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Research Article

Multiattribute Decision-Making Method Based on Hesitant Triangular Fuzzy Power Average Operator

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In the decision-making process, it often happens that decision makers hesitate between several possible preference values, so the multiattribute decision-making (MADM) problem of hesitant triangle fuzzy elements (HTFEs) has been widely studied. In related research works, different operators are used to fuse information, and the weighting model is used to represent the degree of difference between information fusion on various indicators, but the mutual influence between information is often not considered. In this sense, the purpose of this paper is to study the MADM problem of the hesitant triangular fuzzy power average (HTFPA) operator. First, the hesitant triangular fuzzy power-weighted average operator (HTFPWA) and the hesitant triangular fuzzy power-weighted geometric (HTFPWG) operator are given, their properties are analyzed and special cases are discussed. Then, a MADM method based on the HTFPWA operator and the HTFPWG operator is developed, and an example of selecting futures products is used to illustrate the results of applying the proposed method to practical problems. Finally, the effectiveness and feasibility of the HTFPA operator are verified by comparative analysis with existing methods.

1. Introduction

At the age of Internet, the decision-making information not only presents the huge amount of data but also presents complex relationship. Recent years, the development of intuitionistic fuzzy theory has solved the fuzziness and uncertainty between attributions in the MADM problem. Since the proposition of Atanassov-defined intuitionistic fuzzy set (IFS) theory [1, 2], many scholars have studied it in a deep going way. Torra et al. [3, 4] described the membership degree of IFS with a set of precise numbers that can represent the hesitation degree of decision makers, and then extended IFS to hesitant fuzzy set (HFS), and studied the relationship between HFS and IFS. Akram et al. [5] designed hesitant polar fuzzy sets, which is a hybrid model composed of HFSs and m polar fuzzy sets. Chen et al. [6] combined HFS with interval value and put forward a MADM method of interval hesitant fuzzy preference relationship. Chen et al. [7] did research for the formula of the correlation coefficient between HFSs, and applied the formula into cluster analysis. Tong and Yu [8] put forward algorithm and information aggregation operators relevant to HFS. Akram et al. [9, 10] constructed a hesitant fuzzy N-soft ELECTRE II method and an Elimination and Choice Translating REality-II technique to deal with the different opinions of decision makers on MADM problems in hesitant fuzzy environments.

The research on HFS information aggregation operator is an important part of the HFS theory. Xu and Xia [11] studied a series of HFS information aggregation operators and the relationship between them under hesitant fuzzy environment. Xia et al. [12] combined HFS and IFS to study the quasi hesitant fuzzy weighted aggregation operator, the hesitant fuzzy modular weighted averaging operator, etc. Wei et al. [13] combined HFS with interval values to study the information aggregation operators related to hesitation intervals, such as hesitant interval-valued fuzzy weighted averaging operator and hesitant interval-valued fuzzy ordered weighted averaging operator, and proved their idempotence, monotonicity, boundedness, and invariance. Akram et al. [14, 15] pointed out that the application of HFS and related aggregation operators in MADM can be better described by maximum deviation method and extended TOPSIS method. Zhao et al. [16] studied HFS and triangular fuzzy number together, proposed hesitate triangular fuzzy sets (HTFS), and then combined HTFS with Einstein information aggregation operator to study hesitant triangular fuzzy Einstein weighted averaging (HTFEWA) operator, hesitant triangular fuzzy Einstein weighted geometric (HTFEWG) operator, and related properties.

At present, there are many research works on the application of PA operators in hesitant fuzzy environments and MADM problems. Wei et al. [17] extended the PA aggregation operator to the Pythagorean fuzzy environment and proposed the Pythagorean fuzzy PA aggregation operator. Zhang [18] defined three types of hesitant fuzzy PA aggregators and studied the relationship between them. Lin et al. [19] studied the hesitant fuzzy language PA aggregator and applied it to MADM problems. The PA operator proposed by Yager [20] allowed attributes to support and strengthen each other in the form of weight vectors during fusion, thereby eliminating the influence of subjective weights on the fusion results. Liang et al. [21] studied several uncertain information fusion operators based on interval fuzzy preference information and the PA operator. Xu [22] extended the PA operator to the intuitionistic fuzzy environment, combined with IFS to study the intuitionistic fuzzy power average operator, the intuitionistic fuzzy power-weighted average operator, and the intuitionistic fuzzy power geometric operator, etc. Zhou et al. [23] further extended the PA operator and studied the generalized power ordered weighted average operator, the uncertain generalized power average operator, the uncertain generalized power ordered weighted average operator and their properties, so that the theoretical range of the PA operator was extended, and the effect was well applied in the MADM.

The information fusion operators in the above research work mainly consider the situation that the attributes are independent of each other. In MADM problems, decision makers often have subjective preferences, and attribute values have a certain degree of correlation (preference, complementarity, redundancy, etc.) [24, 25]. For example, the quality and price of alternative projects are included in the investment evaluation, generally the project with better quality tends to have higher price. Therefore, the information fusion operator considering the correlation between attributes is obviously more able to meet the needs of practical decision-making. At present, the HFS theory is widely used in MADM problems because of its ability to formally express uncertain data [26], and decision-making information is given by HTFS more often for better describing the hesitation degree of decision makers. However, the HTFS fusion operator whose attributes mutually support has rarely been studied, and few studies have paid attention to the MADM situation where the decision information is HTFS.

Considering that the PA operator can reflect the relationship between attributes, this paper introduces the PA operator into the hesitant triangle fuzzy environment to make MADM. Firstly, according to the hesitant fuzzy environment, the HTFPA operator, the HTFPWA operator, and the HTFPWG are proposed, and the related properties of these operators are discussed. Then, the specific steps of applying the HTFPWA operator and the HTFPWG to the MADM problem are explained. Finally, the effectiveness of the proposed operators is proved by numerical example and methods comparison. This paper's method can be applied to real-life MADM situations such as risk investment decision-making, risk management, and financial risk decision-making.

2. Basic Theory

2.1. PA Operator

Definition 1. Let the real number set be $\{a_1, a_2, \ldots, a_n\}$, then the operator is defined as

$$PA(a_1, a_2, \ldots, a_n) = \frac{\sum_{i=1}^{n} [(1 + T(a_i)a_i)]}{\sum_{i=1}^{n} (1 + T(a_i))}, \quad (1)$$

where $T(a_i) = \sum_{j=1, j \neq i}^{n} \mathrm{Sup}(a_i, a_j)$ and $\mathrm{Sup}(a_i, a_j)$ is support of a_j for a_i satisfying the following condition:

(1) $\mathrm{Sup}(a_i, a_j) \in [0, 1]$
(2) $\mathrm{Sup}(a_i, a_j) = \mathrm{Sup}(a_j, a_i)$
(3) If $|a_i, a_j| < |a_s, a_t|$, then $\mathrm{Sup}(a_i, a_j) > \mathrm{Sup}(a_s, a_t)$. Based on the PA operator and the geometric mean operator, Xu and Yager [27] defined the power geometric (PG) operator.

$$PG(a_1, a_2, \ldots, a_n) = \prod_{i=1}^{n} a_1^{\left(1+T(a_i)/\sum_{i=1}^{n} 1+T(a_i)\right)}. \quad (2)$$

2.2. Hesitant Triangular Fuzzy Sets

Definition 2. Let X be a given set, call $A = \{\langle x, f_A(x)\rangle | x \in X\}$ the HFS on X, where $f_A(x)$ is the set of distinct exact numbers on interval [0, 1], and $f_A(x)$ is the hesitant fuzzy element.

Chen et al. [6] combined HFS to propose interval-valued hesitant fuzzy set (IVHFS).

Definition 3. Let X be a given set, call $D = \{\langle x, g_D(x)\rangle | x \in X\}$ the IVHFS on X, where $g_D(x) = (\gamma^L, \gamma^U)$ $(0 \leq \gamma^L \leq \gamma^U \leq 1)$ represents a set of several possible membership degrees of element x belonging to D.

In reference [16], HFS and triangular fuzzy numbers were studied together, the HTFS definition was given. The HTFEWA operator, HTFEWG operator and related properties were given.

Definition 4. Let X be a given set, call $A = \{\langle x, h_A(x)\rangle | x \in X\}$ the HTFS on X. Among them, $h_A(x) = (\gamma^L, \gamma^M, \gamma^R)$ is a set of mutually different triangular fuzzy numbers, which means that the element x belongs to a set of several possible membership degrees of A, and $h_A(x)$ is the HTFE.

2.3. HTFS Algorithm

Definition 5. Let $h_1(x)=(\gamma_1^L,\gamma_1^M,\gamma_1^R)$ and $h_2(x)=(\gamma_2^L,\gamma_2^M,\gamma_2^R)$ be any two HTFEs, $\lambda>0$, then their calculation methods are defined as follows:

$$\begin{aligned}
h_1^{\lambda} &= \cup_{\gamma_1\in h_1}\left\{\left(\left(\gamma_1^L\right)^{\lambda},\left(\gamma_1^M\right)^{\lambda},\left(\gamma_1^R\right)^{\lambda}\right)\right\},\\
h_1\otimes h_2 &= \cup_{\gamma_1\in h_1,\gamma_2\in h_2}\left\{\left(\gamma_1^L\gamma_2^L,\gamma_1^M\gamma_2^M,\gamma_1^R\gamma_2^R\right)\right\},\\
\lambda h_1 &= \cup_{\gamma_1\in h_1}\left\{\left(1-\left(1-\gamma_1^L\right)^{\lambda},1-\left(1-\gamma_1^M\right)^{\lambda},1-\left(1-\gamma_1^R\right)^{\lambda}\right)\right\},\\
h_1\oplus h_2 &= \cup_{\substack{\gamma_1\in h_1\\ \gamma_2\in h_2}}\left\{\left(\gamma_1^L+\gamma_2^L-\gamma_1^L\gamma_2^L,\gamma_1^M+\gamma_2^M-\gamma_1^M\gamma_2^M,\gamma_1^R+\gamma_2^R-\gamma_1^R\gamma_2^R\right)\right\}.
\end{aligned}\tag{3}$$

2.4. HTFS Score Function

Definition 6. Let any HTFE be h, then the score function of h is

$$S(h)=\frac{1}{\#h}\sum_{\gamma\in h}\gamma,\tag{4}$$

where $\#h$ is the number of elements in the HTFE h, and for any two HTFE h_1 and h_2, if $S(h_1)\geq S(h_2)$, then $h_1\geq h_2$.

Definition 7 (see [23]): Let $h_1=(\gamma_1^L,\gamma_1^M,\gamma_1^R)$ and $h_2=(\gamma_2^L,\gamma_2^M,\gamma_2^R)$ be any two HTFEs, then call Equation (4) the Hamming distance between h_1,h_2.

$$d(h_1,h_2)=\frac{1}{n}\sum_{i=1}^{n}\left|h_1^{n(i)}-h_2^{n(i)}\right|,\tag{5}$$

where $h_1^{n(i)},h_2^{n(i)}$ represent the largest elements of HTFE h_1,h_2, respectively.

The PA operator is often used in an environment where the attributes have mutual support. For the MADM problem given by the attribute value in HTFE, based on the HTFE algorithm and Equation (1), Equation (6) is the definition of the HTFPWA operator.

$$HTFPWA(h_1,h_2,\ldots,h_n)=\frac{\oplus_{i=1}^{n}(w_i(1+T(h_i))h_i)}{\sum_{i=1}^{n}(w_i(1+T(h_i)))},\tag{6}$$

where $T(h_i)=\sum_{j=1,j\neq i}^{n}w_i\mathrm{Sup}(h_i,h_j)=\sum_{j=1,j\neq i}^{n}w_i(1-d(h_i,h_j))$, $Sup(h_i,h_j)$ is the support of h_j for h_i, satisfying the following condition.

(1) $Sup(h_i,h_j)\in[0,1]$

(2) $\mathrm{Sup}(h_i,h_j)=\mathrm{Sup}(h_j,h_i)$

(3) If $d(h_i,h_j)<d(h_s,h_t)$, then $\mathrm{Sup}(h_i,h_j)>\mathrm{Sup}(h_s,h_t)$, where d is the distance defined in Equation (5).

Theorem 1. *Let h_i $(i=1,2,\ldots,n)$ be a set of HTFEs, then the result of integration by Equation (6) is still HTFE, and*

$$\begin{aligned}
HTFPWA(h_1,h_2,\ldots,h_n) &= \frac{\oplus_{i=1}^{n}(w_i(1+T(h_i))h_i)}{\sum_{i=1}^{n}(w_i(1+T(h_i)))}\\
&= \cup_{\gamma_1\in h_1,\gamma_2\in h_2,\ldots,\gamma_n\in h_n}\left\{\left(1-\prod_{i=1}^{n}\left(1-\gamma_i^L\right)^{w_i(1+T(h_i))/\sum_{i=1}^{n}w_i(1+T(h_i))},1-\prod_{i=1}^{n}\left(1-\gamma_i^M\right)^{w_i(1+T(h_i))/\sum_{i=1}^{n}w_i(1+T(h_i))},\right.\right.\\
&\left.\left.1-\prod_{i=1}^{n}\left(1-\gamma_i^R\right)^{w_i(1+T(h_i))/\sum_{i=1}^{n}w_i(1+T(h_i))}\right)\right\}.
\end{aligned}\tag{7}$$

The proof process is shown in Appendix A.

Obviously, when $w=(1/n,1/n,1/n,\cdots,1/n)$, (6) is degraded to the HTFPA operator:

$$HTFPA(h_1,h_2,\cdots,h_n)=\frac{\oplus_{i=1}^{n}\left((1+T(h_i))h_i\right)}{\sum_{i=1}^{n}((1)+T(h_i)))}=\cup_{\gamma_1\in h_1,\gamma_2\in h_2,\ldots,\gamma_n\in h_n}\left\{\left(1-\prod_{i=1}^{n}\left(1-\gamma_i^L\right)^{\left((1+T(h_i))/\sum_{i=1}^{n}(1+T(h_i))\right)},\right.\right.$$
$$\left.\left.1-\prod_{i=1}^{n}\left(1-\gamma_i^M\right)^{\left((1+T(h_i))/\sum_{i=1}^{n}(1+T(h_i))\right)},1-\prod_{i=1}^{n}\left(1-\gamma_i^R\right)^{\left((1+T(h_i))/\sum_{i=1}^{n}(1+T(h_i))\right)}\right)\right\}, \tag{8}$$

where $T(h_i)=(1/n)\sum_{j=1,j\neq i}^{n}\mathrm{Sup}(h_i,h_j)$.

It can be easily proved that HTFPWA has the following properties.

2.5. HTFPWA Properties

Theorem 2. *Idempotency*

Let HTFE $h_i(x)=h=(\gamma^L,\gamma^M,\gamma^R)$ for every $i=1,2,\ldots,n$ has (9).

$$HTFPWA(h_1,h_2,\ldots,h_n)=HTFPWA(h,h,\ldots,h)=h. \tag{9}$$

Theorem 3. *Replacement invariance*

Let $(h_1,h_2,\ldots,h_n)$ be a set of HTFE and $(\overline{h}_1,\overline{h}_2,\cdots,\overline{h}_n)$ be any replacement of $(h_1,h_2,\ldots,h_n)$, then

$$HTFPWA(h_1,h_2,\ldots,h_n)=HTFPWA\left(\overline{h}_1,\overline{h}_2,\ldots,\overline{h}_n\right). \tag{10}$$

Theorem 4. *Monotonicity*

Let $h_i=(\gamma_i^L,\gamma_i^M,\gamma_i^R)$ and $\widetilde{h}_i=(\widetilde{\gamma}_i^L,\widetilde{\gamma}_i^M,\widetilde{\gamma}_i^R)(i=1,2,\ldots,n)$ be a set of two HTFEs, if $\gamma_i^L\geq\widetilde{\gamma}_i^L,\gamma_i^M\geq\widetilde{\gamma}_i^M,\gamma_i^R\geq\widetilde{\gamma}_i^R$, for any i gives

$$HTFPWA(h_1,h_2,\cdots,h_n)\geq HTFPWA\left(\widetilde{h}_1,\widetilde{h}_2,\cdots,\widetilde{h}_n\right). \tag{11}$$

Theorem 5. *Boundedness*

Let $h^-\leq HTFPWA(h_1,h_2,\ldots,h_n)\leq h^+$ be a set of HTFEs, then

$$h^-\leq HTFPWA(h_1,h_2,\ldots,h_n)\leq h^+, \tag{12}$$

where $h^-=\cup_{\gamma_i\in h_i}\min\{\gamma_i\}$ and $h^+=\cup_{\gamma_i\in h_i}\max\{\gamma_i\}$.

Inspired by reference [20], the HTFPWG operator is given based on the HTFE and the geometric mean operator.

Definition 8. Let $h_i(i=1,2,\cdots,n)$ be a set of HTFEs, then the HTFPWG operator is

$$HTFPWG(h_1,h_2,\ldots,h_n)=\frac{\otimes_{1=1}^{n}(w_i(1+T(h_i))h_i)}{\sum_{i=1}^{n}(w_i(1+T(h_i)))}, \tag{13}$$

where $T(h_i)=\sum_{j=1,j\neq i}^{n}w_i\,\mathrm{Sup}(h_i,h_j)=\sum_{j=1,j\neq i}^{n}w_i(1-d(h_i,h_j))$, $Sup(h_i,h_j)$ are the support of h_j to h_i, satisfying the following conditions.

(1) $Sup(h_i,h_j)\in[0,1]$

(2) $Sup(h_i,h_j)=Sup(h_j,h_i)$

(3) If $d(h_i,h_j)<d(h_s,h_t)$, then $Sup(h_i,h_j)>Sup(h_s,h_t)$, where d is the distance defined in Equation (5).

Theorem 6. *Let $h_i(i=1,2,\cdots,n)$ be a set of HTFEs, then the result of integration by Equation (13) is still HTFE, and*

$$HTFPWG(h_1,h_2,\ldots,h_n)=\frac{\otimes_{i=1}^{n}(w_i(1+T(h_i))h_i)}{\sum_{i=1}^{n}(w_i(1+T(h_i)))}$$
$$=\cup_{\gamma_1\in h_1,\gamma_2\in h_2,\cdots,\gamma_n\in h_n}\left\{\left(\prod_{i=1}^{n}\left(\gamma_i^L\right)^{\left(w_i(1+T(h_i))/\sum_{i=1}^{n}(w_i(1+T(h_i)))\right)},\prod_{i=1}^{n}\left(\gamma_i^M\right)^{\left(w_i(1+T(h_i))/\sum_{i=1}^{n}(w_i(1+T(h_i)))\right)},\right.\right.$$
$$\left.\left.\prod_{i=1}^{n}\left(\gamma_i^R\right)^{\left(w_i(1+T(h_i))/\sum_{i=1}^{n}(w_i(1+T(h_i)))\right)}\right)\right\}. \tag{14}$$

The proof process is similar to Appendix A, which is omitted here.

When $w=(1/n,1/n,1/n,\cdots,1/n)$, equation (14) degrades to the HTFPG operator.

$$HTFPG(h_1, h_2, \ldots, h_n) = \frac{\otimes_{i=1}^{n}((1+T(h_i))h_i)}{\sum_{i=1}^{n}((1+T(h_i)))}$$
$$= \cup_{\gamma_1\in h_1,\gamma_2\in h_2,\ldots,\gamma_n\in h_n}\left\{\left(\prod_{i=1}^{n}\left(\gamma_i^L\right)^{\left((1+T(h_i))/\sum_{i=1}^{n}(1+T(h_i))\right)}, \prod_{i=1}^{n}\left(\gamma_i^M\right)^{\left((1+T(h_i))/\sum_{i=1}^{n}(1+T(h_i))\right)},\right.\right.$$
$$\left.\left.\prod_{i=1}^{n}\left(\gamma_i^R\right)^{\left((1+T(h_i))/\sum_{i=1}^{n}(1+T(h_i))\right)}\right)\right\}, \tag{15}$$

where $T(h_i) = 1/n\sum_{j=1,j\neq i}^{n} Sup(h_i, h_j)$.

Like the HTFPWA operator, the HTFPWG operator also has the properties of idempotency, replacement invariance, monotonicity, and boundedness.

Lemma 1. *Let* $x_i > 0$, $\lambda_i > 0$, $i = 1, 2, \ldots, n$, *at the same time* $\sum_{i=1}^{n}\lambda_i = 1$, *then*

$$\prod_{i=1}^{n}(x_i)^{\lambda_i} \leq \sum_{i=1}^{n}\lambda_i x_i. \tag{16}$$

If and only if $x_1 = x_2 = \cdots = x_n$, take the equal sign [28].

Theorem 7. *Let* $h_i\,(i = 1, 2, \ldots, n)$ *be a set of HTFEs, then*

$$HTFPWG(h_1, h_2, \ldots, h_n) \leq HTFPWA(h_1, h_2, \ldots, h_n). \tag{17}$$

The proof process is shown in Appendix B.

Theorem 7 states that the HTFE obtained by the fusion of the HTFPWG operator is less than or equal to the HTFE obtained by the fusion of the HTFPWA operator.

3. MADM Method Based on HTFPWA Operator

Based on the HTFPWA operator, this paper proposes a MADM method with an attribute value of HTFE. Suppose there is a MADM problem, set the scheme set $A = \{A_1, A_2, \ldots, A_t\}$, attribute set $C = \{C_1, C_2, \ldots, C_n\}$, decision set $D = \{d_1, d_2, \ldots, d_m\}$. Also, $w = [w_1, w_2, \ldots, w_n]^T$ is the weight vector of each attribute, $w_k \in [0, 1]\,(k = 1, 2, \ldots, m)$.

Specific decision steps are as follows:

Step 1: suppose that the evaluation value given by the decision expert to the scheme A_i under the attribute C_j is HTFE, and the decision matrix is obtained as $D = (h_{ij})_{n\times t}$.

Step 2: calculate the support $\mathrm{Sup}(h_i, h_j)$ between HTFE attributes,

$$\mathrm{Sup}\left(h_i, h_j\right) = 1 - d\left(h_i, h_j\right), \tag{18}$$

where $d(h_i, h_j)$ can be obtained from Equation (5), and then by (19) obtained $T(h_{ij})$.

$$T\left(h_{ij}\right) = \sum_{j=1,j\neq i}^{n} w_i \mathrm{Sup}\left(h_i, h_j\right) = \sum_{j=1,j\neq i}^{n} w_i\left(1 - d\left(h_i, h_j\right)\right). \tag{19}$$

Step 3: the comprehensive evaluation value of the i-th scheme under the j-th attribute is obtained by the HTFPWA operator (Equation (7)), which is

$$HTFPWA(h_1, h_2, \ldots, h_n) = \cup_{\gamma_1\in h_1,\gamma_2\in h_2,\cdots,\gamma_n\in h_n}$$
$$\left\{\left(1 - \prod_{i=1}^{n}\left(1-\gamma_i^L\right)^{\left(w_i(1+T(h_i))/\sum_{i=1}^{n}w_i(1+T(h_i))\right)}, 1 - \prod_{i=1}^{n}\left(1-\gamma_i^M\right)^{\left(w_i(1+T(h_i))/\sum_{i=1}^{n}w_i(1+T(h_i))\right)},\right.\right.$$
$$\left.\left.1 - \prod_{i=1}^{n}\left(1-\gamma_i^R\right)^{\left(w_i(1+T(h_i))/\sum_{i=1}^{n}w_i(1+T(h_i))\right)}\right)\right\}. \tag{20}$$

Or the HTFPWG operator (14) gets the comprehensive evaluation value of the i-th scheme under the j-th attribute, which is

$$HTFPWG(h_1, h_2, \cdots, h_n) = \cup_{\gamma_1 \in h_1, \gamma_2 \in h_2, \cdots, \gamma_n \in h_n} \left\{ \left(\prod_{i=1}^{n} (\gamma_i^L)^{(w_i(1+T(h_i))/\sum_{i=1}^{n} w_i(1+T(h_i)))}, \prod_{i=1}^{n} (\gamma_i^M)^{(w_i(1+T(h_i))/\sum_{i=1}^{n} w_i(1+T(h_i)))}, \prod_{i=1}^{n} (\gamma_i^R)^{(w_i(1+T(h_i))/\sum_{i=1}^{n} w_i(1+T(h_i)))} \right) \right\}. \tag{21}$$

Step 4: use Equation (4) to calculate the score function value, and rank the pros and cons of the schemes according to the HTFE sorting method.

Step 5: get the best solution.

4. Application of HTFPWA Operator in Futures Selection

An investment group intends to invest in several futures products. Known futures market has five futures products A_1–A_5, the group's decision makers are based on futures-related evaluation indicators for the selection of products. Evaluation indicators are C_1 (product yield), C_2 (product potential), C_3 (investment risk factor), C_4 (product stability coefficient), the weight vector of the indicator is $w = [0.1,0.3,0.2,0.4]^T$. Decision-making experts' satisfaction evaluation to five future products under four evaluation indicators is presented by HTFE, as shown in Table 1. The HTFPWA operator proposed in this paper is used for the selection of candidate products.

Step 1. use Equation (5) to calculate the Hamming distance $d(h_i, h_j)$ between different attributes, and use Equation (18) to find the mutual support between attributes, then using Equation (19) to calculate $T(h_{ij})(i = 1, 2, 3, 4, j = 1, 2, 3, 4, 5)$.

$$T(h_{ij}) = \begin{pmatrix} 2.300 & 1.100 & 2.100 & 2.100 \\ 2.350 & 1.650 & 2.350 & 2.350 \\ 2.400 & 1.500 & 2.400 & 2.400 \\ 1.950 & 1.950 & 0.450 & 1.950 \\ 1.950 & 2.250 & 2.250 & 1.650 \end{pmatrix}. \tag{22}$$

Step 2. the comprehensive evaluation value of the i-th scheme under the j-th attribute is obtained from the HTFPWA operator (Equation (7)).

$$\begin{aligned} HTFPWA(h_{11}, h_{12}, h_{13}, h_{14}) &= (0.4008, 0.5116, 0.6475), \\ HTFPWA(h_{21}, h_{22}, h_{23}, h_{24}) &= (0.4756 , 0.5957, 0.7001), \\ HTFPWA(h_{31}, h_{32}, h_{33}, h_{34}) &= (0.4576, 0.5581, 0.6589), \\ HTFPWA(h_{41}, h_{42}, h_{43}, h_{44}) &= (0.6154, 0.7212, 0.8315), \\ HTFPWA(h_{51}, h_{52}, h_{53}, h_{54}) &= (0.3985, 0.5019, 0.6073). \end{aligned} \tag{23}$$

Step 3. the fractional function of HTFE $S(h_i)(i = 1, 2, 3, 4, 5)$ is calculated from Equation (3) and HTFPWA operator. $S(h_1) = 0.5200, S(h_2) = 0.5905, S(h_3) = 0.5582, S(h_4) = 0.7227, S(h_5) = 0.5026$

Step 4. sort all candidate futures products $A_i (i = 1, 2, 3, 4, 5)$ according to the score function $S(h_i)(i = 1, 2, 3, 4, 5)$ to get $A_4 \succ A_2 \succ A_3 \succ A_1 \succ A_5$. Therefore, the best futures product is A_4.

The following uses the HTFPWG operator proposed in this paper for comparative analysis.

Calculate $T(h_{ij})(i = 1, 2, 3, 4, j = 1, 2, 3, 4, 5)$ same as step 1, and then the comprehensive evaluation value of the i-th scheme under the j-th attribute is obtained from the HTFPWAG operator (11).

$$\begin{aligned} HTFPWG(h_{11}, h_{12}, h_{13}, h_{14}) &= (0.3307, 0.4373, 0.5681), \\ HTFPWG(h_{21}, h_{22}, h_{23}, h_{24}) &= (0.4203 , 0.5688, 0.6709), \\ HTFPWG(h_{31}, h_{32}, h_{33}, h_{34}) &= (0.5361, 0.6385, 7403), \\ HTFPWG(h_{41}, h_{42}, h_{43}, h_{44}) &= (0.5282, 0.6477, 0.7573), \\ HTFPWG(h_{51}, h_{52}, h_{53}, h_{54}) &= (0.3500, 0.4575, 0.5621). \end{aligned} \tag{24}$$

Use (2) and HTFPWG operator to calculate the score function $S'(h_i)(i = 1, 2, 3, 4, 5)$ of HTFE h.

$S'(h_1) = 0.4454, S'(h_2) = 0.5534, S'(h_3) = 0.6383, S'(h_4) = 0.6444, S'(h_5) = 0.4565$. Sort all candidate futures products $A_i (i = 1, 2, 3, 4, 5)$ according to the score function $S'(h_i)(i = 1, 2, 3, 4, 5)$ to get $A_4 \succ A_3 \succ A_2 \succ A_1 \succ A_5$. Therefore, the best futures product is A_4, but there exists

Table 1: Hesitant triangle fuzzy decision matrix.

	C_1	C_2	C_3	C_4
A_1	{(0.3,0.4,0.5)}	{(0.6,0.7,0.8), (0.7,0.8,0.9)}	{(0.2,0.3,0.5)}	{(0.2,0.3,0.4), (0.3,0.4,0.5)}
A_2	{(0.6,0.7,0.8)}	{(0.2,0.4,0.5)}	{(0.3,0.4,0.5), (0.6,0.7,0.8)}	{(0.2,0.3,0.5), (0.4,0.5,0.6), (0.5,0.6,0.7)}
A_3	{(0.5,0.6,0.7)}	{(0.7,0.8,0.9), (0.8,0.9,1.0)}	{(0.2,0.3,0.4), (0.4,0.5,0.6)}	{(0.5,0.6,0.7)}
A_4	{(0.7,0.8,0.9)}	{(0.6,0.7,0.8), (0.7,0.8,0.9)}	{(0.1,0.2,0.3)}	{(0.3,0.4,0.5), (0.6,0.7,0.8)}
A_5	{(0.6,0.7,0.8)}	{(0.2,0.3,0.4), (0.5,0.6,0.7)}	{(0.4,0.5,0.6)}	{(0.2,0.3,0.4)}

Table 2: Comparative analysis of operators.

	Operator	Ranking
Proposed method I	HTFPWA operator	$A_4 \succ A_2 \succ A_3 \succ A_1 \succ A_5$
Proposed method II	HTFPWG operator	$A_4 \succ A_3 \succ A_2 \succ A_1 \succ A_5$
The method in reference [16]	HTFEWA operator	$A_3 \succ A_4 \succ A_1 \succ A_2 \succ A_5$
The method in reference [16]	HTFEWG operator	$A_3 \succ A_4 \succ A_2 \succ A_1 \succ A_5$
The method in reference [29]	GTHF aggregation operator	$A_4 \succ A_3 \succ A_2 \succ A_1 \succ A_5$
The method in reference [30]	GTHFBAM operator	$A_3 \succ A_4 \succ A_2 \succ A_1 \succ A_5$
The method in reference [30]	GTHFBGM operator	$A_4 \succ A_3 \succ A_2 \succ A_1 \succ A_5$

some difference between the orders of pros and cons of A_2 and A_3. The scores of the candidate futures products obtained by the HTFPWG operator fusion from Theorem 6 are less than or equal to the score function values obtained by the fusion of the HTFPWA operator. It can be seen from the experiment that the difference between the scores of the HTFPWG operator is not obvious, so that the rank of the merits does not have high sensitivity.

As shown in Table 2, the methods proposed in this paper are compared with other methods. According to the HTFEWA operator and HTFEWG operator proposed by Zhao et al. [16], the sorting results of the alternative futures products are obtained as $A_3 \succ A_4 \succ A_1 \succ A_2 \succ A_5$ and $A_3 \succ A_4 \succ A_2 \succ A_1 \succ A_5$, respectively. By comparison, it can be seen that the results of these two methods are different from the methods proposed in this paper. The HTFEWA operator and the HTFEWG operator do not fully consider the importance of the relevant membership degrees when calculating.

According to the generalized trapezoidal hesitant fuzzy (GTHF) aggregation operator, the generalized trapezoidal hesitant fuzzy Bonferroni arithmetic mean (GTHFBAM) operator, and the generalized trapezoidal hesitant fuzzy Bonferroni geometric mean (GTHFBGM) operator proposed by Deli et al. [29, 30], the sorting results of the alternative futures products are obtained as $A_4 \succ A_3 \succ A_2 \succ A_1 \succ A_5$, $A_3 \succ A_4 \succ A_2 \succ A_1 \succ A_5$ and $A_4 \succ A_3 \succ A_2 \succ A_1 \succ A_5$ respectively. Among them, the order of GTHF aggregation operator and GTHFBGM operator is the same as that of HTFPWG operator, and the order of other operators is different to some extent, which is not unrelated to the consideration of membership degree by the methods proposed in this paper.

The HTFPWA operator and HTFPWG operator proposed in this paper consider the correlation between attributes and the importance of related information, reduce the randomness of decision-making, and involve fewer parameters, overcome the subjectivity of decision-making, and make the results more comprehensive and scientific.

5. Conclusions

For MADM in hesitant and fuzzy, the decision attributes are often related to each other to a certain extent, which leads to mutual interference of decision results, and even the problem of discussing the weight of the same factor for several times, thus affecting the stability of decision making. In order to eliminate the interference of subjective weights on the information fusion results and achieve the stability of decision making, this paper studies the HTFPA operator, the HTFPWA operator, and the HTFPWG operator, analyzes the relevant properties of these operators, and discusses the process of special cases. Then, the application methods of the HTFPWA operator and the HTFPWA operator in MADM problem are given, and the validity and correctness of the proposed methods are shown by the example of futures products selection. Finally, by comparing the existing researches, the proposed operators comprehensively consider the mutual support between the decision attributes, and realize the objective weighting operation according to the difference between the individual and the whole information fusion, which makes the decision analysis closer to the actual situation and the decision results more reasonable, providing a new idea for solving MADM problems. In the future, we plan to extend our research work to VIKOR, QUALIFLEX, deblurring techniques, ELECTRE I method, ELECTRE II method, ELECTRE III method, etc.

Appendix

A. The proof process of Theorem 6

Let $h_i\,(i = 1, 2, \ldots, n)$ a set of HTFEs, to prove that

$$= \cup_{\gamma_1\in h_1,\gamma_2\in h_2,\ldots,\gamma_n\in h_n}\left\{\left(1-\prod_{i=1}^{n}\left(1-\gamma_i^L\right)^{\left(w_i(1+T(h_i))/\sum_{i=1}^{n}w_i(1+T(h_i))\right)}, 1-\prod_{i=1}^{n}\left(1-\gamma_i^M\right)^{w_i(1+T(h_i))/\sum_{i=1}^{n}w_i(1+T(h_i))}, 1-\prod_{i=1}^{n}\left(1-\gamma_i^R\right)^{w_i(1+T(h_i))/\sum_{i=1}^{n}w_i(1+T(h_i))}\right)\right\}, \tag{A.1}$$

is true, first prove by mathematical induction

$$\begin{aligned}&(w_i(1+T(h_i))h_i)\\&= \cup_{\gamma_1\in h_1,\gamma_2\in h_2,\ldots,\gamma_n\in h_n}\left\{\left(1-\prod_{i=1}^{n}\left(1-\gamma_i^L\right)^{w_i(1+T(h_i))}, 1-\prod_{i=1}^{n}\left(1-\gamma_i^M\right)^{w_i(1+T(h_i))}, 1-\prod_{i=1}^{n}\left(1-\gamma_i^R\right)^{w_i(1+T(h_i))}\right)\right\},\end{aligned} \tag{A.2}$$

is true.

When $n = 2$, according to Definition 5,

$$\begin{aligned}w_1(1+T(h_1))h_1 &= \cup_{\gamma_1\in h_1}\left\{\left(1-\left(1-\gamma_1^L\right)^{w_1(1+T(h_1))}, 1-\left(1-\gamma_1^M\right)^{w_1(1+T(h_1))}, 1-\left(1-\gamma_1^R\right)^{w_1(1+T(h_1))}\right)\right\},\\ w_2(1+T(h_2))h_2 &= \cup_{\gamma_1\in h_2}\left\{\left(1-\left(1-\gamma_2^L\right)^{w_2(1+T(h_2))}, 1-\left(1-\gamma_2^M\right)^{w_2(1+T(h_2))}, 1-\left(1-\gamma_2^R\right)^{w_2(1+T(h_2))}\right)\right\}.\end{aligned} \tag{A.3}$$

Then, get

$$\begin{aligned}&= \cup_{\gamma_1\in h_1,\gamma_2\in h_2}\left\{\left(\begin{array}{c}1-\left(1-\gamma_1^L\right)^{w_1(1+T(h_1))}+1-\left(1-\gamma_2^L\right)^{w_2(1+T(h_2))}-\left(1-\left(1-\gamma_1^L\right)^{w_1(1+T(h_1))}\right)\left(1-\left(1-\gamma_2^L\right)^{w_2(1+T(h_2))}\right),\\ 1-\left(1-\gamma_1^M\right)^{w_1(1+T(h_1))}+1-\left(1-\gamma_2^M\right)^{w_2(1+T(h_2))}-\left(1-\left(1-\gamma_1^M\right)^{w_1(1+T(h_1))}\right)\left(1-\left(1-\gamma_2^M\right)^{w_2(1+T(h_2))}\right),\\ 1-\left(1-\gamma_1^R\right)^{w_1(1+T(h_1))}+1-\left(1-\gamma_2^R\right)^{w_2(1+T(h_2))}-\left(1-\left(1-\gamma_1^R\right)^{w_1(1+T(h_1))}\right)\left(1-\left(1-\gamma_2^R\right)^{w_2(1+T(h_2))}\right),\end{array}\right)\right\}\\ &= \cup_{\gamma_1\in h_1,\gamma_2\in h_2}\left\{\begin{array}{c}\left(1-\left(1-\gamma_1^L\right)^{w_1(1+T(h_1))}\cdot\left(1-\gamma_2^L\right)^{w_2(1+T(h_2))}\right),\\ \left(1-\left(1-\gamma_1^M\right)^{w_1(1+T(h_1))}\cdot\left(1-\gamma_2^M\right)^{w_2(1+T(h_2))}\right),\\ \left(1-\left(1-\gamma_1^R\right)^{w_1(1+T(h_1))}\cdot\left(1-\gamma_2^R\right)^{w_2(1+T(h_2))}\right)\end{array}\right\}.\end{aligned} \tag{A.4}$$

Suppose that when $n = k$, there is

$$\begin{aligned}&\overset{k}{\underset{i=1}{\oplus}}\left(w_i\left(1+T(h_i)\right)h_i\right)\\&=\cup_{\gamma_1\in h_1,\gamma_2\in h_2,\cdots,\gamma_k\in h_k}\left\{\left(1-\prod_{i=1}^{k}\left(1-\gamma_i^L\right)^{w_i(1+T(h_i))},1-\prod_{i=1}^{k}\left(1-\gamma_i^M\right)^{w_i(1+T(h_i))},1-\prod_{i=1}^{k}\left(1-\gamma_i^R\right)^{w_i(1+T(h_i))}\right)\right\}.\end{aligned}\tag{A.5}$$

Then, when $n=k+1$,

$$\begin{aligned}&\overset{k+1}{\underset{i=1}{\oplus}}\left(w_i\left(1+T(h_i)\right)h_i\right)=\left(\overset{k}{\underset{i=1}{\oplus}}\left(w_i\left(1+T(h_i)\right)h_i\right)\right)\oplus\left(w_{k+1}\left(1+T(h_{k+1})\right)h_{k+1}\right)\\&=\cup_{\gamma_1\in h_1,\gamma_2\in h_2,\ldots,\gamma_k\in h_k,\gamma_{k+1}\in h_{k+1}}\left\{\left(\begin{array}{l}\left(1-\prod_{i=1}^{k}\left(1-\gamma_i^L\right)^{w_i(1+T(h_i))}\right)+\left(1-\left(1-\gamma_{k+1}^L\right)^{w_{k+1}(1+T(h_{k+1}))}\right)-\left(1-\prod_{i=1}^{k}\left(1-\gamma_i^L\right)^{w_i(1+T(h_i))}\right)\cdot\left(1-\left(1-\gamma_{k+1}^L\right)^{w_{k+1}(1+T(h_{k+1}))}\right),\\\left(1-\prod_{i=1}^{k}\left(1-\gamma_i^M\right)^{w_i(1+T(h_i))}\right)+\left(1-\left(1-\gamma_{k+1}^M\right)^{w_{k+1}(1+T(h_{k+1}))}\right)-\left(1-\prod_{i=1}^{k}\left(1-\gamma_i^M\right)^{w_i(1+T(h_i))}\right)\cdot\left(1-\left(1-\gamma_{k+1}^M\right)^{w_{k+1}(1+T(h_{k+1}))}\right),\\\left(1-\prod_{i=1}^{k}\left(1-\gamma_i^R\right)^{w_i(1+T(h_i))}\right)+\left(1-\left(1-\gamma_{k+1}^R\right)^{w_{k+1}(1+T(h_{k+1}))}\right)-\left(1-\prod_{i=1}^{k}\left(1-\gamma_i^R\right)^{w_i(1+T(h_i))}\right)\cdot\left(1-\left(1-\gamma_{k+1}^R\right)^{w_{k+1}(1+T(h_{k+1}))}\right)\end{array}\right)\right\}\\&=\cup_{\gamma_1\in h_1,\gamma_2\in h_2,\ldots,\gamma_k\in h_k,\gamma_{k+1}\in h_{k+1}}\left\{\left(1-\prod_{i=1}^{k+1}\left(1-\gamma_i^L\right)^{w_i(1+T(h_i))},1-\prod_{i=1}^{k+1}\left(1-\gamma_i^M\right)^{w_i(1+T(h_i))},1-\prod_{i=1}^{k+1}\left(1-\gamma_i^R\right)^{w_i(1+T(h_i))}\right)\right\}.\end{aligned}\tag{A.6}$$

That is, when $n=k+1$, holds.

By the HTFE algorithm,

$$\begin{aligned}&HTFPA(h_1,h_2,\ldots,h_n)=\frac{\oplus_{i=1}^{n}w_i\left(1+T(h_i)\right)h_i}{\sum_{i=1}^{n}w_i\left(1+T(h_i)\right)},\\&=\cup_{\gamma_1\in h_1,\gamma_2\in h_2,\ldots,\gamma_n\in h_n}\left\{\left(\begin{array}{c}1-\left(1-\left(1-\prod_{i=1}^{n}\left(1-\gamma_i^L\right)^{1+T(h_i)}\right)\right)^{1/\sum_{i=1}^{n}w_i(1+T(h_i))},1-\left(1-\left(1-\prod_{i=1}^{n}\left(1-\gamma_i^M\right)^{1+T(h_i)}\right)\right)^{1/\sum_{i=1}^{n}w_i(1+T(h_i))},\\1-\left(1-\left(1-\prod_{i=1}^{n}\left(1-\gamma_i^R\right)^{1+T(h_i)}\right)\right)^{1/\sum_{i=1}^{n}w_i(1+T(h_i))}\end{array}\right)\right\},\\&=\cup_{\gamma_1\in h_1,\gamma_2\in h_2,\cdots,\gamma_n\in h_n}\left\{\left(\begin{array}{c}1-\left(1-\prod_{i=1}^{n}\left(1-\gamma_i^L\right)^{1+T(h_i)}\right)^{1/\sum_{i=1}^{n}w_i(1+T(h_i))},1-\left(1-\prod_{i=1}^{n}\left(1-\gamma_i^M\right)^{1+T(h_i)}\right)^{1/\sum_{i=1}^{n}w_i(1+T(h_i))},\\1-\left(1-\prod_{i=1}^{n}\left(1-\gamma_i^R\right)^{1+T(h_i)}\right)^{1/\sum_{i=1}^{n}w_i(1+T(h_i))}\end{array}\right)\right\},\\&=\cup_{\gamma_1\in h_1,\gamma_2\in h_2,\ldots,\gamma_n\in h_n}\left\{\left(1-\prod_{i=1}^{n}\left(1-\gamma_i^L\right)^{\left(w_i(1+T(h_i))/\sum_{i=1}^{n}w_i(1+T(h_i))\right)},1-\prod_{i=1}^{n}\left(1-\gamma_i^M\right)^{\left(w_i(1+T(h_i))/\sum_{i=1}^{n}w_i(1+T(h_i))\right)},\right.\right.\\&\left.\left.1-\prod_{i=1}^{n}\left(1-\gamma_i^R\right)^{\left(w_i(1+T(h_i))/\sum_{i=1}^{n}w_i(1+T(h_i))\right)}\right)\right\}.\end{aligned}\tag{A.7}$$

So, Theorem 1 is proved.

B. The proof process of Theorem 7

Let $h_i\,(i = 1, 2, \ldots, n)$ be a set of HTFEs, for any $\gamma_1 \in h_1$, $\gamma_2 \in h_2, \cdots, \gamma_n \in h_n$, because $\sum_{i=1}^{n} w_i\,(1 + T\,(h_i))/\sum_{i=1}^{n} w_i\,(1 + T\,(h_i)) = (\sum_{i=1}^{n} w_i\,(1 + T\,(h_i))/\sum_{i=1}^{n} w_i\,(1 + T\,(h_i))) = 1$, Lemma 1 gives

$$\prod_{i=1}^{n} (\gamma_i)^{w_i\,(1+T\,(h_i))/\sum_{i=1}^{n} w_i\,(1+T\,(h_i))} \leq \sum_{i=1}^{n}\left(\frac{w_i\,(1 + T\,(h_i))}{\sum_{i=1}^{n} w_i\,(1 + T\,(h_i))}\gamma_i\right) \leq 1 - \prod_{i=1}^{n} (1 - \gamma_i)^{w_i\,(1+T\,(h_i))/\sum_{i=1}^{n} w_i\,(1+T\,(h_i))}. \quad \text{(B.1)}$$

So, Theorem 7 is proved.

Data Availability

Previously reported data were used to support this study. The prior study is cited within the text as [11].

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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