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Research Article

Spin Polarization Oscillations and Coherence Time in the Random Interaction Approach

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We study the time evolution of the survival probability and the spin polarization of a dissipative nondegenerate two-level system in the presence of a magnetic field in the Faraday configuration. We apply the Extended Gaussian Orthogonal Ensemble approach to model the stochastic system-environment interaction and calculate the survival and spin polarization to first and second order of the interaction picture. We present also the time evolution of the thermal average of these quantities as functions of the temperature, the magnetic field, and the energy-levels density, for $\rho(\epsilon) \propto \epsilon^s$, in the subohmic, ohmic, and superohmic dissipation forms. We show that the behavior of the spin polarization calculated here agrees rather well with the time evolution of spin polarization observed and calculated, recently, for the electron-nucleus dynamics of Ga centers in dilute (Ga,N)As semiconductors.

1. Introduction

The necessity to maintain the information for long periods of time and the efforts to control and manipulate the electronic spin degrees of freedom led to extensive research activities, such as, among others, the search for the best conditions to keep the spin polarization, as well as new mechanisms to enhance the spin coherence time [1–6]. In spite of abundant empirical knowledge of the spin depolarization rates and the spin coherence times, there is little knowledge of the explicit time evolution of these quantities, as functions of the relevant system-environment interaction parameters. In this paper, we focus on this problem and present a simple calculation of the time evolution of the spin polarization driven by the stochastic interaction of the system and its environment, with good results and few assumptions.

The physics of the two-state systems have been studied since the early days of the quantum theory, and various models and approaches have been proposed and published [7–18]. Attempts to solve completely the models for dissipative two-state systems are generally faced with mathematical complexities. Examples vary from entangled differential equations in master equation approaches [11, 19] to perturbative calculations in the 'spin-boson' [14] and the rotating-wave" approximation [16]. Recently the master equation approach [19] was applied to describe the evolution of the electronic and nuclear spin polarizations of interstitial gallium defects, which behave as paramagnetic centers in dilute (Ga,N)As semiconductor that selectively capture electrons with opposite spin, and block the recombination of conduction band electrons with the same spin (which lead to an increase of the lifetime of conduction electrons and bound electrons from picoseconds to nanoseconds). The intricacy of this approach expressed through almost a hundred of coupled nonlinear differential equations (with assumptions on the dissipative interactions) reminds us of the 'much too intractable (intermediate results) of the spin-boson model' [15]. In the physics of complex systems, it has been frequently found that some processes are insensitive to the details of the interaction, being only a few "gross properties" relevant to describe them. This feature, which is not new to many body problems, has often been used to construct successful and enlightening approaches in terms of ensembles of stochastic interactions [17, 20–27] that make possible satisfying evaluations of ensemble averages for relevant quantities. We will present here a Gaussian stochastic spin-environment interaction approach that strengthens this idea.

In the master equation approach in [28, 29], the electronic and nuclear spin dynamics, obtained through rather detailed

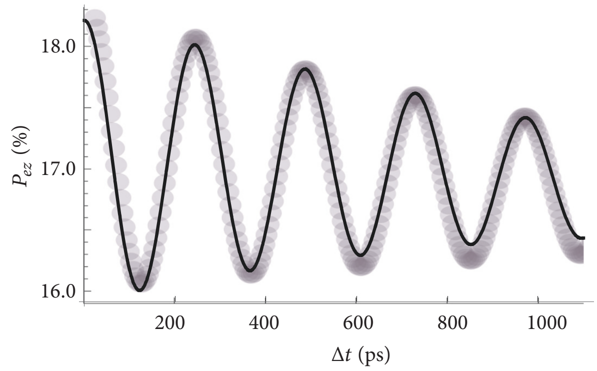

FIGURE 1: The spin polarization calculated in [28] (gray circles) and our results (black curve), to second order of the interaction picture, for level density of the bath $\rho_{\upsilon}(\varepsilon) = 10^9 \epsilon^s$, with $s = 0.28$, and frequency $\omega_o = 2.73556 \times 10^{10} \mathrm{s}^{-1}$. The gray circles graph published with author's permission.

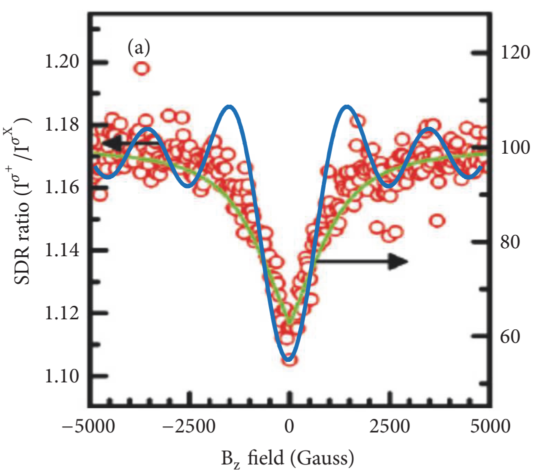

FIGURE 2: Experimental results of the field effect on the spin-dependent recombination ratio reported in Ref. [5] and, on top of these data, we plot (blue curve) the spin polarization behavior as function of the magnetic field. The blue curve is for $\rho_{\upsilon}(\varepsilon) = 10^8 \epsilon^s$, with $s = 2.5$, frequency $\omega_o = 2.279 \times 10^{10} B\mathrm{s}^{-1}$, and $t = 1.410^{-9}$s. Reprinted figure with permission from [5]. Copyright (2013) by the American Physical Society.

calculations, oscillate as shown in Figure 1 (gray-circles curve). This behavior has been explained as caused by oscillations of the hyperfine interaction of bound electrons with the nuclei. In the stochastic interaction model presented here, we obtain, to second order of the interaction picture, the spin polarization oscillations shown also in Figure 1 and plotted with the continuous (black) curve on top of the gray-circles curve. The agreement of these results supports the suggestion that the loss of coherence and the oscillating time evolution of the spins might be caused by the stochastic nature of trapping and recombination. The complexity of the system makes it also difficult to establish to which extent the observed time evolution is a consequence of a new mechanism or of multiple stochastic and spin-field interactions. In Figure 2 we show the experimental results of the field effect, in the Faraday geometry, on the spin-dependent recombination ratio reported in [5], where the hyperfine interaction was uphold to explain this behavior when interstitial Ga^{2+} atoms are present in dilute (Ga,N)As semiconductors. On top of these data we plot also our results (see blue curve) for the spin polarization, as function of the magnetic field, driven by the stochastic interaction. The oscillatory behavior of our results might be behind the large dispersion of data in the experimental results shown in Figure 2.

We will show here that the gross properties of the spin dynamics and spin polarization oscillations, observed in the above mentioned examples, result when a two-state system interacts stochastically with its environment. To take into account the magnetic field in the Faraday configuration, we will consider the Hamiltonian,

$$H = \frac{1}{2}\Delta_o \sigma_x + H_E + \sigma_z V = H_o + \sigma_z V, \tag{1}$$

where $(1/2)\omega_o \sigma_x$ represents a particle of spin 1/2 in the magnetic field $\mathbf{B} = (\Delta_o / g_S \mu_B)\hat{\mathbf{x}}$ and H_E describes the environment (also referred as the bath), characterized by a level density $\rho(\epsilon)$. The potential $\sigma_z V$ represents the system-environment interaction where the operator V represents the environment, with matrix elements modelled as statistically independent Gaussian variables. The Hamiltonians H_B and V belong to Gaussian orthogonal ensembles (GOEs) of random matrices with dimension $N \times N$, large enough that the order relations in (14) are fulfilled. A similar Hamiltonian with the terms σ_x and σ_z interchanged with each other was studied before [18]. In that case, it was possible to evaluate the whole series of the survival probability and the spin polarization, in the interaction representation, for times much larger than the collision time t_{coll} and much smaller than the Poincarè recurrence time t_P. When the basis is chosen such that $|1\rangle$ and $|-1\rangle$ are the eigenstates of σ_x, the Hamiltonian describes the spin flip processes. If, instead, $|1\rangle$ and $|-1\rangle$ are the eigenstates of σ_z, the spin-flip processes are understood as tunneling process in the fictitious spin 1/2 picture [15]. Another system with a similar Hamiltonian $\sigma_z/2 + H_e + \lambda\sigma_z V$, and the two-state system in an eigenstate of σ_x, at $t = 0$, was *numerically* studied in [30], showing also that the decoherence depends greatly on the nature of the random environment H_e and the interaction V. A result that agrees with the results obtained here and other previous papers where it was shown, *analytically*, that the survival probabilities depend explicitly on the density of levels $\rho(\epsilon)$ of the bath. The authors in [30] considered three cases: random matrices that belong to finite-dimension GOE, to BEGOE, and to FEGOE, being the last two embedded ensembles generated by k-body interactions of spinless fermions and bosons, respectively. The dynamics and complexity of this system are different from the ones studied here and in [18]. For the purpose of this paper it is good enough to calculate the survival and spin polarization to second order of the interaction picture, and for times t such that $\Delta_o t/\hbar$ is of the order of 1. This means that when $t \simeq$ 1-10ns the magnetic fields are of the order of 10mT.

The Hamiltonian in (1) is similar to that of the spin-boson model for a two-state system in the fictitious spin 1/2 picture.

In the spin-boson model the environment is modeled as a set of harmonic oscillators and the Hamiltonian describes effectively the tunneling between wells of a double well potential. Some quantities were calculated when appropriate approximations were introduced and the level density was assumed $\propto E^s$ (which, depending on whether the exponent s is equal to 1, < 1 or > 1, corresponds to the so-called ohmic, subohmic and superohmic dissipation forms, respectively).

In the next section we will present the dissipative two-level model. We will present results for the first- and second-order terms of the interaction representation picture in Section 3. In Section 4 we will show some results for the survival probability and for the spin polarization. We discuss some conclusions at the end.

2. The Random Matrix Model and the Survival Probability

As mentioned before, we will consider the Hamiltonian in (1), where the environment interacts with a spin 1/2 particle. The interaction operator V is chosen such that its matrix elements are statistically independent Gaussian variables with zero mean and covariance given by [17, 21–23]

$$\langle V_{ab} V_{cd} \rangle = v_a^2 (E_{ab}) (\delta_{ad}\delta_{bc} + \delta_{ac}\delta_{bd}) W_{\Delta_a} (E_a - E_b), \quad (2)$$

The angular brackets denote the ensemble average, $v_a^2(E_{ab})$ is the strength of the interaction, E_{ab} is the centroid of E_a and E_b, and W_{Δ_a} is a Lorentzian weight factor of width Δ_a. This means that the interaction connects states of the bath within an energy range of order of the Lorentzian width, which defines the collision time $t_{coll} \sim \hbar/\Delta_a$, i.e., the duration of each application of the spin-bath interaction that should be distinguished from the time between successive collisions. We will assume that at $t \leq 0$ the system is held in the eigenstate $|1\rangle$ od σ_z, while the bath is in thermal equilibrium described by the canonical ensemble:

$$p_a = \frac{1}{Z} e^{-\beta E_a}. \quad (3)$$

Here E_a denotes the energy eigenvalue of H_E in the state $|a\rangle$, with levels density $\rho(E)$, and Z is the partition function. The states $|a\alpha\rangle$, with $|\alpha\rangle$ an eigenstate of σ_z, form a complete set. If, at time $t = 0$, the interaction is switched on, we pose the problem of calculating the probability $P_{1\longrightarrow 1}(t)$ that at time $t > 0$ the system remains in the state $|1\rangle$, regardless the state of the bath, i.e., the problem of calculating the thermal average of the survival probability,

$$\langle P_{1\longrightarrow 1}(t) \rangle_\beta = \sum_{a,a_i} p_{a_i} \left\langle \left| \left\langle 1a \left| e^{-iHt/\hbar} \right| 1a_i \right\rangle \right|^2 \right\rangle, \quad (4)$$

where $\langle \cdots \rangle$ stands for the ensemble average on the Gaussian variables and $\langle \cdots \rangle_\beta$ for the thermal average. Concerning the Gaussian ensemble, it is worth mentioning that a wide research on random matrix ensembles underwent a rapid development leading to various modified versions, in particular the embedded ensembles (EE) generated by random k-body interactions, $k = 2$ being the most important; see [26, 27]. The statistical assumptions that we need in our calculations below are completely characterized by (2) and (12), which, according to [21], are compatible with the postulates of matrix elements given by the two-body random ensemble. Before we calculate these averages, we write the time evolution operator $e^{-iHt/\hbar}$ in the interaction representation as

$$\begin{aligned} e^{-iHt/\hbar} &= e^{-iH_o t/\hbar} \sum_n \left(-\frac{i}{\hbar}\right)^n \int_0^t V(t_n)\, dt_n \\ &\cdot \int_0^{t_n} V(t_{n-1})\, dt_{n-1} \cdots \int_0^{t_3} V(t_2)\, dt_2 \\ &\cdot \int_0^{t_2} V(t_1)\, dt_1. \end{aligned} \quad (5)$$

Here

$$\begin{aligned} V(t_j) &= e^{iH_o t_j/\hbar} \sigma_x V e^{-iH_o t_j/\hbar} \\ &= \left(\sigma_z \cos\frac{\Delta_o t}{\hbar} + \sigma_y \sin\frac{\Delta_o t}{\hbar}\right) e^{iH_E t_j/\hbar} V e^{-iH_E t_j/\hbar} \end{aligned} \quad (6)$$

that can be written as

$$\begin{aligned} V(t_j) &= \left(\sigma_z \cos\frac{\Delta_o t_j}{\hbar} + \sigma_y \sin\frac{\Delta_o t_j}{\hbar}\right) \\ &\cdot \sum_{ab} \langle a| e^{iH_E t_j/\hbar} V e^{-iH_E t_j/\hbar} |b\rangle \langle b|, \\ &= \left(\sigma_z \cos\frac{\Delta_o t_j}{\hbar} + \sigma_y \sin\frac{\Delta_o t_j}{\hbar}\right) \\ &\cdot \sum_{ab} V_{ab} e^{(i\varepsilon_a - \varepsilon_b)t_j/\hbar} |a\rangle \langle b|. \end{aligned} \quad (7)$$

To simplify the notation, from here onwards we will define $\varepsilon = E/\hbar$ and $\omega_o = \Delta_o/\hbar$. Using these quantities in (5), the amplitude of the survival probability becomes

$$\begin{aligned} \langle 1a| e^{-iHt} |1a_i\rangle &= \langle 1a| e^{-iH_o t} \sum_{n=0}^{\infty} (-i)^n \sum_{b_1,\ldots,b_n} |b_n\rangle \int_0^t dt_n \\ &\cdot \left(\sigma_z \cos\omega_o t_n + \sigma_y \sin\omega_o t_n\right) \\ &\cdot V_{b_n b_{n-1}} e^{i(\varepsilon_{b_n} - \varepsilon_{b_{n-1}})t_n} \cdots \times \int_0^{t_3} dt_2 \\ &\cdot \left(\sigma_z \cos\omega_o t_2 + \sigma_y \sin\omega_o t_2\right) \\ &\cdot V_{b_2 b_1} e^{i(\varepsilon_{b_2} - \varepsilon_{b_1})t_2} \times \int_0^{t_2} dt_1 \\ &\cdot \left(\sigma_z \cos\omega_o t_1 + \sigma_y \sin\omega_o t_1\right) \\ &\cdot V_{b_1 a_i} e^{i(\varepsilon_{b_1} - \varepsilon_{a_i})t_1} |1\rangle. \end{aligned} \quad (8)$$

This amplitude can straightforwardly be written as

$$\begin{aligned}\langle 1a| e^{-iHt} |1a_i\rangle &= e^{-iE_at}\sum_{n=0}^{\infty}(-i)^n \\ &\cdot \sum_{b_1,\ldots,b_{n-1}} V_{ab_{n-1}}\cdots V_{b_2b_1}V_{b_1a_i}\int_0^t dt_n e^{i(\varepsilon_a-\varepsilon_{b_{n-1}})t_n}\cdots \\ &\cdot \int_0^{t_3} dt_2 e^{i(\varepsilon_{b_2}-\varepsilon_{b_1})t_2}\int_0^{t_2} dt_1 e^{i(\varepsilon_{b_1}-\varepsilon_{a_i})t_1}\langle 1| \\ &\cdot e^{-i(\omega_o/2)\sigma_x t}\left[|1\rangle\cos\omega_o\left(t_n-t_{n-1}+\cdots-(-)^n t_1\right)\right. \\ &\left.+|-1\rangle\sin\omega_o\left(t_n-t_{n-1}+\cdots-(-)^n t_1\right)\right].\end{aligned} \tag{9}$$

Therefore,

$$\begin{aligned}\left\langle|1a| e^{-iHt} |1a_i\rangle\right|^2 &= P_{1\longrightarrow 1}(t) = \sum_{m,n=0}^{\infty}(-i)^m(i)^n \\ &\cdot \sum_{a,b_1,b_2,\ldots,b_{n-1}}\sum_{b_1',b_2',\ldots,b_{m-1}'} V_{ab_{n-1}}\cdots V_{b_2b_1}V_{b_1a_i}V_{a_ib_1'}V_{b_1'b_2'}\cdots V_{b_{m-1}'a} \\ &\cdot \int_0^t dt_n\cdots\int_0^{t_3}dt_2\int_0^{t_2}dt_1\int_0^t dt_m'\cdots\int_0^{t_3'}dt_2'\int_0^{t_2'}dt_1' \\ &\cdot e^{i[(\varepsilon_a-\varepsilon_{b_{n-1}})t_n+\cdots+(\varepsilon_{b_2}-\varepsilon_{b_1})t_2+(\varepsilon_{b_1}-\varepsilon_{a_i})t_1]}e^{-i[(\varepsilon_{a_i}-\varepsilon_{b_1'})t_1'+\cdots+(\varepsilon_{b_{m-1}'}-\varepsilon_a)t_m']} \\ &\cdot \cos\omega_o\left(\frac{t}{2}-t_n+\cdots+(-1)^n t_1\right)\cos\omega_o \\ &\cdot \left(\frac{t}{2}-t_m'+\cdots+(-1)^m t_1'\right).\end{aligned} \tag{10}$$

For the calculation of the ensemble average of the survival probability $P_{1\longrightarrow 1}(t)$, we follow the procedure and the assumptions explained in detail in [17, 18, 21]. In the calculation of the average

$$\left\langle V_{a_ib_1}V_{b_1b_2}\cdots V_{b_{n-1}a}V_{ab_{m-1}'}\cdots V_{b_2'b_1'}V_{b_1'a_i}\right\rangle, \tag{11}$$

using the statistical assumptions given in (2), one has to take into account that only the covariance of V_{ab} with itself or with V_{ba} is nonzero. This property implies that the average of a product factorizes into products of averages of pairs of matrix elements, more specifically into a number of configurations, such as $\langle V_{a_ib_1}V_{b_1b_2}\rangle\cdots\langle V_{b_{n-1}a}V_{ab_{m-1}'}\rangle\cdots\langle V_{b_2'b_1'}V_{b_1'a_i}\rangle$. The Lorentzian weight factor $W_{\Delta_a}(\epsilon_a-\epsilon_b)$ in (2) is assumed to have the property

$$W_{\Delta_a}(\epsilon_a-\epsilon_b)\approx\begin{cases}1, & |\epsilon_a-\epsilon_b|\le\Delta(\overline{\epsilon}_w)\\ 0, & |\epsilon_a-\epsilon_b|>\Delta(\overline{\epsilon}_w)\end{cases} \tag{12}$$

which, as mentioned above, restricts the scope of the interaction V to connect eigenstates of H_E within the energy interval $\Delta_a = \Delta(\overline{\epsilon}_w) = \Delta(\epsilon_{ab})$. Here $\epsilon_{ab}\equiv\overline{\epsilon}_a = (\epsilon_a+\epsilon_b)/2$. The quantity

$$t_{coll}\sim\frac{1}{\Delta_a} \tag{13}$$

has time dimensions and as mentioned before is associated with the collision time, the duration of one application of the interaction V. We assume that Δ_a contains many band levels. In the following we will write Δ for Δ_a. If D is the mean level spacing of the energy eigenvalues of H_E, we will also assume that the characteristic tunneling frequencies, which are of the order of ω_o, are much larger than D. Therefore, the times involved in the calculation satisfy the inequalities

$$\frac{1}{\Delta}\sim t_{coll}\ll t\ll t_P\sim\frac{1}{D} \tag{14}$$

where t_P is the Poincarè recurrence time. Notice that, in the same way as ε and ω_o, the energies Δ, Δ_a, and D are in units of $\hbar$. The assumptions in (14) are taken into account in the calculation of the ensemble average of (10). In this calculation, we meet with quantities like

$$\sum_{b_j,b_{j-1}}\left\langle V_{b_{j+1}b_j}V_{b_jb_{j-1}}\right\rangle e^{i((\varepsilon_{j+1}-\varepsilon_j)t_{j+1}+(\varepsilon_j-\varepsilon_{j-1})t_j)} \tag{15}$$

which become

$$\begin{aligned}&\int_0^{\infty}d\varepsilon_j\rho(\overline{\varepsilon}_j)\nu^2(\overline{\varepsilon}_j)W_\Delta e^{i(\varepsilon_{j+1}-\varepsilon_j)(t_{j+1}-t_j)} \\ &\quad=\nu^2(\overline{\varepsilon}_j)\rho(\overline{\varepsilon}_j)e^{-\Delta|t_{j+1}-t_j|}\end{aligned} \tag{16}$$

where $\nu^2(\overline{\varepsilon}_j)$, $\rho(\overline{\varepsilon}_j)$, and Δ are assumed to vary slowly with the energy. Since the product of $\nu^2(\overline{\varepsilon})$ and $\rho(\overline{\varepsilon})$ appear systematically and the energies ε_j and ε_{j+1} vary almost continuously along the bath spectrum, we define the density $\rho_\nu(\varepsilon) = \nu^2(\varepsilon)\rho(\varepsilon)$, in units of $(E^2/\hbar^2)(\hbar/E) = 1/\mathrm{s}$. The values of m and n that determine the terms (m,n) of the sum in (10) determine also the order of the contribution to the survival probability, which is given by $\nu = (m+n)/2$. For easy reference we write the survival probability as

$$\left\langle P_{1\longrightarrow 1}(t)\right\rangle = \sum_{m,n=0}^{\infty}\left\langle P_{1\longrightarrow 1}^{m,n}(t)\right\rangle = \sum_{\nu=0}^{\infty}\left\langle P_{1\longrightarrow 1}^{(\nu)}(t)\right\rangle \tag{17}$$

with

$$\begin{aligned}\left\langle P_{1\longrightarrow 1}^{m,n}(t)\right\rangle &= (-i)^m(i)^n\sum_{a,b_1,b_2,\ldots,b_{n-1}}\sum_{b_1',b_2',\ldots,b_{m-1}'}\left\langle V_{a_ib_1}V_{b_1b_2}\right. \\ &\left.\cdots V_{b_{n-1}a}V_{ab_{m-1}'}\cdots V_{b_2'b_1'}V_{b_1'a_i}\right\rangle\int_0^t dt_n\cdots\int_0^{t_3}dt_2\int_0^{t_2}dt_1 \\ &\cdot\int_0^t dt_m'\cdots\int_0^{t_3'}dt_2'\int_0^{t_2'}dt_1' \\ &\cdot e^{i[(\varepsilon_{a_i}-\varepsilon_{b_1})t_1+\cdots+(\varepsilon_{b_{n-1}}-\varepsilon_a)t_n]}e^{-i[(\varepsilon_{a_i}-\varepsilon_{b_1'})t_1'+\cdots+(\varepsilon_{b_{m-1}'}-\varepsilon_a)t_m']} \\ &\cdot\cos\omega_o\left(\frac{t}{2}-t_n+\cdots-(-1)^n t_1\right)\cos\omega_o\left(\frac{t}{2}-t_m'\right. \\ &\left.+\cdots-(-1)^m t_1'\right)\end{aligned} \tag{18}$$

3. Time Evolution of the Survival Probability and the Spin Polarization

In this section we present, in detail, results to zeroth and first order of the interaction picture (IP) of the survival probability described in previous section for a spin 1/2 system interacting with a magnetic field in the Faraday configuration and randomly with its environment. The second-order terms of the survival probability are calculated, in detail, in the appendix.

3.1. Survival and Spin Polarization to Zeroth Order of the IP. The zeroth order contribution describes the time evolution of the isolated spin 1/2 system in the Faraday geometry. This contribution is given by the term $\langle P^{0,0}_{1\longrightarrow 1}(t)\rangle$ in (18). The survival probability to zeroth order is then the well-known quantity

$$\left\langle P^{(0)}_{1\longrightarrow 1}(t)\right\rangle = \frac{1}{2} + \frac{1}{2}\cos\omega_o t. \qquad (19)$$

This means that the probability to find the isolated spin 1/2 system in the eigenstate $|-1\rangle$ of σ_z, in the Faraday geometry, is

$$\left\langle P^{(0)}_{1\longrightarrow -1}(t)\right\rangle = 1 - \left\langle P^{(0)}_{1\longrightarrow 1}(t)\right\rangle = \frac{1}{2} - \frac{1}{2}\cos\omega_o t. \qquad (20)$$

Therefore, as expected, the spin polarization of the isolated spin 1/2 system, is

$$\left\langle \Pi_z(t)\right\rangle = \left\langle P^{(0)}_{1\longrightarrow -1}(t)\right\rangle - \left\langle P^{(0)}_{1\longrightarrow 1}(t)\right\rangle = \cos\omega_o t. \qquad (21)$$

It is clear that, in this case, the thermal average is

$$\left\langle \Pi_z(t)\right\rangle_\beta = \frac{1}{Z}\sum_{a_i} p_{a_i}\left\langle \Pi_z(t)\right\rangle = \cos\omega_o t. \qquad (22)$$

3.2. Survival and Spin Polarization to First Order of the IP. We calculate now the survival probability and spin polarization of the spin 1/2 system in the Faraday geometry, to first order of the interaction picture. In this calculation, we use the property

$$\left\langle P^{(n,m)}_{1\longrightarrow 1}(t)\right\rangle = \left\langle P^{(m,n)}_{1\longrightarrow 1}(t)\right\rangle^{*}. \qquad (23)$$

For the first-order contribution, we need to evaluate $\langle P^{0,2}_{1\longrightarrow 1}(t)\rangle$, $\langle P^{1,1}_{1\longrightarrow 1}(t)\rangle$, and $\langle P^{2,0}_{1\longrightarrow 1}(t)\rangle$. The term $P^{0,2}_{1\longrightarrow 1}\langle(t)\rangle$ is given by

$$\begin{aligned}\left\langle P^{0,2}_{1\longrightarrow 1}(t)\right\rangle &= (i)^2\int_0^t dt_2\int_0^{t_2} dt_1\sum_{a,b_1}\left\langle V_{a_i b_1}V_{b_1 a}\right\rangle \\ &\cdot e^{i[(\varepsilon_{a_i}-\varepsilon_{b_1})t_1+(\varepsilon_{b_1}-\varepsilon_a)t_2]}\cos\omega_o\left(\frac{t}{2}-t_2+t_1\right)\cos\omega_o\frac{t}{2}, \\ &= -\int_0^t dt_2\int_0^{t_2} dt_1\rho_\upsilon(\varepsilon)\,e^{-\Delta(t_2-t_1)}\cos\omega_o \\ &\cdot\left(\frac{t}{2}-t_2+t_1\right)\cos\omega_o\frac{t}{2}.\end{aligned} \qquad (24)$$

Neglecting terms of order $1/\Delta$ and smaller, we have

$$\left\langle P^{0,2}_{1\longrightarrow 1}(t)\right\rangle = -\rho_\upsilon(\varepsilon)\frac{t\Delta^2}{\Delta^2+\omega_o^2}\cos\frac{\omega_o t}{2}. \qquad (25)$$

The term $(1,1)$ is given by

$$\begin{aligned}\left\langle P^{1,1}_{1\longrightarrow 1}(t)\right\rangle &= (-i)(i)\int_0^t dt_1\int_0^t dt_1' \\ &\cdot\sum_a\left\langle V_{a_i a}V_{aa_i}\right\rangle e^{i[(\varepsilon_{a_i}-\varepsilon_a)t_1]}e^{-i[(\varepsilon_{a_i}-\varepsilon_a)t_1']}\cos\omega_o \\ &\cdot\left(\frac{t}{2}-t_1\right)\cos\omega_o\left(\frac{t}{2}-t_1'\right).\end{aligned} \qquad (26)$$

To order $1/\Delta$ we have

$$\left\langle P^{1,1}_{1\longrightarrow 1}(t)\right\rangle = \rho_\upsilon(\varepsilon)\frac{t\Delta^2\omega_o+\Delta^2\sin\omega_o t}{\Delta^2\omega_o+\omega_o^3}. \qquad (27)$$

Taking into account these contributions and the zeroth-order contribution $\langle P^{(0)}_{1\longrightarrow 1}(t)\rangle$, we have, to first order of the interaction picture and for $\Delta \gg \omega_o$, the survival probability

$$\begin{aligned}\left\langle P^{(1)}_{1\longrightarrow 1}(t)\right\rangle &= \frac{1}{2}+\frac{1}{2}\left(1-2t\rho_\upsilon(\varepsilon)\right)\cos t\omega_o \\ &+\frac{\rho_\upsilon(\varepsilon)}{\omega_o}\sin t\omega_o,\end{aligned} \qquad (28)$$

and the spin polarization

$$\left\langle \Pi_z^{(1)}(t)\right\rangle = \left(1-2t\rho_\upsilon(\varepsilon)\right)\cos t\omega_o + \frac{2\rho_\upsilon(\varepsilon)}{\omega_o}\sin t\omega_o. \qquad (29)$$

These quantities depend on the level density $\rho_\upsilon(\varepsilon)$ and, through the frequency ω_o, on the magnetic field. We show in Figure 3 the time evolution of the survival probability (upper panel) and of the spin polarization (lower panel), for two values of the density of levels $\rho_\upsilon(\varepsilon)$ each. For the blue curves we considered $\rho_{\upsilon 1}(\varepsilon) = 2\times 10^8\ \mathrm{s}^{-1}$ while for the red curves we have $\rho_{\upsilon 2}(\varepsilon) = 5\times 10^8\ \mathrm{s}^{-1}$. In all of these graphs, we consider the frequency $\omega_o = 2.73556\times 10^{10}\ \mathrm{s}^{-1}$. In Figure 4 we plot the thermal averages

$$\left\langle P^{(1)}_{1\longrightarrow 1}(t)\right\rangle_\beta = \frac{1}{Z}\sum_{a_i} p_{a_i}\left\langle P^{(1)}_{1\longrightarrow 1}(t)\right\rangle \qquad (30)$$

and

$$\left\langle \Pi_z^{(1)}(t)\right\rangle_\beta = \frac{1}{Z}\sum_{a_i} p_{a_i}\left\langle \Pi_z^{(1)}(t)\right\rangle, \qquad (31)$$

assuming that the density of levels ρ_υ is proportional to ϵ^s. The graphs are plotted for $T = 300\mathrm{K}$ and for $s = 0.2$, $s = 1$ and $s = 1.5$, which correspond to the subohmic, ohmic, and superohmic dissipation forms, in the spin-boson model [15].

In Figure 5, we have the thermal average of the spin polarization calculated in [28, 29], and on top of it the thermal

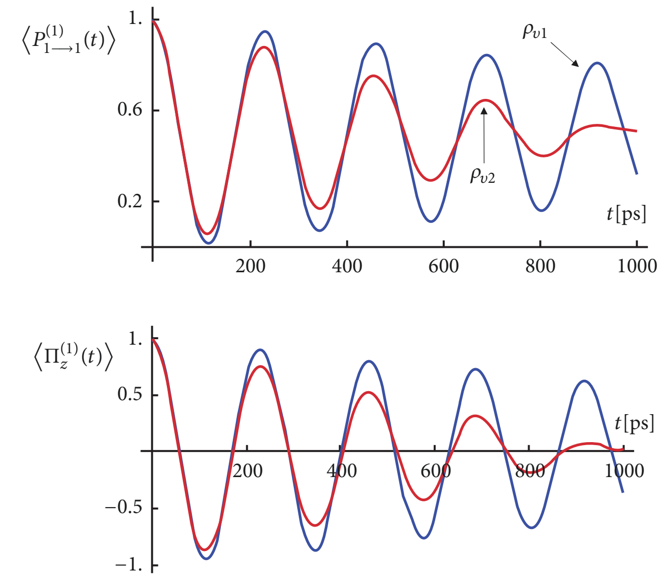

FIGURE 3: The survival probability (upper panel) and the spin polarization (lower panel) for two values of $\rho_v(\varepsilon)$, in arbitrary units. The blue curve corresponds to $\rho_{v1}(\varepsilon) = 2\times 10^8 s^{-1}$ and the red curve to $\rho_{v2}(\varepsilon) = 5\times 10^8 s^{-1}$. The frequency in both cases is $\omega_o = 2.73556\times 10^{10} s^{-1}$.

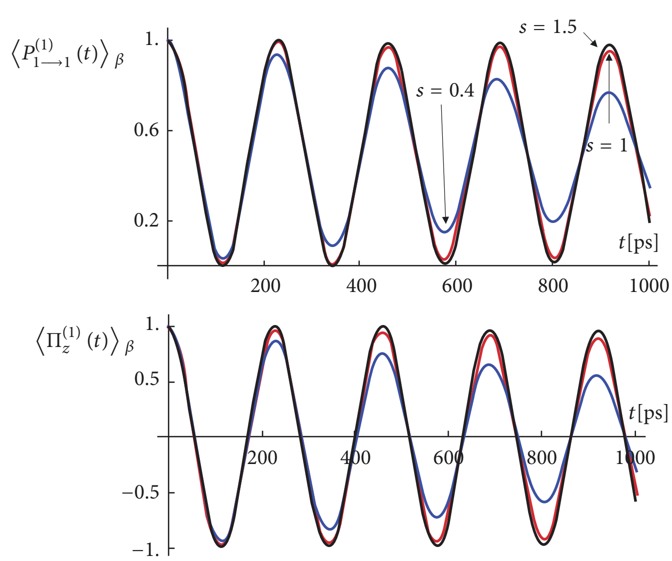

FIGURE 4: The thermal averages of the survival probability (upper panel) and the spin polarization (lower panel) assuming that the density of levels $\rho_v(\varepsilon)$ is $\propto \varepsilon^s$. The plots shown here are for the subohmic $s = 0.2$, ohmic $s = 1$ and superohmic $s = 1.5$ dissipation forms and for $T = 300$K.

average of the spin polarization in (31). For this graph we considered $\rho = 10^9\times \epsilon^s s^{-1}$, with s=0.36 and $\omega_o = 2.58\times 10^{10} s^{-1}$. The agreement is good up to times of the order 500ps. In the appendix we obtain the polarization

$$\left\langle \Pi^{(2)}_z(t)\right\rangle = -t^2\rho_v^2 + \left(1 - 2t\rho_v + t^2\rho_v^2\right)\cos t\omega_o + 2\frac{\rho_v}{\omega_o}\left(1 - t\rho_v\right)\sin t\omega_o, \tag{32}$$

whose thermal average

$$\left\langle \Pi^{(2)}_z(t)\right\rangle_\beta = \frac{1}{Z}\sum_{a_i} p_{a_i}\left\langle \Pi^{(2)}_z(t)\right\rangle, \tag{33}$$

was plotted in Figure 1 on top of the spin polarization calculated in [19]. Our calculation of the spin polarization in Figure 1 was for $T = 300$K, $\rho = 10^9\times\epsilon^s s^{-1}$, with $s = 0.28$, and $\omega_o = 2.58\times 10^{10} s^{-1}$. As mentioned before, the agreement is rather good and strengthens the idea that in complex systems some processes are insensitive to the

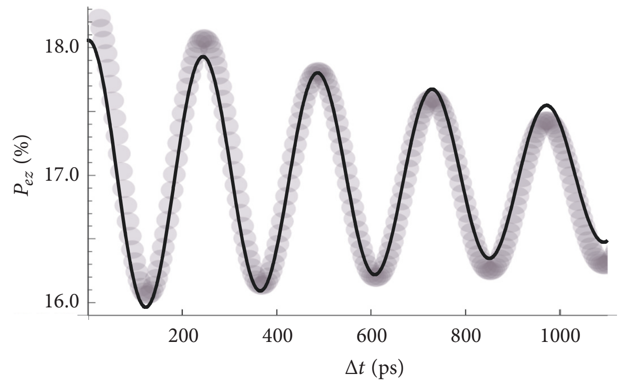

Figure 5: The spin polarization calculated in Ref. [28] (gray circles) and our results (black curve), to first order of the interaction picture, for level density $\rho_v(\varepsilon) = 10^9\epsilon^s$, with $s = 0.36$, and frequency $\omega_o = 2.73556 \times 10^{10}\text{s}^{-1}$. The gray circles graph published with author's permission.

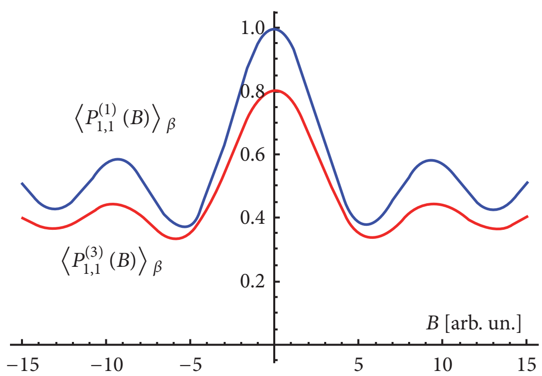

Figure 6: The survival probabilities $\langle P^{(1)}_{1\longrightarrow 1}(t)\rangle_\beta$ and $\langle P^{(3)}_{1\longrightarrow 1}(t)\rangle_\beta$ at $t = 1.2 \times 10^{-9}$, $T = 300\text{K}$ and $\rho = \epsilon^s \times 10^9\text{s}^{-1}$, with $s = 0.2$.

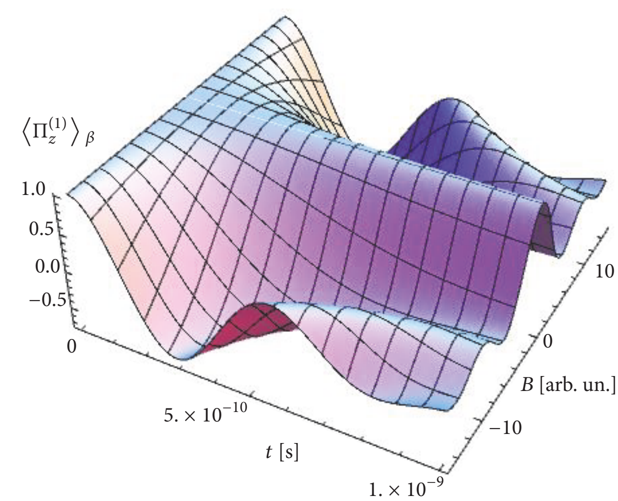

Figure 7: The spin polarization $\langle \Pi_z(t)\rangle_\beta$ as function of the magnetic field and time for $T = 300\text{K}$ and $\rho = \epsilon^s \times 10^9\text{s}^{-1}$, with $s = 0.2$.

details of the interaction, and only few "gross properties" are relevant and can be described by suitable statistical models.

The analytical expressions of the survival probabilities and spin polarizations reported here allow also exploring the behavior of these quantities as functions of the magnetic field. This is the purpose in the next section.

4. The Field Effect on the Spin Polarization

A great amount of experimental research has been published to show the behavior of the spin polarization as function of the magnetic field. Nevertheless, we shall present here the behavior of the thermal averages of the survival probability $\langle P^{(n)}_{1\longrightarrow 1}(t)\rangle_\beta$ and the spin polarization $\langle \Pi_z^{(n)}(t)\rangle_\beta$, as functions of the magnetic field and time.

In Figure 6 we plot the magnetic field behavior of the survival probabilities $\langle P^{(1)}_{1\longrightarrow 1}(t)\rangle_\beta$ and $\langle P^{(3)}_{1\longrightarrow 1}(t)\rangle_\beta$ as functions of the magnetic field at $t = 1.210^{-9}$ and for level density $\rho_v(\varepsilon) = 10^9\epsilon^s$, with $s = 0.2$. As expected, the survival probability tends to a probability of 1/2 as the magnetic field increases.

In Figure 7 we show the spin polarization $\langle \Pi_z^{(1)}(t)\rangle_\beta$ as function of time and of the magnetic field for a level density $\rho_v(\varepsilon) = 10^9\epsilon^s$, with $s = 0.2$. The oscillating behavior shown in this graph is compatible with the previous results, not only as function of time but also as function of the magnetic field.

5. Conclusions

We presented here a simple model to study the behavior of a two-level system interacting stochastically with its environment in the presence of a magnetic field in the Faraday configuration. We calculated the survival and spin polarization to first and second order of the interaction picture. We have shown the oscillating evolution of the thermal average of these quantities as function of time and the magnetic field, for different values of the temperature and for the level density $\rho(\epsilon) \propto \epsilon^s$, in the subohmic, ohmic, and superohmic dissipation forms. We have shown that the spin polarization behavior agrees rather well with

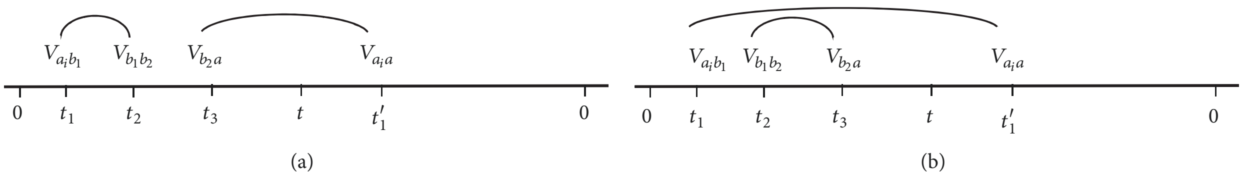

FIGURE 8: Two possible configurations for the pairing of matrix elements V_{ab} that contribute to the second order of the survival probability $\langle P^{(2)}_{1\longrightarrow 1}(t)\rangle$.

the time evolution of the spin polarization observed and calculated, recently, for the electron-nucleus dynamics of Ga centers in dilute (Ga,N)As semiconductors. The calculation of the higher order terms, in the interaction picture, is ongoing and we hope to obtain more accurate results and a better understanding of the oscillating behaviors reported here.

Appendix

Survival Probability and Spin Polarization to Second Order

For the second-order contribution, one has to evaluate the following terms $\langle P^{0,4}_{1\longrightarrow 1}(t)\rangle$, $\langle P^{1,3}_{1\longrightarrow 1}(t)\rangle$, and $\langle P^{2,2}_{1\longrightarrow 1}(t)\rangle$. For the term $P^{0,4}_{1\longrightarrow 1}(t)\rangle$ we have to evaluate

$$\begin{aligned}\left\langle P^{0,4}_{1\longrightarrow 1}(t)\right\rangle &= (i)^4\int_0^t dt_4\int_0^{t_4}dt_3\int_0^{t_3}dt_2\int_0^{t_2}dt_1 \\ &\cdot \sum_{a,b_1,b_2,b_3}\left\langle V_{a_ib_1}V_{b_1b_2}V_{b_2b_3}V_{b_3a}\right\rangle \\ &\cdot e^{i[(\varepsilon_{a_i}-\varepsilon_{b_1})t_1+(\varepsilon_{b_1}-\varepsilon_{b_2})t_2+(\varepsilon_{b_2}-\varepsilon_{b_3})t_3+(\varepsilon_{b_3}-\varepsilon_a)t_4]}\cos\omega_o \\ &\cdot\left(\frac{t}{2}-t_4+t_3-t_2+t_1\right)\cos\omega_o\frac{t}{2},\end{aligned} \tag{A.1}$$

with ensemble average

$$\begin{aligned}\left\langle P^{0,4}_{1\longrightarrow 1}(t)\right\rangle = \int_0^t dt_4\int_0^{t_4}dt_3\int_0^{t_3}dt_2\int_0^{t_2}dt_1\rho_v^2(\varepsilon) \\ \cdot e^{-\Delta(t_4-t_3)}e^{-\Delta(t_2-t_1)}\cos\omega_o \\ \cdot\left(\frac{t}{2}-t_4+t_3-t_2+t_1\right)\cos\omega_o\frac{t}{2}.\end{aligned} \tag{A.2}$$

For the term $\langle P^{1,3}_{1\longrightarrow 1}(t)\rangle$, there are two possible configurations for the pairing of the matrix elements V_{ab} that contribute to the same order in the leading order result. In Figure 8, we show graphically these pairing in configurations A and B. The corresponding averages are

$$\begin{aligned}\left\langle P^{1,3}_{1\longrightarrow 1}(t)\right\rangle_A &= \int_0^t dt_3\int_0^{t_3}dt_2\int_0^{t_2}dt_1\int_0^t dt_1'\rho_v^2(\varepsilon) \\ &\cdot e^{-\Delta|t_3-t_1'|}e^{-\Delta(t_2-t_1)}\cos\omega_o\left(\frac{t}{2}-t_3+t_2-t_1\right)\cos\omega_o \\ &\cdot\left(\frac{t}{2}-t_1'\right),\end{aligned} \tag{A.3}$$

$$\begin{aligned}\left\langle P^{1,3}_{1\longrightarrow 1}(t)\right\rangle_B &= \int_0^t dt_3\int_0^{t_3}dt_2\int_0^{t_2}dt_1\int_0^t dt_1'\rho_v^2(\varepsilon) \\ &\cdot e^{-\Delta|t_1-t_1'|}e^{-\Delta(t_3-t_2)}\cos\omega_o\left(\frac{t}{2}-t_3+t_2-t_1\right)\cos\omega_o \\ &\cdot\left(\frac{t}{2}-t_1'\right).\end{aligned} \tag{A.4}$$

Similarly, for the term $\langle P^{2,2}_{1\longrightarrow 1}(t)\rangle$, the pairing of the matrix elements V_{ab} that contribute to the same order of the survival probability are shown in Figure 9, in configurations A and B. The corresponding averages are

$$\begin{aligned}\left\langle P^{1,3}_{1\longrightarrow 1}(t)\right\rangle_A &= \int_0^t dt_2\int_0^{t_2}dt_1\int_0^t dt_2'\int_0^{t_2'}dt_1'\rho_v^2(\varepsilon) \\ &\cdot e^{-\Delta(t_2-t_1}e^{-\Delta(t_2'-t_1')}\cos\omega_o\left(\frac{t}{2}-t_2+t_1\right)\cos\omega_o \\ &\cdot\left(\frac{t}{2}-t_2'+t_1'\right),\end{aligned} \tag{A.5}$$

$$\begin{aligned}\left\langle P^{1,3}_{1\longrightarrow 1}(t)\right\rangle_B &= \int_0^t dt_2\int_0^{t_2}dt_1\int_0^t dt_2'\int_0^{t_2'}dt_1'\rho_v^2(\varepsilon) \\ &\cdot e^{-\Delta|t_2-t_2'|}e^{-\Delta|t_1-t_1'|}\cos\omega_o\left(\frac{t}{2}-t_2+t_1\right)\cos\omega_o \\ &\cdot\left(\frac{t}{2}-t_2'+t_1'\right).\end{aligned} \tag{A.6}$$

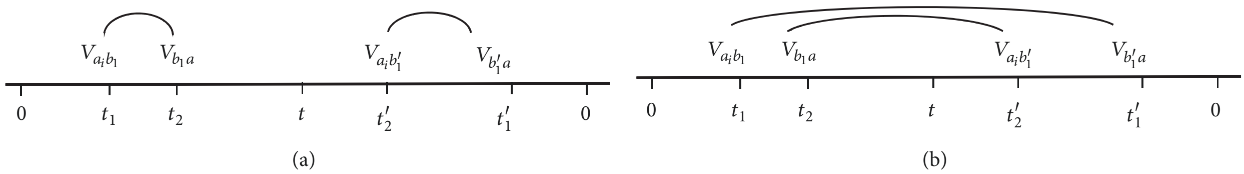

FIGURE 9: Two possible configurations for the pairing of matrix elements V_{ab} that contribute to the second order of the survival probability $\langle P^{(2)}_{1\longrightarrow 1}(t)\rangle$.

Neglecting terms of order $1/\Delta$ and higher, adding the contributions of all these terms and taking into account also the first-order term $\langle P^{(1)}_{1\longrightarrow 1}(t)\rangle$, we have

$$\left\langle P^{(2)}_{1\longrightarrow 1}(t)\right\rangle = \frac{1}{2} - \frac{t^2\rho_v^2}{2} + \frac{1}{2}\left(1 - 2t\rho_v + t^2\rho_v^2\right)\cos t\omega_o + \frac{\rho_v}{\omega_o}\left(1 - t\rho_v\right)\sin t\omega_o. \tag{A.7}$$

Hence the probability that at time t the particle is in the spin state $|-1\rangle$ is

$$\left\langle P^{(2)}_{1\longrightarrow -1}(t)\right\rangle = \frac{1}{2} + \frac{t^2\rho_v^2}{2} - \frac{1}{2}\left(1 - 2t\rho_v + t^2\rho_v^2\right)\cos t\omega_o - \frac{\rho_v}{\omega_o}\left(1 - t\rho_v\right)\sin t\omega_o. \tag{A.8}$$

Therefore the polarization, to second order of the interaction picture, is given by

$$\left\langle \Pi^{(2)}_z(t)\right\rangle = -t^2\rho_v^2 + \left(1 - 2t\rho_v + t^2\rho_v^2\right)\cos t\omega_o + 2\frac{\rho_v}{\omega_o}\left(1 - t\rho_v\right)\sin t\omega_o. \tag{A.9}$$

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The author declares that there are no conflicts of interest regarding the publication of this paper.

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