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Research Article

2-Quasitotal Fuzzy Graphs and Their Total Coloring

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Coloring of fuzzy graphs has many real-life applications in combinatorial optimization problems like traffic light system, exam scheduling, and register allocation. The coloring of total fuzzy graphs and its applications are well studied. This manuscript discusses the description of 2-quasitotal graph for fuzzy graphs. The proposed concept of 2-quasitotal fuzzy graph is explicated by several numerical examples. Moreover, some theorems related to the properties of 2-quasitotal fuzzy graphs are stated and proved. The results of these theorems are compared with the results obtained from total fuzzy graphs and 1-quasitotal fuzzy graphs. Furthermore, it defines 2-quasitotal coloring of fuzzy total graphs and which is justified.

1. Introduction

As of its emerging, the graph theory rapidly moved into the mainstream of mathematics. It has diverse applications in the fields of science and technology [1, 2]. In 1965, the total coloring of the graph was introduced by Behazad [3], which is followed by Harary, who contributed the concept of total graphs [4]. Jayaraman studied the total chromatic number of total graphs [5]. Besides, Sastry and Raju defined quasitotal graphs [6], and Sirnivasarao and Rao introduced 1-quasitotal graphs and bounds for its total chromatic number [7]. Nowadays, many real-world problems cannot be properly modeled by a crisp graph theory as the problems contain uncertain information. The fuzzy set theory, anticipated by Zadeh [8], is used to handle the phenomena of uncertainty and real-life situation. Coloring of fuzzy graphs plays a vital role in both theory and practical applications. It is mainly studied in combinatorial optimization problems such as traffic light control, exam scheduling, and register allocation.

After Zadeh's paper on fuzzy sets [9], Rosenfeld introduced fuzzy graphs [10]. Later, Bhattacharya [11] gave some remarks on fuzzy graphs. Some operations on fuzzy graphs were introduced by Mordeson and Peng [12]. As an advancement, the fuzzy coloring of the fuzzy graph was defined by Eslahchi and Onagh in 2004 and later developed by themselves as fuzzy vertex coloring in 2006 [13]. Lavanay and Sattanathan extended the concept of fuzzy vertex coloring into a family of fuzzy sets [14]. Kavitha [15] defined the total fuzzy graph and studied the total chromatic number of total graphs of fuzzy graphs [1]. Kavitha derived fuzzy chromatic numbers for various graphs of complete fuzzy graphs [15]. Nevethana studied about fuzzy total coloring and its chromatic number of complete fuzzy graphs [16]. Sitara and Akram studied fuzzy graph structures and their applications [17]. The total coloring of 1-quasitotal graph for crisp graph was studied. Recently Fekadu and SrinivasaRao Repalle have established the definition of 1-quasitotal fuzzy graph and its total coloring [18]. Koam and Akram described decision making analysis in the real-life applications like marine crimes and road crimes by using graph structures [19]. Akram and Sitara introduced the concept of Residue Product of Fuzzy Graph Structures and studied their properties [20]. Akram covers both theories and applications of introduction to m-polar fuzzy graphs and m-polar fuzzy hypergraphs [21].

This paper is being organized as follows: In Section 2, some basic definitions and elementary concepts of the fuzzy set, fuzzy graph, and coloring of fuzzy graphs have been reviewed. In Section 3, 2-quasitotal fuzzy graph is defined and the concept is justified with numerous examples. Section

4 describes and proves some properties of 2-quasitotal fuzzy graphs and compares the result with the properties of total fuzzy graphs and 1-quasitotal fuzzy graphs. Furthermore, Section 5 introduces the concept of 2-quasitotal fuzzy coloring and deliberates some of its properties. Finally, the paper is concluded in Section 6.

2. Preliminaries

In this section, some basic definitions that are necessary for this paper have been included. Unless otherwise mentioned, the concepts are from Mordeson and Nair (see [22]).

Definition 1. Fuzzy Graph

A fuzzy graph is defined as an ordered triple f, where V is the set of vertices $\{v_1, v_2, \ldots, v_n\}$, σ is a fuzzy subset of V, such that σ: $V \longrightarrow [0,1]$ and μ are a fuzzy relation on σ with μ: $V \longrightarrow [0,1]$ and that μ: $V \times V \longrightarrow [0,1]$ such that. $\mu(u,v) \leq \sigma(u) \Lambda \sigma(v) \forall u, v \in V$.

Definition 2. Crisp Graph

The underlying crisp graph of the fuzzy graph $G = (V, \sigma, \mu)$ is denoted by $G^* = (V, E)$, where $E \subseteq V \times V$. The crisp graph (V, E) is a special fuzzy graph G with each vertex, and each edge of G has the same degree of membership equal to 1.

Definition 3. Order and Size of Fuzzy Graph

Let $G = (V, \sigma, \mu)$ be a fuzzy graph with the underlying set V. Then, the order of G denoted by Order (G) is defined as follows:

$$\text{Order}\,(G) = \sum_{u \in V} \sigma(u), \tag{1}$$

and the size of G denoted by Size(G) and defined as follows:

$$\text{Size}\,(u) = \sum_{u,\, v \in V} \mu(u,v). \tag{2}$$

Definition 4. Degree of a Vertex.

Let $G = (V, \sigma, \mu)$ be a fuzzy graph. The degree of a vertex $u \in V$ is defined as follows:

$$d_G(u) = \sum_{v \neq u, v \in V} \mu(u,v). \tag{3}$$

Definition 5. Busy Value of a Vertex.

Let $G = (V, \sigma, \mu)$ be a fuzzy graph. The busy value of the vertex v in G is $D(v) = \sum_i \sigma(v) \Lambda \sigma(v_i)$ where v_i are neighbors of v and the busy value of G is $D(G) = \sum_i D(v_i)$ where v_i are the vertexes of G.

Definition 6. Adjacent Vertices

If $\mu(u,v) > 0$, then u and v are said to be adjacent to each other and lie on the edge, $e = (u,v)$. A path ρ in a fuzzy graph $G = (V, \sigma, \mu)$ is a sequence of distinct nodes $v_0, v_1, v_2, \ldots, v_n$ such that $\mu(v_{i-1}, v_i) > 0$, $1 \leq i \leq n$. Here n is called the length of the path.

Definition 7 (see [23]). Path in Fuzzy Graph

A path P in a fuzzy graph $G = (\sigma, \mu)$ is a sequence of distinct vertices $u_0, u_1, \ldots, u_n$ (except possibly u_0 and u_n) such that $\mu(u_{i-1}, u_i) > 0$, $i = 1, 2, \ldots, n$. Here, n is called the length of the path.

Definition 8. Connected Vertices

If u, v are vertices in G and if they are connected by means of a path, then the strength of that path is defined as $\wedge_n^{i=1} \mu(v_{i-1}, v_i)$. If u, v are connected by means of paths of length k, then

$$\mu^k(u,v) = \sup\{\mu(u,v_1) \Lambda \mu(v_1,v_2) \Lambda \mu(v_2,v_3) \Lambda, \\ \ldots, \Lambda \mu(v_{k-1},v) : u, v_1, v_2, \ldots, v_{k-1}, v \in V\}, \tag{4}$$

If $u, v \in V$, then, the strength of connectedness between u and v $\mu^\infty(u,v) = \sup\{\mu^k(u,v)\text{: } k = 1, 2, \ldots,\}$

Definition 9. Connected Fuzzy Graph

Let $G = (V, \sigma, \mu)$ be a fuzzy graph. Then, G is said to be connected if $\mu^\infty(u,v) > 0$ for all $u, v \in \sigma^*$. An arc (u,v) is said to be a strong arc if $\mu(u,v) \geq \mu^\infty(u,v)$ and a node u is said to be an isolated node, if. $\mu(u,v) = 0$, for all $u \neq v$.

Definition 10 (see [24]) Cyclic Fuzzy Graph

$G = (V, \sigma, \mu)$ is a fuzzy cycle if and only if (σ^*, μ^*) is a cycle and there does not exist a unique $(x,y) \in \mu^*$ such that $\mu(x,y) = \Lambda\{\mu(u,v)\text{: } (u,v) \in \mu^*\}$.

Definition 11 (see [25]). Total Coloring

A family $\Gamma = \{\gamma_1, \gamma_2, \gamma_3, \ldots, \gamma_k\}$ of fuzzy sets on $V \cup E$ is called a $k-$ fuzzy total coloring of $G = (V, \sigma, \mu)$, if

(a) Max$\{\gamma_i(v)\} = \sigma(v)$ for all $v \in V$ and Max$\{\gamma_i(u,v)\} = \mu(u,v)$ for all edges $(u,v) \in E$

(b) $\gamma_i \Lambda \gamma_j = 0$

(c) For every adjacent vertex u, v of G, Min$\{\gamma_i(u), \gamma_i(v)\} = 0$

The least value of k for which there exists a $k-$ fuzzy coloring is called the fuzzy total chromatic number of G and is denoted by $\chi_T^f(G)$.

Definition 12 (see [18]). 1-Quasitotal Fuzzy Graph

Let $G = (V, \sigma, \mu)$ be a fuzzy graph with its underlying set V and crisp graph $G^* = (\sigma^*, \mu^*)$. The pair $Q_1T_f(G) = (\sigma_{Q_1T_f}, \mu_{Q_1T_f})$ of the fuzzy graph G is defined as follows:

Let the node set of $Q_1T_f(G)$ be $V \cup E$, where V is the vertex set and E is the edge set of the underlying crisp graph. The fuzzy subset $\sigma_{Q_1T_f}$ is defined on $V \cup E$ as follows:

$$\sigma_{Q_1T_f}(u) = \sigma(u), \quad \text{if } u \in V, \\ \sigma_{Q_1T_f}(e) = \mu(e), \quad \text{if } e \in E. \tag{5}$$

The fuzzy relation $\mu_{Q_1T_f}$ is defined on $(V \cup E) \times (V \cup E)$, called edges of $Q_1T_f(G)$ as follows:

$$\mu_{Q_1T_f}(u,v)=\mu(u,v),\quad \text{if } u,v\in V$$
$$\mu_{Q_1T_f}(e_i,e_j)=\mu(e_i)\wedge\mu(e_j),\quad \text{if } e_i \text{ and } e_j \text{ have a node in common between them}$$
$$=0,\quad \text{Otherwise.} \tag{6}$$

By definition, $\mu_{Q_1T_f}(u,v)\le\sigma_{Q_1T_f}(u)\wedge\sigma_{Q_1T_f}(v)$ for all $u,v\in V\cup E$. Hence, $\mu_{Q_1T_f}$ is a fuzzy relation on the fuzzy subset $\sigma_{Q_1T_f}$. Thus, the pair $Q_1T_f(\sigma_{Q_1T_f},\mu_{Q_1T_f})$ is a fuzzy graph, and it is termed as 1-Quasitotal fuzzy graph of G.

3. 2-Quasitotal Fuzzy Graph

This section introduces the definition of 2-quasitotal fuzzy graph and sketches the 2-quasitotal fuzzy graph of a given fuzzy graph.

Definition 13. Let $G=(V,\sigma,\mu)$ be a fuzzy graph with its underlying set V and crisp graph $G^*=(\sigma^*,\mu^*)$. The pair $Q_2T_f(G)=(\sigma_{Q_2T_f},\mu_{Q_2T_f})$ of the fuzzy graph G is defined as follows:

Let the node set of $Q_2T_f(G)$ be the union of the vertex set and the edge set of the underlying crisp graph. That is $V\cup E$.

Let the fuzzy subset $\sigma_{Q_2T_f}$ be defined on $V\cup E$ as follows:

$$\sigma_{Q_2T_f}(u)=\sigma(u),\quad \text{if } u\in V,$$
$$\sigma_{Q_2T_f}(e)=\mu(e),\quad \text{if } e\in E. \tag{7}$$

Let the fuzzy relation $\mu_{Q_2T_f}$ be defined on $(V\cup E)\times(V\cup E)$, called edges of $Q_2T_f(G)$ as follows:

$$\mu_{Q_2T_f}(u,v)=\mu(u,v),\quad \text{if } u,v\in V$$
$$\mu_{Q_2T_f}(u,e)=\sigma(u)\wedge\mu(e),\quad \text{if } u\in V\ e\in E \text{ and the node } u \text{ lies on the edge } e$$
$$=0,\quad \text{otherwise.} \tag{8}$$

By the definition of the fuzzy graph, $\mu_{Q_2T_f}(u,v)\le\sigma_{Q_2T_f}(u)\wedge\sigma_{Q_2T_f}(v)$ for all $u,v\in V\cup E$. Hence, $\mu_{Q_2T_f}$ is a fuzzy relation on the fuzzy subset $\sigma_{Q_2T_f}$. Therefore, the pair $Q_2T_f(\sigma_{Q_2T_f},\mu_{Q_2T_f})$ is a fuzzy graph, and it is termed as 2-Quasitotal Fuzzy Graph of G.

Example 1. Let $G=(V,\sigma,\mu)$ be a fuzzy graph with its underlying crisp graph $G^*=(V,E)$, where $V=\{v_1,v_2,v_3\}$ and edge set $E=\{v_1v_2,v_2v_3,v_3v_1\}$. Let the fuzzy vertex set defined on V be as σ: $S\longrightarrow[0,1]$ such that

$$\sigma(v_1)=\frac{1}{3},$$
$$\sigma(v_2)=\frac{1}{2}, \tag{9}$$
$$\sigma(v_3)=\frac{1}{4}.$$

Let the fuzzy relation defined on the fuzzy edge set be as μ: SXS $\longrightarrow[0,1]$ such that

$$\mu(v_1,v_2)=\frac{1}{3},$$
$$\mu(v_2,v_3)=\frac{1}{5}, \tag{10}$$
$$\mu(v_3,v_1)=\frac{1}{4}.$$

However,

$$\frac{1}{3}=\mu(v_1,v_2)\le\sigma(v_1)\wedge\sigma(v_2)=\frac{1}{3}\wedge\frac{1}{2}=\frac{1}{3},$$
$$\frac{1}{5}=\mu(v_2,v_3)\le\sigma(v_2)\wedge\sigma(v_3)=\frac{1}{2}\wedge\frac{1}{4}=\frac{1}{4}, \tag{11}$$
$$\frac{1}{4}=\mu(v_3,v_1)\le\sigma(v_3)\wedge\sigma(v_1)=\frac{1}{4}\wedge\frac{1}{3}=\frac{1}{4}.$$

Then, we have $\mu(v_i,v_j)\le\sigma(v_i)\wedge\sigma(v_j)$ for all $v_i,v_j\in V$, and hence the graph $G=(V,\sigma,\mu)$ is a fuzzy graph and its graph is as shown in Figure 1.

Now, let us construct the 2-quasitotal fuzzy graph of the fuzzy graph in Example 1 as follows.

That is, $Q_2T_f(\sigma_{Q_2T_f},\mu_{Q_2T_f})$ of the fuzzy graph G, where the node set of Q_2T_f is $V\cup E$, which is the set $\{v_1,v_2,v_3,v_1v_2,v_2v_3,v_3v_1\}$. Hence, we define the fuzzy subset $\delta_{Q_2T_f}$ as follows:

$$\sigma_{Q_2T_f}(u)=\sigma(u),\quad \text{if } u\in V,$$
$$\sigma_{Q_2T_f}(e)=\mu(e),\quad \text{if } e\in E. \tag{12}$$

Thus, we have the following fuzzy subsets σ_{Q_2T}:

$$\begin{aligned}
\sigma_{Q_2T_f}(v_1) &= \sigma(v_1) = \frac{1}{3},\\
\sigma_{Q_2T_f}(v_2) &= \sigma(v_2) = \frac{1}{2},\\
\sigma_{Q_2T_f}(v_3) &= \sigma(v_3) = \frac{1}{4},\\
\sigma_{Q_2T_f}(v_1v_2) &= \mu(v_1, v_2) = \frac{1}{3},\\
\sigma_{Q_2T_f}(v_2v_3) &= \mu(v_2, v_3) = \frac{1}{5},\\
\sigma_{Q_2T_f}(v_3v_1) &= \mu(v_3, v_1) = \frac{1}{4}.
\end{aligned} \tag{13}$$

The fuzzy relations $\mu_{Q_2T_f}$ will be as follows:

$$\begin{aligned}
&\mu_{Q_2T_f}(u, v) = \mu(u, v), \quad \text{if } u, v \in V\\
&\mu_{Q_2T_f}(u, e) = \sigma(u) \wedge \mu(e), \quad \text{if } u \in V,\ e \in E \text{ and } u \text{ lies on the edge } e\\
&\quad = 0, \quad \text{otherwise.}
\end{aligned} \tag{14}$$

Hence,

$$\begin{aligned}
\mu_{Q_2T_f}(v_1, v_2) &= \mu(v_1, v_2) = \frac{1}{3},\\
\mu_{Q_2T_f}(v_2, v_3) &= \mu(v_2, v_3) = \frac{1}{5},\\
\mu_{Q_2T_f}(v_3, v_1) &= \mu(v_3, v_1) = \frac{1}{4},\\
\mu_{Q_2T_f}(v_1, v_1v_2) &= \sigma(v_1) \wedge \mu(v_1, v_2) = \frac{1}{3},\\
\mu_{Q_2T_f}(v_1, v_3v_1) &= \sigma(v_1) \wedge \mu(v_3, v_1) = \frac{1}{4},\\
\mu_{Q_2T_f}(v_1, v_2v_3) &= 0,\\
\mu_{Q_2T_f}(v_2, v_2v_3) &= \sigma(v_2) \wedge \mu(v_2, v_3) = \frac{1}{5},\\
\mu_{Q_2T_f}(v_2, v_2v_1) &= \sigma(v_2) \wedge \mu(v_2, v_1) = \frac{1}{3},\\
\mu_{Q_2T_f}(v_2, v_3v_1) &= 0,\\
\mu_{Q_2T_f}(v_3, v_3v_1) &= \sigma(v_3) \wedge \mu(v_3, v_1) = \frac{1}{4},\\
\mu_{Q_2T_f}(v_3, v_3v_2) &= \sigma(v_3) \wedge \mu(v_3, v_2) = \frac{1}{5},\\
\mu_{Q_2T_f}(v_3, v_1v_2) &= 0.
\end{aligned} \tag{15}$$

However,

$$\begin{aligned}
\frac{1}{3} &= \mu_{Q_2T_f}(v_1, v_2) \le \sigma(v_1) \wedge \sigma(v_2) = \frac{1}{3},\\
\frac{1}{5} &= \mu_{Q_2T_f}(v_2, v_3) \le \sigma(v_2) \wedge \sigma(v_3) = \frac{1}{4},\\
\frac{1}{4} &= \mu_{Q_2T_f}(v_3, v_1) \le \sigma(v_3) \wedge \sigma(v_1) = \frac{1}{4},\\
\frac{1}{3} &= \mu_{Q_2T_f}(v_1, v_1v_2) \le \sigma(v_1) \wedge \mu(v_1, v_2) = \frac{1}{3},\\
\frac{1}{4} &= \mu_{Q_2T_f}(v_1, v_3v_{1,}) \le \sigma(v_1) \wedge \mu(v_3, v_1) = \frac{1}{4},\\
\frac{1}{5} &= \mu_{Q_2T_f}(v_2, v_2v_3) \le \sigma(v_2) \wedge \mu(v_2, v_3) = \frac{1}{5},\\
\frac{1}{3} &= \mu_{Q_2T_f}(v_2, v_2v_1) \le \sigma(v_2) \wedge \mu(v_2, v_1) = \frac{1}{3},\\
\frac{1}{4} &= \mu_{Q_2T_f}(v_3, v_3v_1) \le \sigma(v_3) \wedge \mu(v_3, v_1) = \frac{1}{4},\\
\frac{1}{5} &= \mu_{Q_2T_f}(v_3, v_3v_2) \le \sigma(v_3) \wedge \mu(v_3, v_2) = \frac{1}{5}.
\end{aligned} \tag{16}$$

Thus, we conclude that $\mu_{Q_2T_f}(v_i, v_j) \le \sigma_{Q_2T_f}(v_i) \wedge \sigma_{Q_2T_f}(v_j)$ for all $v_i, v_j \in V \cup E$; thus the graph $Q_2T_f(\sigma_{Q_2T_f}, \mu_{Q_2T_f})$ is a fuzzy graph and is called 2-quasitotal fuzzy graph of the fuzzy graph G in Example 1.

Now, based on the node sets $V \cup E$, fuzzy subsets $\sigma_{Q_2T_f}$, and fuzzy relations $\mu_{Q_2T_f}$, the 2-quasitotal fuzzy graph of G is as shown in Figure 2.

Example 2. Consider the following graph $G = (V, \sigma, \mu)$ with the fuzzy vertex set:

$\sigma(v_1) = 1$, $\sigma(v_2) = 0.75$, $\sigma(v_3) = 1$, $\sigma(v_4) = 0.25$ and fuzzy edge set:

$$\begin{aligned}
\mu(v_1, v_2) &= 0.5,\\
\mu(v_2, v_3) &= 0.5,\\
\mu(v_3, v_4) &= 0.25,\\
\mu(v_4, v_1) &= 0.25.
\end{aligned} \tag{17}$$

Clearly, $\mu(v_i, v_j) \le \sigma(v_i) \wedge \sigma(v_j)$ for all $v_i, v_j \in V$, the graph $G = (V, \sigma, \mu)$ is a fuzzy graph and its graph is as shown in Figure 3.

Now, the construction of 2-quasitotal fuzzy graph $Q_2T_f(\sigma_{Q_2T_f}, \mu_{Q_2T_f})$ of the graph G in Example 2 will be as follows.

(i) The node set of $\sigma_{Q_2T_f}$ will be as follows:

$$V \cup E = \{v_1, v_2, v_3, v_4, v_1v_2, v_2v_3, v_3v_4, v_4v_1\}. \tag{18}$$

(ii) The fuzzy subset $\sigma_{Q_2T_f}(G)$ will be as follows:

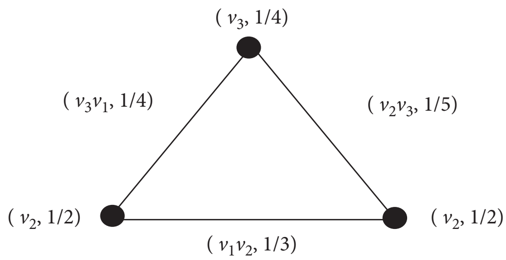

Figure 1: Fuzzy graph G.

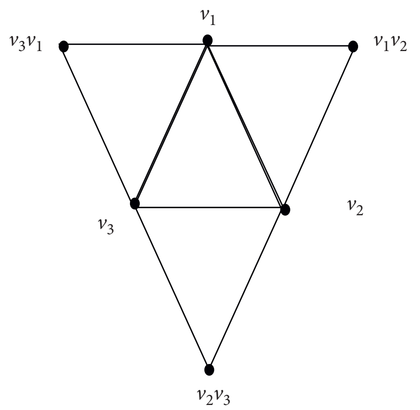

Figure 2: 2-Quasitotal fuzzy graph of G.

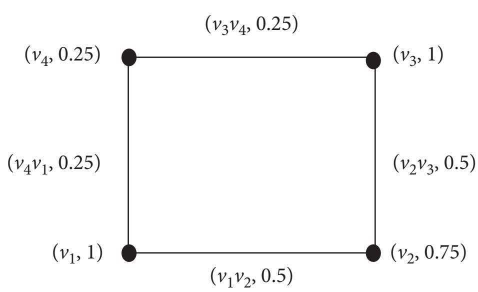

Figure 3: Fuzzy graph.G

$$\sigma_{Q_2T_f}(u) = \sigma(u), \quad \text{if } u \in V,$$
$$\sigma_{Q_2T_f}(e) = \mu(e), \quad \text{if } e \in E. \tag{19}$$

Hence,

$$\begin{aligned}
\sigma_{Q_2T_f}(v_1) &= \sigma(v_1) = 1,\\
\sigma_{Q_2T_f}(v_2) &= \sigma(v_2) = 0.75,\\
\sigma_{Q_2T_f}(v_3) &= \sigma(v_3) = 1,\\
\sigma_{Q_2T_f}(v_4) &= \sigma(v_4) = 0.25,\\
\sigma_{Q_2T_f}(v_1v_2) &= \mu(v_1, v_2) = 0.5,\\
\sigma_{Q_2T_f}(v_2v_3) &= \mu(v_2, v_3) = 0.5,\\
\sigma_{Q_2T_f}(v_3v_4) &= \mu(v_3, v_4) = 0.25,\\
\sigma_{Q_2T_f}(v_4v_1) &= \mu(v_4, v_1) = 0.25.
\end{aligned} \tag{20}$$

(iii) The fuzzy relation $\mu_{Q_2T_f}$ will be as follows:

$$\begin{aligned}
\mu_{Q_2T_f}(u, v) &= \mu(u, v), \quad \text{if } u, v \in V,\\
\mu_{Q_2T_f}(u, e) &= \sigma(u) \Lambda \mu(e), \quad \text{if } u \text{ lies on the edge of } e,\\
&= 0, \quad \text{Otherwise.}
\end{aligned} \tag{21}$$

Hence,

$$\begin{aligned}
\mu_{Q_2T_f}(v_1, v_2) &= \mu(v_1, v_2) = 0.5,\\
\mu_{Q_2T_f}(v_2, v_3) &= \mu(v_2, v_3) = 0.5,\\
\mu_{Q_2T_f}(v_3, v_4) &= \mu(v_3, v_4) = 0.25,\\
\mu_{Q_2T_f}(v_4, v_1) &= \mu(v_4, v_1) = 0.25,\\
\mu_{Q_2T_f}(v_1, v_1v_2) &= \sigma(v_1) \Lambda \mu(v_1, v_2) = 1 \Lambda 0.5 = 0.5,\\
\mu_{Q_2T_f}(v_1, v_2v_3) &= 0,\\
\mu_{Q_2T_f}(v_1, v_3v_4) &= 0,\\
\mu_{Q_2T_f}(v_1, v_4v_1) &= \sigma(v_1) \Lambda \mu(v_4, v_1) = 1 \Lambda 0.25 = 0.25,\\
\mu_{Q_2T_f}(v_2, v_2v_3) &= \sigma(v_2) \Lambda \mu(v_2, v_3) = 0.75 \Lambda 0.5 = 0.5,\\
\mu_{Q_2T_f}(v_2, v_3v_4) &= 0,\\
\mu_{Q_2T_f}(v_2, v_4v_1) &= 0,\\
\mu_{Q_2T_f}(v_2, v_1v_2) &= \sigma(v_2) \Lambda \mu(v_1, v_2) = 0.75 \Lambda 0.5 = 0.5,\\
\mu_{Q_2T_f}(v_3, v_3v_4) &= \sigma(v_3) \Lambda \mu(v_3, v_4) = 1 \Lambda 0.25 = 0.25,\\
\mu_{Q_2T_f}(v_3, v_4v_1) &= 0,\\
\mu_{Q_2T_f}(v_2, v_1v_2) &= 0,\\
\mu_{Q_2T_f}(v_3, v_2v_3) &= \sigma(v_3) \Lambda \mu(v_2, v_3) = 1 \Lambda 0.5 = 0.5,\\
\mu_{Q_2T_f}(v_4, v_4v_1) &= \sigma(v_4) \Lambda \mu(v_4, v_1) = 0.25 \Lambda 0.25 = 0.25,\\
\mu_{Q_2T_f}(v_4, v_1v_2) &= 0,\\
\mu_{Q_2T_f}(v_4, v_2v_3) &= 0,\\
\mu_{Q_2T_f}(v_4, v_3v_4) &= \sigma(v_4) \Lambda \mu(v_3, v_4) = 0.25 \Lambda 0.25 = 0.25.
\end{aligned} \tag{22}$$

Clearly, $\mu_{Q_2T_f}(v_i, v_j) \leq \sigma_{Q_2T_f}(v_i) \Lambda \sigma_{Q_2T_f}(v_j)$ for all $v_i, v_j \in V \cup E$ and hence the graph $Q_2T_f(\delta_{Q_2T_f}, \mu_{Q_2T_f})$ is a fuzzy graph and it is a 2-quasitotal fuzzy graph of a graph in the above Example 2, and its graph is as shown in Figure 4.

4. Properties of 2-Quasitotal Fuzzy Graph

Theorem 1. *Let $G = (V, \sigma, \mu)$ be a fuzzy graph. Then,*

$$\text{Order}(Q_2T_f(G)) = \text{Order}(G) + \text{Size}(G). \tag{23}$$

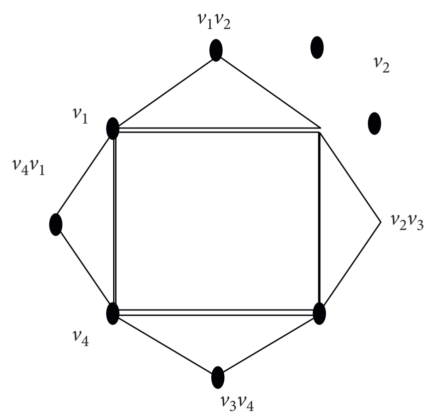

FIGURE 4: 2-Quasitotal fuzzy graph of G.

Proof. From the definition of 2-quasitotal fuzzy graph, we have the node set of $Q_2T_f(G)$ as VUE and the fuzzy subset $\sigma_{Q_2T_f}(u) = \sigma(u)$, if $u \in V$ and $\sigma_{Q_2T_f}(e) = \mu(e)$, if $e \in E$.

Now,

$$\text{Order}\left(Q_2T_f(G)\right) = \sum_{u\in V\cup E} \sigma_{Q_2T_f}(u), \quad (24)$$

(by the definition of the order of G).

$$= \sum_{u\in V} \sigma_{Q_2T_f}(u) + \sum_{u\in E} \sigma_{Q_2T_f}(u) = \sum_{u\in V} \sigma(u) + \sum_{u\in E} \sigma(u), \quad \text{by definition of } \sigma_{Q_2T_f}(u),$$
$$= \text{Order}(G) + \text{Size}(G),$$
$$\text{Order}\left(Q_2T_f(G)\right) = \text{Order}(G) + \text{Size}(G). \quad (25)$$

□

Note 1. For any fuzzy graph, $G = (V, \sigma, \mu)$,

(1) Order $(T(G))$ = Order (G) + Size (G), where $T(G)$ is the total fuzzy graph of G.

(2) Order $(Q_1T_f(G))$ = Order (G) + Size (G), where $Q_1T_f(G)$ is 1-quasitotal fuzzy graph of G:

$$\text{Order}\left(Q_2T_f(G)\right) = \text{Order}(G) + \text{Size}(G). \quad (26)$$

(3) Order $(T(G))$ = Order $(Q_1T_f(G))$ = Order $(Q_2T_f(G))$ = Order (G) + Size (G).

Theorem 2. *Let $G = (V, \sigma, \mu)$ be a fuzzy graph, then*

$$\text{Size}\left(Q_2T_f(G)\right) = \text{Size}(G) + \sum_{u\in V, e\in E} (\sigma(u) \wedge \mu(e)). \quad (27)$$

Proof. By the definition of the size of a fuzzy graph, we have the following:

$$\text{Size}\left(Q_2T_f(G)\right) = \sum_{u,v\in V\cup E} \mu_{Q_2T_f}(u, v)$$
$$= \sum_{u,v\in V} \mu_{Q_2T_f}(u, v) + \sum_{u\in V, e\in E} \mu_{Q_2T_f}(u, e) + \sum_{e_i,e_j\in E} \mu_{Q_2T_f}\left(e_i, e_j\right)$$
$$= \sum_{u,v\in V} \mu_{Q_2T_f}(u, v) + \sum_{u\in V, e\in E} \mu_{Q_2T_f}(u, e) + 0. \quad (28)$$

(The third summation is zero since there is no fuzzy relation between $e_i, e_j \in E$ in 2-quasitotal fuzzy graph)

$$= \sum_{u,v\in V} \mu(u, v) + \sum_{u\in V, e\in E} \mu_{Q_2T_f}(u, e),$$
$$= \sum (u, v) + \sum \mu(u, e),$$
$$= \text{Size}(G) + \sum (\sigma(u) \wedge \mu(e)), \quad (29)$$
$$\text{Size}\left(Q_2T_f(G)\right) = \text{Size}(G) + \sum (\sigma(u) \wedge \mu(e)).$$

□

Note 2. For any fuzzy graph $G = (V, \sigma, \mu)$,

(1) Size $(T(G))$ = 3 Size (G) + $\sum_{e_i,e_j\in E} \mu(e_i) \wedge \mu(e_j)$, where $T(G)$ is total fuzzy graph of G

(2) Size $(Q_1T_f(G))$ = Size (G) + $\sum_{e_i,e_j\in E} \mu(e_i) \wedge \mu(e_j)$, where Q_1T_f is 1-quasitotal fuzzy graph of G

(3) Size $(Q_2T_f(G))$ = Size (G) + $\sum \sigma(u) \wedge \mu(e)$, where Q_2T_f is 2-quasitotal fuzzy graph of G

Theorem 3. *Let $G = (V, \sigma, \mu)$ be a fuzzy graph; then,*

$$d\left(Q_2T_f(u)\right) = d_G(u) + \sum_{u\in V, e\in E} (\sigma(u) \wedge \mu(e)), \quad \text{if } u \in V,$$
$$= \sum_{e_{i\in E, u\in V}} (\mu(e_i) \wedge \sigma(u)), \quad \text{if } e_i \in E. \quad (30)$$

Proof. By the definition of the degree of a vertex of a fuzzy graph, we have the following two cases to prove the theorem. □

Case 1. Let $u \in V$. Then,

$$d\left(Q_2T_{f(G)}(u)\right) = \sum_{u,v\in V} \mu_{Q_2T_{f(G)}}(u, v) + \sum_{u\in V, e\in E} \mu_{Q_2T_{f(G)}}(u, e), \quad (31)$$

(where u lies on the edge of e in the second summation)

$$= \sum_{u,v\in V} \mu(u, v) + \sum_{u\in V, e\in E} \mu(u, e),$$
$$= d_G(u) + \sum_{u\in V, e\in E} (\sigma(u) \wedge \mu(e)), \quad (32)$$
$$d\left(Q_2T_{f(G)}(u)\right) = d_G(u) + \sum_{u\in V, e\in E} (\sigma(u) \wedge \mu(e)).$$

Case 2. Let $e_i \in E$; then,

$$d\left(Q_2T_{f(G)}(e_i)\right) = \sum_{u\in V} \mu_{Q_2T_{f(G)}}(e_i, u) + \sum_{e_j\in E} \mu_{Q_2T_{f(G)}}\left(e_i, e_j\right) = \sum_{u\in V} \mu(e_i, u) + 0. \quad (33)$$

(The second summation is zero since there is no fuzzy relation between $e_i, e_j \in E$ in 2-quasitotal fuzzy graph)

$$= \sum_{u\in V} \mu(e_i, u) = \sum_{u\in V} (\mu(e_i) \Lambda \sigma(u)),$$
$$d\left(Q_2T_{f(G)}(e_i)\right) = \sum_{u\in V} (\mu(e_i) \Lambda \sigma(u)). \quad (34)$$

Note 3. For any fuzzy graph $G = (V, \sigma, \mu)$,

(1) $d(T_G(u)) = 2d_G(u)$, if $u \in V$, where $T_G(u)$ is the total fuzzy graph of $G =$ busy value of e_i in $T(G)$, if $u \in E$

(2) $d(Q_1T_{f(G)}(u)) = d_G(u)$, if $u \in V =$ bussy value of e_i in $Q_1T_{f(G)}$, if $u \in E$, and where $Q_1T_{f(G)}$ is 1-quasitotal fuzzy graph of G

(3) $d(Q_2T_f(u)) = d_G(u) + \sum_{u\in V, e\in E}(\sigma(u) \Lambda \mu(e))$, *if* $u \in V = \sum_{u\in V}(\mu(e_i) \Lambda \sigma(u))$ and if $e_i \in E$ and Q_2T_f is 2-quasitotal fuzzy graph of G

Theorem 4. *2-quasitotal fuzzy graph of any connected fuzzy graph is a connected graph.*

Proof. Let $G = (V, \sigma, \mu)$ be a fuzzy graph.

The fuzzy vertex set of $Q_2T_f(G)$ consists of $V \cup E$ of G. The fuzzy relation $\mu_{Q_2T_{f(G)}}$ is defined only for $\mu_{Q_2T_f}(u, v)$, where $u, v \in V$ and $\mu_{Q_2T_f}(u, e)$, where $u \in V$, $e \in E$ and u lies on the edge of E.

Since G is connected and every edge of G is also considered as a node for $Q_2T_f(G)$, there is at least one path that connects every vertex u and v in $Q_2T_f(G)$ and $\mu^{\infty}_{Q_2T_f}(u, v) \neq 0$.

Hence, $Q_2T_f(G)$ is a connected fuzzy graph. □

5. 2-Quasitotal Fuzzy Coloring

In this section, we introduce the concept of 2-quasitotal fuzzy total coloring and discuss some of its properties.

Definition 14. A family $\Gamma = \{\gamma_1, \gamma_2, \ldots, \gamma_k\}$, of a fuzzy set on $V \cup E$ is called a 2-quasi k-fuzzy total coloring of fuzzy graph $G = (V, \sigma, \mu)$, if the following three conditions are met.

(i) $\text{Max}\{\gamma_i(v)\} = \sigma(v)$ for all $v \in V$ and $\text{Max}\{\gamma_i(u, v)\} = \mu(u, v)$, for all edges $(u, v) \in E.\gamma_i \Lambda \gamma_j = 0$

(ii) For every adjacent vertex u, v of $Q_2T_f(G)$, $\text{Min}\{\gamma_i(u), \gamma_i(v)\} = 0$.

The least number of colors possible is called 2-quasitotal fuzzy chromatic number of $Q_2T_f(G)$ and it is denoted by $\chi^f_{Q_2}(G)$.

Example 3. Consider a fuzzy graph $G = (V, \sigma, \mu)$ as shown in Figure 5.

From the graph, we have the vertex set $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ and edge set $E = \{v_1v_2, v_2v_3, v_3v_4, v_4v_5, v_5v_6, v_6v_1\}$, whose membership functions can be expressed as follows from the graph:

$$\sigma(v_i) = \begin{cases} 0.2, & \text{for } i = 1, \\ 0.7, & \text{for } i = 2, \\ 0.5, & \text{for } i = 3, \\ 0.4, & \text{for } i = 4, \\ 0.6, & \text{for } i = 5, \\ 0.3, & \text{for } i = 6, \end{cases}$$
$$\mu(v_i, v_j) = \begin{cases} 0.2, & \text{for } (i, j) = (1, 2), \\ 0.5, & \text{for } (i, j) = (2, 3), \\ 0.1, & \text{for } (i, j) = (3, 4), \\ 0.4, & \text{for } (i, j) = (4, 5), \\ 0.3, & \text{for } (i, j) = (5, 6), \\ 0.1, & \text{for } (i, j) = (6, 1). \end{cases} \quad (35)$$

The family of fuzzy sets $\Gamma = \{\gamma_1, \gamma_2\}$ on $V \cup E$ will be as follows:

$$\gamma_1(v_i) = \begin{cases} 0.2, & \text{for } i = 1, \\ 0.5, & \text{for } i = 3, \\ 0.6, & \text{for } i = 5, \\ 0, & \text{Otherwise}, \end{cases}$$
$$\gamma_2(v_i) = \begin{cases} 0.7, & \text{for } i = 2 \\ 0.4, & \text{for } i = 4 \\ 0.3, & \text{for } i = 6 \\ 0, & \text{Otherwise}, \end{cases}$$
$$\gamma_1(v_i, v_j) = \begin{cases} 0.2, & \text{for } (i, j) = (1, 2), \\ 0.1, & \text{for } (i, j) = (3, 4), \\ 0.3, & \text{for } (i, j) = (5, 6), \\ 0, & \text{Otherwise}, \end{cases}$$
$$\gamma_2(v_i, v_j) = \begin{cases} 0.5, & \text{for } (i, j) = (2, 3), \\ 0.4, & \text{for } (i, j) = (4, 5), \\ 0.1, & \text{for } (i, j) = (6, 1), \\ 0, & \text{Otherwise}. \end{cases} \quad (36)$$

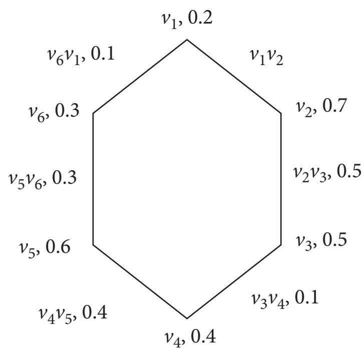

Figure 5: Fuzzy graph of G.

Table 1: Example of the total coloring of a fuzzy graph $G = (V, \sigma, \mu)$.

v and **e**	γ_1	γ_2	Max.	$\gamma_1 \Lambda \gamma_2$	Min.
v_1	0.2	0	0.2	0	$\text{Min}\{\gamma_i(v_1), \gamma_i(v_2)\} = 0$
v_2	0	0.7	0.7	0	$\text{Min}\{\gamma_i(v_2), \gamma_i(v_3)\} = 0$
v_3	0.5	0	0.5	0	$\text{Min}\{\gamma_i(v_3), \gamma_i(v_4)\} = 0$
v_4	0	0.4	0.4	0	$\text{Min}\{\gamma_i(v_4), \gamma_i(v_5)\} = 0$
v_5	0.6	0	0.6	0	$\text{Min}\{\gamma_i(v_5), \gamma_i(v_6)\} = 0$
v_6	0	0.3	0.3	0	$\text{Min}\{\gamma_i(v_6), \gamma_i(v_1)\} = 0$
v_1v_2	0.2	0	0.2	0	
v_2v_3	0	0.5	0.5	0	
v_3v_4	0.1	0	0.1	0	
v_4v_5	0	0.4	0.4	0	
v_5v_6	0.3	0	0.3	0	
v_6v_1	0	0.1	0.1	0	

To justify that the family of fuzzy sets $\Gamma = \{\gamma_1, \gamma_2\}$ defined as above satisfies the definition of the total coloring of the fuzzy graph and determines its total chromatic number, $\chi_T^f(G)$, we use Table 1 to check for the three conditions of the total coloring of a fuzzy graph.

From Table 1, we observe that the family of the fuzzy set $\Gamma = \{\gamma_1, \gamma_2\}$ satisfies the definition of the total coloring of a fuzzy graph G. Hence, $\chi_T^f(G) = 2$.

When we come to our point of concern, we need to determine the chromatic number of 2-quasitotal fuzzy graph of the fuzzy graph in Example 3.

Now, to construct a 2-quasitotal fuzzy graph $Q_{Q_2T_f}(G) = (V \cup E, \sigma_{Q_2T_f}, \mu_{Q_2T_f})$, where. $V \cup E = \{v_1, v_2, v_3, v_4, v_5, v_6, v_1v_2, v_2v_3, v_3v_4, v_4v_5, v_5v_6, v_6v_1\}$. The fuzzy subset of $Q_2T_f(G)$ will be as follows:

$$\sigma_{Q_2T_f}(v_i) = \begin{cases} 0.2, & \text{for } i = 1, \\ 0.7, & \text{for } i = 2, \\ 0.5, & \text{for } i = 3, \\ 0.4, & \text{for } i = 4, \\ 0.6, & \text{for } i = 5, \\ 0.3, & \text{for } i = 6, \end{cases}$$
$$\sigma_{Q_2T_f}(v_i, v_j) = \begin{cases} 0.2, & \text{for } (i, j) = (1, 2), \\ 0.5, & \text{for } (i, j) = (2, 3), \\ 0.1, & \text{for } (i, j) = (3, 4), \\ 0.4, & \text{for } (i, j) = (4, 5), \\ 0.3, & \text{for } (i, j) = (5, 6), \\ 0.1, & \text{for } (i, j) = (6, 1). \end{cases} \quad (37)$$

The fuzzy relation will be as follows:

$$\mu_{Q_2T_f}(v_i, v_j) = \begin{cases} 0.2, & \text{for } (i, j) = (1, 2), \\ 0.5, & \text{for } (i, j) = (2, 3), \\ 0.1, & \text{for } (i, j) = (3, 4), \\ 0.4, & \text{for } (i, j) = (4, 5), \\ 0.3, & \text{for } (i, j) = (5, 6), \\ 0.1, & \text{for } (i, j) = (6, 1), \end{cases}$$
$$\mu_{Q_2T_f}(v_i, v_iv_j) = \begin{cases} 0.2, & \text{for } (i, ij) = (1, 12), \\ 0.1, & \text{for } (i, ij) = (1, 61), \\ 0.5, & \text{for } (i, ij) = (2, 23), \\ 0.2, & \text{for } (i, ij) = (2, 12), \\ 0.1, & \text{for } (i, ij) = (3, 34), \\ 0.5, & \text{for } (i, ij) = (3, 23), \\ 0.4, & \text{for } (i, ij) = (4, 45), \\ 0.4, & \text{for } (i, ij) = (4, 34), \\ 0.3, & \text{for } (i, ij) = (5, 56), \\ 0.4, & \text{for } (i, ij) = (5, 45), \\ 0.1, & \text{for } (i, ij) = (6, 61), \\ 0.3, & \text{for } (i, ij) = (6, 56). \end{cases} \quad (38)$$

Hence, the 2-quasitotal fuzzy graph $Q_{Q_2T_f}(G) = (V \cup E, \sigma_{Q_2T_f}, \mu_{Q_2T_f})$ of the fuzzy graph G in Example 3 is as shown in Figure 6.

Let $\Gamma = \{\gamma_1, \gamma_2\}$ be a family of fuzzy subset defined on $V \cup E$ as follows:

(i) For the vertex set:

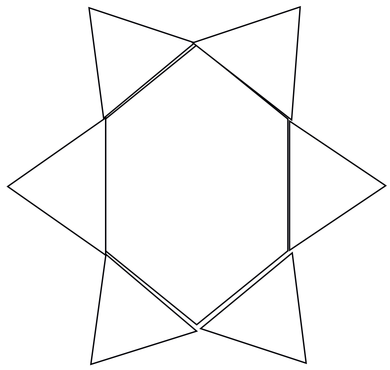

Figure 6: Total fuzzy graph of fuzzy graph G.

$$\gamma_1(v_i) = \begin{cases} 0.2, & \text{for } i = 1, \\ 0.5, & \text{for } i = 3, \\ 0.6, & \text{for } i = 5, \\ 0, & \text{Otherwise,} \end{cases}$$

$$\gamma_1(v_i v_j) = \begin{cases} 0.2, & \text{for } ij = 12, \\ 0.1, & \text{for } ij = 34, \\ 0.3, & \text{for } ij = 56, \\ 0, & \text{Otherwise,} \end{cases}$$

$$\gamma_2(v_i) = \begin{cases} 0.7, & \text{for } i = 2, \\ 0.4, & \text{for } i = 4, \\ 0.3, & \text{for } i = 6, \\ 0, & \text{Otherwise,} \end{cases} \quad (39)$$

$$\gamma_2(v_i v_j) = \begin{cases} 0.5, & \text{for } (i,j) = (2,3), \\ 0.4, & \text{for } (i,j) = (4,5), \\ 0.1, & \text{for } (i,j) = (6,1), \\ 0, & \text{Otherwise.} \end{cases}$$

(ii) For the edge set:

$$\gamma_1(v_i, v_j) = \begin{cases} 0.2, & \text{for } (i,j) = (1,2), \\ 0.1, & \text{for } (i,j) = (3,4), \\ 0.3, & \text{for } (i,j) = (5,6), \\ 0, & \text{Otherwise,} \end{cases}$$

$$\gamma_1(v_i, v_i v_j) = \begin{cases} 0.2, & \text{for } (i,ij) = (1,12), \\ 0.5, & \text{for } (i,ij) = (2,23), \\ 0.1, & \text{for } (i,ij) = (3,34), \\ 0.4, & \text{for } (i,ij) = (4,45), \\ 0.3, & \text{for } (i,ij) = (5,56), \\ 0.1, & \text{for } (i,ij) = (6,61), \\ 0, & \text{otherwise,} \end{cases}$$

Table 2: Example of 2-quasitotal coloring of a fuzzy graph $G = (V, \sigma, \mu)$

v and **e**	γ_1	γ_2	Max.	$\gamma_1 \Lambda \gamma_2$	Min.
v_1	0.2	0	0.2	0	$\text{Min}\{\gamma_i(v_1), \gamma_i(v_2)\} = 0$
v_2	0	0.7	0.7	0	$\text{Min}\{\gamma_i(v_2), \gamma_i(v_3)\} = 0$
v_3	0.5	0	0.5	0	$\text{Min}\{\gamma_i(v_3), \gamma_i(v_4)\} = 0$
v_4	0	0.4	0.4	0	$\text{Min}\{\gamma_i(v_4), \gamma_i(v_5)\} = 0$
v_5	0.6	0	0.6	0	$\text{Min}\{\gamma_i(v_5), \gamma_i(v_6)\} = 0$
v_6	0.3		0.3	0	$\text{Min}\{\gamma_i(v_6), \gamma_i(v_1)\} = 0$
v_1v_2	0.2	0	0.2	0	$\text{Min}\{\gamma_i(v_1v_2), \gamma_i(v_2v_3)\} = 0$
v_2v_3	0	0.5	0.5	0	$\text{Min}\{\gamma_i(v_2v_3), \gamma_i(v_3v_4)\} = 0$
v_3v_4	0.1	0	0.1	0	$\text{Min}\{\gamma_i(v_3v_4), \gamma_i(v_4v_5)\} = 0$
v_4v_5	0	0.4	0.4	0	$\text{Min}\{\gamma_i(v_4v_5), \gamma_i(v_5v_6)\} = 0$
v_5v_6	0.3	0	0.3	0	$\text{Min}\{\gamma_i(v_5v_6), \gamma_i(v_6v_1)\} = 0$
v_6v_1	0	0.1	0.1	0	$\text{Min}\{\gamma_i(v_6v_1), \gamma_i(v_1v_2)\} = 0$
(v_1, v_2)	0.2	0	0.2	0	
(v_2, v_3)	0	0.5	0.5	0	
(v_3, v_4)	0.1	0	0.1	0	
(v_4, v_5)	0	0.4	0.4	0	
(v_5, v_6)	0.3	0	0.3	0	
(v_6, v_1)	0	0.1	0.1	0	
(v_1, v_1v_2)	0.2	0	0.2	0	
(v_1, v_6v_1)	0	0.1	0.1	0	
(v_2, v_2v_3)	0.5	0	0.5	0	
(v_2, v_1v_2)	0	0.2	0.2	0	
(v_3, v_3v_4)	0.4	0	0.4	0	
(v_3, v_2v_3)	0	0.5	0.5	0	
(v_4, v_4v_5)	0.4	0	0.4	0	
(v_4, v_3v_4)	0	0.4	0.4	0	
(v_5, v_5v_6)	0.3	0	0.3	0	
(v_5, v_4v_5)	0	0.4	0.4	0	
(v_6, v_6v_1)	0.1	0	0.1	0	
(v_6, v_5v_6)	0	0.3	0.3	0	

$$\gamma_2(v_i v_j) = \begin{cases} 0.5, & \text{for } (i,j) = (2,3), \\ 0.4, & \text{for } (i,j) = (4,5), \\ 0.1, & \text{for } (i,j) = (6,1), \\ 0, & \text{Otherwise,} \end{cases}$$

$$\gamma_2(v_i, v_i v_j) = \begin{cases} 0.1, & \text{for } (i,ij) = (1,61), \\ 0.2, & \text{for } (i,ij) = (2,12), \\ 0.5, & \text{for } (i,ij) = (3,23), \\ 0.4, & \text{for } (i,ij) = (4,34), \\ 0.4, & \text{for } (i,ij) = (5,45), \\ 0.3, & \text{for } (i,ij) = (6,56), \\ 0, & \text{otherwise.} \end{cases} \quad (40)$$

Using Table 2 below, we can check whether Γ satisfies the definition of 2-quasitotal fuzzy coloring of G.

As shown in Table 1, $\Gamma = \{\gamma_1, \gamma_2\}$ satisfies the definition of 2-quasitotal fuzzy coloring of a fuzzy graph G.

Therefore, $\chi^f_{Q_1 T}(G) = 2$.

Note 4. Unfortunately, $\chi^f_{Q_1T}(G) = \chi^f_{Q_2T}(G) = 2$ for this example and no evidence that it is always true in this manuscript.

6. Conclusion

This article has introduced the new concept of 2-quasitotal fuzzy graph for a given fuzzy graph. The concept is clearly explained with the particle examples by giving a fuzzy graph and its 2-quasitotal fuzzy graph. Some properties of the 2-quasitotal fuzzy graphs have been proposed and proved. Further, the theorems and results obtained for 2-quasitotal fuzzy graphs are compared with the existing properties of total fuzzy graphs and 1-quasitotal fuzzy graphs. Lastly, it has been defined 2-quasitotal coloring for a fuzzy graph and its total coloring is exemplified.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Authors' Contributions

Both authors contributed equally and significantly in writing this article. Both authors read and approved the final manuscript.

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