Hindawi
Advances in Fuzzy Systems
Volume 2021, Article ID 6697060, 8 pages
https://doi.org/10.1155/2021/6697060

Research Article

Solving Constrained Flow-Shop Scheduling Problem through Multistage Fuzzy Binding Approach with Fuzzy Due Dates

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Received 28 December 2020; Revised 6 February 2021; Accepted 22 February 2021; Published 4 March 2021

Academic Editor: Antonin Dvorák

This paper deals with constrained multistage machines flow-shop (FS) scheduling model in which processing times, job weights, and break-down machine time are characterized by fuzzy numbers that are piecewise as well as quadratic in nature. Avoiding to convert the model into its crisp, the closed interval approximation for the piecewise quadratic fuzzy numbers is incorporated. The suggested method leads a noncrossing optimal sequence to the considered problem and minimizes the total elapsed time under fuzziness. The proposed approach helps the decision maker to search for applicable solution related to real-world problems and minimizes the total fuzzy elapsed time. A numerical example is provided for the illustration of the suggested methodology.

1. Introduction

Scheduling contains the sequence of jobs following the resource as well as time constraints, with a specific objective. The job scheduling and controlling through a production is a significant role in any industrial manufacturing unit. The FS scheduling model is the simple version where all jobs are operated on all the machines in order, is one of the recent issues in the field of production control, and is to determine the job sequence on the machines to minimize the makespan. The scheduling model usually consists of three components: time of transportation, job weight, and machine time for the break down.

Job scheduling problems, normally, occur such as programs for running on a sequence using some computer operators and to order the jobs for processing in a plant of manufacturing. Numerous researchers studied various FS scheduling problem and job scheduling problems and proposed algorithms in the crisp environment [1, 2]. A new heuristic algorithm was introduced by Aggarwal et al. [3] for obtaining an optimal (near-optimal) sequence to bicriteria three-stage FS scheduling based on heuristic technique, which was further discussed by Patider et al. [4]. Abdullah and Abdolrazzagh-Nezhad [5] developed an algorithm for solving theatrical models for fuzzy job-shop scheduling.

The FS scheduling model under the fuzzy processing time has been formulated by Ishibuchi et al. [6]. Afterwards, several researchers considered the machine sequence-dependent processing times. Ahonen and de Alvarenga [7] formulated and proposed a solution for the FS scheduling model, considering the recirculation and machine sequence-varying processing time. Qu et al. [8] proposed an algorithm to solve the no-wait FS scheduling problem based on the hormone modulation mechanism. Komaki et al. [9] introduced a consolidated survey of assembly FS models with their solution approach. Belabid et al. [10] proposed three methods for resolution of a permutation FS problem with independent setup time: mixed-integer LP model and two heuristics so as to minimize the maximum of job competition time.

In literature, authors, such as Zadeh [11] and Dubois and Prade [12], considered the FS problem with the consideration transportation cost. Hnaien et al. [13] presented the makespan minimization problem by describing the two-machine FS under a constraint related to availability of the first machine. A two-stage multiprocessor FS scheduling problem was considered under the deterioration of maintenance in a cleaner production [14]. Khatami and Zegordi [15] suggested the flexible maintenance time intervals.

Yang et al. [16] studied the FS scheduling of many production lines for precast production. Toumi et al. [17] presented the branch-and-bound technique for the solution of blocking FS scheduling problem under the assumption of makespan criterion. Yu et al. [18] presented the iterative method for batching and scheduling problem for the minimization of total job tardiness in two-stage hybrid FS. Shahvari and Logendran [19] presented a comparison of hybrid algorithm for a batch scheduling problem in hybrid FS under the assumption of learning effect. They used a clustering-genetic algorithm-based technique.

A particular kind of FS problem is called the permutation FS scheduling problem, where the job processing order is the same for each subsequent step of the processing [20]. Over the years in literature, several authors studied the permutation FS problem. Damodaran et al. [21] proposed the particle swarm optimization procedure for solving the permutation FS. They considered the scheduling batch processing machines in the model. Some multiobjective methods were also suggested by many researchers. Li and Ma [22] presented an artificial bee colony algorithm for multiobjective permutation FS problem with sequence varying with setup times. Chaouch et al. [23] presented a modified method of ant colony optimization algorithm to determine the optimal scheduling for the distributed job shop problem. Khalifa [24] analyzed the single-machine preparation issue in a fuzzy date setting.

Several researchers studied the fuzzy methods for solving the permutation FS problem, for instance, Tirkolaee et al. [25], Sioud and Gagne [26], and Kumar [27]. Tirkolaee et al. [25] studied a multitrip green capacitated arc routing problem with an application to urban services. They used the hybrid genetic algorithm. Sioud and Gagne [26] proposed a special type solution method based on the enhanced migrating birds to permutation FS problem with the assumption of sequence-dependent setup times. Goli et al. [28] proposed a FS scheduling problem with outsourcing option on subcontractors. They considered the just-in-time criteria in model formulation. Tirkolaee et al. [29] investigated the pollution-routing problem with cross-dock selection. They used the Pareto-based algorithm to deal with the multiobjective optimization problem. Afterwards, Khalifa and Kumar [30] proposed the fuzzy solution approach to fully neutrosophic linear programming problem. They also presented an application to stock portfolio selection. Very recently, Tirkolaee et al. [31] presented a FS scheduling problem with outsourcing option. They used fuzzy programming and artificial fish swarm algorithm. Goli et al. [32] investigated a fuzzy production scheduling model. They considered the automated guided vehicles as well as human factors.

In this paper, a novel method called multistage fuzzy binding for solving the problem under consideration in which jobs processing time, weights, and break-down machine are characterized as piecewise quadratic fuzzy numbers is proposed. Here, it is assumed that there is no power break up for dealing with break-down power as it has been assumed that the unit of production is still a small-scale one. The suggested method depends on the binding method applied by Pandian and Rajendran [33] which provides a noncrossing optimal sequence to the considered problem with the minimizing total fuzzy elapsed time.

The rest of the research work is organized as follows: the basic concept and arithmetic operations related to fuzzy numbers and their arithmetic operations are described in Section 2. Section 3 describes some of the assumptions and notations required in the proposed problem mathematical formulation. Section 4 formulates fuzzy constrained multistage FS scheduling problems. Section 5 proposes multistage fuzzy binding approach for obtaining a noncrossing optimal sequence. In Section 6, a numerical example to illustrate the methodology is introduced. Finally, some concluding remarks are reported in Section 7.

2. Preliminaries

This section introduces some of the basic concepts, and results related to fuzzy numbers, piecewise quadratic fuzzy numbers, and their arithmetic operations are recalled.

Definition 1 (see [34]). A piecewise quadratic fuzzy number (PQFN) is denoted by $\tilde{a}_{PQ} = (a_1, a_2, a_3, a_4, a_5)$, where $a_1 \le a_2 \le a_3 \le a_4 \le a_5$ are real numbers, and is defined by if its membership function $\mu_{\tilde{a}_{PQ}}$ as follows (as in Figure 1):

$$\mu_{\tilde{a}_{PQ}} = \begin{cases} 0, & x < a_1; \\ \dfrac{1}{2}\dfrac{1}{(a_2 - a_1)^2}(x - a_1)^2, & a_1 \le x \le a_2; \\ \dfrac{1}{2}\dfrac{1}{(a_3 - a_2)^2}(x - a_3)^2 + 1, & a_2 \le x \le a_3; \\ \dfrac{1}{2}\dfrac{1}{(a_4 - a_3)^2}(x - a_3)^2 + 1, & a_3 \le x \le a_4; \\ \dfrac{1}{2}\dfrac{1}{(a_5 - a_4)^2}(x - a_5)^2, & a_4 \le x \le a_5; \\ 0, & x > a_5. \end{cases} \tag{1}$$

The interval of confidence at level α for the PQFN is defined as follows:

$$\left(\tilde{a}_{PQ}\right)_\alpha = [a_1 + 2(a_2 - a_1)\alpha, a_5 - 2(a_5 - a_4)\alpha]; \quad \forall \alpha \in [0, 1]. \tag{2}$$

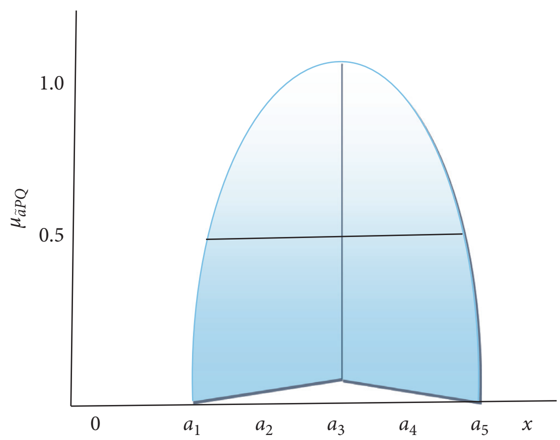

Figure 1: Graphical representation of a piecewise quadratic fuzzy number (PQFN).

Definition 2 (see [34]). An interval approximation $[A] = [(a_\alpha)^L, (a_\alpha)^U]$ of a PQFN $\widetilde{A}$ is called closed interval approximation if

$$(a_\alpha)^L = \inf\{x \in \mathbb{R}: \mu_{\widetilde{A}} \geq 0.5\},$$
$$(a_\alpha)^U = \sup\{x \in \mathbb{R}: \mu_{\widetilde{A}} \geq 0.5\}. \quad (3)$$

Definition 3 (see [35, 36]). An interval on $\mathbb{R}$ is defined as

$$A = [a^L, a^R] = \{a :, a^L \leq a \leq a^R, a \in \mathbb{R}\}, \quad (4)$$

where a^L is the left limit and a^R is the right limit of A.

Definition 4 (see [37]). The interval is also defined as

$$A = a_C, a_W = \{\mathbf{a}: a_C - a_W \leq \mathbf{a} \leq a_C + a_W, \mathbf{a} \in \mathbb{R}\}, \quad (5)$$

where $a_C = (1/2)(a^R + a^L)$ is the center and $a_W = (1/2)(a^R - a^L)$ is the width of A.

Definition 5 The associated ordinary numbers of PQFN corresponding to the closed interval approximation $[A] = [(a_\alpha)^L, (a_\alpha)^U]$ are $\widehat{A} = ((a_\alpha)^L + (a_\alpha)^U)/2$.

Definition 6 The associated ordinary (crisp) number corresponding to the PQFN $\tilde{a}_{PQ} = (a_1, a_2, a_3, a_4, a_5)$ is defined as

$$\hat{a}_{PQ} = \frac{a_1 + a_2 + 4a_3 + a_4 + a_5}{8}. \quad (6)$$

Definition 7 (see [34]). Let $[A] = [(a_\alpha)^L, (a_\alpha)^U]$ and $[B] = [(b_\alpha)^L, (b_\alpha)^U]$ be two interval approximations of PQFN. Then, the arithmetic operations are defined as follows:

(1) Addition: $[A]\oplus[B] = [(a_\alpha)^L + (b_\alpha)^L, (a_\alpha)^U + (b_\alpha)^U]$.

(2) Subtraction: $[A]\ominus[B] = [(a_\alpha)^L - (b_\alpha)^U, (a_\alpha)^U - (b_\alpha)^L]$.

(3) Scalar multiplication: $\alpha[A] = \begin{cases} [\alpha(a_\alpha)^L, \alpha(a_\alpha)^U], & \alpha > 0, \\ [\alpha(a_\alpha)^U, \alpha(a_\alpha)^L], & \alpha < 0. \end{cases}$

(4) Multiplication: $[A]\otimes[B]$:

$$\begin{bmatrix} \min\left((a_\alpha)^L.(b_\alpha)^L, (a_\alpha)^L.(b_\alpha)^U, (a_\alpha)^U.(b_\alpha)^L, (a_\alpha)^U.(b_\alpha)^U\right), \\ \max\left((a_\alpha)^L.(b_\alpha)^L, (a_\alpha)^L.(b_\alpha)^U, (a_\alpha)^U.(b_\alpha)^L, (a_\alpha)^U.(b_\alpha)^U\right). \end{bmatrix}. \quad (7)$$

(5) Division: $[A]\oslash[B]$:

$$\begin{bmatrix} \min\left(\frac{(a_\alpha)^L}{(b_\alpha)^L}, \frac{(a_\alpha)^L}{(b_\alpha)^U}, \frac{(a_\alpha)^U}{(b_\alpha)^L}, \frac{(a_\alpha)^U}{(b_\alpha)^U}\right), \\ \max\left(\frac{(a_\alpha)^L}{(b_\alpha)^L}, \frac{(a_\alpha)^L}{(b_\alpha)^U}, \frac{(a_\alpha)^U}{(b_\alpha)^L}, \frac{(a_\alpha)^U}{(b_\alpha)^U}\right), \end{bmatrix} \quad [B] > 0. \quad (8)$$

(6) Maximum: $[A]\vee[B] = [(a_\alpha)^L \vee (b_\alpha)^L, (a_\alpha)^U \vee (b_\alpha)^U]$.

(7) Minimum: $[A]\wedge[B] = [(a_\alpha)^L \wedge (b_\alpha)^L, (a_\alpha)^U \wedge (b_\alpha)^U]$.

3. Notation and Assumptions

3.1. Notation. The following notations can be used in the proposed FS scheduling problem.

S_k: sequence resulted by applying Johnson's procedure, $k (k = 1, 2, \ldots, m)$.

i: job $(i = 1, 2, \ldots, n)$.

M_j: machine j $(j = 1, 2, \ldots, m)$.

$\widetilde{M}_{ij}$: quadratic piecewise fuzzy processing time of the ith job on machine $M_j (i = 1, 2, \ldots, n; j = 1, 2, \ldots, m)$.

$\widetilde{P}_i$: processes that require uninterrupted power supply and no break-down time are permitted.

$\widetilde{Q}_i$: processes that require power supply and break-down time are permitted.

$\widetilde{M}_i$: processes that do not require power supply and may be continued during break-down time.

$\widetilde{F}$: fuzzy performance measure $(i = 1, 2, \ldots, n)$, $\widetilde{F} = ((\sum_{i=1}^{n} (\tilde{w}_i \otimes \tilde{f}_i))/(\sum_i^m \tilde{w}_i))$.

$\tilde{f}_i$: flow time of the job i $(i = 1, 2, \ldots, n)$.

$\tilde{w}_i$: fuzzy weights $(i = 1, 2, \ldots, n)$.

3.2. Assumptions. In this FS scheduling problem, the following assumptions are made:

(i) No passing is permitted.

(ii) All the jobs are available for processing at time zero.

(iii) All jobs are available at the beginning of scheduling time horizon.

(iv) The machines setup times are negligible.

(v) All jobs have deterministic processing times.

(vi) Due dates are PQF numbers.

(vii) Machine may be idle.

(viii) Processing times are independent of the schedule.

(ix) To feed a job on a second machine, it must be completed on the first machine.

(x) Each job has m operations.

(xi) Each job must be completed once it is started.

4. Problem Statement

The aim of the problem is to minimize the total piecewise quadratic fuzzy elapsed time that is to find the optimal sequence of the jobs. Assume that job $i\,(i = 1, 2, \ldots, n)$ is to be processed on machine $j\,(j = 1, 2, \ldots, m)$ in the existence of specified rental policy. Let $\widetilde{M}_{ij}\,(i = 1, 2, \ldots, n;\ j = 1, 2, \ldots, m)$ be the processing time of job i on machine j characterized by PCF numbers, which may be classified into three categories:

(1) The processes require uninterrupted power supply, and no break-down is permitted (say, $\widetilde{P}_1, \widetilde{P}_2, \ldots$).

(2) The processes require power supply, and break down is permitted (say, $\widetilde{Q}_1, \widetilde{Q}_2, \ldots$).

(3) The processes do not require power supply and can be continued during the break-down time. Let them be $\widetilde{M}_1, \widetilde{M}_2, \ldots$.

In addition, let job $i\,(i = \overline{1, n})$ be assigned having fuzzy weights $\widetilde{w}_i$ relative to the importance of performance in the sequence. The measure of the fuzzy performance is defined as

$$\widetilde{G} = \frac{\sum_{i=1}^{n} \widetilde{w}_i \otimes \widetilde{f}_i}{\sum_{i=1}^{n} \widetilde{w}_i}, \quad (9)$$

where $\widetilde{f}_i$ is the flow time of the i^{th} job. Let the fuzzy break-down approximate interval be $[\widetilde{a}, \widetilde{b}]$. Our aim is to determine the optimal sequence of jobs to minimize the total fuzzy elapsed time. The problem can be illustrated as in Table 1.

Assume that the considered problem satisfies one or both the following conditions:

$$\begin{aligned} &\min_i \widetilde{M}_{1i} \pm \max_i \widetilde{M}_{ij}, \quad j = 2, 3, \ldots, m-1, \\ &\text{or/and } \min_i \widetilde{M}_{mj} \pm \max_i \widetilde{M}_{ij}, \quad j = 2, 3, \ldots, m-1. \end{aligned} \quad (10)$$

5. Proposed Approach

The steps of the approach are as follows:

Step 1: consider the piecewise quadratic fuzzy constrained multistage machines FS scheduling (PQFCMFSS) problem.

Step 2: convert the PQFCMFSS problem into the corresponding approximated closed-interval CMFSS problem.

Step 3: convert the CMFSS problem into a two-machine FS scheduling problem by introducing two fictitious machines H_1 and H_2 with

Table 1: Piecewise quadratic fuzzy processing times $\widetilde{M}_{ij} = (a_1, a_2, a_3, a_4, a_5)$.

Job	Machines with PQF processing times					PCF weights of job
	M_1	M_2	M_3	...	M_m	$\widetilde{w}_i$
i	$\widetilde{M}_{i1}$	$\widetilde{M}_{i2}$	$\widetilde{M}_{i3}$	...	$\widetilde{M}_{im}$	—
1	$\widetilde{M}_{11}$	$\widetilde{M}_{12}$	$\widetilde{M}_{13}$	...	$\widetilde{M}_{1m}$	$\widetilde{w}_1$
...	...	...	...	...	...	
N	$\widetilde{M}_{n1}$	$\widetilde{M}_{n2}$	$\widetilde{M}_{n3}$	...	$\widetilde{M}_{nm}$	$\widetilde{w}_n$

$$\begin{aligned} \widetilde{H}_1^i &= \sum_{j=1}^{m-1} \widetilde{M}_{ij}, \quad i = 1, 2, \ldots, n, \\ \widetilde{H}_2^i &= \sum_{j=2}^{m} \widetilde{M}_{ij}, \quad i = 1, 2, \ldots, n. \end{aligned} \quad (11)$$

Here, $\widetilde{H}_1^i$ and $\widetilde{H}_2^i$ are the closed-interval processing time for job i on machines H_1 and H_2, respectively.

Step 4: applying the method introduced by Pandian and Rajendran [33] to obtain the optimal sequence.

Step 5: identify the effect of break-down interval $[[a^L, a^U], [b^L, b^U]]$ or $[\widetilde{a}, \widetilde{b}]$ on different jobs. If the affected jobs come under $\widetilde{M}_1, \widetilde{M}_2, \ldots$, there is no need to be modified and can be neglected.

Step 6: identify the modified processing time on different jobs under categories $\widetilde{P}_1, \widetilde{P}_1, \ldots$, and $\widetilde{Q}_1, \widetilde{Q}_2, \ldots$.

Step 7: modify the fuzzy processing time after categorizing the jobs as follows:

Let $[t^L, t^U]$ be the existing interval processing time and $[u^L, t^U]$ be a new interval processing time.
Let $[a^L, a^U]$ be interval processing time span begin and $[b^L, b^U]$ break-down time span interval end.
Let $[s_1^L, s_1^U]$ be interval existing processing time span begin and $[s_2^L, s_2^U]$be existing interval processing time span end.

(i) Category 1: if the process is a continuous one not to be interrupted in any case as welding and forging, then add $([b^L, b^U] - [s_1^L, s_1^U])$ to the interval processing time $[t^L, t^U]$ to get $[u^L, t^U]$.

(ii) Category 2: if the process need not be a continuous one and is not affected by any interrupts such as packing, drilling, and threading, and then the existing interval processing time $[t^L, t^U]$ is converted to the new interval processing time $[u^L, t^U]$. There are two cases:

Case 1: if the break-down starts or/and stars and ends in between, $[b^L - a^U, b^U - a^L]$ is added to the interval processing time.
Case 2: if the break-down ends in between or/and stars before and ends after the interval processing time span, $[b^L - s_1^U, b^U - s_1^L]$ is added to the interval processing time.

Step 8: determine the minimum total elapsed time and the weighted men-flow for the FS scheduling problem.

Table 2: Piecewise quadratic fuzzy processing times $\widetilde{M}_{ij} = (a_1, a_2, a_3, a_4, a_5)$.

Job	Machines with PQF processing times					PCF weights of jobs
i	$\widetilde{P}_1$	$\widetilde{M}_1$	$\widetilde{Q}_1$	$\widetilde{Q}_2$	$\widetilde{P}_2$	$\widetilde{w}_i$
1	(12, 13, 14, 15, 16)	(0, 1, 2, 3, 4)	(5, 6, 7, 8, 9)	(0, 1, 2, 3, 4)	(3, 4, 5, 6, 7)	(1, 2, 3, 4, 5)
2	(13, 14, 15, 16, 17)	(0, 1, 2, 3, 4)	(3, 4, 6, 7, 8)	(3, 4, 5, 6, 7)	(7, 8, 9, 10, 11)	(0, 1, 2, 3, 4)
3	(11, 12, 13, 14, 15)	(0, 1, 2, 3, 4)	(1, 2, 3, 4, 5)	(2, 3, 4, 5, 6)	(3, 4, 5, 6, 7)	(1, 2, 3, 4, 5)
4	(12, 13, 14, 15, 16)	(3, 4, 5, 6, 7)	(0, 1, 2, 3, 4)	(0, 1 2, 3, 4)	(4, 5, 6, 7, 8)	(2, 3, 4, 5, 6)
5	(13, 14, 15, 16, 17)	(1, 2, 3, 4, 5)	(3, 4, 5, 6, 7)	(1, 2, 3, 4, 5)	(5, 6, 7, 8, 9)	(0, 1, 2, 3, 4)

6. Numerical Example

In this section, we solve a numerical example to illustrate the suggested approach.

Step 1: consider the following PQFCMFSS problem as in Table 2.

Consider the break-down interval is $[\widetilde{a}, \widetilde{b}] = [(29, 30, 31, 32, 33)$ to $(33, 34, 35, 36, 37)]$.

Step 2: use the approximated closed intervals corresponding to the piecewise quadratic fuzzy numbers as in Table 3.

$\text{Min}P_1 = [12, 14] \geq \text{Max}\{[M_1], [Q_1], [Q_2]\} = \{[4, 6], [6, 8], [4, 6]\} = [6, 8]$ is satisfied. Therefore, convert the problem into two machines problem.

Step 3: convert the problem into two machines problem as in Table 4.

Step 4: using the binding method introduced by Pandian and Rajendran [33]; the modified processing times are as in Table 5.

By applying Johnson's algorithm, the PQF constrained multistage machines FS scheduling problem is given by the following sequence:

$$2 \longrightarrow 5 \longrightarrow 1 \longrightarrow 3 \longrightarrow 4. \quad (12)$$

Hence, the PQF elapsed time is (105, 106, 107, 108, 109).

Step 5: the break-down interval [(30, 31, 32, 33, 34), (34, 35, 36, 37, 38)] at affected jobs is listed in Table 6.

Step 6: we observe that $\widetilde{M}_1$ for job 2: (34, 34, 34, 34, 34) to (35, 36, 37, 38, 39) is neglected.

Step 7: modify the processing time all in Table 7, except the one in Step 6, for job 2 and job 5, respectively. $\widetilde{Q}_2$: (30, 30, 30, 30, 30) to (33, 34, 35, 36, 37) and $\widetilde{P}_1$: (17, 17, 17, 17, 17) to (30, 31, 32, 33, 34); the break down is started in between, and 3 is added to the PCF processing time. The new PQF processing times become

$$\widetilde{Q}_2 = (6, 7, 8, 9, 10), \quad \widetilde{P}_1 = (16, 17, 18, 19, 20). \quad (13)$$

Also, for job 2, and job 1, respectively, $\widetilde{P}_2$: (37, 37, 37, 37, 37) to (44, 45, 46, 47, 48) and P_1: (34, 34, 34, 34, 34) to (46, 47, 48, 49, 50), the break down is end in between, and the processing time is started by adding 1 to the PQF processing time. The new PQF processing time becomes

$$\widetilde{P}_2: (8, 9, 10, 11, 12), \quad \widetilde{P}_1: (13, 14, 15, 16, 17). \quad (14)$$

Based on Definition 6, Table 8 changes to Table 9 as follows.

It is obvious that the optimal sequence in fuzzy environment is

$$2 \longrightarrow 5 \longrightarrow 1 \longrightarrow 3 \longrightarrow 4. \quad (15)$$

Accordingly, Table 9 changes to Table 10 as follows.

Table 3: Approximate closed interval processing times.

Job	Machines with PQF processing times					PCF weights of jobs
i	$[P_1]$	$[M_1]$	$[Q_1]$	$[P_2]$	$[Q_2]$	$[w_i]$
1	[13, 15]	[1, 3]	[6, 8]	[1, 3]	[4, 6]	[2, 4]
2	[14, 16]	[1, 3]	[4, 7]	[4, 6]	[8,10]	[1, 3]
3	[12, 14]	[1, 3]	[2, 4]	[3, 5]	[4, 6]	[2, 4]
4	[13, 15]	[4, 6]	[1, 3]	[1, 3]	[4, 7]	[3, 5]
5	[14, 16]	[2, 4]	[4, 6]	[2, 4]	[6, 8]	[1, 3]

Table 4: Two machines FS problem.

Job	Machine with processing time		Weights of jobs
i	$[H_1^i]$	$[H_2^i]$	$[w_i]$
1	[25, 35]	[12, 20]	[2, 4]
2	[23, 32]	[17, 26]	[1, 3]
3	[18, 26]	[10, 18]	[2, 4]
4	[19, 27]	[10, 19]	[3, 5]
5	[22, 30]	[14, 22]	[1, 3]

Table 5: Modified machine processing times.

Job	Machine with processing time	
i	$[H_1^{i'}]$	$[H_2^{i'}]$
1	[25, 35]	[14, 24]
2	[23, 32]	[18, 29]
3	[18, 26]	[12, 22]
4	[19, 27]	[13, 24]
5	[22, 30]	[15, 25]

Table 6: Break-down effect on jobs.

Jobs i	2	**5**	1
Beak down	$\overline{Q}_2$ (30, 30, 30, 30, 30) to (33, 34, 35, 36, 37) $\overline{P}_2$ (37, 37, 37, 37, 37) to (44, 45, 46, 47, 48)	$\overline{P}_1$ (17, 17, 17, 17, 17) to (30, 31, 32, 33, 34) $\widetilde{M}_1$ (34, 34, 34, 34, 34) to (35, 36, 37, 38, 39)	$\overline{P}_1$ (34, 34, 34, 34, 34) to (46, 47, 48, 49, 50) —

Table 7: Piecewise quadratic fuzzy elapsed time.

Job	Machines with PQF processing times					PCF weights of jobs
i	$\widetilde{P}_1$	$\widetilde{M}_1$	$\widetilde{Q}_1$	$\widetilde{Q}_2$	$\widetilde{P}_2$	$\widetilde{w}_i$
2	(0, 0, 0, 0, 0) to (13, 14, 15, 16, 17)	(17, 17, 17, 17, 17) to (18, 19, 20, 21, 22)	(22, 22, 22, 22, 22) to (25, 26, 28, 29, 30)	(30, 30, 30, 30, 30) to (33, 34, 35, 36, 37)	(37, 37, 37, 37, 37) to (44, 45, 46, 47, 48)	(1, 2, 3, 4, 5)
5	(17, 17, 17, 17, 17) to (30, 31, 32, 33, 34)	(34, 34, 34, 34, 34) to (35, 36, 37, 38, 39)	(39, 39, 39, 39, 39) to (42, 43, 44, 45, 46)	(46, 46, 46, 46, 46) to (47, 47, 49, 50, 51)	(47, 47, 47, 47, 47) to (52, 53, 54, 55, 56)	(0, 1, 2, 3, 4)
1	(34, 34, 34, 34, 34) to (46, 47, 48, 49, 50)	(50, 50, 50, 50, 50) to (51, 52, 53, 54, 55)	(55, 55, 55, 55, 55) to (60, 61, 62, 63, 64)	(64, 64, 64, 64, 64) to (65, 66, 67, 68, 69)	(69, 69, 69, 69, 69) to (72, 73, 74, 75, 76)	(1, 2, 3, 4, 5)
3	(50, 50, 50, 50, 50) to (61, 62, 63, 64, 65)	(65, 65, 65, 65, 65) to (66, 67, 68, 69, 70)	(70, 70, 70, 70, 70) to (71, 72, 73, 74, 75)	(75, 75, 75, 75, 75) to (77, 78, 79, 80, 81)	(81, 81, 81, 81, 81) to (84, 85, 86, 87, 88)	(2, 3, 4, 5, 6)
4	(65, 65, 65, 65, 65) to (77, 78, 79, 80, 81)	(81, 81, 81, 81, 81) to (84, 85, 86, 87, 88)	(88, 88, 88, 88, 88) to (89, 90, 91, 92, 93)	(94, 94, 94, 94, 94) to (95, 96, 97, 98, 99)	(100, 100, 100, 100, 100) to (105, 106, 107, 108, 109)	(0, 1, 2, 3, 4)

Table 8: Modified crisp break-down time of FS scheduling problem.

Job	Machines with PQF processing times					PCF weights of jobs
i	$\widetilde{P}_1$	$\widetilde{M}_1$	$\widetilde{Q}_1$	$\widetilde{Q}_2$	$\widetilde{P}_2$	$\widetilde{w}_i$
1	(13, 14, 15, 16, 17)	(0, 1, 2, 3, 4)	(5, 6, 7, 8, 9)	(0, 1, 2, 3, 4)	(3, 4, 5, 6, 7)	(1, 2, 3, 4, 5)
2	(13, 14, 15, 16, 17)	(0, 1, 2, 3, 4)	(3, 4, 6, 7, 8)	(6, 7, 8, 9, 10)	(8, 9, 10, 11, 12)	(0, 1, 2, 3, 4)
3	(11, 12, 13, 14, 15)	(0, 1, 2, 3, 4)	(1, 2, 3, 4, 5)	(2, 3, 4, 5, 6)	(3, 4, 5, 6, 7)	(1, 2, 3, 4, 5)
4	(12, 13, 14, 15, 16)	(3, 4, 5, 6, 7)	(0, 1, 2, 3, 4)	(0, 1 2, 3, 4)	(4, 5, 6, 7, 8)	(2, 3, 4, 5, 6)
5	(16, 17, 18, 19, 20)	(1, 2, 3, 4, 5)	(3, 4, 5, 6, 7)	(1, 2, 3, 4, 5)	(5, 6, 7, 8, 9)	(0, 1, 2, 3, 4)

Table 9: Modified PQF break-down time of FS scheduling problem.

Job	Machines with PQF processing times					PCF weights of jobs
i	$\widetilde{P}_1$	$\widetilde{M}_1$	$\widetilde{Q}_1$	$\widetilde{Q}_2$	$\widetilde{P}_2$	$\widetilde{w}_i$
1	15	2	7	2	5	(1, 2, 3, 4, 5)
2	15	2	5.75	8	10	(0, 1, 2, 3, 4)
3	13	2	3	4	5	(1, 2, 3, 4, 5)
4	14	5	2	2	6	(2, 3, 4, 5, 6)
5	18	3	5	3	7	(0, 1, 2, 3, 4)

Table 10: PQF elapsed time of the scheduling problem.

Job	Machines with PQF processing times					PCF weights of jobs
i	$\widetilde{P}_1$	$\widetilde{M}_1$	$\widetilde{Q}_1$	$\widetilde{Q}_2$	$\widetilde{P}_2$	$\widetilde{w}_i$
2	(0, 0, 0, 0, 0) to (13, 14, 15, 16, 17)	(17, 17, 17, 17, 17) to (18, 19, 20, 21, 22)	(22, 22, 22, 22, 22) to (25, 26, 28, 29, 30)	(30, 30, 30, 30, 30) to (36, 37, 38, 39, 40)	(40, 40, 40, 40, 40) to (48, 49, 50, 51, 52)	(1, 2, 3, 4, 5)
5	(17, 17, 17, 17, 17) to (33, 34, 35, 36, 37)	(37, 37, 37, 37, 37) to (39, 41, 43, 45, 47)	(39, 39, 39, 39, 39) to (42, 43, 44, 45, 46)	(46, 46, 46, 46, 46) to (47, 47, 49, 50, 51)	(47, 47, 47, 47, 47) to (52, 53, 54, 55, 56)	(0, 1, 2, 3, 4)
1	(40, 40, 40, 40, 40) to (53, 54, 55, 56, 57)	(50, 50, 50, 50, 50) to (51, 52, 53, 54, 55)	(55, 55, 55, 55, 55) to (60, 61, 62, 63, 64)	(64, 64, 64, 64, 64) to (65, 66, 67, 68, 69)	(69, 69, 69, 69, 69) to (72, 73, 74, 75, 76)	(1, 2, 3, 4, 5)
3	(50, 50, 50, 50, 50) to (61, 62, 63, 64, 65)	(65, 65, 65, 65, 65) to (66, 67, 68, 69, 70)	(70, 70, 70, 70, 70) to (71, 72, 73, 74, 75)	(75, 75, 75, 75, 75) to (77, 78, 79, 80, 81)	(81, 81, 81, 81, 81) to (84, 85, 86, 87, 88)	(2, 3, 4, 5, 6)
4	(65, 65, 65, 65, 65) to (77, 78, 79, 80, 81)	(81, 81, 81, 81, 81) to (84, 85, 86, 87, 88)	(88, 88, 88, 88, 88) to (89, 90, 91, 92, 93)	(94, 94, 94, 94, 94) to (95, 96, 97, 98, 99)	(100, 100, 100, 100, 100) to (105, 106, 107, 108, 109)	(0, 1, 2, 3, 4)

Step 8. The total PQF elapsed time is (105, 106, 107, 108, 109), and hence, we have the following results:

For job 2: $f_2 = (48, 49, 50, 51, 52)$.
For job 5: $f_5 = (42, 43, 44, 45, 46)$.
For job 1: $f_1 = (35, 36, 37, 38, 39)$.
For job 4: $f_4 = (30, 31, 32, 33, 34)$.

Therefore, the closed interval weighted mean flow is $([315, 770]/[8, 19]) = [(315/19), (770/8)]$ hours.

The total PQF elapsed time and the weight flow by the proposed method is less than comparing to the ones obtained by Thangaraj and Rajendran [38]. All the calculations are entertained by MATLAB 2020a under Windows 10. The CPU frequency of the computer is 2.3 GHz, and the memory size is 8 GB.

7. Conclusions

In this research article, a new approach, namely, multistage fuzzy binding method has applied for solving the PQF constrained multistage FS scheduling problems, where the processing times and the jobs weight are characterized by PQF numbers. The advantage of the approach is that there is no risk for the decision maker, it is more applicable for real-world problems, it is easy and simple for understanding, and it is an important tool to the managers who are dealing with the flow-job problems so as to provide a noncrossing optimal sequence. The main findings are particularly useful for a fuzzy FS scheduling problem, while the processing times and the jobs weight are fuzzy parameters. Some practical implications and managerial insights can be drawn from this proposed study, under fuzzy due dates. In industry and business sector, the decision maker can apply to schedule the flow-shop of the machines in the workshop under fuzzy due dates. This would optimize the usages of the machines and hence the revenue of the company. For future research, the proposed problem may be extended by considering the stochastic random variable, for the processing times as well as the jobs weight.

Data Availability

The data used to support the findings of this research are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

Acknowledgments

The authors would like to thank the Dean-SASL of VIT Bhopal University as well as Qassim University.

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