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Research Article

The Gap and the Upper Critical Field H_{c2} as Function of Doping for High-T_c Cuprates

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The relation between the d-wave superconducting gap Δ_0 and the specific heat obtained with the Volovik effect is used to determine the upper critical field H_{c2} as doping function, for high-temperature superconductors. A two-components model with d-wave symmetry, within the BCS framework, is introduced to describe the superconducting state. Generalized Fermi surface topologies are used in order to increase the density of states at the Fermi level, allowing the high-T_c values observed. The electron-phonon interaction is considered the most relevant mechanism for the high-T_c cuprates, where the available phonon energy is provided by the half-breathing modes. The energy gap values Δ_0 calculated with this model are introduced to describe the variation of the upper critical field H_{c2} as function of doping, for $La_{2-x}Sr_xCuO_4$.

1. Introduction

The high-temperature superconductors (HTSC) have stimulated intense research since their discovery in 1986 and continue to defy theoretical explanation; that is, a microscopic theory of the mechanism of high-temperature superconductivity remains missing. However, great progress has been made in understanding high-temperature cuprate superconductors, both in experiment and theory.

The HTSC materials exhibit a number of exotic behaviors, both in the superconducting state below the critical temperature T_c and in the normal state above T_c, as a pseudogap phase in the underdoped region and a strange metal, or non-Fermi liquid phase in the region above the maximum of the superconducting dome. Experimental data accumulated so far has given some useful clues to unravel the fundamental ingredients responsible for the high transition temperature T_c. However, the underlying physical process remains unknown. The mystery of the superconductor phase is the electron pairing mechanism.

The complexities of the cuprates and the rapid proliferation and refinement of experimental techniques have often determined the direction of theoretical research. These efforts were essentially phenomenologically based. Although there is a lack of consensus for an overall theory there has been some very interesting and imaginative theoretical research in this area. However, there are controversial experimental results which disagree with theoretical models. In this context, it seems crucial to study new ideas that use simplified schematic models to isolate the mechanism(s) that generate high-T_c superconductivity, in which the most relevant experimental evidence must be considered.

An important element toward understanding the mechanism of high-T_c superconductivity is the pairing symmetry. For many cuprate superconductors it is generally accepted that the pairing symmetry is d-wave for hole-doped cuprate superconductors [1] as for electron doped cuprates [2, 3]. Several experiments with Raman scattering and ARPES have shown that the gap structure on high-T_c cuprate superconductors, as a function of the angle, is similar to a d-wave gap [4]. The small but nonvanishing isotope effects in high-T_c cuprates have been shown compatible with d-wave superconductivity [5].

Additionally, it is important to clarify the relationship between the pseudogap and the superconductivity, in particular whether the pseudogap is associated with precursor incoherent pairing of the superconductivity or an ordered state competing with the superconductivity.

For these reasons the pseudogap region has received important theoretical attention since it was introduced. The term pseudogap was first introduced by J. Friedel to explain the lowering of T_c on the underdoped region by the development of a suppression of the electronic density of states (DOS) as doping was decreased, in this region [6]. In a variety of experiments, in the underdoped region of most cuprates, a suppression of the DOS is seen below a temperature $T^* > T_c$ marking the gradual onset of the pseudogap, as well as T_c which marks the appearance of the condensate. T^* decreases as doping is increased in the underdoped region and disappears into the superconducting dome near or above optimal doping.

Two distinct energy scales have been introduced, namely, the gap near the node characterized by Δ_0 and the pseudogap Δ^* in the antinodal region. Dichotomy between them has been demonstrated from the material, doping, and temperature dependence. The antinodal gap Δ^* does not close at T_c in contrast to the gap near the node Δ_0, which closes at T_c, reminiscent of the BCS theory, although the exact functional form may be different. This indicates that the gap near the node Δ_0 is primarily of superconductivity origin [7].

Theoretical works based on the kinetic energy driven superconducting mechanism consider the coexistence of the superconducting gap and the normal state pseudogap in the whole superconducting dome [8, 9]. Also a phenomenological theory of pseudogap state is proposed [10]. They introduce an ansatz for the form of the self-energy for a two-dimensional lightly doped resonant valence bond (RVB) spin liquid.

Yoshida et al. [11] present an overview of ARPES studies of high-T_c cuprate superconductors aiming at elucidating the relationship among the superconductivity, the pseudogap, and the Fermi arc. From their results, they conclude that the observed behavior below T^* is different from the expectation of the superconducting state, suggesting that the transition from the true normal state above T^* to the pseudogap state has a different origin from Cooper pairing, and consider that the pseudogap is a broken-symmetry state below T^*. This pseudogap most likely competes with superconductivity, leaving the form of the interaction between the pseudogap and superconductivity as an open question. They suggest an effective superconducting gap Δ_{sc} which scales with T_c through $2\Delta_{sc} \approx 4k_BT_c$.

Nevertheless, ARPES and Andreev reflection measurements suggest that the gap Δ_0 in the nodal direction is the gap associated with superconductivity [7]. However, the discussion about the relation between Δ^*, Δ_0, and Δ_{sc} remains.

To describe the high-T_c superconductor state, the determination of the most important parameters is fundamental, in particular for type II superconductors; the behavior of the upper critical field H_{c2} is significant. The temperature dependence of H_{c2} has been extensively studied around T_c, but there is still ambiguity about the H_{c2} definition at this temperature. In torque magnetometry experiments it is defined as the field below which vortices appear in the sample; in magnetoresistivity measurements H_{c2} is defined as the field at which the temperature dependent resistance is 90% of the normal state resistance [12]. In these experiments of resistivity and low-field susceptibility measurements H_{c2} goes to zero at $T = T_c$ [12–14]; on the contrary, in torque magnetometry experiments with the other definition, H_{c2} does not become zero through T_c [15]. At $T = 0$ there is no disagreement; however, the behavior of H_{c2} at low temperature is not yet fully understood [13], because the normal state can only be accessed in very high magnetic fields, in this temperature range.

Several works have been carried out to determine the doping dependence of H_{c2} through direct measurement. However, the results do not agree with each other, yet there is no agreement on its magnitude and behavior with doping in cuprate superconductors, because there are important obstacles to determine H_{c2} by resistive transport or magnetization measurements: the high magnitude of H_{c2} at $T = 0$ and the large superconducting fluctuations [15].

Nevertheless, there are some other possible ways to evaluate H_{c2} at $T = 0$: from the electronic specific heat (SH) [15, 16], as well as by the use of the thermal conductivity [17]. Grissonnanche et al. [17] show that the thermal conductivity can be used to directly detect H_{c2} in the cuprates and maps out H_{c2} across the doping phase diagram. To detect H_{c2}, they use the fact that electrons are scattered by vortices and monitor their mobility as they enter the superconducting state by measuring the thermal conductivity of a sample as a function of the magnetic field H.

The procedure from the electronic SH is based on the implicit relation between the DOS in the mixed state and the correspondence to the normal state. With the calculation of the electronic SH in the mixed state and the values of the normal-state SH it is possible to obtain H_{c2} and the gap Δ_0. Volovik pointed that for a d-wave superconductor in a magnetic field, the most prominent contribution to the SH arises from the extended states outside the vortex core [18] and the change on the excitation energy of those states, due to the fact that the supercurrent around the core (Volovik effect) is comparable to the gap near the nodes.

From the theoretical point of view, the Werthamer-Helfand-Hohenberg (WHH) theory, for conventional low-T_c superconductors, has been used to calculate $H_{c2}(0)$ through the slope dH_{c2}/dT in the vicinity of T_c, but this method gives only a rough estimation for $H_{c2}(0)$ [13, 15], for this reason it is important to develop theoretical models which describe and predict the upper critical field $H_{c2}(0)$.

The determination of H_{c2} is also relevant because it is correlated with the coherence length ξ and the size of the Cooper pairs. ξ is one of the determining characteristic parameters of superconductors; it cannot be measured directly, but it is possible to calculate it from zero temperature upper critical field $H_{c2}(0)$ by the expression $H_{c2} = \Phi_0/2\pi\xi^2$, where Φ_0 is the magnetic flux quantum [19].

The improvement of the experimental techniques and advances in theoretical methods brought evidence of the importance of the electron-phonon interaction (EPI) in HTSC. For under doped cuprates experimental as well as theoretical results on spectroscopy show the role of the EPI in formation of the spectral properties [20]. Measurements of ARPES [7, 21, 22] as well as tunneling [23] provided

enough evidence for the relevance of phonons in HTSC. High precision experiments show that sudden changes in electron dispersion curves (kinks) occur from electronic coupling to optical phonon modes, like half-breathing oxygen modes [5]. The isotope shift of the kink energy gives direct evidence of the EPI in the origin of the kink. However, the most obvious and undebatable evidence for the EPI can be found in phonon spectra where the EPI is manifested in the softening and broadening of particular phonons [24–26].

ARPES and scanning tunneling spectroscopy (STM) experiments also provide evidence for two components in cuprate superconductors [27]. In two-band superconductors the two components may arise from Cooper pairing in different bands; from two groups of charge carriers associated with free or localized states; or from some lattice symmetry breaking [28]. The combined results of energy distribution curves and momentum distribution curves suggest that the system has two electronic components and each has similar k-space dispersion [21]. Experimental results with extended X-ray absorption fine structure (EXAFS) also support a two-component pairing model for the interpretation of anomalous electronic and transport properties of cuprate superconductors [29]. Additionally, data of unconventional isotope effects and local lattice responses experiments show strong support for a multiband superconductivity in cuprate superconductors, and multiband models with polaronic coupling have been introduced to describe the experimental data [30].

Another element to consider is the behavior of the superconducting parameters with doping. The bell of the critical temperature as a function of doping is well known, but regarding the upper critical magnetic field, it is known that there is no agreement on its magnitude and doping dependence [17, 31, 32]. For the foregoing reasons, it is interesting to propose a theoretical model in which the doping dependence of several superconducting parameters can be studied.

Many properties are affected by the pseudogap order and superconductivity in a not yet determined way. For various doping levels ARPES results show that the gap function near the node approximately follows a simple d-wave form over the entire doping range; however the gap function near the antinodal region deviates from this momentum dependence in the deeply underdoped; in the more overdoped regime the deviation is not as evident at the lowest temperatures. At $T \ll T_c$, the gap in the antinodal region is affected by the pseudogap order which is distinct from superconductivity, whereas the d-wave gap in the near-nodal region is dominantly determined by the superconductivity [33]. A superconducting gap that maintains a simple d-wave form to the lowest dopings is an argument for a one-gap picture.

In this work the relation between the d-wave gap and the specific heat obtained with the Volovik effect [18] is used to determine the upper critical field H_{c2} as doping function for HTSC. Generalized Fermi surface topologies are introduced via a two-band model in order to increase the DOS at the Fermi level, allowing the high T_c values observed. A two-component model with d-wave symmetry, within the BCS framework, is introduced to describe the superconducting state. This model considers that the EPI is the most relevant mechanism of the high-T_c superconductivity, where the phonon energy available is provided by the half-breathing modes. The order parameter and the pairing interaction have d-wave symmetry.

The superconducting order parameter Δ_0, which closes at T_c calculated previously, as function of doping with this model for $La_{2-x}Sr_xCuO_4$ [34], as well as the available experimental data, is introduced. Finally, with the upper critical field H_{c2}, using the relation between H_{c2} and ξ at $T = 0$, the behavior of the coherence length $\xi(0)$ with doping is obtained.

2. Theoretical Considerations

To describe the superconductor state, we propose a two-dimensional (2D) model with generalized Fermi surface, which take into account a two-component system with the same dispersion relation. The following generalized Fermi sea has been introduced

$$n_k = \Theta\left(\alpha k_F - k\right) + \Theta\left(\alpha k_F - k\right)\Theta\left(k - \beta k_F\right), \quad (1)$$

with k_F being the Fermi momentum and $0 < \beta < \alpha < 1$. In order to keep the average number of electron states constant, the parameters are related in the 2D system by the following equation:

$$2\alpha^2 - \beta^2 = 1; \quad (2)$$

then only one parameter is independent. The distribution in momentum induces one in energy, $E_\beta < E_\alpha$, where $E_\beta = \beta^2 E_F$ and $E_\alpha = \alpha^2 E_F$. For physical consistency, the anomalous occupancy parameter (AOP) must be restricted, because the energy scale of the anomaly $(1 - \alpha^2)E_F$ must be not larger than the phonon available energy, E_{ph}, which means $(1 - \alpha^2)E_F \leq E_{\text{ph}}$. The minimum of α^2 from this condition can be obtained from

$$\left(1 - \alpha^2\right)E_F = E_{\text{ph}}. \quad (3)$$

This model is introduced in the gap equation

$$\Delta\left(k'\right) = \sum_k V\left(k, k'\right)\Delta\left(k\right)\frac{\tanh\left(E_k/2k_BT\right)}{2E_k}, \quad (4)$$

with $V(k, k')$ being the pairing interaction, k_B being the Boltzmann constant, and $E_k^2 = \epsilon_k^2 + \Delta_k^2$, where $\epsilon_k = \hbar^2k^2/2m$ are the self-consistent single-particle energies.

The electron-phonon interaction is considered as $V(k, k') = V_0\psi(k)\psi(k')$, with V_0 being a constant, when $|\epsilon_k|$ and $|\epsilon_{k'}| \leq E_{\text{ph}}$ and 0 elsewhere. E_{ph} is the phonon available energy; the attractive BCS interaction is nonzero only for unoccupied orbitals in the neighborhood of the Fermi level E_F. In the last equation, $\psi(k) = \cos(2\phi_k)$ for $d_{x^2-y^2}$ symmetry; $\phi_k = \tan^{-1}(k_y/k_x)$ is the angular direction of the momentum in the ab plane. The superconducting order parameter is $\Delta(k) = \Delta(T)\psi(k)$ if $|\epsilon_k| \leq E_{\text{ph}}$ and 0 elsewhere.

In this framework the summation in (4) is changed to an integration which is done over the generalized Fermi surface defined above. One gets

$$1 = \frac{\lambda}{2}\int_{E_\alpha - E_{\mathrm{ph}}}^{E_\alpha + E_{\mathrm{ph}}}\int_0^{2\pi} d\phi \cos^2(2\phi)\tanh\left(\frac{\sqrt{\Xi_k}}{2k_BT}\right)\frac{d\epsilon_k}{\sqrt{\Xi_k}} + \frac{\lambda}{2}\int_{E_\beta}^{E_F}\int_0^{2\pi} d\phi \cos^2(2\phi)\tanh\left(\frac{\sqrt{\Xi_k}}{2k_BT}\right)\frac{d\epsilon_k}{\sqrt{\Xi_k}}. \quad (5)$$

In this equation $\Xi_k = (\epsilon_k - E_F)^2 + \Delta(T)^2\cos^2(2\phi)$, the coupling parameter is $\lambda = V_0 D(E)$, with $D(E)$ being the electronic density of states, which will be taken as a constant for the $2D$ system in the integration range. $E_F = (\hbar^2\pi/m)n_{2D}$, with n_{2D} being the carriers density per CuO_2 layer. The two integrals correspond to the model proposed by (1). $\Delta(T)$ in this model is the gap that closes at $T = T_c$.

The integration over the surface at E_α for the first component is restricted to states in the interval $E_\alpha - E_{\mathrm{ph}} \leq E_k \leq E_\alpha + E_{\mathrm{ph}}$. For the second component, in order to conserve the particle number, the integration is restricted to the interval $E_\beta \leq E_k \leq E_F$, if $E_\alpha + E_{\mathrm{ph}} > E_F$, with $E_\beta = (2\alpha^2 - 1)E_F$, according to (2) while $E_F - E_\alpha \leq E_{\mathrm{ph}}$ implies that the energy difference between the anomalously occupied states must be provided by the material itself. At $T = 0$ K, $\Delta(0) = \Delta_0$; at $T = T_c$, $\Delta(T_c) = 0$.

Δ_0 is the maximum value of d-wave superconducting gap; this gap is consistent with the one-gap picture that maintains a simple d-wave form in all the studied doping range. Equation (5) is evaluated at $T = 0$ K, integrating over all the momentum space, considering the contribution of two components, and α^2 values consistent with the model are obtained numerically using experimental data as input parameters. The doping dependence of the gap $\Delta_0(x)$ is introduced through the Fermi temperature. To arrive to the normal state at $T = 0$ for each doping x an energy greater than $\Delta_0(x)$ or a depairing field $H_{c2}(x)$ is required.

When a magnetic field is introduced, from the Volovik effect, the relation between the d-wave gap and the specific heat can be used to determine the upper critical field H_{c2} as doping function [18]. Volovik pointed out that, in a d-wave superconductor, in the mixed state, the entropy and the DOS may be dominated by contributions from extended states outside the cores, rather than the bound states located inside the vortex cores, found in conventional s-wave superconductors. For a d-wave superconductor, in a magnetic field, the quasiparticle excitation spectrum experiences a Doppler shift due to the supercurrent flow circulating around the vortex cores, and in a nodal superconductor, this Doppler shift will be comparable to the gap near the nodes. Hence the DOS is strongly affected and a finite residual DOS appears at the Fermi level.

In the presence of a magnetic field H, the electronic SH in the mixed state increases by $\gamma(H)T$. The electronic SH coefficient $\gamma = C_{\mathrm{el}}/T$ in this state has a residual value at $T = 0$ K, reflecting the residual DOS (E_F), which increases with the increase of H. The field-induced increase of the SH has been quantitatively determined as $\gamma(H)T \approx H^{1/2}T$. From the Volovik effect, the electronic SH of the CuO_2 planes, calculated by averaging the effect of the Doppler shift over a single vortex-lattice unit cell, is given by [18]

$$\gamma(H) = \frac{C_{\mathrm{el}}}{T} = \frac{4k_B^2}{3\hbar}\sqrt{\frac{\pi}{\Phi_0}}\frac{nV_{\mathrm{mol}}}{d}\frac{a}{v_\Delta}H^{1/2} = AH^{1/2}, \quad (6)$$

where V_{mol} is the volume of one mole of the sample, n is the number of CuO_2 planes per unit cell, d is the c-axis lattice parameter, Φ_0 is the flux quantum, $a = 0.4654$ for a triangular vortex lattice, and v_Δ is the gap slope parameter:

$$v_\Delta = \frac{1}{\hbar}\frac{\partial\Delta_k}{\partial k} = 2\sqrt{2}\Delta_0 a \sin k_{0x}a. \quad (7)$$

The parameter A is inversely proportional to the slope of the gap dispersion around the nodes v_Δ and to the gap maximum Δ_0; that is. $A \approx 1/\Delta_0$ [15].

The change in the SH coefficient γ can be obtained taking into account that the DOS should extrapolate the normal state coefficient γ_N at $H = H_{c2}$, the upper critical magnetic field. At $T = 0$ the SH coefficient is γ_N in a nondiamagnetic state (without vortex). Then

$$\frac{\gamma(H)}{\gamma_N} = \left(\frac{8}{\pi a}\right)^{1/2}\left(\frac{H}{H_{c2}}\right)^{1/2}, \quad (8)$$

using that $\gamma(H) = AH^{1/2}$ the upper critical magnetic field can be calculated by

$$H_{c2} = \frac{8a^2\gamma_N^2}{\pi A^2}. \quad (9)$$

The parameter A can be obtained as function of doping $A(x)$ through the doping dependence of $\Delta_0(x)$. From $H_{c2} \approx (\gamma_N\Delta_0)^2$, with γ_N from the literature and Δ_0 obtained from (5), the $H_{c2}(0)$ values can be calculated with (9).

The model presented in this section and the Volovik effect can be used to determine the relation between the d-wave gap and the critical magnetic field. The AOP α^2 and other relevant parameters are calculated. In any case a specific material must be selected to introduce the available experimental data. Ranges for the coupling parameter λ in the intermediate coupling region, and the AOP α^2, consistent with the model and the experimental data can be obtained for each material, in order to determine the gap Δ_0. From these results and the electronic SH relation, the behavior of $H_{c2}(0)$ with doping can be obtained. The relation between ξ and H_{c2} at $T = 0$ allows us to calculate the behavior of the coherence length ξ with doping.

3. Results

In order to get numerical results with our model, the La-based compounds were selected. Several doping concentrations x, ranging from the underdoped to the overdoped regime, are considered. The single layer cuprate superconductor $La_{2-x}Sr_xCuO_4$ (LSCO) is very attractive for both theoretical and experimental studies because it has one of the simplest crystal structures among the high-T_c superconductors.

High quality single crystals of this material are available with several doping concentrations which are required for experimental studies. The LSCO carrier concentration is nearly unambiguously determined, because the hole concentration for CuO_2 plane, n_{2D}, is equal to the x value, that is, to the Sr concentration, as long as the oxygen is stoichiometric [35]. Additionally, for several LSCO samples in the underdoped regime, there are reliable data for T_c and measurements of the coherent d-wave superconducting gap and its momentum dependence by ARPES [4].

On the other hand, neutron scattering experiments [36] and first-principles phonon calculations of the role of phonons and the electron-phonon interaction, in the photoemission spectra of LSCO [37, 38], indicate a few phonon modes that are likely to be most relevant for the electron-phonon coupling. Among these, the 70 meV half-breathing mode was selected for our calculations. The energy scale of the anomaly $(1-\alpha^2)E_F$ in our model must be lower or equal to these phonon modes, because they provide an energy scale accessible to the lattice.

To calculate the H_{c2} from the Volovik effect it is important to recognize that when a magnetic field is applied along the direction perpendicular to the CuO_2 plane, the electronic SH coefficient γ changes with H: $\gamma \approx H^{1/2}$ [15]. As H keeps rising, a phase transition from the mixed to the normal state is obtained; this means that the field-induced DOS increases and eventually the normal state DOS is recovered; that is, $\gamma(H)$ rises and saturates to the electronic SH normal-state coefficient γ_N at $H = H_{c2}$.

In a systematic differential calorimetry study Matsuzaki et al. [39] have estimated γ_N at $T \to 0$ K for LSCO across the entire phase diagram. With the normal state SH and the field-induced increase of the SH, quantitatively determined in the mixed state, Wang and Wen [15] extracted the H_{c2}, in a wide doping range for LSCO. They found that H_{c2} falls from its maximum value at optimal doping, forming a similar dome shape as T_c. Their estimation of H_{c2} agrees quantitatively with other methods from SH [15].

Felner et al. [13] determined recently the upper critical magnetic field $H_{c2}(0)$ for $La_{1.85}Sr_{0.15}CuO_4$, from the slope dH_{c2}/dT in the vicinity of T_c; they use the WHH relation to justify their results.

Zhao et al. [8] based on the $t-J$ model, with specific parameters values, have reproduced qualitatively the upper critical field, considering the interplay between the SC gap and the normal-state pseudogap. Similar to the result obtained by Wang and Wen [15], Zhao found that H_{c2} follows the domelike shape of the doping dependence of the superconducting transition temperature.

Wang et al. [32] have determined the upper critical field H_{c2} as function of x in bismuth-based cuprates by scaling of Nernst profiles. They report that as doping decreases, H_{c2} increases to high values, for Bi2212 the range of H_{c2} is from 50 to 165 T, and for Bi2201 the range goes from 42 to 65 T.

Taking into account the experimental data, different values of the coupling constant and the AOP α^2 consistent with our model are obtained for LSCO, at several x values. With (5), Δ_0 can be calculated in the whole doping range studied in this work. The continuous curve in Figure 1 shows the prediction of the model for the superconducting gap Δ_0, as function of x in the range $0.11 \le x \le 0.22$; several experimental measurements are reproduced as it is shown in [34]. In this doping range the electron-phonon coupling λ is obtained in the range $0.55 \le \lambda \le 0.75$; at $x \le 0.11$ a higher coupling is required. The dashed curve shows the d-wave BCS result. In the overdoped region the Δ_0 curve obtained in this work converges to the BCS result.

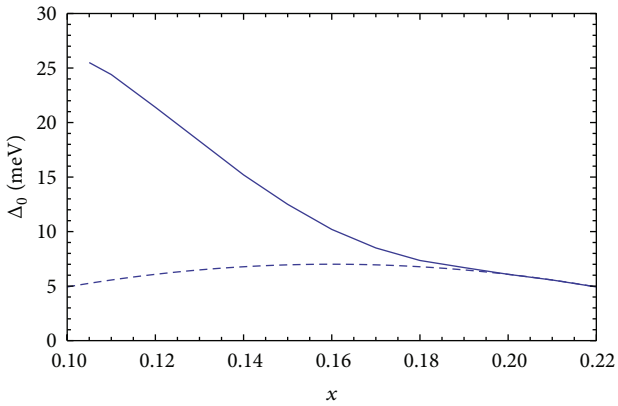

Figure 1: The superconducting gap Δ_0 of $La_{2-x}Sr_xCuO_4$ at $T = 0$, as function of doping in the range $0.11 \le x \le 0.22$. The continuous curve shows the results of our model; the dashed curve is the d-wave BCS results.

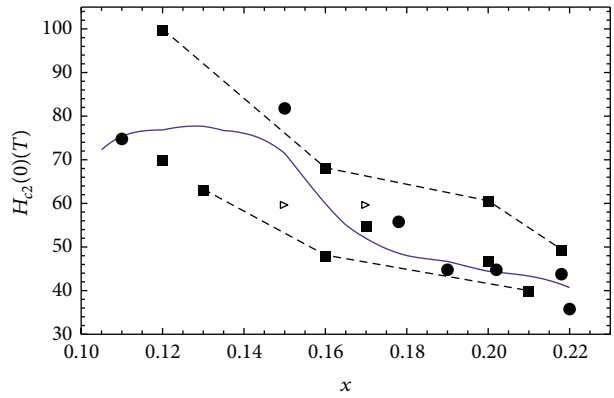

Figure 2: Upper critical field $H_{c2}(0)$ as function of doping. The continuous curve shows the results obtained with our model. The circles correspond to the Wang and Wen calculations [15] from SH; the right triangles correspond to the Ando et al. calculations [40] from resistive transport; the squares are obtained from the Nernst effect [31]. The dashed lines are results for Bi-based superconductors, the upper one for Bi2212 and the lower one for Bi2201 [32].

With these results and the data reported in the literature for γ_N, an interpolated function was calculated in order to obtain H_{c2} as function of doping; the continuous curve in Figure 2 shows our results. The H_{c2} behavior agrees qualitatively with the calculated data obtained with different methods for LSCO. The circles correspond to the Wang and Wen calculations [15] from SH; the right triangles correspond to the Ando et al. calculations [40] from resistive transport;

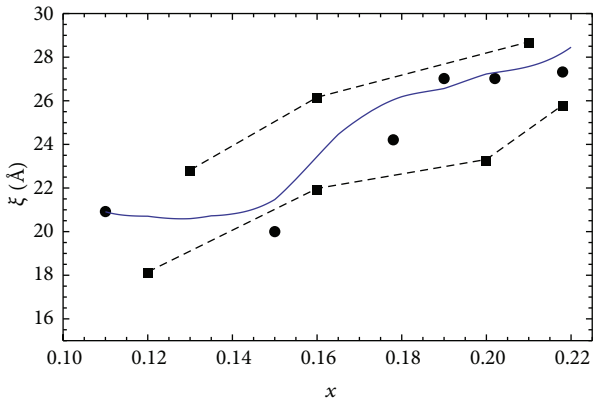

FIGURE 3: The coherence length for $La_{2-x}Sr_xCuO_4$ as function of doping. The full curve shows our results; the full circles are the results for the same material from [15]. The dashed curves show the Wang et al. results [32], the upper curve for B12201 and the lower one for Bi2212.

the squares are obtained from the Nernst effect [31]. The maximal upper critical field at the optimal doping ($x = 0.15$) obtained by Zhao et al. [8] ($H_{c2} \approx 150$ T) is out of the range shown in Figure 2.

In order to compare with other cuprate superconductors the results for bismuth-based cuprates obtained by Wang et al. from the Nernst effect are also shown. The dashed lines are results for Bi-based superconductors, the upper one for Bi2212 and the lower one for Bi2201 [32]. The general behavior of H_{c2} with doping is similar for these materials.

Finally from $H_{c2} = \Phi_0/2\pi\xi^2$ the doping dependence of the coherence length ξ can be obtained. In Figure 3 our $\xi(x)$ results are shown, as well as several data reported in literature. The continuous line shows the results of this work; the circles are the reports of Wang and Wen [15] for LSCO and the dashed lines are results for Bi-based superconductors [32], the upper one for Bi2212 and the lower one for Bi2201. The general behavior of $\xi(x)$ with doping is also similar to these cuprates.

There is a doping regime where superconductivity might suppress the pseudogap order not just in a portion of the Fermi surface, but completely, that is, in the overdoped region. The stronger deviation from a simple d-wave form observed [33] in the deeply underdoped regime cannot be explained with our one-gap model; for this reason we begin our calculation at $x > 0.11$.

In conclusion we determine the upper critical field H_{c2} as doping function for HTSC using the relation between the d-wave gap and the specific heat obtained with the Volovik effect. A two-component model with generalized Fermi surface topologies is introduced, within the BCS framework, to describe the superconducting state. The phonon energy available is provided by the half-breathing modes. The order parameter and the pairing interaction have d-wave symmetry. The electron-phonon interaction is obtained in the intermediate coupling range. The calculated superconducting order parameter Δ_0 as function of doping, as well as the available experimental data, was used. The agreement with other values reported for the upper critical field H_{c2} is shown in the figures for LSCO and also compared with Bi-based superconductors.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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