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Research Article

Coding B-Frames of Color Videos with Fuzzy Transforms

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We use a new method based on discrete fuzzy transforms for coding/decoding frames of color videos in which we determine dynamically the GOP sequences. Frames can be differentiated into intraframes, predictive frames, and bidirectional frames, and we consider particular frames, called Δ-frames (resp., R-frames), for coding P-frames (resp., B-frames) by using two similarity measures based on Lukasiewicz t-norm; moreover, a preprocessing phase is proposed to determine similarity thresholds for classifying the above types of frame. The proposed method provides acceptable results in terms of quality of the reconstructed videos to a certain extent if compared with classical-based F-transforms method and the standard MPEG-4.

1. Introduction

A video can be considered as a sequence of frames of sizes $N \times M$; a frame is an image that can be compressed by using a lossy compression method. We can classify each frame as intraframe (for short, I-frame), predictive frame (for short, P-frame), and bidirectional frame (for short, B-frame) which is more compressible than I-frame. A B-frame can be predicted or interpolated from an earlier and/or later frame. In order to avoid a growing propagation error, a B-frame is not used as a reference to make further predictions in most encoding standards except in AVC [1]. A frame can be considered as a P-frame if it is "similar" to the previous I-frame in the frame sequence; otherwise, it must be considered as a new I-frame. This similarity relation between a P-frame and the previous I-frame is fundamental in video-compression processes because a P-frame has values in its pixels very close to the pixels of the previous I-frame. This suggests to define a frame containing differences between a P-frame and the previous I-frame, called Δ-frame which has a low quantity of information and hence it can be coded with a low compression rate. A P-frame is decoded via the previous I-frame and the Δ-frame. In the MPEG-4 method [2, 3], that adopts the JPEG technique [4] for coding/decoding frames, the I-frames, P-frames, and B-frames are arranged in a Group of Picture (for short, GOP) sequence. A B-frame is reconstructed by using either the previous or successive I-frame. Here the results of [5] are improved by using a technique based on F-transforms for coding B-frames. For convenience, we assume that the first frame of a video is an I-frame. We assign an ID number to each frame of the video. Then we can say that the kth frame is a B-frame or a P-frame if it is "very similar" to the previous ith I-frame in the sense that its similarity $\mathrm{Sim}(i, k)$ a parameter defined on the Lukasiewicz t-norm (see formula (12)) is greater than a threshold value $\mathrm{Sim}P$ [5]; otherwise the kth frame is assumed to be a new I-frame as the first frame of the successive GOP sequence.

The first algorithm is used for determining the GOP sequences; the second algorithm is used for determining the type of P-frame or B-frame. The first frame of the GOP sequence is always an I-frame and the last frame is a P-frame. The function "analyze GOP sequence (ID1, ID2)" reported in Algorithm 1 describes this process, where ID1 is the ID of the first I-frame and ID2 is the ID of the last P-frame in the GOP sequence. This function is used for determining if the kth frame in the GOP sequence, where $\mathrm{ID1} < k < \mathrm{ID2}$, is a B-frame or a P-frame. We define a threshold similarity $\mathrm{Sim}B$, and we compare it with the frame whose ID is formed from the integer $[Ms]$ contained in the mean Ms of the previous I-frame or P-frame and the kth frame by obtaining a similarity value $\mathrm{Sim}(k, [Ms])$. In the array element $\mathrm{NP}[k]$ we insert the ID number of the last frame after the kth frame for which

Sim(k, [Ms]) < Sim$B$ holds. The variable i contains the ID number of the previous I-frame or P-frame; it is initially called ID1; the variable w points to the last frame in the GOP sequence; it is called ID2.

Algorithm 1 (analyze GOP sequence (ID1, ID2)). Pseudo-code for determining a GOP sequence

(1) i = ID of the first I-frame //i is the ID of first frame of the video

(2) w = number of frames //w is the ID of the last P-frame of the video

(3) $k = i + 1$

(4) IF $k < w$

(5) Calculate the similarity Sim(i, k) between the kth frame and the ith frame

(6) If Sim$(i, k) <$ Sim$P$,

- (a) the kth frame is a B-frame or a P-frame and is inserted in the GOP sequence
- (b) $k = k + 1$

(7) Else

- (a) analyse GOP sequence $(i, k - 1)$
- (b) $i = k$
- (c) go to (3)

(8) End.

Algorithm 2. Pseudo-code for determining type of frames

(1) i = ID1//i is the ID of the first frame of the GOP sequence

(2) w = ID2//w is the ID of the last P-frame of the GOP sequence

(3) For each k in $[i + 1, w - 1]$
NP[k] = k//Initialize NP[k]

(4) $s = k + 1$

(5) Create the [Ms]th frame as a new frame whose normalized pixels are obtained as the mean between the normalized pixels of the ith and sth frames

(6) Calculate the similarity Sim(k, [Ms]) between the kth and [Ms]th frames. If Sim(k, [Ms]) < Sim$B$,

- (a) NP[k] = $s - 1$
- (b) Else $s = s + 1$
- (c) go to step (6)

(7) next for

(8) NPMin = min(NP[k])

(9) The frames between the ith and NPMin-th frames are labelled as B-frames

(10) The NPMin-th frame is labelled as a P-frame

(11) If NPmin < w then

- (a) i = NPMin,
- (b) go to step (2)

(12) End.

In our approach we determine a GOP sequence at each step. The frame after the last P-frame is the I-frame of the new GOP sequence. After determining the GOP sequences of the color video, we use the F-transforms [5, 7–10] for compressing the frames. The F-transform method has been developed in [5]. In this paper each frame is converted in the YUV space. Indeed, since the human eye perceives an image mostly in the Y band (brightness) with respect to the U and V bands (chrominance), we can use a stronger compression rate for coding the image in U and V bands with respect to that one used for coding the image in the Y band, without loss of information in the reconstructed image. In [5] the authors show that the quality of the reconstructed images is better than the one obtained using the F-transform method directly in the RGB space (see also [11, 12]). The proposed method is widely discussed in Section 4. In Sections 2 and 3 the theory of F-transforms and its application are recalled for image compression, respectively. In Section 5 the results are deduced on a large color videos dataset.

2. Fuzzy Transforms

We recall from [9] some essential definitions. Let $n \geq 3$ and $x_1, x_2, \ldots, x_n$ be points (nodes) of $[a, b]$ such that $x_1 = a < x_2 < \cdots < x_n = b$. The fuzzy sets $A_1, \ldots, A_n : [a, b] \to [0, 1]$ form a fuzzy partition of $[a, b]$ if

(1) $A_i(x_i) = 1$ for any $i = 1, 2, \ldots, n$;

(2) $A_i(x) = 0$ if $x \notin (x_{i-1}, x_{i+1})$, where $i = 1, 2, \ldots, n$ and $x_0 = x_1 = a$, $x_{n+1} = x_n = b$;

(3) $A_i(x)$ is a continuous function on $[a, b]$;

(4) $A_i(x)$ is strictly increasing on the interval $[x_{i-1}, x_i]$ for $i = 2, \ldots, n$ and is strictly decreasing on the interval $[x_i, x_{i+1}]$ for $i = 1, \ldots, n - 1$;

(5) for any $x \in [a, b]$, $\sum_{i=1}^{n} A_i(x) = 1$.

We say that $\{A_1, A_2, \ldots, A_n\}$ constitute a symmetric fuzzy partition if the following hold:

(6) equidistance of the nodes, that is, $x_i = a + h \cdot (i - 1)$ for $i = 1, 2, \ldots, n$, where $h = (b - a)/(n - 1)$;

(7) $A_i(x_i - x) = A_i(x_i + x)$ for any $x \in [0, h]$ and $i = 2, \ldots, n - 1$;

(8) $A_{i+1}(x) = A_i\,(x - h)$ for any $x \in [x_i, x_{i+1}]$ and $i = 1, 2, \ldots, n - 1$.

Considering functions f taking values on a finite set $P = \{p_1, \ldots, p_m\} \subseteq [a, b]$, $f : P \to [0, 1]$, we suppose that P is sufficiently dense with respect to a fuzzy partition $\{A_1, A_2, \ldots, A_n\}$ of $[a, b]$, that is, if $m > n$ and for each $i = 1, \ldots, n$ there exists an index $j \in \{1, \ldots, m\}$ such that $A_i(p_j) > 0$. Now let $n, m \geq 3$, $y_1, y_2, \ldots, y_m \in [c, d]$ be other m assigned nodes such that $y_1 = c < \cdots < y_m = d$. Let $C_1, \ldots, C_m : [c, d] \to [0, 1]$ be another fuzzy partitions of $[c, d]$. Let $f : P \times Q \to [0, 1]$ be a function defined on the finite set $P \times Q = \{p_1, \ldots, p_N\} \times \{q_1, \ldots, q_M\} \subseteq [a, b] \times [c, d]$, with $N > n$ and $M > m$, where P (resp., Q) is sufficiently dense with respect to some fuzzy partition

$\{A_1, A_2, \ldots, A_n\}$ of $[a, b]$ (resp., $\{C_1, \ldots, C_m\}$ of $[c, d]$). Then $[F_{kl}]$, $F_{kl} \in [0,1]$, $k = 1, \ldots, n$ and $l = 1, \ldots, m$, is the fuzzy matrix which is defined as discrete F-transform of f with respect to $\{A_1, A_2, \ldots, A_n\}$ and $\{C_1, \ldots, C_m\}$ if the following holds:

$$F_{kl} = \frac{\sum_{j=1}^{M} \sum_{i=1}^{N} f\left(p_i, q_j\right) A_k\left(p_i\right) C_l\left(q_j\right)}{\sum_{j=1}^{M} \sum_{i=1}^{N} A_k\left(p_i\right) C_l\left(q_j\right)}. \quad (1)$$

Afterwards we define $f_{nm}^F : P \times Q \to [0,1]$ to be the inverse F-transform of f with respect to $\{A_1, A_2, \ldots, A_n\}$ and $\{C_1, \ldots, C_m\}$ as

$$f_{nm}^F\left(p_i, q_j\right) = \sum_{k=1}^{n}\sum_{l=1}^{m} F_{kl} A_k\left(p_i\right) C_l\left(q_j\right). \quad (2)$$

The following theorem holds.

Theorem 3. *Let $f : P \times Q \to [0,1]$ be a function assigned on $P \times Q = \{p_1, \ldots, p_N\} \times \{q_1, \ldots, q_M\} \subseteq [a, b] \times [c, d]$. Then for every $\varepsilon > 0$, there exist two integers $n(\varepsilon)$, $m(\varepsilon)$ with $n(\varepsilon) < N$, $m(\varepsilon) < M$ and some fuzzy partitions $\{A_1, A_2, \ldots, A_{n(\varepsilon)}\}$ of $[a, b]$ and $\{C_1, C_2, \ldots, C_{m(\varepsilon)}\}$ of $[c, d]$ for which P and Q are sufficiently dense with respect to these partitions, respectively, and such that the following inequality holds for every $i = 1, \ldots, N$, $j = 1, \ldots, M$:*

$$\left| f\left(p_i, q_j\right) - f_{n(\varepsilon)m(\varepsilon)}^F\left(p_i, q_j\right) \right| < \varepsilon. \quad (3)$$

3. The Coding/Decoding Process

Let R be an image of sizes $N \times M$, considered as a fuzzy relation $R : (i, j) \in \{1, \ldots, N\} \times \{1, \ldots, M\} \to [0,1]$; that is, $R(i, j) = P(i, j)/L_t$, with $P(i, j)$ being the normalized value of the pixel with respect to the length Lt of the scale used. For simplicity, let $p_i = i$, $q_j = j$, $a = c = 1$, $b = N$, and $d = M$. Let the fuzzy sets $A_1, \ldots, A_n : [1, N] \to [0,1]$ and $C_1, \ldots, C_m : [1, M] \to [0,1]$, with $n < N$ and $m < M$, form a fuzzy partition of $[1, N]$ and $[1, M]$, respectively. Following [8], R is subdivided in submatrices R_B of sizes $N(R_B) \times M(R_B)$, $R_B : (i, j) \in \{1, \ldots, N(R_B)\} \times \{1, \ldots, M(R_B)\} \to [0,1]$, called blocks, coded to matrices of sizes $n(R_B) \times m(R_B)$, $(n(R_B) < N(R_B), m(R_B) < M(R_B))$ via the following discrete F-transforms $[F_{kl}^B]$ for every $(k, l) \in \{1, \ldots, n(R_B)\} \times \{1, \ldots, m(R_B)\}$ as

$$F_{kl}^B = \frac{\sum_{j=1}^{M(R_B)} \sum_{i=1}^{N(R_B)} R_B(i, j) A_k(i) C_l(j)}{\sum_{j=1}^{M(R_B)} \sum_{i=1}^{N(R_B)} A_k(i) C_l(j)}, \quad (4)$$

and decode $[F_{kl}^B]$ via $R_{n(R_B)m(R_B)}^F : (i, j) \in \{1, \ldots, N(R_B)\} \times \{1, \ldots, M(R_B)\} \to [0,1]$ defined as

$$R_{n(R_B)m(R_B)}^F = \sum_{j=1}^{M(R_B)} \sum_{i=1}^{N(R_B)} F_{kl}^B A_k(i) C_l(j) \quad (5)$$

which approximates R_B in the sense of Theorem 3; that is, there exist, for every $\varepsilon > 0$, two integers $n(R_B, \varepsilon)$, $m(R_B, \varepsilon)$ such that the following holds for every $(i, j) \in \{1, \ldots, N(R_B)\} \times \{1, \ldots, M(R_B)\}$:

$$\left| R_B(i, j) - R_{n(R_B,\varepsilon)m(R_B,\varepsilon)}^F(i, j) \right| < \varepsilon. \quad (6)$$

Unfortunately the previous theorem does not suggest a method for finding such integers, and then we try to assign values to $n(R_B) = n(R_B, \varepsilon)$ and $m(R_B) = m(R_B, \varepsilon)$ for getting compression rates given by

$$\rho(R_B) = \frac{n(R_B) \cdot m(R_B)}{N(R_B) \cdot M(R_B)} \quad (7)$$

which are useful to code any original block R_B. The recomposition of the blocks $R_{n(R_B)m(R_B)}^F$ gives the image R^F whose PSNR with respect to the original image R is calculated via the following well-known formula:

$$\operatorname{PSNR}\left(R, R^F\right) = 20\log_{10} \frac{Lt}{\sqrt{\sum_{i=1}^{N} \sum_{j=1}^{M} \left(R(i, j) - R^F(i, j)\right)^2 / N \times M}}. \quad (8)$$

In accordance with [8], in the proposed experiments the best results are deduced with the symmetric fuzzy partitions $A_1, \ldots, A_{n(R_B)} : [1, N(R_B)] \to [0,1]$ and $C_1, \ldots, C_{m(R_B)} : [1, M(R_B)] \to [0,1]$ defined as

$$A_1(i) = \begin{cases} 0.5\left(\cos\frac{\pi}{h}(i-1)+1\right) & \text{if } 1 \le i \le x_2, \\ 0 & \text{else}, \end{cases}$$

$$A_k(i) = \begin{cases} 0.5\left(\cos\frac{\pi}{h}(i-x_k)+1\right) & \text{if } x_k \le i \le x_{k+1}, \\ 0 & \text{else}, \end{cases}$$

$$A_{n(R_B)}(i) = \begin{cases} 0.5\left(\cos\frac{\pi}{h}\left(i-x_{n(R_B-1)}\right)+1\right) & \text{if } x_{n(B)-1} \le i \le N(R_B), \\ 0 & \text{else}, \end{cases} \quad (9)$$

where $k = 2, \dots, n(R_B) - 1$, $h = (N(R_B) - 1)/(n(R_B) - 1)$, $x_k = 1 + h \cdot (k - 1)$, and

$$C_1(j) = \begin{cases} 0.5\left(\cos\frac{\pi}{s}(j-1)+1\right) & \text{if } 1 \le j \le y_2, \\ 0 & \text{else,} \end{cases}$$
$$C_t(j) = \begin{cases} 0.5\left(\cos\frac{\pi}{s}(j-y_t)+1\right) & \text{if } y_{t-1} \le j \le x_{t+1}, \\ 0 & \text{else,} \end{cases}$$
$$C_{m(R_B)}(j) = \begin{cases} 0.5\left(\cos\frac{\pi}{s}\left(j-y_{m(R_B)-1}\right)+1\right) & \text{if } y_{m(R_B)-1} \le j \le M(R_B), \\ 0 & \text{else,} \end{cases} \quad (10)$$

where $t = 2, \dots, m(R_B) - 1$, $s = (M(R_B) - 1)/(m(R_B) - 1)$, and $y_t = 1 + s \cdot (t - 1)$.

4. Our Proposal

The proposed process includes the following steps:

(1) each color frame, seen as a fuzzy relation, is converted from the space *RGB* to the space *YUV*;

(2) a classification of the frames is made via the previous algorithms;

(3) the compression rate $\rho_I = \rho_I(R_B)$ of the I-frames is the mean of three (possibly different) compression rates used in the three bands, that is, if any block R_B of an I-frame has sizes (say) $N_{IY}(R_B) \times M_{IY}(R_B)$ in the band Y and is coded to a block of sizes (say) $n_{IY}(R_B) \times m_{IY}(R_B)$ for which the related compression rate is given by $\rho_{IY} = \rho_{IY}(R_B) = (n_{IY}(R_B) \cdot m_{IY}(R_B)) \cdot (N_{IY}(R_B) \cdot M_{IY}(R_B))^{-}$ and the analogous meaning has the symbols ρ_{IU}, ρ_{IV}. Of course we have $\rho_I = (\rho_{IY} + \rho_{IU} + \rho_{IV})/3$. A similar meaning can be given to $\rho_\Delta = \rho_\Delta(R_B)$ (resp., $\rho_R = \rho_R(R_B)$) for Δ-frames (resp., R-frames).

A color image in the *RGB* space with pixels normalized in $[0, 1]$ is converted to *YUV* space via the formula [5]

$$\begin{bmatrix} Y \\ U \\ V \end{bmatrix} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ -0.169 & -0.332 & 0.500 \\ 0.500 & -0.419 & -0.0813 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix} + \begin{bmatrix} 0 \\ 0.5 \\ 0.5 \end{bmatrix}. \quad (11)$$

Since no misunderstanding can arise, a frame is denoted by a capital letter instead of its ID number in a sequence of a video. In step (2), the similarity measure adopted in [5] is used for classifying the type of frame. It is based on the Lukasiewicz t-norm between two frames F and G, with $F, G : (i, j) \in \{1, 2, \dots, N\} \times \{1, 2, \dots, M\} \to [0, 1]$, defined as

$$\operatorname{Sim}(F,G) = \left(\sum_{i=1}^{N}\sum_{j=1}^{M}\{1 - \max\{F(i,j), G(i,j)\} + \min\{F(i,j), G(i,j)\}\}\right) \times (N \times M)^{-1}. \quad (12)$$

In the μth band ($\mu \in \{Y, U, V\}$) we will use the symbol $\operatorname{Sim}_\mu(F, G)$. The authors [5] have shown that Lukasiewicz t-norm provides the best results with respect to other t-norms as the classical Min and the arithmetical product. For convenience, we assume that the first frame of a video is an I-frame. For determining a GOP sequence in a single band, it can be verified if the successive frame G is a B-frame or a P-frame, that is, if it is "very similar" to the preceding I-frame F in the sense that $\operatorname{Sim}(F, G) < \operatorname{Sim}B$, with $\operatorname{Sim}B \in [0, 1]$ being a prefixed threshold value; otherwise G is assumed to be a new I-frame. We determine a GOP sequence in an assigned band using (12) with the following process:

(1) we consider the first frame F as an I-frame;

(2) we compare F with the successive frame G;

(3) if $\operatorname{Sim}(F, G) < \operatorname{Sim}P$, the frame G is a B-frame or a P-frame and is enclosed in the GOP sequence. Then we consider the successive frame G and go to step (2); otherwise G is a new I-frame. The previous frame is a P-frame and represents the last frame of the GOP sequence.

After determining the GOP sequence, we check if each frame of the sequence is a B-frame or a P-frame by using the previous algorithms. In step (3) we finally compress the frames. In order to reduce the mean compression rate for a P-frame, in [5] and references therein, the authors introduce a "difference" frame D, called Δ-frame, between a P-Frame G and I-frame F by defining $D : (i, j) \in \{1, 2, \dots, N\} \times \{1, 2, \dots, M\} \to [0, 1]$ as

$$D(i,j) = \frac{[F(i,j) - G(i,j) + 1]}{2}. \quad (13)$$

The usage of the Δ-frame has the advantage of using a stronger compression rate for the P-frames with respect to the I-frames; indeed a P-frame G has values in its pixels very close to the pixels of the previous I-frame. Hence the Δ-frame D in (13) has a low quantity of information and it can be coded with a low compression rate. Then, if D' and F' are the frames obtained after coding/decoding F and D, the frame G' (reconstruction of the frame G), with $D', F', G' : (i, j) \in \{1, 2, \dots, N\} \times \{1, 2, \dots, M\} \to [0, 1]$, is deduced from the membership values of F' and D' via the following formula:

$$G'(i,j) = \frac{\max\{0, F'(i,j) - 2D'(i,j) + 1\}}{\max\{1, F'(i,j) - 2D'(i,j) + 1\}}. \quad (14)$$

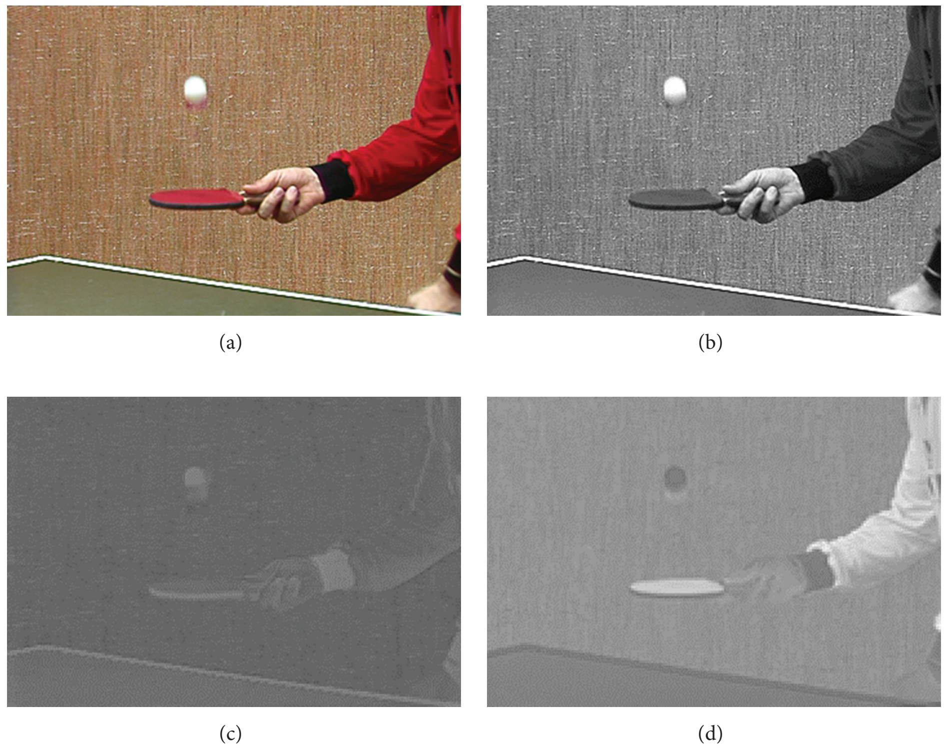


Figure 1: (a) Frame 1 of "tennis2" [6], (b) Frame 1 in Y band, (c) Frame 1 in U band, and (d) Frame 1 in V band.

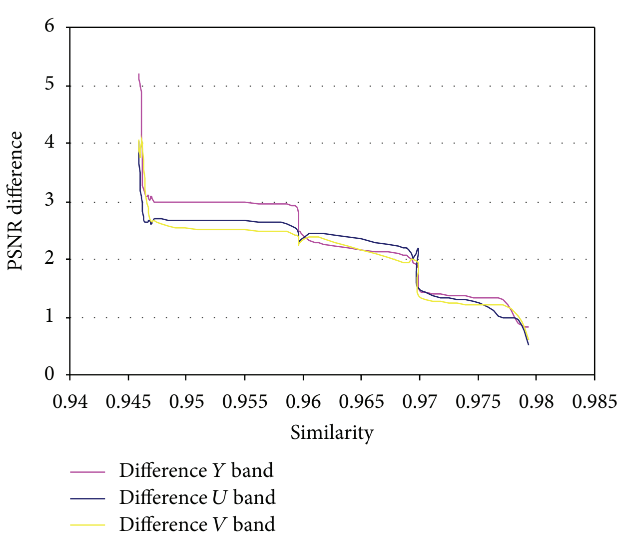


Figure 2: Diff(PSNR) with the similarity in Y, U, and V bands.

Now we present a new schema for coding/decoding a B-frame which is inserted in a GOP between an I-frame F and a P-frame G. Then we consider a frame R given by

$$R(i,j) = \frac{[(F(i,j) + G(i,j))/2 - B(i,j) + 1]}{2} \tag{15}$$

and we code it. Let R' be the frame obtained after decoding R, with $R' : (i,j) \in \{1,2,\ldots,N\} \times \{1,2,\ldots,M\} \rightarrow [0,1]$. All the coding/decoding processes are realized via the F-transforms with the symmetric fuzzy partition given in Section 3. We reconstruct the B-frame, say B', by combining the membership values of F', G', and R' via the following formula:

$$B'(i,j) = \frac{\max\left\{0, \left[F'(i,j) + G'(i,j)\right]/2 - 2R'(i,j) + 1\right\}}{\max\left\{1, \left[F'(i,j) + G'(i,j)\right]/2 - 2R'(i,j) + 1\right\}}. \tag{16}$$

We use the formulas (14) and (16) for reconstructing the P-frames and the B-frames in the videos, respectively. In accordance with [5], we convert each image in the RGB space by using the formula

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1.4075 \\ 1 & -0.3455 & -0.7169 \\ 1 & 1.7790 & 0 \end{bmatrix} \begin{bmatrix} Y \\ U \\ V \end{bmatrix} + \begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \end{bmatrix}. \tag{17}$$

For simplicity of presentation, in our tests here we adopt $M(R_B) = N(R_B)$, $m(R_B) = n(R_B)$. In [5] a preprocessing phase is adopted for determining the threshold Sim$P$ calculated with the following steps:

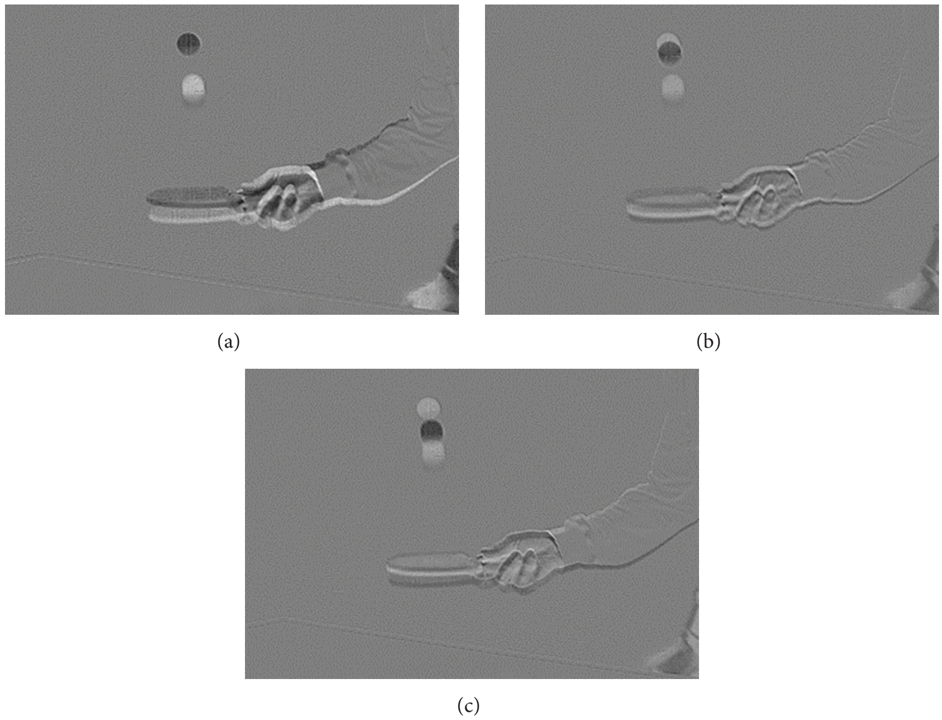


FIGURE 3: (a) Δ-frame from Frame 4 in Y band, (b) R-frame from Frame 2 in Y band, and (c) R-frame from Frame 3 in Y band.

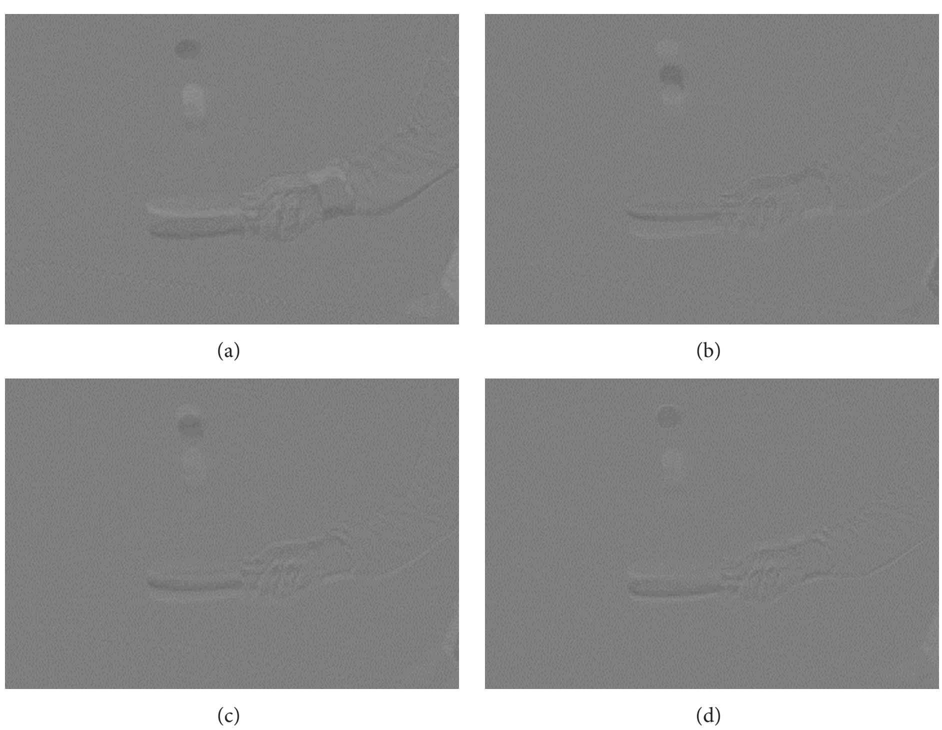


FIGURE 4: (a) Δ-frame from Frame 6 in U band, (b) R-frame from Frame 2 in U band, (c) R-frame from Frame 3 in U band, and (d) R-frame from Frame 4 in U band.

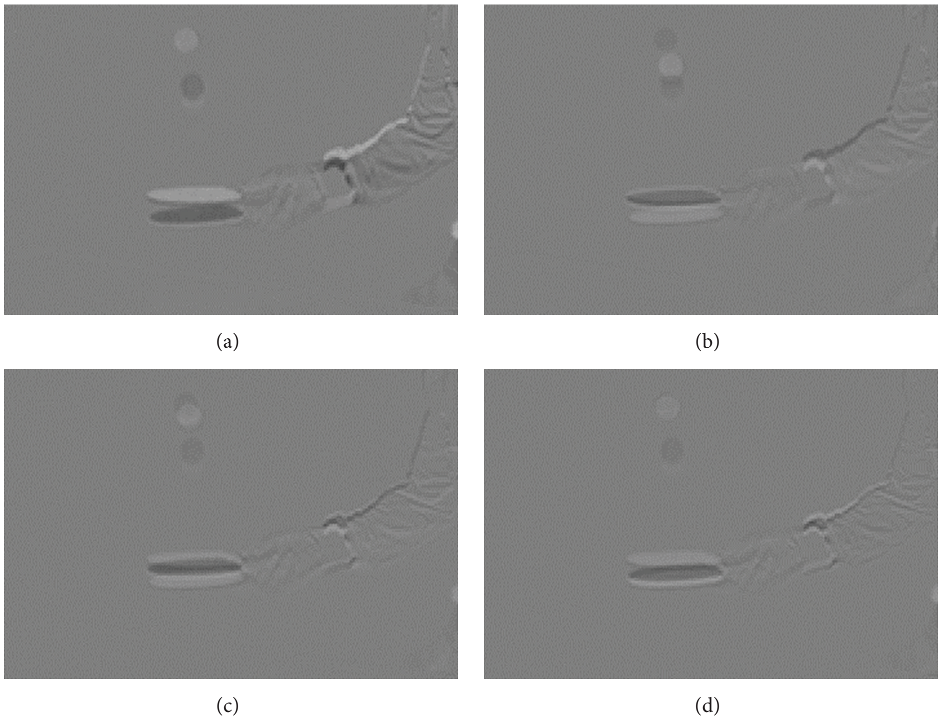


Figure 5: (a) Δ-frame from Frame 5 in V band, (b) R-frame from Frame 2 in V band, (c) R-frame from Frame 3 in V band, and (d) R-frame from Frame 4 in V band.

(1) if the initial frame F is considered as an I-frame, we compress F in the μth band ($\mu \in \{Y, U, V\}$) with compression rate $\rho_{I\mu}$; each successive frame is a P-frame G and we archive the similarity value $\mathrm{Sim}_\mu(F, G)$ calculated with formula (12); we compress the Δ-frame D in the μth band with compression rate equal to $\rho_{P\mu}$ (less than of $\rho_{I\mu}$) and if D' is the related decompressed frame, we derive the P-frame G' via (14);

(2) each P-frame G is also coded in the μth band with compression rate $\rho_{P\mu}$ and let G'' be the decoded P-frame by using directly the F-transforms; then we determine the difference diff(PSNR) $= |\mathrm{PSNR}(G'', G) - \mathrm{PSNR}(G', G)|$;

(3) the trend of diff(PSNR) is plotted with respect to the similarity $\mathrm{Sim}_\mu(F, G)$ in each band of the image. As similarity threshold, we assume that value of $\mathrm{Sim}_\mu(F, G)$ such that diff(PSNR) does not exceed a prefixed limit is equal to 3 (cf. [5] for details);

(4) then the threshold Sim$P$ is given by

$$\mathrm{Sim}P = \max_{G \in \mathrm{GOP}} \left\{ \max \left\{ \mathrm{Sim}_\mu (F, G) : \mu \in \{Y, U, V\} \right\} \right\} \quad (18)$$

with F being the first I-frame of the GOP sequence. In our tests, in addition we put Sim$B$ = Sim$P$ in the preprocessing phase.

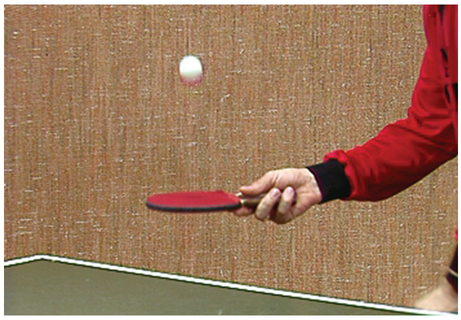
5. The Results

For brevity of discussion, we show the results obtained for the color video "tennis2" [6]. We present all the results by assuming $\rho_I \approx 0.262$ for the I-frames, $\rho_\Delta \approx 0.027$ for the Δ-frames, and $\rho_R \approx 0.020$ for the R-frames. Figures 1(a)–1(d) show the first frame of the video and the corresponding single-band images in the YUV space, respectively. As example of Diff(PSNR), Figure 2 contains the plots of Diff(PSNR) ≤ 3 for the similarity values obtained in Y, U, and V bands for which we choose $\mathrm{Sim}_Y(F, G) > 0.948 = \mathrm{Sim}P$ (as average). As examples we show some Δ-frames and R-frames in each band.

(i) *Y Band.* The first P-frame is given by the fourth frame. Figure 3(a) contains the Δ-frame obtained by using (13) from the fourth frame and the first frame (an I-frame). The second and the third frames are B-frames. Figure 3(b) (resp., Figure 3(c)) shows the R-frame obtained by using (15) from the second (resp., third) frame, the first frame (an I-frame), and the fourth frame (a P-frame).

(ii) *U Band.* The first P-frame is given by the sixth frame. Figure 4(a) contains the Δ-frame obtained by using (13) from the sixth frame and the first frame (an I-frame). The frames 2, 3, and 4 are B-frames. Figures 4(b)–4(d) show the R-frames obtained by using (15) from the first frame (an I-frame), the B-frames 2, 3, and 4, and the sixth frame (a P-frame), respectively.

(iii) *V Band.* The first P-frame is given by the fifth frame. Figure 5(a) contains the Δ-frame obtained by using (13) from



(a)

(b)

(c)

Figure 6: (a) Frame 2 in the proposed method, (b) Frame 2 in F-transforms, and (c) Frame 2 in MPEG-4.

Table 1: Results for "tennis2" [6] in the proposed method.

Parameters	Y band	U band	V band
Number of I-frames	15	7	8
Number of P-frames	31	23	23
Number of B-frames	54	70	69
Mean compression rate $\rho(B)$	0.1128	0.0236	0.0245
Mean PSNR for I-Frames	27.011	25.545	25.812
Mean PSNR for P-Frames	24.816	23.710	23.815
Mean PSNR for B-Frames	24.734	22.819	23.026

the sixth frame and the first frame (an I-frame). The frames 2, 3, and 4 are B-frames. Figures 5(b)–5(d) show the R-frames obtained by using (15) from the first frame (an I-frame), the B-frames 2, 3, and 4, and the fifth frame (a P-frame), respectively.

All the results obtained for the video "tennis2" are synthetized in Table 1.

Figures 6(a)–6(c) contain Frame 2 decoded with the proposed method, classical F-transforms, and MPEG-4, respectively.

In Table 2 we report the final PSNR index in the three methods.

6. Conclusions

We present a new method for coding/decoding color videos, in which we classify a frame in I-frame, P-frame, and B-frame using similarity measures for determining the GOP sequences and the type of frames. Our method seems to be fully comparable with classical F-transforms and MPEG-4 for similar mean compression rates to a certain extent.

Table 2: Comparison with other methods for "tennis2" [6].

Parameters	Proposed method	F-transforms	MPEG-4
Mean compression rate	0.053	0.058	0.055
Mean PSNR	23.915	22.801	23.431

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