following an unemployment shock. Thereafter, inflation stabilizes at a slightly higher level than before the unemployment shock. Although the specification is not exact, this is qualitatively similar to predictions made by the NAIRU theory.

These results are largely robust to the choice of prior and number of lags (i.e., $p = 2$ or $p = 3$). One small, notable difference emerges in the response of unemployment to inflation shocks (Figure 3, column 2, row 1). In contrast to the slight four-period reduction in unemployment in response to an

inflation shock under the independent normal-Wishart priors, both diffuse (Jeffrey's) and Minnesota priors depict a negligible response of unemployment to inflation shocks in the short-run, which may weaken support for short-run employment policy using the Phillips curve.

The impulse response functions generated from applying BVAR to the NAIRU theory are in Figure 4.

Changes in inflation now exhibit a pattern akin to serial correlation, with shocks to the change in inflation causing subsequent fluctuation of changes in inflation around zero lasting about two periods before reversal, with the shock wearing off after about eight periods (Figure 4, column 1, row 1). Shocks to the unemployment gap cause a disinflation persisting roughly four periods, thereafter having a near-zero response (Figure 4, column 1, row 2). Shocks to changes in the inflation rate seem to cause a small short-run fluctuation in the unemployment gap which may loosely be interpreted as a decline in the unemployment gap, but the effect is ambiguous (Figure 4, column 2, row 1).

These effects are robust to the choice of lag (i.e., $p = 2$ or $p = 3$). They are less robust to the choice of prior. Under Minnesota priors, the unemployment gap experiences a deep, long-lasting decline following a shock to the change in inflation. The response of the change in the inflation rate to shocks to the unemployment gap is also much larger in the negative direction. Under diffuse priors, the former effect is similar to results with independent normal-Wishart priors, while the latter effect is similar to results with Minnesota priors.

5.4. Wavelet Analysis.

In our wavelet analysis (our application follows that of Grinsted et al. [28]), we apply the cross-wavelet transform and calculate the wavelet coherence for
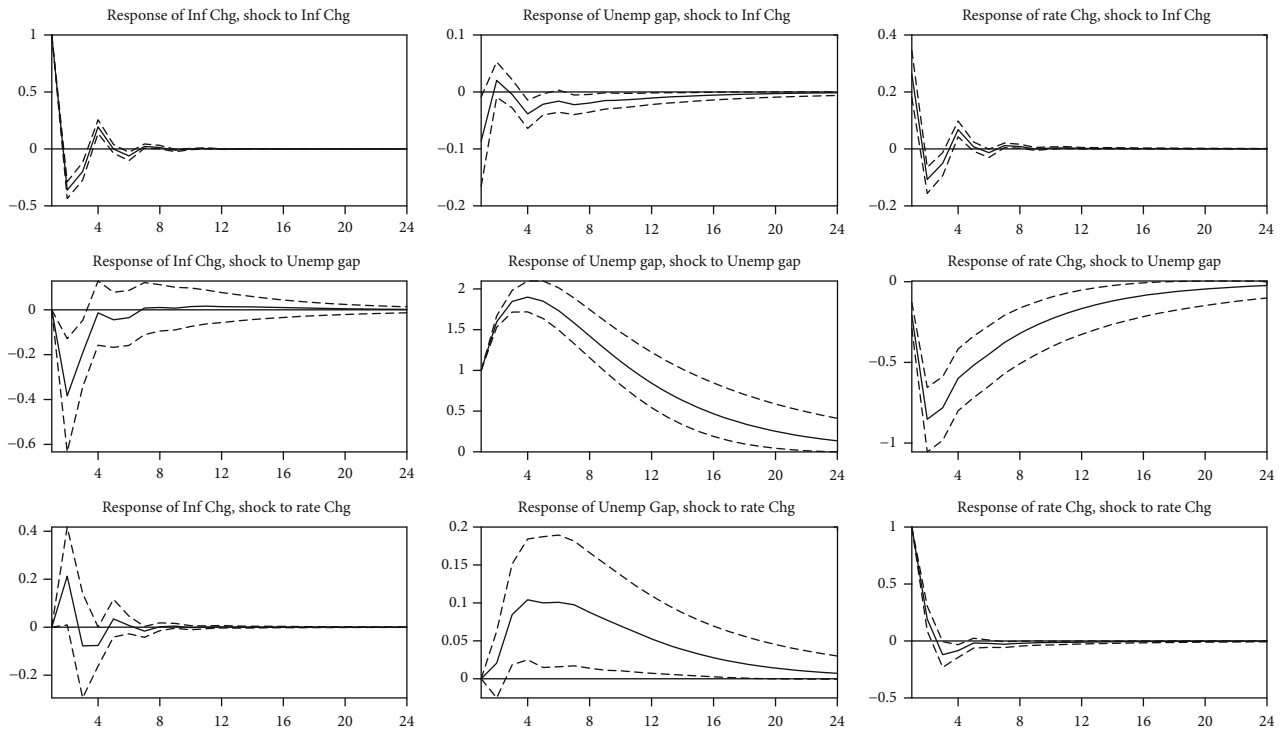
Figure 4: The NAIRU theory impulse response functions.

the following time series pairs to find regions in the time-frequency space where the two time series covary:

(i) Inflation rate– ("INF"–) unemployment rate ("UE")

(ii) Change in inflation– ("dINF"–) unemployment gap ("UE Gap").

The first time series pair is used to assess the conventional Phillips curve, while the latter pair is used to assess the NAIRU theory.

Reading across Figure 5, one may surmise the power of the transformation over time. Reading down provides the power at a given time for different scales. As is clear in Figure 5, both the conventional Phillips curve and NAIRU theory relationships share regions of statistically and economically significant coherence in the 10 to 40 quarter period. This suggests the Phillips curve and NAIRU theory primarily reflect medium-term relationships, consistent with the findings of Gallegati et al. [9], Fratianni et al. [24], and Mutascu [25]. The phase difference predictably indicates a negative relationship, but the direction of the lead-lag relationship is ambiguous over time and across time series. This may reflect feedback between inflation and unemployment, rather than a unidirectional relationship.

Consistent with the findings of Koop and Korobilis [27], Gallegati et al. [9], Fratianni et al. [24], Aguiar-Conraria et al. [26], and Mutascu [25], the short-run Phillips curve is intermittently significant, suggesting a time-dependent relationship, and the long-run relationship (greater than 10 years) is largely vertical. The same is true of the NAIRU theory relationship. Studies that seek to estimate a short-run relationship over small windows of

time may discover problems with external validity due to sampling bias.

It is worth noting that the scales at which the Phillips curve and NAIRU theory relationships are highly coherent have changed since the early 1990's, with the medium-run region of high coherence disappearing for about a decade and then reemerging within a narrower region. Notice further that the NAIRU theory relationship experiences a brief phase change in the significant medium-run region after the mid-2000's. This may partially corroborate the findings of Karlsson and Osterholm [23] vis-à-vis the flattening of the Phillips curve during 2005-2013, while still illuminating that this is only part of the story.

Gallegati et al. [9] emphasize the need to explain why the medium-run relationship between unemployment, wage inflation, and price inflation appears to change after the mid-1990's. They cite the work of Akerlof et al. [29, 30] regarding near-rational wage and price-setting during periods of high and low inflation. These authors suggest there is an asymmetry in the response of wage and price setters to high and low inflation. As inflation increases, the cost of ignoring inflation increases, causing wage and price setters to be increasingly alert to such changes. When inflation is low, however, it is virtually ignored in wage and price-setting decisions. In the wake of a low-inflation environment of the Great Moderation (roughly 1984 to 2007), wage and price setters may simply have responded little to economic shocks.

Despite alternative conclusions, Aguiar-Conraria et al. [26] acknowledge that the New Keynesian theory also predicts less frequent price adjustments and stronger nominal rigidities during a period in which credible monetary policy and anchored inflation expectations lead to low and stable inflation, which would imply a flatter Phillips tradeoff (see
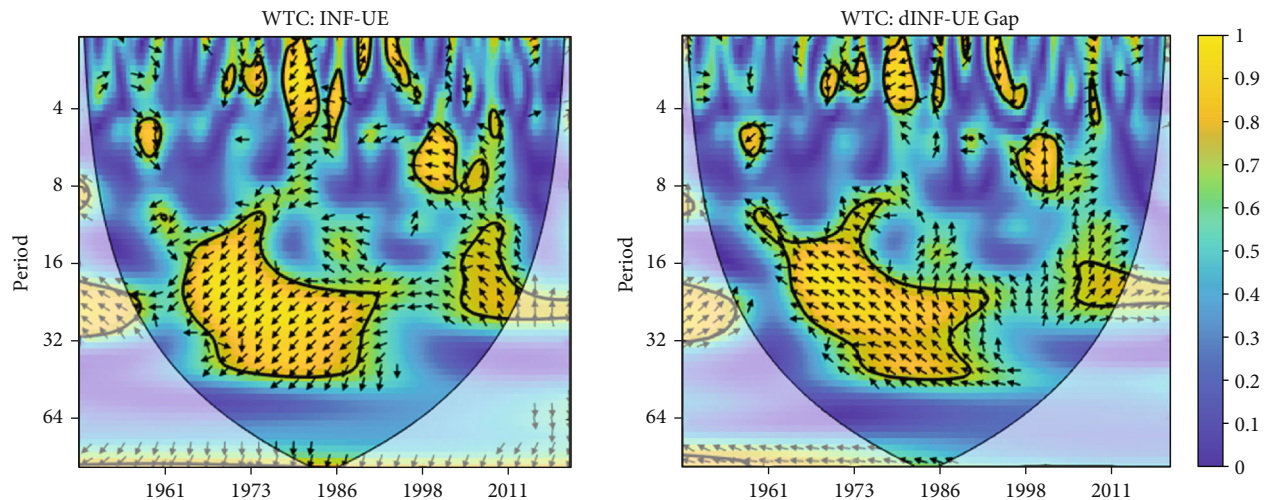
FIGURE 5: Wavelet coherence. The x-axis represents time space while the y-axis represents frequency space (defined by quarters). The color coding indicates coherence, with blue and yellow representing low and high coherence, respectively. The regions enclosed by a black line denote statistical significance at the 5% significance level relative to a null hypothesis of a nearly process-independent background power spectrum. The arrows describe phase difference, with arrows pointing leftward signifying antiphase and rightward signifying in-phase. The arrows' tilt up or down reflects the lead-lag relationship. The cone of influence, represented by the lightly shaded region from the cone outline to the axes, shows areas that may be impacted by edge effects (i.e., effects arising from wavelets stretched beyond the edges of the observation interval); the observed representation of the data in this shaded region should be interpreted with caution.

also Carrera and Ramirez-Rondan [31] and Lopez-Villavicencio and Mignon [32] for empirical work suggesting a flatter Phillips curve when inflation is below a certain threshold). In the context of the conventional Phillips curve and NAIRU theory analysis performed in this section, we find it plausible that an environment of low and stable inflation could explain the lack of a medium-run Phillips tradeoff in our results from the early-1990's to mid-2000's.

There is the additional question regarding *why* the conventional Phillips curve and NAIRU theory relationships are primarily medium-run phenomena. The NAIRU theory provides a sufficient explanation for the lack of a long-run relationship but does not necessarily explain concentrated power in the medium-run over the short-run, or why the short-run relationship is only intermittently observed. While various theories may provide context for these observations, none represent consensus. We mention interesting and potentially relevant literature for the reader's benefit in Appendix G.

6. Conclusion

The market power analysis reveals potentially interesting differences in the relationship between wage growth and employment for different skill levels. Greater skill, as defined by educational attainment, appears to stabilize the Phillips curve as originally defined, leading to more predictable effects of the unemployment rate on wage growth. Such predictability may enhance the bargaining power of laborers with Bachelor's degrees in wage setting and employment decisions relative to lower-skill groups with more unpredictable changes in wages.

The point of wage stability is heavily dependent on the functional form imposed by Phillips [8] but may indicate labor market segmentation in which groups of laborers with different skill levels experience different natural rates of

unemployment. These results may prove an artifact of the relatively small sample, use of annual data, choice of educational groups and country analyzed, and the time period selected. Specifically, a single observation in the Less Than High School group causes extreme uncertainty in the associated Phillips curve, while the time period selected and use of annual data resulted in no unemployment rate observations greater than 5% for the Bachelors group.

The use of simultaneous equations in the BVAR analysis generally revealed a short-term tradeoff between inflation and unemployment in the U.S. for the years 1948 through 2019. These results are robust to the choice prior and consistent with results from Egger et al. [21] and Koop and Korobilis [27].

The NAIRU theory, while it does not appear to be supported in its simplest form, enjoys support from the more sophisticated analysis allowing for endogeneity and other explanatory variables. There is evidence that shocks to the unemployment gap cause a decline in the change in inflation, suggesting a short-run Phillips curve. This effect does not, however, persist over the longer run. There is also evidence that the unemployment gap falls in response to unexpected inflation, though this response is small, ambiguous, and not robust to the choice of prior.

Using wavelet analysis, we make a stronger case that the long-run relationship between inflation and unemployment is vertical. We further discover that the short-run relationship estimated using simultaneous equations may be time-dependent. Rather, the Phillips curve and NAIRU theory relationships are statistically and economically significant in the medium-run (2-10 years); though the region of statistical and economic significance has narrowed in recent decades to roughly the 4-7-year period. Phase difference indicates the predicted negative relationships but provides an ambiguous signal regarding the lead-lag relationship.

Appendix

A. NAIRU Theory Graphical Representation

On the initial short-run Phillips curve, policy may move the economy from point A to point B. Once inflation expectations are recalibrated to the new inflation environment, the curve will shift right to the new short-run Phillips curve. The new point C has the same level of unemployment as point A, but the equilibrium level of inflation is now higher than before (Figure 6).

B. Phillips' Famous "Loops"

Figure 2 in Phillips [6], representing the Phillips curve in the U.K. from 1861 to 1868 (Figure 7).

C. LHS Restatement

Restating the fitted mean curves after eliminating this single observation from 2010 for the Less Than High School group changes the results to those in Table 3 (all data points were tested to determine the impact of removal, but none had nearly the same magnitude of impact on results as the 2010 observation. This analysis was included merely to demonstrate the extent of the impact of the sample size and the interesting pattern that emerges following removal of this data point) (Figure 8).

An interesting pattern is now formed across groups (Figure 9).

Of course, there is no economic reason for excluding the 2010 observation for the Less Than High School group. Rather than to arbitrarily drop this observation, we can use empirical Bayesian methods to impose stability on the Less Than High School curve. We thus apply a moderately informed prior to the Less Than High School group driven by the initial results for all three groups. Specifically, $\log(\alpha_{LHS}) \sim N(2.5,1)$, $\beta_{LHS} \sim N(-1,1)$, and $\tau_{LHS} \sim G(0.25,1)$. This produces a similar pattern as above (Figure 10).

The convexity of each of the curves is now more similar, although the HPDI is still significantly tighter for the Bachelors group relative to the other two groups, supporting our initial conclusion regarding curve stability.

The Bachelors curve now intersects with the x-axis at a materially lower unemployment rate than the other two curves, which would indicate that labor groups with different skill levels may have different natural rates of unemployment.

D. HPDI (MarketPower Analysis)

The fitted (97.5%) and fitted (2.5%) curves represent the curve using parameter values associated with that confidence level, i.e., 2.5% indicates 2.5% confidence that the true curve will fall below this curve. The fitted (97.5%) curve for the "Less Than High School" group is so high it cannot be comfortably shown alongside the other two curves, underscoring the instability of the Phillips curve relationship for this group (Figure 11).

E. Impulse Response Function Comparison

The impulse response functions here are displayed in transpose position relative to Figure 3, with the response of inflation to the three variables across the top rather than down the left side (Figures 12 and 13).

The impulse response functions here reflect the noninformative prior for the time period 1953Q1 to 2006Q3 in the U.S.

F. Methodology

We present here a deeper dive into the methodology applied in this paper.

Our initial analysis is performed through a Bayesian lens. In its simplest form, Bayes' theory makes use of the fact that, for two events A and B, $P(A, B) = P(A \mid B)P(B) = P(B \mid A)P(A)$. Rearranging provides the useful identity

$$P(B \mid A) = \frac{P(A \mid B)P(B)}{P(A)}. \tag{F.1}$$

Restating this identity for econometrics purposes, we get

$$P(\theta \mid Y) = \frac{P(Y \mid \theta)P(\theta)}{P(Y)}, \tag{F.2}$$

where Y represents the data being analyzed and θ is a vector or matrix containing parameters for a model seeking to explain Y. The term $P(Y \mid \theta)$ is the conditional probability density of the data given the parameters. When Y is from actual data (rather than a random variable), this term is called the likelihood function. The term $P(\theta)$ is referred to as the prior probability distribution for θ. The term $P(Y)$ is the marginal likelihood of the data and is found by marginalizing the likelihood function with respect to θ.

F.1. Market Power Analysis. In Equation (12), we assume $\log(\dot{y}_j - a) \mid \alpha_j, \beta_j, \tau^2 \sim N(\log(\alpha_j) + \beta_j[\log(x_j)], (1/\tau^2))$ where $\tau^2 = 1/\sigma^2$ is the error precision. Note that log represents the natural logarithm. This implies a likelihood function with general form

$$P(y, x \mid \beta, \tau) = L(\beta, \tau; y, x) \propto \left(\frac{\tau}{2\pi}\right)^{\frac{n}{2}} \exp$$
$$\cdot \left[-\frac{\tau}{2}(y - x\beta)'(y - x\beta)\right], \tag{F.3}$$

where β is inclusive of α, x is exogenous, and y represents $\dot{y}$.

We apply diffuse independent normal-Gamma priors

$$P(\beta, \tau) = P(\beta)P(\tau) \propto \exp\left[-\frac{1}{2}(\beta - \beta_0)'(V_0)^{-1}(\beta - \beta_0)\right]\tau^{\frac{v_0}{2}-1}\exp\left[-\tau\frac{v_0}{2}s_0^{-2}\right], \tag{F.4}$$

where $\beta \sim N(\beta_0, V_0)$ and $\tau \sim G(s_0^{-2}, v_0)$. Consistent with diffuse priors, we set $\beta_0 = 0$, $V_0 = 1/0.0001$, $s_0^{-2} = 1$, and $v_0 = 0.0002$.
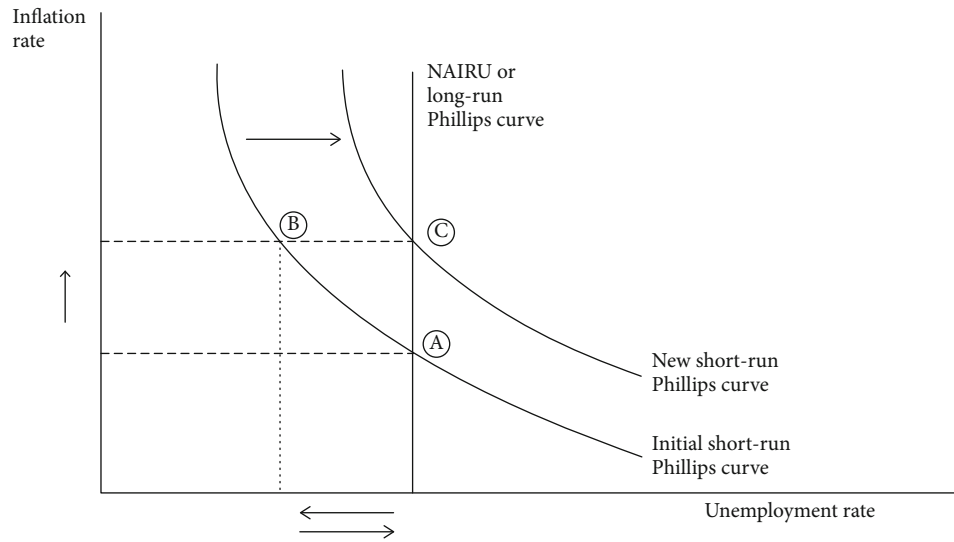
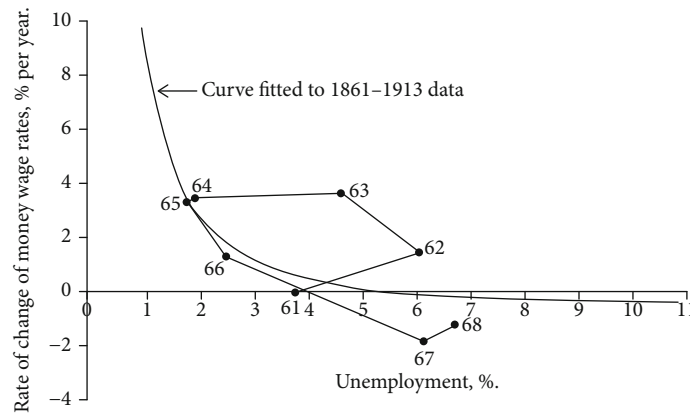FIGURE 6: NAIRU theory graphical representation (this chart comes from Dritsaki and Dritsaki [13]).

FIGURE 7: Phillips' famous "loops."

TABLE 3: 95% HPDIs (excluding 2010 for LHS).

Parameters	2.50%	Mean	97.50%
Log(α) (LHS)	0.833	2.758	4.612
β (LHS)	-1.763	-0.865	0.068
σ^2 (LHS)	0.335	0.697	1.411

This implies the posterior distribution

$$P(\beta, \tau \mid y) \propto \exp\left[-\frac{1}{2}\tau(y - x\beta)'(y - x\beta) \right.$$
$$\left. + (\beta - \beta_0)'(V_0)^{-1}(\beta - \beta_0)\right]\tau^{\frac{n+v_0}{2}}\exp\left[-\tau\frac{v_0}{2}s_0^{-2}\right]. \tag{F.5}$$

This posterior distribution does not have an explicit analytical solution for key features like mean and variance. To estimate these features, we apply a Gibbs sampler. Gibbs sampling allows for determination of posterior distributions via simulation. Successful use requires the full set of conditional distributions to be available. Assuming they are, one
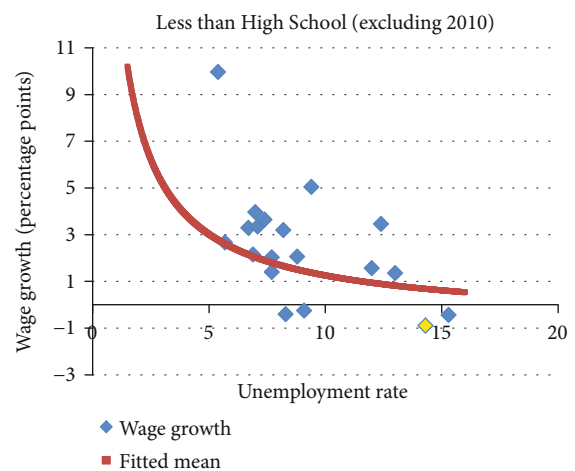

FIGURE 8: Less Than High School (excluding 2010).

may draw successively from each of the conditionals until the desired number of iterations is obtained. For example, let $x = (x_1, x_2)$. The sampling kernel is given by $p(x, y) = f(y_1 \mid x_2)f(y_2 \mid y_1)$. Then, $\int p(x, y)f(x)dx = \int\int f(y_1 \mid x_2)f(y_2 \mid$
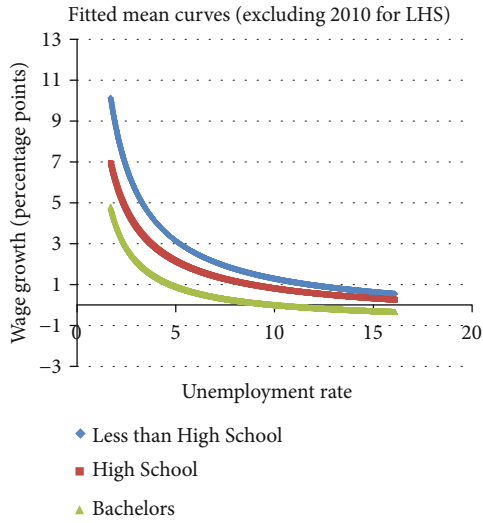
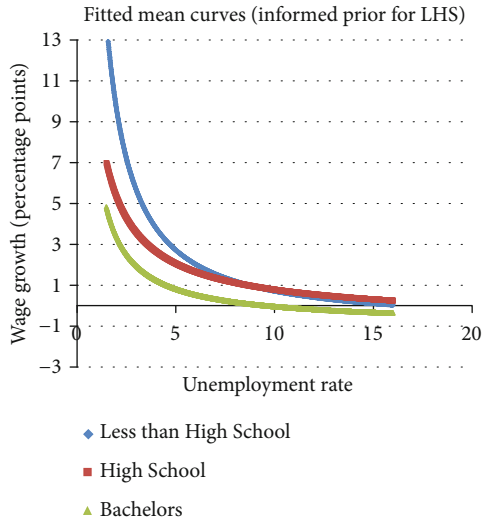FIGURE 9: Fitted mean curves (excluding 2010 for LHS).


FIGURE 10: Fitted mean curves (informed prior for LHS).

$y_1)f(x_1, x_2)dx_1dx_2 = f(y_2 \mid y_1) \int f(y_1 \mid x_2)f(x_2)dx_2 = f(y_2 \mid y_1)f(y_1) = f(y)$.

For independent normal-gamma priors, conditional distributions are available of the form

$$P(\beta \mid y, \tau) \sim N(\bar{\beta}, \bar{V}),$$
$$P(\tau \mid y, \beta) \sim G(\bar{s}^{-2}, \bar{v}),$$
(F.6)

where $\bar{V} = (V_0^{-1} + \tau x'x)^{-1}$, $\bar{\beta} = \bar{V}(V_0^{-1}\beta_0 + \tau x'y)$, $\bar{v} = n + v_0$, and $\bar{s}^2 = (y - x\beta)'(y - x\beta) + v_0 s_0^2/\bar{v}$. We discard the first 1000 burn-in draws and utilize the next 49,000 draws for estimation of the full joint posterior.
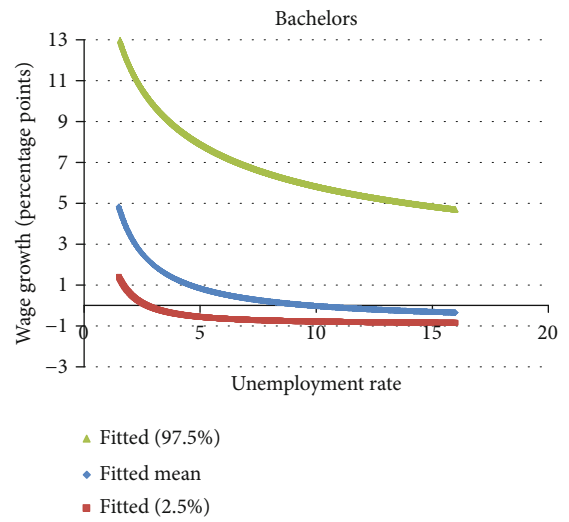
F.2. NAIRU Theory Analysis. The likelihood function, priors, and posterior distribution for Equation (13) are identical to those from Appendix F ("Market Power Analysis") above (see Equations (F.3), (F.4), and (F.5)). We
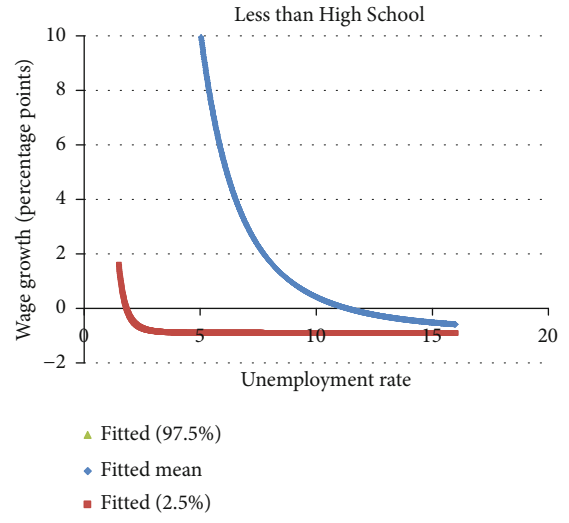
(a)
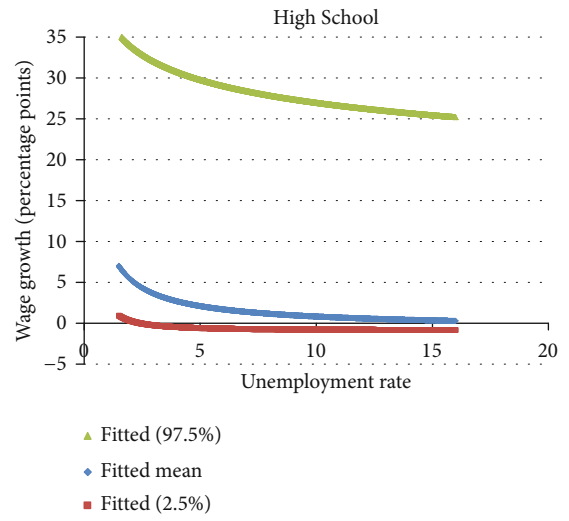
(b)

(c)

FIGURE 11: Relationship between wage growth and unemployment rate for Less Than High School, High School, and Bachelors group.

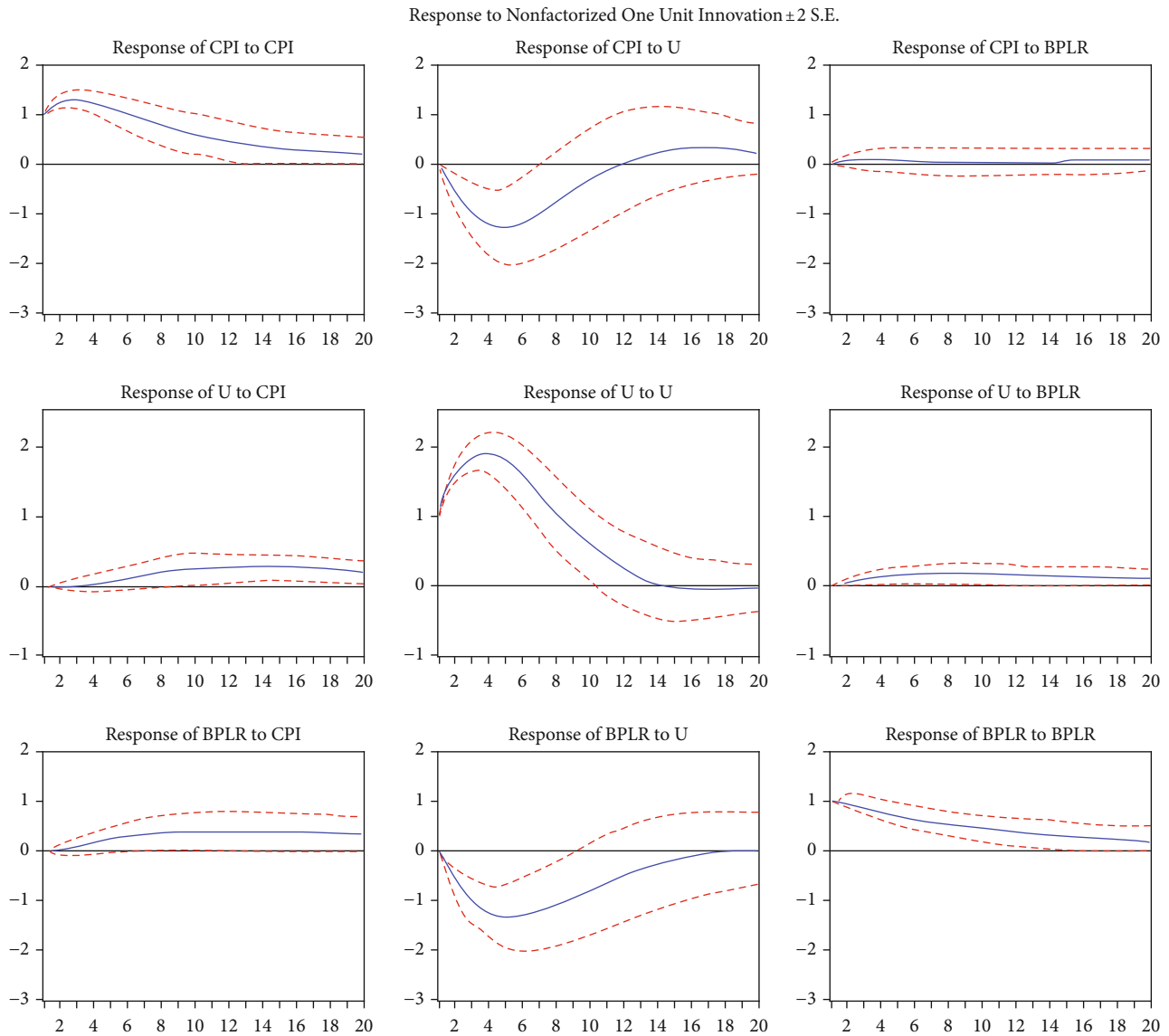Response to Nonfactorized One Unit Innovation ± 2 S.E.

FIGURE 12: Impulse response functions (this chart comes from Egger et al. [21]).

again apply the Gibbs sampling with conditional posteriors found in Equation (F.6). We drop the assumption that the unemployment gap is exogenous by restating the NAIRU theory using BVAR in the Appendix F ("Simultaneous Equations") below.

F.3. Simultaneous Equations. Incorporating initial condition $Y_0 = (\dot{y}_0 \quad x_0 \quad z_0)'$, the likelihood function for Equation (19) is

$$p(Y \mid Y_0, A, \Sigma) = \left(\frac{1}{2\pi}\right)^{\frac{3T}{2}} |\Sigma|^{-\frac{T}{2}} \exp\left[-\frac{1}{2}\operatorname{trace}\left(\Sigma^{-1}\widehat{S}\right)\right]$$

$$\cdot \exp\left[-\frac{1}{2}\operatorname{trace}\left[\left(A - \widehat{A}\right)'\left(X'X\right)\left(A - \widehat{A}\right)\right]\right],$$

$$(F.7)$$

where $\widehat{A} = (X'X)^{-1}(X'Y)$ is the OLS estimate of A and $\widehat{S} = (Y - XA\wedge)'(Y - X\widehat{A})$.

We rely on independent normal-Wishart priors, although a robustness analysis regarding choice of prior is briefly discussed in "Results and Interpretation." Consistent with the literature, $A \sim N(0_{KM}, 10I_{KM})$ and $\Sigma \sim W(M + 1, I_{MM}^{-1})$ where K is the number of parameters in each equation, M is the total number of equations, and I is the identity matrix with the subscripted dimensions.

The independent normal-Wishart prior does not provide for a posterior distribution with analytical solutions for key features. Thus, we again apply a Gibbs sampler to perform statistical inference. For independent normal-Wishart priors, conditional distributions of the form $P(A \mid Y, \Sigma^{-1}) \sim N(\bar{\alpha}, \bar{V})$ and $P(\Sigma^{-1} \mid Y, A) \sim W(\bar{S}, \bar{v})$ are employed. We discard the first 2000 burn-in draws and utilize the next 10,000 draws for estimation of the full joint posterior.
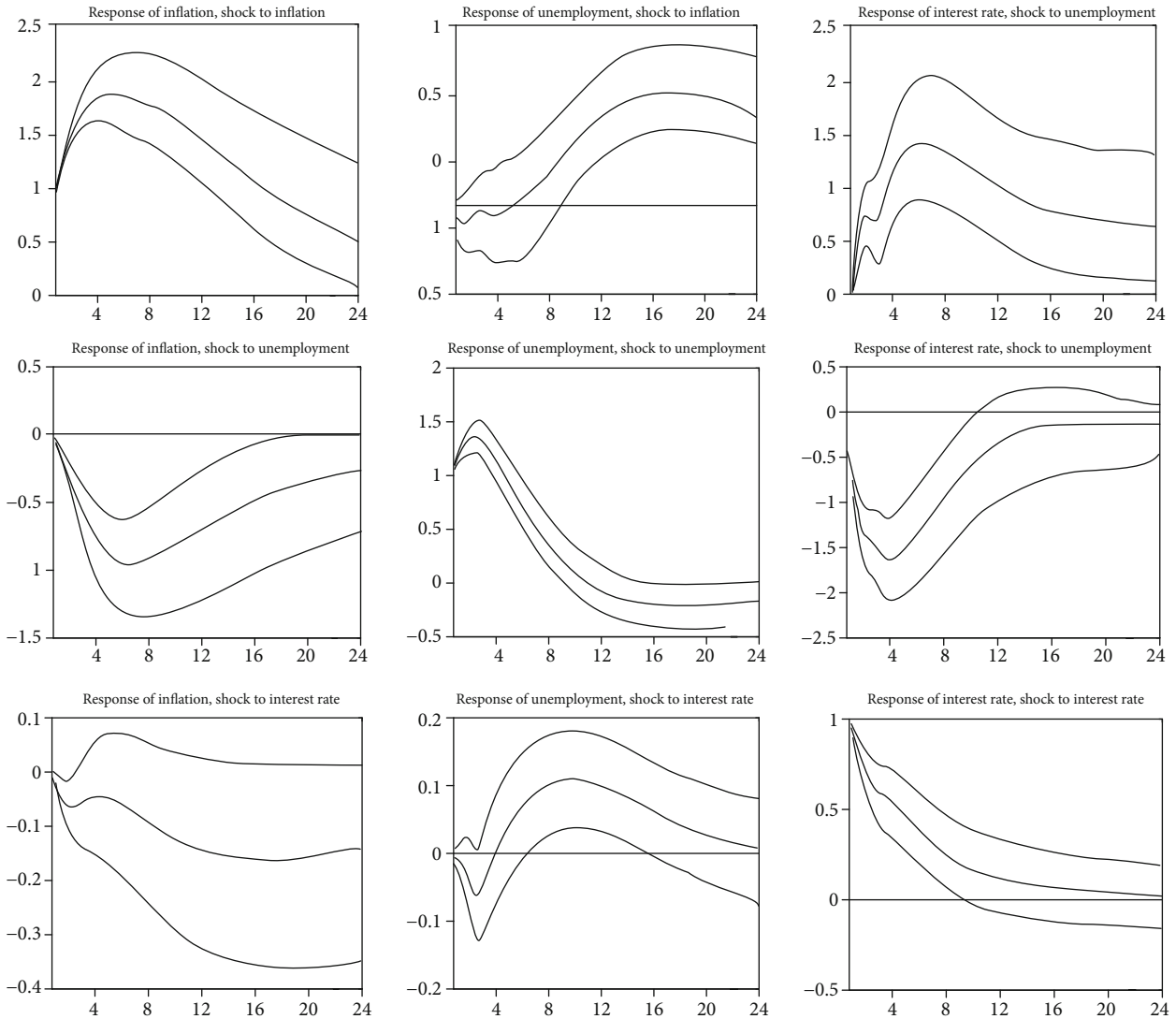
FIGURE 13: Impulse response functions (this chart comes from Koop and Korobilis [27]).

To recover the structural model, we rely on a Choleski decomposition of B_0 as follows:

$$B_0 = \begin{bmatrix} b_{11}^0 & 0 & 0 \\ b_{21}^0 & b_{22}^0 & 0 \\ b_{31}^0 & b_{32}^0 & b_{33}^0 \end{bmatrix}. \tag{F.8}$$

This implies that shocks to the inflation rate may have a contemporaneous impact on the inflation rate, but only a lagged impact on the unemployment rate and interest rates; shocks to the unemployment rate may have a contemporaneous impact on both the inflation rate and the unemployment rate, but only a lagged impact on the interest rate; and shocks to the interest rate may have a contemporaneous impact on all variables.

F.4. Wavelet Analysis. Wavelet analysis, a type of multiresolution decomposition, involves transforming and analyzing time series in the time-frequency space. While many traditional approaches to analyzing the frequency space assume time stationarity (e.g., Fourier analysis), wavelet transforms allow for examination of localized, time-dependent periodicities.

Wavelets are mathematical functions characterized by their degree of time and frequency localization. Wavelet graphs oscillate up and down the time axis, thus integrating to zero. Wavelets can be used to decompose a function $f(\bullet)$ into frequency component functions that inherit information about $f(\bullet)$. Through this decomposition, one may assess which modes of variation dominate across time. Specifically, a wavelet is a kernel function $\varphi(\bullet)$ localized at location τ with scale $\lambda > 0$. By design, wavelet functions have zero mean and normalized scale. In other words, $\int_{-\infty}^{\infty} \varphi(t)dt = 0$ and $\int_{-\infty}^{\infty} \varphi(t)^2 dt = 1$. This normalization ensures wavelet transforms are comparable to each other at various scales.

The continuous wavelet transform of a discrete time series $\{x_i\}_{i=1}^N$ represents the convolution of $\{x_i\}$ with the normalized wavelet $\varphi_{\lambda,\tau}(t) \equiv \varphi(x(t) - \tau/\lambda)$, providing a time-

frequency representation of the time series. $\varphi_{\lambda,\tau}(t)$ is referred to as the "mother" wavelet, in turn being stretched and shifted into a set of "daughter" wavelets $W_\varphi(\tau, \lambda)$. Treating $\{x_i\}$ as a continuous function of time $x(t)$ and applying the continuous wavelet transform with scale (as will be explained below, scale is approximately 1/frequency in our application; thus, a large scale implies a low frequency and vice versa) parameter λ and location parameter τ, we get

$$W_\varphi(\tau, \lambda) = \frac{1}{\sqrt{\lambda}} \int x(t)\varphi\left(\frac{x(t) - \tau}{\lambda}\right) dt. \qquad (F.9)$$

The motivation underlying the continuous wavelet transform is to use wavelets as a means to attenuate frequencies of a certain range (i.e., use wavelets as a band-pass filter for time series). After applying the wavelet transform, one may analyze scale variations of the time series locally, rather than globally. Consequently, nonstationarity is accommodated naturally without the need to detrend or difference the data. As the wavelet scale is shifted and translated along the localized time index, one may view the amplitude-scale relationship across time.

In our analysis, we apply the Morlet wave (see Grossman and Morlet [31]) for φ:

$$\varphi(t) = \pi^{-\frac{1}{4}} e^{i\omega_0 t} e^{-\frac{t^2}{2}}, \qquad (F.10)$$

where ω_0 is dimensionless frequency and t is dimensionless time. Setting $\omega_0 = 6$ to satisfy admissibility conditions (see Farge [32]), this complex wavelet provides a desirable time-frequency localization balance, making it a common choice in economic applications. Using this wavelet, the Fourier frequency is approximately equal to $1/\lambda$, facilitating the scale-frequency relation, hence the time-frequency interpretation of the (τ, λ)-plane.

The square of the transform in modulus, $|W_\varphi(\tau, \lambda)|^2$, provides the local variance, also known as power, of the time series $x(t)$ in the time-frequency space. Wavelet power yields information as to the association between the wavelet (given a scale) and the data array (given a location).

The cross-wavelet transform is an extension of the continuous wavelet transform to two time series. This transformation reveals regions of common power and relative phase in the time-frequency space. Consider two signals $x(t)$ and $y(t)$ and their continuous wavelet transforms W_x and W_y. The cross-wavelet transform is defined as

$$W_{x,y} = W_x W_y^*, \qquad (F.11)$$

where W_y^* denotes the complex conjugate of W_y. The cross-wavelet power is then $|W_{x,y}|$, which we may interpret as the local covariance between the two time series at each time and frequency. The cross-wavelet power distribution is

$$D\left(\frac{|W_{x,y}|}{\sigma_x \sigma_y} < p\right) = \frac{Z_\nu(p)}{\nu}\sqrt{P_x P_y}, \qquad (F.12)$$

where P_x and P_y are the background power spectra for the two time series and $Z_\nu(p)$ is the confidence level for probability p of a distribution for the square root of the product of two chi-squared distributions with degrees of freedom ν. Derivation of this distribution allows one to identify statistically significant regions of cross-wavelet power, indicating rejection of the null hypothesis of a signal generated from a stationary, normally distributed AR (1) noise with a given background power spectrum. (The use of a normally distributed AR (1) noise as the null hypothesis makes the distribution of the underlying data important for determining regions of statistical significance. The AR (1) coefficients are those which best fit the underlying data, so a Gaussian AR (1) process must be a decent fit for the underlying data. Otherwise, the null hypothesis is trivially rejected and the significance of the coherence is spurious. However, consistent with the central limit theorem, the distribution will tend converge to a normal distribution as we convolute with increasingly long waves, making the data distribution more important at shorter scales.)

Wavelet coherence $R_{x,y}$ is defined such that

$$R_{x,y}^2 = \frac{\left|S\left(\lambda^{-1} W_{x,y}\right)\right|^2}{S\left(\lambda^{-1} |W_x|^2\right) S\left(\lambda^{-1} |W_y|^2\right)}, \qquad (F.13)$$

where $S = S_{\text{scale}}(S_{\text{time}}(W))$ is a smoothing operator with S_{scale} and S_{time} denoting smoothing along the wavelet scale axis and in time, respectively. $R_{x,y}^2$ ranges between zero and one. The squared wavelet coherence between two continuous wavelet transformations is a measure of significant local correlation between the two underlying two time series in the time-frequency space, regardless of common power. This measure is useful for determining scales at which a relationship is statistically significant from those which are not. Note that transforming data at the beginning and end of the time series involves missing values which must be arbitrarily specified. These "edge effects" may impact the integrity of the output. The region in which one must be wary of edge effects is referred to as the "cone of influence."

Phase difference θ, representing the angle of the wavelet coherence, is calculated as the imaginary to real ratio of the wavelet coherence. Formally, phase difference is calculated as

$$\theta_{x,y} = \arctan\left(\frac{\mathfrak{I}\left[S\left(\lambda^{-1} W_{x,y}\right)\right]}{\mathfrak{R}\left[S\left(\lambda^{-1} W_{x,y}\right)\right]}\right). \qquad (F.14)$$

The phase indicates the series' position within their respective cycles, parameterized in radians ranging from $-\pi$ to π. In other words, this measure describes the relative phase (i.e., lead-lag relationship) between two time series variations. The following schedule may provide a better intuition for interpreting phase difference:

(i) $\theta_{x,y} = 0$ indicates the two time series are synchronous at the specified time and frequency

(ii) $\theta_{x,y}$ in the first quadrant (i.e., phase arrow pointing northeast) indicates the series move in-phase, with y leading x

(iii) $\theta_{x,y}$ in the second quadrant indicates the series are out of phase, with x leading y

(iv) $\theta_{x,y}$ in the third quadrant indicates the series are out of phase, with y leading x

(v) $\theta_{x,y}$ in the fourth quadrant indicates the series move in-phase, with x leading y

(vi) $\theta_{x,y} = \pm\pi$ indicates the series are in antiphase.

Interpretation of phase difference for determining lead-lag relationships should be done with care. A phase arrow pointing straight up indicates y leads x by 90°. This could equally be interpreted as y lagging x by 270° or y lagging $-x$ by 90°.

G. Relevant Literature

In this final appendix, we highlight work that may be of interest to readers following the observations from "Wavelet Analysis." Though many of these papers focus on the dynamics of monetary (non)neutrality, they are stimulating and touch on concepts related to the behavior of wage and price inflation and economic shocks. We do not consider any of the referenced literature a conclusive description of the "Wavelet Analysis" results, but believe it worthwhile to mention theories that explain rigidities and medium-run relationships.

According to the New Keynesian models, the wage-price relationship is a function of the relative degree of wage and price rigidities, which are presupposed to differ across time depending on the prevailing inflation regime and the nature of shocks. Wage increases, beyond that which is supported by productivity increases, are expected to be inflationary (i.e., cost-push inflation).

Bobeica et al. [33] find that there is evidence for shock-induced cost-push inflation in the short- to medium-run using data from the Euro area. The relationship is both state-dependent and shock-dependent, resulting in time-varying responses. Specifically, labor costs are more likely to be passed through to price inflation following demand shocks. This link is systematically less relevant in periods of low inflation. (See Bobeica et al. [33] and references therein for the empirical controversy regarding the direction of pass-through (i.e., do wage increases flow through to prices or vice versa).)

Gorodnichenko [34] developed a model establishing microfoundations that may explain predominantly medium-run inflation following demand shocks. In this model, firms set prices and acquire information in a state-dependent system with imperfect information and internalized menu costs. Firms rely on both private and public signals for measuring nominal demand. Macroeconomic variables, endogenously determined, provide these public signals. Firms are thus slow to adjust prices for two reasons: (a) inter-nalized menu costs associated with price changes and (b) external informational benefits enjoyed by other firms looking to better understand optimal price adjustments. In other words, postponing price adjustments may allow a firm to benefit from the adjustments of other firms. The response of inflation in this model is thus hump-shaped, with adjustments concentrated in the medium-run. (Woodford [35] can also explain a dominant medium-run inflation response in an economy with no menu costs in which information is dispersed, but this model is less general; particularly, it lacks a revealing public signal, weakening the mechanism for converting aggregate demand shocks into pricing decisions. Nonetheless, under relatively restrictive assumptions, the model shows how bounded rationality, uncertainty regarding privately held higher-order expectations, and monopolistic competition can also lead to a sluggish but accelerating response of inflation to demand shocks.)

Hall [36] built upon the classic Keynesian wage-rigidity models to argue that sectoral rigidity imbalances can create economy-wide rigidities. Hall rejects theories that rely on unionization, slow information diffusion, or bargaining power to explain economy-wide wage rigidities, explaining instead how wage rigidity may spill over from the "nonentre-preneurial" sector (this primarily includes government and nonprofit jobs and jobs in highly regulated industries) to the competitive sector. Following a negative shock to aggregate demand, workers will search longer for jobs in the rigid wage sector due to the temporarily increased wage premium between competitive and nonentrepreneurial sectors. This delays wage adjustments in the competitive sector.

Kuester [37] explores the impact of search and matching frictions in staggered-contract price-setting sectors using a New Keynesian model. Matching frictions result in wage negotiations, creating strategic price-setting complements and rigidities. This dulls the adjustment of wages and prices following economic shocks. By assuming workers are directly employed in monopolistically competitive firms producing final goods, increased marginal labor costs are passed through to customers in prices, reducing demand and hours worked (i.e., reducing the incentive to ever increase labor costs in the first place). Through this circular process, wages and prices become rigid.

These New Keynesian approaches assume the prevalence of cost-push inflation or, in the case of negative demand shocks, cost-savings-pull disinflation. (Or, in the case of negative demand shocks, cost-savings-pull disinflation.) While these theories have components that are consistent with the stylized facts presented in this paper, their appropriateness relies on a strong role for unemployment in conveying demand signals to firms, a role that is not well established in the literature. Additionally, Nakamura and Steinsson [38] challenge the prevailing view of price rigidity, arguing that the frequency with which prices change may be an improper proxy for flexibility of the aggregate price level. They suggest a special role for temporary sales, product substitution, the degree of heterogeneity in price changes across sectors, and large idiosyncratic price movements. Specifically, the extent of price rigidity is highly sensitive to the treatment of temporary price discounts, replacement of old

inventory with new, the choice to calibrate to mean or median frequency for the aggregate price level, and nonprice features in producer contracts. They further suggest that price rigidity may be endogenous and state-dependent, with periods of high inflation corresponding to lower rigidity.

Funk and Kromen [39] explore the long-term relationship between inflation, employment, and output under short-term nominal price rigidity, Schumpeterian growth, and quality-improving innovations. They find that unemployment is a hump-shaped function of inflation (with a negative relationship), due to several effects of short-run rigidities that compound over time. They emphasize the relative price distortions caused by price rigidities, in turn impacting real wages, employment, and output growth in the long-run. (While these authors use "long-run" in their paper, one could interpret the relationship implied by their model as "medium-run" as well.)

Data Availability

Available from the author by request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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