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Research Article

Existence and Stability of Solutions of Fuzzy Fractional Stochastic Differential Equations with Fractional Brownian Motions

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The existence, uniqueness, and stability of solutions to fuzzy fractional stochastic differential equations (FFSDEs) driven by a fractional Brownian motion (fBm) with the Lipschitzian condition are investigated. Finally, we investigate the exponential stability of solutions.

1. Introduction

There appears to be confusion of various kinds in the modeling of several real world systems, such as trying to characterize a physical system and opinions on its parameters. To deal with this ambiguity, the fuzzy set theory will be used [1]. It is able to handle such linguistic statements mathematically using this theory, such as "large" and "less." The capacity to investigate fuzzy differential equations (FDEs) in modeling numerous phenomena, including imprecision, is provided by a fuzzy set. In particular, the fuzzy stochastic differential equations (FSDEs), in instance, might be used to investigate a variety of economics and engineering problems that involve two types of uncertainty: randomness and fuzziness.

The fuzzy It $\hat{o}$ stochastic integral was powered in [2, 3]. In [4, 5], the fuzzy stochastic integral is driven by the Wiener process as a fuzzy adapted stochastic process. In [6], Fei et al. studied the existence and uniqueness of solutions to the (FSDEs) under non-Lipschitzian condition. In [7], Jafari et al. study FSDEs driven by fBm. Jialu Zhu et al., in [8], prove existence of solutions to SDEs with fBm. Ding and Nieto [9] investigated analytical solutions of multitime-scale FSDEs driven by fBm. Vas'kovskii et al. [10] prove that the pth moments, $p \geq 1$, of strong solutions of a mixed-type SDEs are driven by a standard Brownian motion and a fBm. Despite the fact that some research exists on the problem of the uniqueness and existence of solutions to SDEs and FSDEs which are disturbed by Brownian motions or semimartingales [4, 11–15], a kind of the FFSDEs driven by an fBm has not been investigated. Agarwal et al. [16, 17] considered the concept of solution for FDEs with uncertainty and some results on FFDEs and optimal control nonlocal evolution equations. Recently, Zhou et al., in [18–20], gave some important works on the stability analysis of such SFDEs. Our results are inspired by the one in [21] where the existence and uniqueness results for the FSDEs with local martingales under the Lipschitzian conditions are studied. The rest of this paper is given as follows. Section 2 summarizes the fundamental aspects. In Section 3, existence and uniqueness of solutions to the FFSDEs are proved. Moreover, the stability of solutions is studied in Section 4. Finally, in Section 5, a conclusion is given.

2. Preliminaries

This part introduces the notations, definitions, and background information that will be utilized throughout the article.

Let $\mathbf{K}(\mathbb{R}^n)$ be the family of nonempty convex and compact subsets of $\mathbb{R}^n$. In $\mathbf{K}(\mathbb{R}^n)$, the distance d_H is defined by

$$\mathrm{d}_H(M,N) = \max\left(\sup_{m\in M}\inf_{n\in N}\|m-n\|, \sup_{n\in N}\inf_{m\in M}\|m-n\|\right), \quad M,N \in \mathbf{K}(\mathbb{R}^n). \tag{1}$$

We denote by $\mathscr{M}(\Omega,\mathscr{A};\mathbf{K}(\mathbb{R}^n))$ the family of $\mathscr{A}$-measurable multifunction, taking value in $\mathbf{K}(\mathbb{R}^n)$.

Definition 1 (see [21, 22]). A multifunction $G \in \mathscr{M}(\Omega, \mathscr{A};\mathbf{K}(\mathbb{R}^n))$ is called $\mathscr{L}^p$-integrably bounded if $\exists h \in \mathscr{L}^p(\Omega,\mathscr{A},\mathbb{P};\mathbb{R}^+)$ such that $|||G||| \leq h\,\mathbb{P}$-a.e, where

$$|||B||| \coloneqq d_H(B,\widehat{0}) = \sup_{b\in B}\|b\|, \quad \text{for } B \in \mathbf{K}(\mathbb{R}^n). \tag{2}$$

We denote by

$$\mathscr{L}^p(\Omega,\mathscr{A},\mathbb{P};\mathbf{K}(\mathbb{R}^n)) \coloneqq \{G \in \mathscr{M}\Omega,\mathscr{A};\mathbf{K}(\mathbb{R}^n): |||G||| \in \mathscr{L}^p(\Omega,\mathscr{A},\mathbb{P};\mathbb{R}^+)\}. \tag{3}$$

Let $\mathbf{E}^n$ denote the set of the fuzzy x: $\mathbb{R}^n \longrightarrow [0,1]$ such that $[x]^\alpha \in \mathbf{K}(\mathbb{R}^n)$, for every $\alpha \in [0,1]$, where $[x]^\alpha: = \{a \in \mathbb{R}^n: x(a) \geq \alpha\}$, for $\alpha \in (0,1]$, and $[x]^0: = cl\{a \in \mathbb{R}^n: x(a) > 0\}$. Let the metric be $\mathrm{d}_\infty(x,y) \coloneqq \sup_{\alpha\in[0,1]} \mathrm{d}_H([x]^\alpha, [y]^\alpha)$, in $\mathbf{E}^n$, $a \in \mathbb{R}$; we have $\mathrm{d}_\infty(x+z,y+z) = \mathrm{d}_\infty(x,y)$, $\mathrm{d}_\infty(x+y,z+w) \leq \mathrm{d}_\infty(x,z) + \mathrm{d}_\infty(y,w)$, and $\mathrm{d}_\infty(ax,ay) = |a|d_\infty(x,y)$.

Definition 2 (see [23]). Let f: $[c,d] \longrightarrow \mathbf{E}^n$; the fuzzy Riemann–Liouville integral of f is given by

$$\left(\mathscr{I}^\alpha_{c^+}f\right)(u) = \frac{1}{\Gamma(\alpha)}\int_c^u (u-v)^{\alpha-1}f(v)\mathrm{d}v. \tag{4}$$

Definition 3 (see [23]). Let $Df \in C([c,d],\mathbf{E}^n) \cap L([c,d],\mathbf{E}^n)$. The fuzzy fractional Caputo differentiability of f is given by

$$^{C}\mathscr{D}^\alpha_{c^+}f(u) = \mathscr{I}^{1-\alpha}_{c^+}(Df)(u) = \frac{1}{\Gamma(1-\alpha)}\int_c^u (u-v)^{-\alpha}(Df)(v)\mathrm{d}v. \tag{5}$$

Now, we define the Henry–Gronwall inequality [24], which can be used in the proof of our result.

Lemma 1. *Let f, g: $[0,T) \longrightarrow \mathbb{R}^+$ be continuous functions. If g is nondecreasing and there exists constants $K \geq 0$ and $\alpha > 0$ as*

$$f(u) \leq g(u) + K\int_0^u (u-v)^{\alpha-1}f(v)\mathrm{d}v, \quad u \in [0,T), \tag{6}$$

then

$$f(u) \leq g(u) + \int_0^u\left[\sum_{m=1}^{\infty}\frac{(K\Gamma(\alpha))^m}{\Gamma(m\alpha)}(u-v)^{n\alpha-1}g(v)\right]dv, \quad u \in [0,T). \tag{7}$$

If $g(u) = b$ is constant on $[0,T)$, the previous inequality is transformed into

$$f(u) \leq bE_\alpha\left(K\Gamma(\alpha)u^\alpha\right), \quad u \in [0,T), \tag{8}$$

where E_α is given by

$$E_\alpha(z) = \sum_{m=0}^{\infty}\frac{z^m}{\Gamma(m\alpha+1)}. \tag{9}$$

Remark 1 (see [24]). For all $u \in [0,T)$, $\exists N^*_K > 0$ does not depend on b such that $f(u) \leq N^*_K b$.

Definition 4 (see [21, 22]).

A function f: $\Omega \longrightarrow \mathbf{E}^n$ is said fuzzy random variable if $[f]^\alpha$ is an $\mathscr{A}$-measurable random variable $\forall\alpha \in [0,1]$

A fuzzy random variable f: $\Omega \longrightarrow \mathbf{E}^n$ is said $\mathscr{L}^p$-integrably bounded, $p \geq 1$, if $[f]^\alpha \in \mathscr{L}^p(\Omega,\mathscr{A},\mathbb{P};\mathbf{K}(\mathbb{R}^n))$, $\forall\alpha \in [0,1]$

Let $\mathscr{L}^p(\Omega,\mathscr{A},\mathbb{P};\mathbf{E}^n)$ denote the set of all fuzzy random variables; they are $\mathscr{L}^p$-integrally bounded.

For the notion of an fBm, we referred to [25].

Let us define a sequence of partitions of $[a,b]$ by $\{\psi_m, m \in \mathbb{N}\}$ such that $|\psi_m| \longrightarrow 0$ as $m \longrightarrow \infty$. If, in $L^2(\Omega,\mathscr{A},\mathbb{P})$, $\sum_{i=0}^{m-1}\phi(t_i^{(m)})(\mathbf{B}^H(t_{i+1}^{(m)}) - \mathbf{B}^H(t_i^{(m)}))$ converge to the same limit for all this sequences $\{\psi_m, m \in \mathbb{N}\}$, then this limit is said a Stratonovich-type stochastic integral and noted by $\int_a^b \phi(s)\mathrm{d}\mathbf{B}^H(s)$. Let $J: = [0,T]$, where $0 < T < \infty$.

Definition 5 (see [21, 22]).

A function f: $J \times \Omega \longrightarrow \mathbf{E}^n$ is called fuzzy stochastic process; if $\forall t \in J$, $f(t,.) = f(t)$: $\Omega \longrightarrow \mathbf{E}^n$ is a fuzzy random variable

A fuzzy stochastic process f is continuous; if $f(.,v)$: $J \longrightarrow \mathbf{E}^n$ are continuous, and it is $\{\mathscr{A}^H_t\}_{t\in J}$-adapted if for every $\alpha \in [0,1]$ and for all $t \in J$, $[f(t)]^\alpha$: $\Omega \longrightarrow \mathbf{K}(\mathbb{R}^n)$ is $\mathscr{A}^H_t$-measurable

Definition 6 (see [21, 22]).

The function f is called measurable if $[f]^{\alpha}: J \times \Omega \longrightarrow \mathbf{K}(\mathbb{R}^n)$ is a $\mathscr{B}(J) \otimes \mathscr{A}$-measurable, for all $\alpha \in [0, 1]$

The function $f: J \times \Omega \longrightarrow \mathbf{E}^n$ is said to be nonanticipating if it is $\{\mathscr{A}_t^H\}_{t \in J}$-adapted and measurable

Remark 2. The process x is nonanticipating if and only if x is measurable with respect to $\mathbf{N}: = \{A \in \mathscr{B}(J) \otimes \mathscr{A}: A^u \in \mathscr{A}_u^H, u \in J\}$, where, for $u \in J$, $A^u = \{v: (u, v) \in A\}$.

Definition 7 (see [21, 22]). A fuzzy process $f: J \times \Omega \longrightarrow \mathbf{E}^n$ is said $\mathscr{L}^p$-integrally bounded if $\exists h \in \mathscr{L}^p(J \times \Omega, \mathbf{N}; \mathbb{R}) / \mathrm{d}_{\infty}(f(s, v), \widehat{0}) \leq h(s, v)$.

We denote by $\mathscr{L}^p(J \times \Omega, \mathbf{N}; \mathbf{E}^n)$ the set of all $\mathscr{L}^p$-integrally bounded and nonanticipating fuzzy stochastic processes.

Proposition 1 (see [4]). *For $f \in \mathscr{L}^p(J \times \Omega, \mathbf{N}; \mathbf{E}^n)$ and $p \geq 1$, we have $J \times \Omega \ni (t, v) \longrightarrow \int_0^t f(s, v) ds \in \mathscr{L}^p\ (J \times \Omega, \mathbf{N}; \mathbf{E}^n)$ and d_{∞}-continuous.*

Proposition 2 (see [4]). *For $f, g \in \mathscr{L}^p(J \times \Omega, \mathbf{N}; \mathbf{E}^n)$ and $p \geq 1$, we have*

$$\mathbb{E} \sup_{a \in [0,t]} \mathrm{d}_{\infty}^p \left(\int_0^a f(u) \mathrm{d}u, \int_0^a g(u) \mathrm{d}u \right) \leq t^{p-1} \int_0^t \mathbb{E} \mathrm{d}_{\infty}^p (f(u), g(u)) \mathrm{d}u. \tag{10}$$

Proposition 3 (see [26]). *Let $\psi: J \longrightarrow \mathbb{R}^n$; then, for $t \in J$,*

$$\sup_{a \in [0,t]} \mathbb{E} \left\| \int_0^a \psi(s) \mathrm{d}\mathbf{B}^H(s) \right\|^2 \leq c_{t,H} \int_0^t \|\psi(s)\|^2 \mathrm{d}s. \tag{11}$$

Let us define the embedding of $\mathbb{R}^n$ to $\mathbf{E}^n$ as $\langle . \rangle: \mathbb{R}^n \longrightarrow \mathbf{E}^n$:

$$\langle r \rangle (a) = \begin{cases} 1, & \text{if } a = r, \\ 0, & \text{if } a \neq r. \end{cases} \tag{12}$$

Proposition 4 (see [4]). *Assume that the function $\psi: J \longrightarrow \mathbb{R}^n$ satisfies $\int_0^T \|\psi(v)\|^2 dv < \infty$. Then,*

(*i*) *The fuzzy stochastic Itô integral $\langle \int_0^v \psi(u) d\ \mathbf{B}^H(u) \rangle \in \mathbf{L}^2(J \times \Omega, \mathbf{N}; \mathbf{E}^n)$*

(*ii*) *For $x \in \mathbf{L}^2(J \times \Omega, \mathbf{N}; \mathbf{E}^n)$, we have, for $u \leq v \in J$,*

$$d_{\infty}\left(\int_0^v x(w_1) \mathrm{d}w_1 + \int_0^v \psi(w_2) \mathrm{d}\mathbf{B}^H(w_2), \int_0^u x(w_1) \mathrm{d}w_1 + \int_0^u \psi(w_2) \mathrm{d}\mathbf{B}^H(w_2) \right) = d_{\infty}\left(\int_u^v x(w_1) \mathrm{d}w_1 + \int_u^v \psi(w_2) \mathrm{d}\mathbf{B}^H(w_2), \widehat{0} \right). \tag{13}$$

3. Main Result

Now, we investigate the FFSDEs driven by an fBm given by

$$\begin{cases} {}^C\mathscr{D}^{\alpha} x(s) \overset{J\mathbb{P}.1}{=} f(s, x(s)) \mathrm{d}s + \langle g(s) \mathrm{d}\mathbf{B}^H(s) \rangle, \\ x(0) \overset{\mathbb{P}.1}{=} x_0, \end{cases} \tag{14}$$

where

$$\begin{aligned} f&: J \times \Omega \times \mathbf{E}^n \longrightarrow \mathbf{E}^n, \\ g&: J \longrightarrow \mathbb{R}^n, \\ x_0&: \Omega \longrightarrow \mathbf{E}^n, \end{aligned} \tag{15}$$

and $\{\mathbf{B}^H(s)\}_{s \in J}$ is a fBm defined on $(\Omega, \mathscr{A}, \{\mathscr{A}_s^H\}_{s \in J}, \mathbb{P})$ with Hirst index $H \in (1/2, 1)$.

Definition 8. A process $x: J \times \Omega \longrightarrow \mathbf{E}^n$ is said to be a solution to equation (14) if the following holds:

(i) $x \in \mathbf{L}^2(J \times \Omega, \mathbf{N}; \mathbf{E}^n)$.

(ii) x is d_{∞}-continuous.

(iii) We have

$$x(t) \overset{J\mathbb{P}.1}{=} x_0 + \frac{1}{\Gamma(\alpha)} \int_0^t \frac{f(s, x(s))}{(t-s)^{1-\alpha}} \mathrm{d}s + \langle \frac{1}{\Gamma(\alpha)} \int_0^t \frac{g(s)}{(t-s)^{1-\alpha}} \mathrm{d}\mathbf{B}^H(s) \rangle. \tag{16}$$

We will assume that all through this paper, $f: (J \times \Omega) \times \mathbf{E}^n \longrightarrow \mathbf{E}^n$ is $\mathscr{B}_{d_S} \otimes \mathbf{N} | \mathscr{B}_{d_{\infty}}$-measurable. Let the following assumptions be introduced.

($\mathscr{H}$1) If x_0 is $\mathscr{A}_0$-measurable, we have

$$\mathbb{E} d_{\infty}^2 \left(x_0, \widehat{0} \right) < \infty. \tag{17}$$

($\mathscr{H}$2) For $f(s, \widehat{0})$ and g, we have

$$\max\left\{ \mathrm{d}_{\infty}^2 (f(s, \widehat{0}), \widehat{0}), \|g\| \right\} \leq c, \tag{18}$$

for every $s \in J$.

($\mathscr{H}$3) For all $z, w \in \mathbf{E}^n$,

$$d_{\infty}^2 (f(s, z), f(s, w)) \leq c d_{\infty}^2 (z, w), \tag{19}$$

where c is equal to one in ($\mathscr{H}2$).

Let us now introduce the main theorem in this part.

Theorem 1. *Under assumptions* ($\mathscr{H}1$)–($\mathscr{H}3$) *and* $x_0 \in \mathbf{L}^2(\Omega, \mathscr{A}_0, \mathbb{P}; \mathbf{E}^n)$*, the equation (14) has a unique solution* $x(t)$.

Proof. The method of successive approximations will be used to demonstrate the existence of a solution to (1). Therefore, define a sequence $x_n \colon J \times \Omega \longrightarrow \mathbf{E}^n$ as follows:

$$x_0(t) \overset{\mathbb{P}.1}{=} x_0, \tag{20}$$

and for $n = 1, \ldots,$

$$x_n(t) \overset{J\mathbb{P}.1}{=} x_0 + \frac{1}{\Gamma(\alpha)} \int_0^t \frac{f(s, x_{n-1}(s))}{(t-s)^{1-\alpha}} \mathrm{d}s + \langle \frac{1}{\Gamma(\alpha)} \int_0^t \frac{g(s)}{(t-s)^{1-\alpha}} \mathrm{d}\mathbf{B}^H(s) \rangle. \tag{21}$$

It is clear that x_ns are in $\mathbf{L}^2(J \times \Omega, \mathbf{N}; \mathbf{E}^n)$ and d_∞-continuous. Indeed, we have $x_0 \in \mathbf{L}^2(J \times \Omega, \mathbf{N}; \mathbf{E}^n)$ and x_0 is d_∞-continuous.

Let us define for $n \in \mathbb{N}$ and $t \in J$ $K_n = \sup_{0 \le u \le t} \mathbb{E} d_\infty^2(x_n(u), x_{n-1}(u))$. Then, from Propositions 2 and 3 and ($\mathscr{H}1$)–($\mathscr{H}3$), we have

$$\begin{aligned}
K_1(t) &= \sup_{0 \le u \le t} \mathbb{E}\mathrm{d}_\infty^2 \left(\frac{1}{\Gamma(\alpha)} \int_0^u \frac{f(s, x_0)}{(u-s)^{1-\alpha}} \mathrm{d}s + \langle \frac{1}{\Gamma(\alpha)} \int_0^u \frac{g(s)}{(u-s)^{1-\alpha}} \mathrm{d}\mathbf{B}^H(s) \rangle, \hat{0} \right) \\
&\le 2 \sup_{0 \le u \le t} \left[\mathbb{E}\mathrm{d}_\infty^2 \left(\frac{1}{\Gamma(\alpha)} \int_0^u \frac{f(s, x_0)}{(u-s)^{1-\alpha}} \mathrm{d}s, \hat{0} \right) + \left| \frac{1}{\Gamma(\alpha)} \int_0^u \frac{g(s)}{(u-s)^{1-\alpha}} \mathrm{d}\mathbf{B}^H(s) \right|^2 \right] \\
&\le 2 \sup_{0 \le u \le t} \left[2\mathbb{E}\mathrm{d}_\infty^2 \left(\frac{1}{\Gamma(\alpha)} \int_0^u \frac{f(s, x_0)}{(u-s)^{1-\alpha}} \mathrm{d}s, \frac{1}{\Gamma(\alpha)} \int_0^u \frac{f(s, \hat{0})}{(u-s)^{1-\alpha}} \mathrm{d}s \right) \right] \\
&\quad + 4 \sup_{0 \le u \le t} \mathbb{E}\mathrm{d}_\infty^2 \left(\frac{1}{\Gamma(\alpha)} \int_0^u \frac{f(s, \hat{0})}{(u-s)^{1-\alpha}} \mathrm{d}s, \hat{0} \right) + 2\mathbb{E}\sup_{u \in [0,t]} \left[\frac{c_{T,H}}{\Gamma(\alpha)} \int_0^u \frac{\|g(s)\|^2}{(u-s)^{1-\alpha}} \mathrm{d}s \right] \\
&\le \frac{4T}{\Gamma(\alpha)} \int_0^t \frac{\mathbb{E}d_\infty^2(f(s, x_0), f(s, \hat{0}))}{(t-s)^{1-\alpha}} \mathrm{d}s + \frac{4T}{\Gamma(\alpha)} \int_0^t \frac{\mathbb{E}d_\infty^2(f(s, \hat{0}), \hat{0})}{(t-s)^{1-\alpha}} \mathrm{d}s + \frac{2c_{T,H}}{(t-s)^{1-\alpha}} \int_0^t \frac{\|g(s)\|^2}{(t-s)^{1-\alpha}} \mathrm{d}s \\
&\le \frac{4Tct^\alpha}{\Gamma(\alpha+1)} \mathrm{d}_\infty^2(x_0, \hat{0}) + \frac{4Tt^\alpha c}{\Gamma(\alpha+1)} + \frac{2c^2 c_{T,H} t^\alpha}{\Gamma(\alpha+1)} := \frac{l_1 t^\alpha}{\Gamma(\alpha+1)},
\end{aligned} \tag{22}$$

where $l_1 = 4cTd_\infty^2(x_0, \hat{0}) + 4Tc + 2c^2 c_{T,H}$. Moreover, similarly, we have

$$\begin{aligned}
K_{n+1}(t) &= \sup_{0 \le u \le t} \mathbb{E}\mathrm{d}_\infty^2 \Bigg(\frac{1}{\Gamma(\alpha)} \int_0^u \frac{f(s, x_n(s))}{(u-s)^{1-\alpha}} \mathrm{d}s + \langle \frac{1}{\Gamma(\alpha)} \int_0^u \frac{g(s)}{(u-s)^{1-\alpha}} \mathrm{d}\mathbf{B}^H(s) \rangle, \frac{1}{\Gamma(\alpha)} \int_0^u \frac{f(s, x_{n-1}(s))}{(u-s)^{1-\alpha}} \mathrm{d}s \\
&\quad + \langle \frac{1}{\Gamma(\alpha)} \int_0^u \frac{g(s)}{(u-s)^{1-\alpha}} \mathrm{d}\mathbf{B}^H(s) \rangle \Bigg) \\
&\le 2 \sup_{0 \le u \le t} \mathbb{E}d_\infty^2 \left(\frac{1}{\Gamma(\alpha)} \int_0^u \frac{f(s, x_n(s))}{(u-s)^{1-\alpha}} \mathrm{d}s, \int_0^u \frac{f(s, x_{n-1}(s))}{(u-s)^{1-\alpha}} \mathrm{d}s \right) \\
&\le \frac{2t}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} \mathbb{E}\mathrm{d}_\infty^2(f(s, x_n(s)), f(s, x_{n-1}(s))) \mathrm{d}s \\
&\le \frac{2tc}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} \sup_{u \in [0,s]} \mathbb{E}\mathrm{d}_\infty^2(x_n(s), x_{n-1}(s)) \mathrm{d}s \\
&\le \frac{2Tc}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} K_n(s) \mathrm{d}s.
\end{aligned} \tag{23}$$

Thus, we obtain

$$K_n(t) \leq \frac{l_1}{l_2} \frac{(l_2 t^\alpha)^n}{n!\Gamma(\alpha+1)}, \quad \forall t \in J,\ n \in \mathbb{N}, \tag{24}$$

where $l_2 = 2Tc$.

Hence, from Chebyshev's inequality and (24), we obtain

$$\mathbb{P}\left(\sup_{u\in J} \mathrm{d}_\infty^2 (x_n(u), x_{n-1}(u)) > \frac{1}{4^n} \right) \leq \frac{l_1}{l_2} \frac{(4l_2 T^\alpha)^n}{n!\Gamma(\alpha+1)}, \tag{25}$$

Since the series $\sum_{n\geq 1} (4l_2 T^\alpha)^n / n!$ converges, according to Borel–Cantelli lemma, we obtain

$$\mathbb{P}\left(\sup_{u\in J} \mathrm{d}_\infty (x_n(u), x_{n-1}(u)) > \frac{1}{2^n} \right) = 0. \tag{26}$$

Thus, the sequence $\{x_n(.,v)\}$ is uniformly convergent to $\tilde{x}(.,v)\colon J \longrightarrow \mathbb{R}^n$ for $v \in \Omega_c$, where $\Omega_c \in \mathscr{A}$ and $\mathbb{P}(\Omega_c) = 1$. Then,

$$\lim_{n\longrightarrow\infty} \sup_{t\in J} \mathbb{E}\mathrm{d}_\infty^2 (x_n(t), \tilde{x}(t)) = 0. \tag{27}$$

Let us define $x\colon J \times \Omega \longrightarrow \mathbf{E}^n$ as follows:

$$x(.,v) = \begin{cases} \tilde{x}(.,v), & \text{if } v \in \Omega_c, \\ \text{freely chosen}, & \text{if } v \in v \in \dfrac{\Omega}{\Omega_c}. \end{cases} \tag{28}$$

We can observe that, for each $0 \leq \alpha \leq 1$ and $t \in J$, we have

$$\lim_{n\longrightarrow\infty} \mathrm{d}_H \left([x_n(,v)]^\alpha, [x_{n-1}(,v)]^\alpha \right) = 0. \tag{29}$$

Then, $[x(t,.)]^\alpha\colon \Omega \longrightarrow \mathbf{K}(\mathbb{R}^n)$ is $\mathscr{A}_t$-measurable. Hence, x is nonanticipating. By (27), we have

$$\lim_{n\longrightarrow\infty} \sup\nolimits_{t\in J} \mathbb{E}\mathrm{d}_\infty^2 (x_n(t), x(t)) = 0, \tag{30}$$

which shows that $\exists \lambda > 0$ independent of $n \in \mathbb{N}$ such that

$$\sup_{t\in J} \mathbb{E}\mathrm{d}_\infty^2 (x_n(t), x(t)) \leq \lambda. \tag{31}$$

Since $x_n \in \mathbf{L}^2(J\times\Omega, \mathbf{N}; \mathbf{E}^n)$, we have $x_n(t) \in \mathbf{L}^2(\Omega, \mathscr{A}, \mathbb{P}; \mathbf{E}^n)$. In addition, we can prove that $x \in \mathbf{L}^2(J\times\Omega, \mathbf{N}; \mathbf{E}^n)$.

Indeed, for all $n \in \mathbb{N}$ and $t \in J$, let us denote

$$\psi_n(t) = \sup_{0\leq u\leq t} \mathbb{E}\mathrm{d}_\infty^2 \left(x_0, \hat{0} \right). \tag{32}$$

Then, we obtain

$$\psi_n(t) \leq 3\mathbb{E}\mathrm{d}_\infty^2\left(x_0,\hat{0}\right) + 3 \sup_{0\leq u\leq t} \mathbb{E}\mathrm{d}_\infty^2 \left(\frac{1}{\Gamma(\alpha)} \int_0^u (u-s)^{\alpha-1} f(s, x_{n-1}(s))\mathrm{d}s, \hat{0} \right) + 3\mathbb{E} \sup_{0\leq u\leq t} \left\| \frac{1}{\Gamma(\alpha)} \int_0^u (u-s)^{\alpha-1} g(s)\mathrm{d}\mathbf{B}^H(s) \right\|^2. \tag{33}$$

By the triangle inequality, $(\mathscr{H}1)$–$(\mathscr{H}3)$, and Propositions 2 and 3, we have

$$\begin{aligned} \psi_n(t) &\leq 3\mathbb{E}\mathrm{d}_\infty^2\left(x_0,\hat{0}\right) + \frac{6t}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} \left\{ \mathbb{E}d_\infty^2\left(f(s, x_{n-1}(s)), f(s,\hat{0}) \right) + \mathbb{E}\mathrm{d}_\infty^2 (f(s,\hat{0}),\hat{0}) \right\} \mathrm{d}s \\ &\quad + \frac{3c_{T,H}}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} \|g(s)\|^2 \mathrm{d}s, \\ &\leq 3\mathbb{E}d_\infty^2\left(x_0,\hat{0}\right) + \frac{6ct}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} \mathbb{E}\mathrm{d}_\infty^2\left(x_{n-1}(s),\hat{0}\right)\mathrm{d}s + \frac{6ct^{\alpha+1}}{\Gamma(\alpha+1)} + \frac{3t^\alpha c^2 c_{T,H}}{\Gamma(\alpha+1)}. \end{aligned} \tag{34}$$

We obtain

$$\psi_n(t) \leq A_1 + A_2 \int_0^t (t-s)^{\alpha-1} \psi_{n-1}(s)\mathrm{d}s, \tag{35}$$

where $A_1 = 3\mathbb{E}\mathrm{d}_\infty^2(x_0,\hat{0}) + (6ct^{\alpha+1}/\Gamma(\alpha+1)) + (3t^\alpha c^2 c_{T,H}/\Gamma(\alpha+1))$ and $A_2 = 6ct/\Gamma(\alpha)$.

According to Lemma 1 and Remark 1, there exist a constant $M_{A_2} > 0$ independent of A_1 such that

$$\psi_n(t) \leq M_{A_2} A_1. \tag{36}$$

Due to $(\mathscr{H}1)$, (31), and (36), we obtain

$$\sup_{0\leq s\leq t} \mathbb{E}\mathrm{d}_\infty^2 (x(s),\hat{0}) \leq 2 \sup_{0\leq s\leq t} \mathbb{E}\mathrm{d}_\infty^2 (x(s), x_n(s)) + 2 \sup_{0\leq s\leq t} \mathbb{E}\mathrm{d}_\infty^2 \left(x_n(s),\hat{0}\right), \leq 2\lambda + 2M_{A_2}A_1 < \infty, \tag{37}$$

which implies

$$\int_0^T \mathbb{E}\mathrm{d}_\infty^2(x(s),\widehat{0})\mathrm{d}s \le T\sup_{t\in J}\mathbb{E}\mathrm{d}_\infty^2(x(t),\widehat{0}) < \infty. \quad (38)$$

Thus, we get $x \in \mathbf{L}^2(J\times\Omega,\mathbf{N};\mathbf{E}^n)$.

On the contrary, we have

$$\sup_{t\in J}\mathbb{E}\mathrm{d}_\infty^2\left(x(t), x_0 + \frac{1}{\Gamma(\alpha)}\int_0^t \frac{f(s,x(s))}{(t-s)^{1-\alpha}}\mathrm{d}s + \langle\frac{1}{\Gamma(\alpha)}\int_0^t \frac{g(s)}{(t-s)^{1-\alpha}}\mathrm{d}\mathbf{B}^H(s)\rangle\right) = 0. \quad (39)$$

Indeed, we observe

$$\begin{aligned}
&\sup_{t\in J}\mathbb{E}\mathrm{d}_\infty^2\left(x(t), x_0 + \frac{1}{\Gamma(\alpha)}\int_0^t \frac{f(s,x(s))}{(t-s)^{1-\alpha}}\mathrm{d}s + \langle\frac{1}{\Gamma(\alpha)}\int_0^t \frac{g(s)}{(t-s)^{1-\alpha}}\mathrm{d}\mathbf{B}^H(s)\rangle\right)\\
&\le 3\Bigg[\sup_{t\in J}\mathbb{E}\mathrm{d}_\infty^2(x(t),x_n(t)) + \sup_{t\in J}\mathbb{E}\mathrm{d}_\infty^2\left(x_n(t), x_0 + \frac{1}{\Gamma(\alpha)}\int_0^t \frac{f(s,x_{n-1}(s))}{(t-s)^{1-\alpha}}\mathrm{d}s + \langle\frac{1}{\Gamma(\alpha)}\int_0^t \frac{g(s)}{(t-s)^{1-\alpha}}\mathrm{d}\mathbf{B}^H(s)\rangle\right)\\
&+ \sup_{t\in J}\mathbb{E}\mathrm{d}_\infty^2\left(x_0 + \frac{1}{\Gamma(\alpha)}\int_0^t \frac{f(s,x_{n-1}(s))}{(t-s)^{1-\alpha}}\mathrm{d}s + \langle\frac{1}{\Gamma(\alpha)}\int_0^t \frac{g(s)}{(t-s)^{1-\alpha}}\mathrm{d}\mathbf{B}^H(s)\rangle,\right)\\
&\cdot x_0 + \frac{1}{\Gamma(\alpha)}\int_0^t \frac{f(s,x(s))}{(t-s)^{1-\alpha}}\mathrm{d}s + \langle\frac{1}{\Gamma(\alpha)}\int_0^t \frac{g(s)}{(t-s)^{1-\alpha}}\mathrm{d}\mathbf{B}^H(s)\rangle)\Bigg] := I_1 + I_2 + I_3,
\end{aligned} \quad (40)$$

where $\lim_{n\longrightarrow\infty} I_1 = 0$ and $I_2 = 0$. For I_3, by using Propositions 2 and 3, $(\mathscr{H}3)$, and (30), we have

$$\lim_{n\longrightarrow\infty} I_3 \le \lim_{n\longrightarrow\infty}\left(\frac{T^{\alpha+1}c}{\Gamma(\alpha+1)}\sup_{t\in J}\mathbb{E}\mathrm{d}_\infty^2(x(u),x_{n-1}(u))\mathrm{d}u\right) = 0. \quad (41)$$

Hence, we get (39), which implies (16) holds. Hence, from definition (8), $x(t)$ is a solution to equation (14).

For the uniqueness of a solution x, suppose that $x, z\colon J\times\Omega \longrightarrow \mathbf{E}^n$ are solutions to equation (14). We denote by $K(t) := \sup_{v\in J}\mathbb{E}\mathrm{d}_\infty^2(x(v),z(v))$. So, for each $t\in J$, we obtain

$$\begin{aligned}
K(t) &\le \frac{tc}{\Gamma(\alpha)}\int_0^t \frac{\mathbb{E}\mathrm{d}_\infty^2(x(s),z(s))}{(t-s)^{1-\alpha}}\mathrm{d}s\\
&\le \frac{Tc}{\Gamma(\alpha)}\int_0^t \frac{K(s)}{(t-s)^{1-\alpha}}\mathrm{d}s.
\end{aligned} \quad (42)$$

Thus, by Lemma 1, we have, for $t\in J$, $K(t)\equiv 0$, which implies

$$\sup_{t\in J}\mathrm{d}_\infty(x(t),z(t)) \overset{\mathbb{P}.1}{=} 0. \quad (43)$$

□

4. Stability Result

In this part, we examine the stability of the solution with respect to initial values by using Henry–Gronwall inequality. Indeed, let x and z denote the solutions of the following FFSDEs:

$$\begin{cases} {}^{C}\mathscr{D}^{\alpha}x(s) \overset{J\mathbb{P}.1}{=} f(s,x(s))\mathrm{d}s + \langle g(s)d\mathbf{B}^H(s)\rangle,\\ x(0) \overset{\mathbb{P}.1}{=} x_0, \end{cases} \quad (44)$$

$$\begin{cases} {}^{C}\mathscr{D}^{\alpha}z(s) \overset{J\mathbb{P}.1}{=} f(s,x(s))\mathrm{d}s + \langle g(s)d\mathbf{B}^H(s)\rangle,\\ z(0) \overset{\mathbb{P}.1}{=} x_0, \end{cases} \quad (45)$$

respectively.

Proposition 5. *Suppose that $x_0, z_0 \in \mathbf{L}^2(\Omega,\mathscr{A}_0,\mathbb{P};\mathbf{E}^n)$ and $f\colon J\times\Omega\times\mathbf{E}^n \longrightarrow \mathbf{E}^n$ $g\colon J\longrightarrow \mathbb{R}^n$ satisfy $(\mathscr{H}1)$–$(\mathscr{H}3)$. Then,*

$$\sup_{0\le u\le t}\mathbb{E}\mathrm{d}_\infty^2(x(u),z(u)) \le \lambda_0 M_{\lambda_1}, \quad (46)$$

where $\lambda_0 = 2\mathbb{E}\mathrm{d}_\infty^2(x_0,z_0)$ and $\lambda_1 = 2Tc/\Gamma(\alpha)$. Especially, $x(t)\overset{J\mathbb{P}.1}{=} z(t)$ if $x_0 \overset{\mathbb{P}.1}{=} z_0$.

Proof. Suppose that $x, z\colon J\times\Omega \longrightarrow \mathbf{E}^n$ are solutions to equations (44) and (45), respectively. So, let $K(t) := \mathbb{E}\sup_{0\le u\le t}\mathrm{d}_\infty^2(x(u),z(u))$. Due to Propositions 2 and 3 and $(\mathscr{H}3)$, we obtain

$$\begin{aligned} K(t) &\leq 2\mathbb{E}\mathrm{d}_{\infty}^{2}(x_0, z_0) + \frac{2}{\Gamma(\alpha)}\sup_{u\in[0,t]}\mathbb{E}\mathrm{d}_{\infty}^{2}\left(\int_0^u \frac{f(s,x(s))}{(t-s)^{1-\alpha}}\mathrm{d}s, \int_0^u \frac{f(s,z(s))}{(t-s)^{1-\alpha}}\mathrm{d}s\right) \\ &\leq 2\mathbb{E}\mathrm{d}_{\infty}^{2}(x_0, z_0) + \frac{2Tc}{\Gamma(\alpha)}\int_0^t \frac{\mathbb{E}\mathrm{d}_{\infty}^{2}(x(s), z(s))}{(t-s)^{1-\alpha}}\mathrm{d}s \\ &\leq 2\mathbb{E}\mathrm{d}_{\infty}^{2}(x_0, z_0) + \frac{2Tc}{\Gamma(\alpha)}\int_0^t \sup_{u\in(0,s)}\mathbb{E}\mathrm{d}_{\infty}^{2}(x(u), z(u))(t-s)^{1-\alpha}\mathrm{d}u \\ &= 2\mathbb{E}\mathrm{d}_{\infty}^{2}(x_0, z_0) + \frac{2Tc}{\Gamma(\alpha)}\int_0^t \frac{K(s)}{(t-s)^{1-\alpha}}\mathrm{d}s \\ &:= \lambda_0 + \lambda_1\int_0^t \frac{K(s)}{(t-s)^{1-\alpha}}\mathrm{d}s. \end{aligned} \tag{47}$$

Then, according to Lemma 1 and Remark 1, there exist a constant $M_{\lambda_1} > 0$ independent of λ_0 such that

$$K(t) \leq \lambda_0 M_{\lambda_1}, \quad \forall t \in J. \tag{48}$$

Then, $\lambda_0 = 0$ if $x_0 \overset{\mathbb{P}.1}{=} z_0$. Therefore, we know that $x(t) \overset{J\mathbb{P}.1}{=} z(t)$.

Finally, we examine the exponential stability of solutions to the FFSDEs which disturbed an fBm with respect to f and g. Thus, let x and x_n denote solutions to the following FFSDEs:

$$\begin{cases} {}^{C}\mathscr{D}^{\alpha}x(s) \overset{J\mathbb{P}.1}{=} f(s,x(s))\mathrm{d}s + \langle g(s)\mathrm{d}\mathbf{B}^{H}(s)\rangle, \\ x(0) \overset{\mathbb{P}.1}{=} x_0, \end{cases} \tag{49}$$

$$\begin{cases} {}^{C}\mathscr{D}^{\alpha}x_n(s) \overset{J\mathbb{P}.1}{=} f_n(s,x_n(s))\mathrm{d}s + \langle g_n(s)\mathrm{d}\mathbf{B}^{H}(s)\rangle, \\ x_n(0) \overset{\mathbb{P}.1}{=} x_0, \end{cases} \tag{50}$$

respectively. □

Proposition 6. *Suppose that $x_0 \in \mathbf{L}^2(\Omega, \mathscr{A}_0, \mathbb{P}; \mathbf{E}^n)$ and $f, f_n: J\times\Omega\times\mathbf{E}^n \longrightarrow \mathbf{E}^n$ $g, g_n: J \longrightarrow \mathbb{R}^n$ $(n \in \mathbb{N})$ fulfill $(\mathscr{H}1)$–$(\mathscr{H}3)$. Furthermore, assume that*

$$\lim_{n\longrightarrow\infty}\left(\frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}\mathbb{E}\mathrm{d}_{\infty}^{2}((t,x), f_n(t,x)(t,x))\mathrm{d}t\right) = 0, \tag{51}$$

$$\lim_{n\longrightarrow\infty}\left(\frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}\left\|g_n(s) - g(s)\right\|^2\mathrm{d}s\right) = 0. \tag{52}$$

Then, we have

$$\lim_{n\longrightarrow\infty}\left(\mathbb{E}\sup_{t\in J}\mathrm{d}_{\infty}^{2}(x(t), x_n(t))\right) = 0, \tag{53}$$

where $x, x_n: J\times\Omega \longrightarrow \mathbf{E}^n$ are solutions of equations (49) and (50), respectively.

Proof. According to Theorem 1, the solutions x and x_n are unique and exist. From Propositions 3 and 4, we deduce that, for every $t \in J$,

$$\begin{aligned} \sup_{0\leq u\leq t}\mathbb{E}\mathrm{d}_{\infty}^{2}(x(u), x_n(u)) \leq{}& 2\sup_{0\leq u\leq t}\mathbb{E}\mathrm{d}_{\infty}^{2}\left(\frac{1}{\Gamma(\alpha)}\int_0^u \frac{f(s,x(s))}{(u-s)^{1-\alpha}}\mathrm{d}s, \frac{1}{\Gamma(\alpha)}\int_0^u \frac{f_n(s,x_n(s))}{(u-s)^{1-\alpha}}\mathrm{d}s\right) \\ &+ 2\sup_{0\leq u\leq t}\mathbb{E}\mathrm{d}_{\infty}^{2}\left(\langle\frac{1}{\Gamma(\alpha)}\int_0^u \frac{g(s)}{(u-s)^{1-\alpha}}\mathrm{d}\mathbf{B}^{H}(s)\rangle, \langle\frac{1}{\Gamma(\alpha)}\int_0^u \frac{g_n(s)}{(u-s)^{1-\alpha}}\mathrm{d}\mathbf{B}^{H}(s)\rangle\right) \end{aligned}$$

$$\begin{aligned}
&\leq q4 \sup_{0\leq u\leq t} \mathbb{E}d_{\infty}^{2}\left(\frac{1}{\Gamma(\alpha)}\int_0^u \frac{f_n(s,x(s))}{(u-s)^{1-\alpha}}\mathrm{d}s, \frac{1}{\Gamma(\alpha)}\int_0^u \frac{f_n(s,x_n(s))}{(u-s)^{1-\alpha}}\mathrm{d}s\right)\\
&\quad + 4 \sup_{0\leq u\leq t} \mathbb{E}\mathrm{d}_{\infty}^{2}\left(\frac{1}{\Gamma(\alpha)}\int_0^u \frac{f(s,x(s))}{(u-s)^{1-\alpha}}\mathrm{d}s, \frac{1}{\Gamma(\alpha)}\int_0^u \frac{f_n(s,x(s))}{(u-s)^{1-\alpha}}\mathrm{d}s\right) + \frac{2c_{T,H}}{\Gamma(\alpha)}\int_0^t \frac{\left\|g(s)-g_n(s)\right\|^2}{(t-s)^{1-\alpha}}\mathrm{d}s\\
&\leq \frac{4ct}{\Gamma(\alpha)}\int_0^t \frac{\mathbb{E}d_{\infty}^{2}(x(s),x_n(s))}{(t-s)^{1-\alpha}}\mathrm{d}s + \frac{4t}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}\mathbb{E}\mathrm{d}_{\infty}^{2}(f(s,x(s)),f_n(s,x(s)))\mathrm{d}s\\
&\quad + \frac{2c_{T,H}}{\Gamma(\alpha)}\int_0^t \frac{\left\|g(s)-g_n(s)(s)\right\|^2}{(t-s)^{1-\alpha}}\mathrm{d}s\\
&\leq \beta_1^n + \beta_2\int_0^t \frac{\sup_{0\leq u\leq s}\mathbb{E}\mathrm{d}_{\infty}^{2}(x(u),x_n(s))}{(t-s)^{1-\alpha}}\mathrm{d}s,
\end{aligned} \tag{54}$$

where

$$\beta_1^n := \frac{4T}{\Gamma(\alpha)}\int_0^t \frac{\mathbb{E}\mathrm{d}_{\infty}^{2}(f(s,x(s)),f_n(s,x(s)))}{(t-s)^{1-\alpha}}\mathrm{d}s + \frac{2c_{T,H}}{\Gamma(\alpha)}\int_0^t \frac{\left\|g(s)-g_n(s)\right\|^2}{(t-s)^{1-\alpha}}\mathrm{d}s, \tag{55}$$

$\beta_2 = 4cT/\Gamma(\alpha)$. From Lemma 2 and Remark 1, $\exists M_{\beta_2} > 0$ is independent of β_1^n such that

$$\sup_{u\in[0,t]}\mathbb{E}d_{\infty}^{2}(x(u),x_n(u)) \leq \beta_1^n M_{\beta_2}. \tag{56}$$

Hence, from (51) and (52), we get $\lim_{n\longrightarrow\infty}\beta_1^n = 0$. □

5. Conclusions

In this study, we have proved the existence and uniqueness of solutions to FFSDEs under the Lipschitzian coefficient. On the contrary, the stability of solutions to the FFSDEs is analyzed.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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