Hindawi Publishing Corporation
Advances in High Energy Physics
Volume 2007, Article ID 39676, 17 pages
doi:10.1155/2007/39676

Research Article

Predictions for $\mu \to e\gamma$ in Supersymmetry from Nontrivial Quark-Lepton Complementarity and Flavor Symmetry

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Received 19 July 2007; Revised 18 October 2007; Accepted 31 October 2007

Recommended by Joseph Formaggio

We compute the effect of nondiagonal neutrino mass in $l_i \to l_j\gamma$ in Supersymmetry (SUSY) theories with nontrivial quark-lepton complementarity and a flavor symmetry. The correlation matrix $V_M = U_{\rm CKM}U_{\rm PMNS}$ is such that its (1,3) entry, as preferred by the present experimental data, is zero. We do not assume that V_M is bimaximal. Quark-lepton complementarity and the flavor symmetry strongly constrain the theory and we obtain a clear prediction for the contribution to $\mu \to e\gamma$ and the τ decays $\tau \to e\gamma$ and $\tau \to \mu\gamma$. If the Dirac neutrino Yukawa couplings are degenerate but the low-energy neutrino masses are not degenerate, then the lepton decays are related among them by the V_M entries. On the other hand, if the Dirac neutrino Yukawa couplings are hierarchical or the low-energy neutrino masses are degenerate, then the prediction for the lepton decays comes from the $U_{\rm CKM}$ hierarchy.

1. Introduction

The present experimental situation is such that we are very close to obtain a theory of flavor that is able to explain in a clear way all the standard model masses and mixing. The last but not least experimental ingredient has been the neutrino data and the determination of Δm^2_{12}, $|\Delta m^2_{23}|$, θ_{12}, and θ_{23}.From all these results we are able to extract strong constraints on the flavor structure of the SM. In particular the neutrino data were determinant to clarify the role of the discrete symmetry in flavor physics.

The disparity that nature indicates between quark and lepton mixing angles has been viewed in terms of a "quark-lepton complementarity" (QLC) [1, 2] which can be expressed in the relations

$$\theta_{12}^{\mathrm{PMNS}} + \theta_{12}^{\mathrm{CKM}} \simeq 45^\circ; \qquad \theta_{23}^{\mathrm{PMNS}} + \theta_{23}^{\mathrm{CKM}} \simeq 45^\circ. \tag{1.1}$$

Despite the naive relations between the PMNS and CKM angles, a detailed analysis shows that the correlation matrix $V_M = U_{\mathrm{CKM}} U_{\mathrm{PMNS}}$ is phenomenologically compatible with a tribimaximal pattern, and only marginally with a bimaximal pattern. Future experiments on neutrino physics, and in particular in the determination of θ_{23} and the CP violating parameter J, will be able to better clarify if a trivial quark-lepton complementarity (i.e., V_M bimaximal) is ruled out in favor of a nontrivial quark-lepton complementarity (i.e., V_M tribimaximal or even more structured) [3]. From present experimental evidences a nontrivial quark-lepton complementarity arises [4]. Moreover the clear nontrivial structure of V_M and the strong indication of gauge coupling unification allow us to obtain in a straightforward way constraints on the high-energy spectrum too. Within this framework we get information about flavor physics from the correlation matrix V_M too. It is very impressive that for some discrete flavor symmetries, such as A_4 dynamically broken into Z_3 [5–7] or S_3 softly broken into S_2 [8–10], the tribimaximal structure appears in a natural way.

In supergravity theories if the effective Lagrangian is defined at a scale higher than the grand unification scale, the matter fields have to respect the underlying gauge and flavor symmetry. Hence, we expect quark-lepton correlations among entries of the sfermion mass matrices. In other words, the quark-lepton unification seeps also into the SUSY breaking soft sector [11]. In general we do not get strongly renormalization effects on flavor violating quantities from the heavy neutrino scale to the electroweak scale because of the absence of flavor violation. In fact the remaining flavor violation related to the low-energy neutrino sector gives a negligible contribution with the exception of the case with highly degenerate neutrinos and $\tan\beta > 40$ [12, 13].

In this work we compute the effect of nondiagonal neutrino mass in $l_i \to l_j \gamma$ in SUSY theories with nontrivial quark-lepton complementarity and flavor symmetry. In comparison with previous works (i.e., [14, 15]), where a bimaximal V_M matrix is assumed, in the present work the correlation matrix $V_M = U_{\mathrm{CKM}} U_{\mathrm{PMNS}}$ is such that its (1,3) entry, as preferred by experimental data, is zero. All the other entries are assumed to vary as allowed by the experimental data [3, 4]. Nevertheless we obtain a clear prediction for the contribution to $l_i \to l_j \gamma$. By using the nontrivial quark-lepton complementarity, flavor symmetry, and the see-saw mechanism we will compute the explicit spectrum of the heavy neutrinos. This will allow us to investigate the relevance of the form of V_M in $l_i \to l_j \gamma$. There are three cases which depend on the spectrum of the Dirac neutrino mass matrix and the low-energy neutrinos. We may have the following: (1) hierarchical Dirac neutrino eigenvalues (in this case we have very hierarchical right-handed neutrino masses); (2) degenerate Dirac neutrino eigenvalues, with nondegenerate low-energy neutrino masses (in this case the hierarchy of the right-handed neutrino masses is close to the one of the low-energy spectrum); (3) degenerate Dirac neutrino eigenvalues and low-energy neutrino spectrum (that implies right-handed neutrinos close to degenerate). For each of these cases we have different contributions to $l_i \to l_j \gamma$. We will show that only when Dirac neutrino eigenvalues are degenerate and low-energy neutrino masses are not degenerate, the explicit form of V_M plays an important role.

The plan of the work is as follows. In Section 2 we explain our notations and clarify the meaning of the correlation matrix V_M in flavor theories. In Section 3 we introduce the relation between $l_i \to l_j \gamma$ and the Dirac neutrino matrix. In Section 4 we relate the Dirac neutrino Yukawa coupling to the CKM mixing matrix by using the nontrivial quark-lepton complementarity and flavor symmetry. Then we compute the heavy neutrino spectrum. In Section 5 we compute the value of the contribution to the $l_i \rightarrow l_j \gamma$ processes from a nondiagonal Dirac neutrino Yukawa coupling. Finally in Section 6 we report our conclusions.

2. Notations

In this section we explain the relation between the product $V_M = U_{\text{CKM}} U_{\text{PMNS}}$ and the diagonalization of the right-handed neutrino mass.

2.1. V_M *in theories with see-saw of type I*

Let us fix the notations in the lepton sector. Let Y_l be the Yukawa matrix for charged leptons. It can be diagonalized by

$$Y_l = U_l Y_l^{\Delta} V_l^{\dagger}. \tag{2.1}$$

Let M_R be the Majorana mass matrix for the right-handed neutrino and M_D the Dirac mass matrix. Under the assumption that the low-energy neutrino masses are given by the see-saw of Type I we have that the light neutrino mass matrix is given by

$$M_\nu = M_D \frac{1}{M_R} M_D^T. \tag{2.2}$$

Let us introduce U_0 from the diagonalization of the Dirac mass matrix

$$M_D = U_0 M_D^{\Delta} V_0^{\dagger}, \tag{2.3}$$

then we define V_M by the diagonalization of the light neutrino mass

$$M_\nu = U_\nu M_\nu^{\Delta} U_\nu^T = U_0 V_M M_\nu^{\Delta} (V_M) U_0^T \tag{2.4}$$

with the constraint that $U_0 V_M$ is an unitary matrix. Finally the lepton mixing matrix is

$$U_{\text{PMNS}} = U_l^{\dagger} U_\nu = U_l^{\dagger} U_0 V_M. \tag{2.5}$$

Let us introduce the following symmetric complex matrix $\mathcal{C}$

$$\mathcal{C} = M_D^{\Delta} V_0^{\dagger} \frac{1}{M_R} V_0^{\star} M_D^{\Delta}, \tag{2.6}$$

where V_0 is the mixing matrix that diagonalizes on the right the Dirac neutrino mass matrix in (2.3). From (2.3), (2.4) we see that the inverse of V_M diagonalizes the symmetric matrix $\mathcal{C}$, in fact we have

$$V_M M_\nu^{\Delta} V_M^T = C. \tag{2.7}$$

2.2. Flavor symmetry implies V_M as correlation matrix

In the quark sector we introduce Y_u and Y_d to be the Yukawa matrices for up and down sectors. They can be diagonalized by

$$Y_u = U_u Y_u^{\Delta} V_u^{\dagger}, \qquad Y_d = U_d Y_d^{\Delta} V_d^{\dagger}, \tag{2.8}$$

where the Y^{Δ} are diagonal and the Us and Vs are unitary matrices.

Then the quark mixing matrix is given by

$$U_{\text{CKM}} = U_u^{\dagger} U_d. \tag{2.9}$$

To relate the U_{CKM} with the U_{PMNS} normally, one makes use of GUT models, such as generic SO(10) or E_6, where there are some natural Yukawa unifications. In fact these cases give an interesting relation between the U_{CKM} quark mixing matrix, the U_{PMNS} lepton mixing matrix, and V_M obtained from (2.6). The mixing matrix V_M turns out to be the correlation matrix defined in (2.15). The reason for it is that in SO(10) or E_6 one has intriguing relations between the Yukawa couplings of the quark sector and that of the lepton sector. For instance, in minimal renormalizable SO(10) with Higgs in the **10**, **126**, and **120**, we can have $Y_l \approx Y_d^T$.

However this feature is much more general and may depend on the flavor symmetry instead of the gauge grand unification. The presence of a flavor symmetry usually implies the structure of the Yukawa matrices and the equivalent entries of Y_l and Y_d are of the same order of magnitude. We conclude that, as long as the flavor symmetry fully constraints the mixing matrices that diagonalize the Yukawa matrices, we have $U_l \simeq V_d^{\star}$. Notice that if there is a flavor symmetry that constrains the Yukawa couplings in such a way that the diagonalizing unitary matrices are fixed, then the entries of Y_l can still be very different from the entries of Y_d^T. However both Yukawa matrices are diagonalized by the same mixing matrices. This is exactly the case in the presence of an A_4 discrete flavor symmetry dinamically broken into Z_3 [5–7] and can be partially true in the case of S_3 softly broken into S_2 [8, 10]. From (2.5) we get

$$U_{\text{PMNS}} \simeq V_d^T U_0 V_M. \tag{2.10}$$

If we denote by Y_ν the Yukawa coupling that generates the Dirac neutrino mass matrix M_D, we have also the relation

$$Y_\nu \approx Y_u^T \longrightarrow U_0 \simeq V_u^{\star}. \tag{2.11}$$

This relation, together with the previous one, implies

$$U_{\text{PMNS}} \simeq V_d^T V_u^{\star} V_M. \tag{2.12}$$

If the Yukawa matrices are diagonalized by a similar matrix on the left and on the right, for example, in minimal renormalizable SO(10) with only small contributions from the antisymmetric representations such as **120** or more important in models where the diagonalization is strongly constrained by the flavor symmetry, the previous relationship translates into a relation between U_{PMNS}, U_{CKM}, and V_M. In fact we have

$$Y_u \simeq Y_u^T \longrightarrow V_u^{\star} = U_u, \qquad Y_d \simeq Y_d^T \longrightarrow V_d^{\star} = U_d. \tag{2.13}$$

The first relation tells us that

$$U_{\text{PMNS}} = V_d^T U_u V_M. \tag{2.14}$$

Finally, using the second relation in (2.15) and the definition of the CKM mixing matrix of (2.9) we get

$$V_M = U_{\text{CKM}} \cdot \Omega \cdot U_{\text{PMNS}}, \tag{2.15}$$

where we introduced the matrix

$$\Omega = \text{diag}\left(e^{i\omega_1}, e^{i\omega_2}, e^{i\omega_3}\right) \tag{2.16}$$

to allow us to write the CKM and PMNS matrices in their standard form (i.e., three rotation angles and one phase for the CKM and the equivalent for the PMNS) and to take into account the phase mismatching between quarks and leptons. The form of V_M can be obtained under some assumptions about the flavor structure of the theory. Some flavor models give, for example, a correlation V_M with $(V_M)_{13} = 0$. As a consequence of the from of the nontrivial quark-lepton complementarity, there are some predictions from the model, such as for $\theta_{13}^{\text{PMNS}}$ from [4] and the correlations between CP violating phases and the mixing angle θ_{12} of [3].

3. The Observables

As explained in the introduction, in this work we are interested in extracting informations from nontrivial quark-lepton complementarity and flavor symmetry about the $l_i \to l_j \gamma$ decays. We report here the usual formula obtained in the literature on these processes. It is obtained in the weak-eigenstate neutrino base, where charged lepton and Majorana right-handed neutrino mass matrices and weak interactions are diagonal. These processes depend on $\widetilde{M}_D$, the Dirac neutrino mass in the weak base.

3.1. $l_i \to l_j \gamma$

The contribution at first-order approximation to the process $l_i \to l_j \gamma$ in SUSY models is given by

$$\text{BR}(l_i \longrightarrow l_j \gamma) \propto \frac{\Gamma(l_i \longrightarrow e\nu\nu)}{\Gamma(l_i)} \frac{\alpha^3}{G_f m_s^8 v_u^4} \tan^2\beta \left(\frac{3m_0 + A_0}{8\pi^2}\right)^2 \left|\left(\widetilde{M}_D L \widetilde{M}_D^\dagger\right)_{ij}\right|^2, \tag{3.1}$$

where m_0 is the universal scalar mass, A_0 is the universal trilinear coupling parameter, $\tan\beta$ is the ratio of the vacuum expectation values of the up and down Higgs doublets, and m_s is a typical mass of superparticles with [16] $m_s^8 \approx 0.5 m_0^2 M_{1/2}^2 (m_0^2 + 0.6 M_{1/2}^2)^2$, where $M_{1/2}$ is the gaugino mass. The matrix $L_{ij} = \mathbf{1}_{ij} \log M_x / M_i$ takes into account the RGE effects on the Majorana right-handed neutrino masses. In fact, (3.1) is computed in the base where the Yukawa of the charged lepton and the Majorana neutrino mass are diagonal. Equation (3.1) is valid in the base where right-handed Majorana neutrino mass matrix, charged lepton mass matrix, and weak-gauge interactions are diagonal. The experimental limit for the branching ratio of $\mu \to e\gamma$ is 1.2×10^{-11} at 90% of confidence level [17] and it could go down to 10^{-14} as proposed by MEG collaboration.

4. $\widetilde{M}_D$ from nontrivial quark-lepton complementarity and flavor symmetry

Let us investigate the value of Dirac neutrino mass matrix $\widetilde{M}_D$ in the base where right-handed Majorana neutrino mass matrix, charged leptons mass matrix, and weak-gauge interactions are diagonal. The part of the standard model Lagrangian containing the leptons is

$$\mathcal{L} = \overline{\nu}_L Y_D \nu_R H + \nu_R^T C M_R \nu_R + \bar{l}_L Y_l l_R H + \overline{\nu}_L \,\not{W}\, l_L. \tag{4.1}$$

We want to redefine the fields in such a way that the only source of flavor violation is in the Dirac neutrino Yukawa coupling. We introduce the following definitions:

$$l'_R = V_l^\dagger l_R, \qquad \nu'_R = V_R^T \nu_R, \qquad l'_L = U_l^\dagger l_L, \qquad \nu'_L = U_l^\dagger \nu_L, \tag{4.2}$$

where the unitary matrices V_l, U_l are defined in (2.1). The unitary matrix V_R is defined by the diagonalization of M_R

$$V_R M_R^\Delta V_R^T = M_R. \tag{4.3}$$

Consequently we have

$$\begin{aligned} l_R &= V_l\, l'_R, \qquad \nu_R = V_R^*\, \nu'_R, \qquad l_L = U_l\, l'_L, \\ \nu_R^T &= (\nu'_R)^T V_R^\dagger, \qquad \bar{l}_L = \bar{l}'_L U_l^\dagger, \qquad \overline{\nu}_L = \overline{\nu}'_L U_l^\dagger. \end{aligned} \tag{4.4}$$

In this primed base we get

$$\mathcal{L} = \overline{\nu}'_L U_l^\dagger M_D V_R^* \nu'_R + (\nu'_R)^T C M_R^\Delta \nu'_R + \bar{l}'_L M_l^\Delta l'_R + \overline{\nu}'_L \,\not{W}\, l'_L \tag{4.5}$$

and we define

$$\widetilde{M}_D = U_l^\dagger M_D V_R^\star. \tag{4.6}$$

We want now to relate this $\widetilde{M}_D$ matrix to the CKM mixing matrix by using the nontrivial quark-lepton complementarity and flavor symmetry. First of all we rewrite this matrix as

$$\widetilde{M}_D = U_l^\dagger M_D V_R^\star = U_l^\dagger U_0 M_D^\Delta V_0^\dagger V_R^\star. \tag{4.7}$$

Then we notice that the matrix $V_0^\dagger V_R^\star$ is related via the $\mathcal{C}$ matrix to the diagonal low-energy neutrino mass matrix m_{low}^Δ and to V_M. In fact we have

$$V_M m_{\text{low}}^\Delta V_M^T = \mathcal{C} = M_D^\Delta V_0^\dagger \frac{1}{M_R} V_0^\star M_D^\Delta = M_D^\Delta V_0^\dagger V_R^\star \frac{1}{M_R^\Delta} V_R^T V_0^\star M_D^\Delta, \tag{4.8}$$

where we used the inverse of (4.3):

$$V_R^\star \frac{1}{M_R^\Delta} V_R^\dagger = \frac{1}{M_R}. \tag{4.9}$$

We multiply on the left and on the right both sides of (4.8) by $1/M_D^\Delta$ and we get

$$V_0^\dagger V_R^\star \frac{1}{M_R^\Delta} V_R^T V_0^\star = \frac{1}{M_D^\Delta} V_M m_{\text{low}}^\Delta V_M^T \frac{1}{M_D^\Delta}. \tag{4.10}$$

If one uses the method of [18] one can extract the matrix $V_0^\dagger V_R^\star$ by making the *square root* of the matrices in (4.10). One has

$$V_0^\dagger V_R^\star \sqrt{\frac{1}{M_R^\Delta}} = \frac{1}{M_D^\Delta} V_M \sqrt{m_{\text{low}}^\Delta} R^T, \tag{4.11}$$

where R is a complex orthogonal matrix such that $R^T R = \mathbf{1}$, and one obtains

$$V_0^\dagger V_R^\star = \frac{1}{M_D^\Delta} V_M \sqrt{m_{\text{low}}^\Delta} R^T \sqrt{M_R^\Delta}. \tag{4.12}$$

Finally one concludes that

$$\widetilde{M}_D = U_l^\dagger U_0 M_D^\Delta \frac{1}{M_D^\Delta} V_M \sqrt{m_{\text{low}}^\Delta} R^T \sqrt{M_R^\Delta} \tag{4.13}$$

$$= U_{\text{PMNS}} \sqrt{m_{\text{low}}^\Delta} R^T \sqrt{M_R^\Delta}. \tag{4.14}$$

Notice that in (4.14) the matrix V_M does not appear, and any information from V_M is hidden into the R matrix.

In our discussion, however, (4.10) unequivocally fixes $V_0^\dagger V_R^\star$ and the R matrix, once we know the eigenvalues of the Dirac neutrino mass matrix and the low-energy neutrino spectrum. In fact the V_M matrix is assumed to be known because of the nontrivial quark-lepton complementarity. Once we computed the $V_0^\dagger V_R^\star$ matrix form (4.10), by using (4.7), we get

$$\begin{aligned} \widetilde{M}_D &= U_l^\dagger U_0 M_D^\Delta V_0^\dagger V_R^\star \\ &= U_{\text{PMNS}} V_M^\dagger M_D^\Delta V_0^\dagger V_R^\star \\ &= \Omega^\dagger U_{\text{CKM}}^\dagger M_D^\Delta V_0^\dagger V_R^\star, \end{aligned} \tag{4.15}$$

where in the last line we used the relations in (2.5) and (2.15).

4.1. Full determination of $V_0^\dagger V_R^\star$ *and* M_R^Δ

Equation (4.15) is the equivalent of the general equation (4.14) in presence of nontrivial quark-lepton complementarity and flavor symmetry. We observe that the main modification is the presence of $U_{\text{CKM}}^\dagger$ instead of U_{PMNS}, thanks to the fact that these matrices are related to each other through V_M as shown in (2.15). Moreover the R is absent and is substantially substituted by the *known* $V_0^\dagger V_R^\star$ matrix, computed with (4.10). Let us now compute the $V_0^\dagger V_R^\star$ matrix in a general scenario.

In the following we use the experimental constraint from [4] that says that $(V_M)_{13}$ is zero. With this single constraint on V_M we write

$$V_M = \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12}\cos\theta_{23} & \cos\theta_{12}\cos\theta_{23} & \sin\theta_{23} \\ \sin\theta_{12}\sin\theta_{23} & -\cos\theta_{12}\sin\theta_{23} & \cos\theta_{23} \end{pmatrix} \tag{4.16}$$

and the allowed ranges for $\theta_{12}^{V_M}$ and $\theta_{23}^{V_M}$ are [4]

$$\tan^2\theta_{12}^{V_M} \in [0.3, 1.0], \qquad \tan^2\theta_{23}^{V_M} \in [0.5, 1.4]. \tag{4.17}$$

Let us denote by m_i the complex low-energy neutrino masses obtained after the see-saw ($m_{\text{low}}^{\Delta} = \{m_1, m_2, m_3\}$), and M_i the eigenvalues of the Dirac neutrino mass matrix M_D ($M_D^{\Delta} = \{M_1, M_2, M_3\}$). We have $V_M m_{\text{low}}^{\Delta} V_M^T$ equal to

$$\begin{pmatrix} (m_1 c_{12}^2 + m_2 s_{12}^2) & -(m_1 - m_2)c_{12}c_{23}s_{12} & (m_1 - m_2)c_{12}s_{12}s_{23} \\ -(m_1 - m_2)c_{12}c_{23}s_{12} & (m_1 s_{12}^2 c_{23}^2 + m_2 c_{12}^2 c_{23}^2 + m_3 s_{23}^2) & s_{23}c_{23}(m_3 - m_2 c_{12}^2 - m_1 s_{12}^2) \\ (m_1 - m_2)c_{12}s_{12}s_{23} & s_{23}c_{23}(m_3 - m_2 c_{12}^2 - m_1 s_{12}^2) & s_{23}^2(m_1 s_{12}^2 + m_2 c_{12}^2) + m_3 c_{23}^2 \end{pmatrix} \tag{4.18}$$

and from (4.10) we get

$$\begin{pmatrix} \dfrac{(m_1 c_{12}^2 + m_2 s_{12}^2)}{M_1^2} & \dfrac{-(m_1 - m_2)c_{12}c_{23}s_{12}}{M_1 M_2} & \dfrac{(m_1 - m_2)c_{12}s_{12}s_{23}}{M_1 M_3} \\ \dfrac{-(m_1 - m_2)c_{12}c_{23}s_{12}}{M_1 M_2} & \dfrac{(m_1 s_{12}^2 c_{23}^2 + m_2 c_{12}^2 c_{23}^2 + m_3 s_{23}^2)}{M_2^2} & \dfrac{s_{23}c_{23}(m_3 - m_2 c_{12}^2 - m_1 s_{12}^2)}{M_2 M_3} \\ \dfrac{(m_1 - m_2)c_{12}s_{12}s_{23}}{M_1 M_3} & \dfrac{s_{23}c_{23}(m_3 - m_2 c_{12}^2 - m_1 s_{12}^2)}{M_2 M_3} & \dfrac{s_{23}^2(m_1 s_{12}^2 + m_2 c_{12}^2) + m_3 c_{23}^2}{M_3^2} \end{pmatrix}. \tag{4.19}$$

Equation (4.19) is general and must be specified depending on the explicit form of V_M. For example for V_M tribimaximal we get

$$V_0^{\dagger} V_R^{\star} \frac{1}{M_R^{\Delta}} V_R^{\dagger} V_0^{\star} = \begin{pmatrix} \dfrac{2m_1 + m_2}{3M_1^2} & \dfrac{m_1 - m_2}{3M_1 M_2} & \dfrac{m_1 - m_2}{3M_1 M_3} \\ \dfrac{m_1 - m_2}{3M_1 M_2} & \dfrac{m_1 + 2m_2 + 3m_3}{6M_2^2} & \dfrac{m_1 + 2m_2 - 3m_3}{6M_2 M_3} \\ \dfrac{m_1 - m_2}{3M_1 M_3} & \dfrac{m_1 + 2m_2 - 3m_3}{6M_2 M_3} & \dfrac{m_1 + 2m_2 + 3m_3}{6M_3^2} \end{pmatrix}, \tag{4.20}$$

where we remind the reader that m_i are complex numbers, and their sign is not defined.

4.2. *Hierarchical* M_D

First of all let us investigate the case where the M_D eigenvalues have a hierarchical structure as well as any other Dirac mass matrix M_u, M_d, M_l. As it is well known in this case the heavy neutrino masses are very hierarchical and the lighter one is very light compared to the unification scale. For example if we take the eigenvalues of the Dirac mass matrix M_D to be $M_3\{\lambda^{2n},\lambda^n,1\}$ with n of order 1, we get (We neglect here the cases $m_1 \simeq m_2\tan^2\theta_{12}$ and $m_3\tan^2\theta_{23} \simeq m_1m_2/(m_1\cos\theta_{12}+m_2\sin\theta_{12})$)

$$\frac{1}{M_R^\Delta} = \begin{pmatrix} \frac{m_\alpha}{(\lambda^{4n}M_3^2)} & 0 & 0 \\ 0 & \frac{m_\beta}{(\lambda^{2n}M_3^2)} & 0 \\ 0 & 0 & \frac{m_\gamma}{M_3^2} \end{pmatrix}(1+O(\lambda)),$$
$$V_0^\dagger V_R^\star = \begin{pmatrix} \frac{1-\alpha^2\lambda^{2n}}{2} & \alpha\lambda^n & \beta\lambda^{2n} \\ -\alpha\lambda^n & 1-(\alpha^2+\gamma^2)\lambda^{2n} & \gamma\lambda^n \\ (-\beta+\alpha\gamma)\lambda^{2n} & -\gamma\lambda^n & \frac{1-\gamma^2\lambda^{2n}}{2} \end{pmatrix} + O(\lambda^{3n}), \tag{4.21}$$

where

$$\begin{aligned} m_\alpha &= m_1\cos^2\theta_{12} + m_2\sin^2\theta_{12} + O(\lambda^{2n}), \\ m_\beta &= \frac{m_1m_2}{m_\alpha}\cos^2\theta_{23} + m_3\sin^2\theta_{23} + O(\lambda^{2n}), \\ m_\gamma &= \frac{m_1m_2m_3}{m_\alpha m_\beta}, \\ \alpha &= -\frac{(m_1-m_2)}{2m_\alpha}\sin(2\theta_{12})\cos\theta_{23} + O(\lambda^{2n}), \\ \gamma &= \frac{m_1m_2 - m_3m_\alpha}{2m_\alpha m_\beta}\sin(2\theta_{23}) + O(\lambda^{2n}), \\ \beta &= \frac{(m_1-m_2)}{2m_\alpha}\sin(2\theta_{12})\sin\theta_{23} + O(\lambda^{2n}). \end{aligned} \tag{4.22}$$

The numbers α, β, γ are of order 1 but the corresponding angles must be computed up to order λ^{6n} to obtain the right heavy neutrino masses. The parameters m_α, m_β, m_γ are of order of the low-energy neutrino masses. Notice that the rotation angles $(1,2)$ and $(2,3)$ in $V_0^\dagger V_R^\star$ are of order λ^n while the $(1,3)$ angle is of order λ^{2n}.

We observe that in this scenario, with hierarchical Dirac neutrino eigenvalues, the result depends on the explicit value of the angle $\theta_{12}^{V_M}$ and $\theta_{23}^{V_M}$ only at higher order in λ and via the value of m_α, m_β, m_γ. For example, if the (2,3) angle of V_M is $\pi/4$ (i.e., for V_M maximal) we obtain

$$\begin{aligned}
m_\alpha &= m_1 \cos^2\theta_{12} + m_2 \sin^2\theta_{12} + O(\lambda^{2n}),\\
m_\beta &= \frac{m_1 m_2 + m_3 m_\alpha}{2m_\alpha} + O(\lambda^{2n}),\\
m_\gamma &= \frac{m_1 m_2 m_3}{m_\alpha m_\beta},\\
\alpha &= -\frac{\sqrt{2}(m_1 - m_2)\sin(2\theta_{12})}{4m_\alpha} + O(\lambda^{2n}),\\
\gamma &= 1 - \frac{m_\gamma}{m_3} + O(\lambda^{2n}),\\
\beta &= -\alpha + O(\lambda^{2n}),
\end{aligned} \tag{4.23}$$

and for V_M tribimaximal we get

$$\begin{aligned}
m_\alpha &= \frac{2m_1 + m_2}{3} + O(\lambda^{2n}),\\
m_\beta &= \frac{3m_1 m_2 + 2m_1 m_3 + m_2 m_3}{2(2m_1 + m_2)} + O(\lambda^{2n}),\\
m_\gamma &= \frac{6m_1 m_2 m_3}{3m_1 m_2 + 2m_1 m_3 + m_2 m_3}.
\end{aligned} \tag{4.24}$$

For any V_M, the heavy neutrino spectrum is hierarchical with ratios given mainly by

$$M_1^R : M_2^R : M_3^R \simeq (M_1)^2 : (M_2)^2 : (M_3)^2. \tag{4.25}$$

In fact on one hand we have that, for normal low-energy neutrino hierarchy, m_α is of order m_2, m_β is of order m_3, and m_γ is of order m_1. Then we obtain

$$\frac{|m_\alpha|}{\lambda^{4n}} \gg \frac{|m_\beta|}{\lambda^{2n}} \gg |m_\gamma|. \tag{4.26}$$

On the other hand, for inverted low-energy neutrino hierarchy, m_α is of order m_2, m_β is of order m_1 ($\approx m_2$), and m_γ is of order m_3 ($< m_1, m_2$) and then

$$\frac{|m_\alpha|}{\lambda^{4n}} \gg \frac{|m_\beta|}{\lambda^{2n}} \gg |m_\gamma|. \tag{4.27}$$

Moreover the mixing matrix $V_0^\dagger V_R^\star$ is close to the identity. Notice that the lightest right-handed neutrino has a mass smaller than $M_{\text{Planck}}(M_1/M_3)^2$ if we want the mass of the heaviest right-handed neutrino to be smaller than M_{Planck}.

4.3. Degenerate M_D

Notice that the fact that the nontrivial quark-lepton complementarity can come from a flavor symmetry implies that the Dirac neutrino may have a different hierarchical structure than the up sector, as clarified in Section 2.2. For example the same argument applies to the charged lepton and down sectors, where we know that the hierarchical structure differs from each other. The idea beyond this fact, as explained in Section 2, is that the quark-lepton complementarity comes both from a unified-gauge theory and from a flavor theory. It is supposed that, as the recent progresses shown by [3–10, 19], the nature of the mixing angles and that of the mass come from different type of flavor symmetries. For this reason, the nontrivial quark-lepton complementarity can survive even if there is no Yukawa matrices unification. The important point is that the mixing in the Yukawa are related among them. In Section 2 we assumed these relations, but from recent literature about flavor physics we know that this is the case.

4.3.1. Nondegenerate m_{low}

If the Dirac neutrino mass eigenvalues are degenerate then, from (4.10), we obtain

$$V_0^\dagger V_R^\star \frac{1}{M_R^\Delta} V_R^\dagger V_0^\star \simeq V_M \frac{1}{M_D^\Delta} m_{\text{low}}^\Delta \frac{1}{M_D^\Delta} V_M^T. \tag{4.28}$$

In this case, if the low-energy neutrino masses are not degenerate, $V_0^\dagger V_R^\star$ is close to V_M and $M_R^\Delta \simeq m_{\text{low}}^\Delta/(M_D^\Delta)^2$. Let us define $\delta M_i = M_3 - M_i$. By performing the full computation up to orders $(\delta M_i/M_3)^2$, we get

$$\frac{1}{M_R^\Delta} \simeq \begin{pmatrix} \dfrac{m_\alpha}{M_3^2} & 0 & 0 \\ 0 & \dfrac{m_\beta}{M_3^2} & 0 \\ 0 & 0 & \dfrac{m_\gamma}{M_3^2} \end{pmatrix},$$
$$V_0^\dagger V_R^\star \simeq V_M \begin{pmatrix} \dfrac{1-\alpha^2}{2} & \alpha & \beta \\ -\alpha & 1-(\alpha^2+\gamma^2) & \gamma \\ (-\beta+\alpha\gamma) & -\gamma & \dfrac{1-\gamma^2}{2} \end{pmatrix} \equiv V_M V_\epsilon, \tag{4.29}$$

where

$$\begin{aligned}
m_\alpha &\simeq m_1\left(1-\frac{\delta M_1}{M_3}\left(1+\frac{\cos(2\theta_{12})}{2}\right)+\frac{\delta M_2}{M_3}\left(-\frac{1-\cos(2\theta_{12})}{2}-\cos(2\theta_{23})\sin^2\theta_{12}\right)\right),\\
m_\beta &\simeq m_2\left(1-\frac{\delta M_1}{M_3}\left(1-\frac{\cos(2\theta_{12})}{2}\right)+\frac{\delta M_2}{M_3}\left(-\frac{1-\cos(2\theta_{12})}{2}-\cos(2\theta_{23})\cos^2\theta_{12}\right)\right),\\
m_\gamma &\simeq m_3\left(1-\frac{\delta M_2}{M_3}(1+\cos(2\theta_{23}))\right),\\
\alpha &\simeq -\frac{m_1+m_2}{4(m_1-m_2)}\frac{2\delta M_1-\delta M_2-\delta M_2\cos(2\theta_{23})}{M_3}\sin(2\theta_{12}),\\
\gamma &\simeq \frac{m_2+m_3}{2(m_2-m_3)}\frac{\delta M_2}{M_3}\sin(2\theta_{23})\cos\theta_{12},\\
\beta &\simeq \frac{m_1+m_3}{2(m_1-m_3)}\frac{\delta M_2}{M_3}\sin(2\theta_{23})\sin\theta_{12}.
\end{aligned} \tag{4.30}$$

The parameters m_α, m_β, m_γ are of order of the low-energy neutrino masses. The angles α, β, γ are of order $\delta M_i/M_3$ with the exception of degenerate low-energy neutrino masses. In this case α is enhanced by a factor $m^2/\delta m_{12}^2$ while the other two angles β and γ have a factor $m^2/\delta m_{13}^2$, and our approach here is not valid any more because the three angles can be small only if the degeneracy of the Dirac neutrino eigenvalues is such that $\delta M_i/M < 10^{-5}$. We notice that there is not any substantial difference for normal ($m_1 < m_2 < m_3$) or inverted hierarchy ($m_3 < m_1 < m_2$) of the low-energy neutrino masses, and the only effect is to change the signs of β and γ angles.

From (4.15) we get

$$\widetilde{M}_D = \Omega^\dagger U_{\mathrm{CKM}}^\dagger M_D^\Delta V_M V_\epsilon, \tag{4.31}$$

and $\widetilde{M}_D$ can be computed using the expressions in (4.30) and U_{CKM}. Notice that in this case the resulting $\widetilde{M}_D$ strongly depends on the V_M matrix.

For any V_M, the heavy neutrino spectrum is degenerate. However the mixing matrix $V_0^\dagger V_R^\star$ is close to the V_M matrix.

4.3.2. Degenerate m_{low}

If the low-energy neutrino masses m_i and the Dirac neutrino eigenvalues are degenerate then we get

$$V_0^\dagger V_R^\star \frac{1}{M_R^\Delta} V_R^\dagger V_0^\star \simeq \frac{1}{M_D^\Delta} m_{\mathrm{low}}^\Delta \frac{1}{M_D^\Delta}. \tag{4.32}$$

In this case the value of V_M plays a marginal role. The mixing matrix $V_0^\dagger V_R^\star$ is close to a small rotation in the $(1,3)$ plane and the heavy neutrino spectrum is degenerate too:

$$\begin{aligned}
M_1^R &= \frac{m}{M^2}\left(1 - \frac{\delta M_1}{M}\left(1 + \sqrt{1 - \frac{1}{3}\frac{\sqrt{\delta m_{\mathrm{sol}}^2}/m}{\delta M_1/M} + \frac{\delta m_{\mathrm{sol}}^2/m^2}{(\delta M_1/M)^2}}\right)\right),\\
M_2^R &= \frac{m}{M^2}\left(1 - 2\frac{\delta M_2}{M} + \frac{\sqrt{\delta m_{\mathrm{atm}}^2}}{m}\right),\\
M_3^R &= \frac{m}{M^2}\left(1 - \frac{\delta M_1}{M}\left(1 - \sqrt{1 - \frac{1}{3}\frac{\sqrt{\delta m_{\mathrm{sol}}^2}/m}{\delta M_1/M} + \frac{\delta m_{\mathrm{sol}}^2/m^2}{(\delta M_1/M)^2}}\right)\right).
\end{aligned} \tag{4.33}$$

For any V_M compatible with the experiments, the heavy neutrino spectrum is almost degenerate. Moreover the mixing matrix $V_0^\dagger V_R^\star$ is close to the identity matrix.

5. Contribution to $l_i \to l_j \gamma$

Using the result in (4.15) and the general equation (3.1), we get

$$\mathrm{BR}(l_i \longrightarrow l_j \gamma) \propto \left| \left(\Omega^\dagger U_{\mathrm{CKM}}^\dagger M_D^\Delta V L V^\dagger M_D^\Delta U_{\mathrm{CKM}} \Omega\right)_{ij} \right|^2, \tag{5.1}$$

where $V = V_0^{\dagger} V_R^{\star}$ is the mixing matrix computed with (4.10). Notice that the Ω phase differences $\exp^{i(\phi_i - \phi_j)}$ cancel because we take the absolute value. We want to stress here that the result in (5.1) depends on the quark-lepton complementarity (and the underlying flavor symmetry) assumption only, and not on the explicit form of the correlation matrix V_M.

At zero approximation we neglect the different normalizations for different right-handed neutrinos. We assume that $L = \widehat{L} = 1 \log M_X/M_R$, where M_R is the common heavy neutrino mass. The BR($\mu \to e\gamma$) can be rewritten as

$$\begin{aligned} \mathrm{BR}(\mu \longrightarrow e\gamma) &\propto \frac{\Gamma(\mu \longrightarrow e\nu\nu)}{\Gamma(\mu)} \frac{\alpha^3}{G_f m_s^8 v_u^4} \tan^2\beta \left(\frac{3m_0 + A_0}{8\pi^2}\right)^2 \widehat{L}^2 \left| \left(U_{\mathrm{CKM}}^{\dagger} \left(M_D^{\Delta}\right)^2 U_{\mathrm{CKM}}\right)_{21} \right|^2 \\ &= \frac{\Gamma(\mu \longrightarrow e\nu\nu)}{\Gamma(\mu)} \frac{\alpha^3}{G_f m_s^8 v_u^4} \tan^2\beta \left(\frac{3m_0 + A_0}{8\pi^2}\right)^2 \widehat{L}^2 \\ &\quad \times \left| \left(M_2^2 - M_1^2\right)\lambda\left(1 + O\left(\lambda^2\right)\right) + M_3^2 A^2 (\rho - i\eta)\lambda^5 \left(1 + O\left(\lambda^6\right)\right) \right|. \end{aligned} \tag{5.2}$$

where λ is the sine of the Cabibbo angle, and A, ρ, and η are the other parameters of the unitary CKM matrix. We introduced only the first contribution of each Dirac neutrino eigenvalue. Similarly to the process $\mu \to e\gamma$ we can compute the contribution to the τ decays. For $\tau \to e\gamma$ we get

$$\begin{aligned} \mathrm{BR}(\tau \longrightarrow e\gamma) &\propto \frac{\Gamma(\tau \longrightarrow e\nu\nu)}{\Gamma(\tau)} \frac{\alpha^3}{G_f m_s^8 v_u^4} \tan^2\beta \left(\frac{3m_0 + A_0}{8\pi^2}\right)^2 \widehat{L}^2 \\ &\quad \times \left| \left((1 - (\rho - i\eta))M_1^2 - M_2^2 + M_3^2 (\rho - i\eta)\right) A\lambda^3 \left(1 + O\left(\lambda^2\right)\right) \right|^2. \end{aligned} \tag{5.3}$$

The other τ decay process that violates the individual lepton number is such that

$$\begin{aligned} \mathrm{BR}(\tau \longrightarrow \mu\gamma) &\propto \frac{\Gamma(\tau \longrightarrow \mu\nu\nu)}{\Gamma(\tau)} \frac{\alpha^3}{G_f m_s^8 v_u^4} \tan^2\beta \left(\frac{3m_0 + A_0}{8\pi^2}\right)^2 \widehat{L}^2 \\ &\quad \times \left| \left(-M_1^2\lambda^2 - M_2^2 + M_3^2\right) A\lambda^2 \left(1 + O\left(\lambda^2\right)\right) \right|^2. \end{aligned} \tag{5.4}$$

To understand the main contribution we must make some assumptions about the hierarchy of the Dirac neutrino masses M_i. Moreover to include the effect of nondegeneration for heavy neutrino masses we must include V, whose form depends also on the hierarchy of the low-energy neutrino masses.

5.1. Hierarchical M_D

For hierarchical M_D the factor L in (5.1) cannot be neglected. If we introduce the full form of L then the form of V is relevant. Under the assumption of hierarchical M_D, V is close to the identity and we get

$$\begin{aligned} \mathrm{BR}(\mu \longrightarrow e\gamma) &\propto \frac{\Gamma(\mu \longrightarrow e\nu\nu)}{\Gamma(\mu)} \frac{\alpha^3}{G_f m_s^8 v_u^4} \tan^2\beta \left(\frac{3m_0 + A_0}{8\pi^2}\right)^2 \\ &\quad \times \left| \left(M_2^2 \log \frac{M_X}{M_2^R} - M_1^2 \log \frac{M_X}{M_3^R} \right)\lambda + M_3^2 \log \frac{M_X}{M_1^R} A^2 (\rho - i\eta)\lambda^5 \right|^2, \end{aligned} \tag{5.5}$$

where we introduced the structure of L to take into account the hierarchical structure of heavy neutrino masses too. For example if we assume that

$$M_1 : M_2 : M_3 \propto m_u : m_b : m_t \tag{5.6}$$

at the unification scale, then we obtained in Section 4.3 that

$$M_1^R : M_2^R : M_3^R \propto m_u^2 : m_c^2 : m_t^2. \tag{5.7}$$

For the BR we have

$$\begin{aligned} \mathrm{BR}(\mu \longrightarrow e\gamma) \propto & \frac{\Gamma(\mu \longrightarrow e\nu\nu)}{\Gamma(\mu)} \frac{\alpha^3}{G_f m_s^8 v_u^4} \tan^2\beta \left(\frac{3m_0 + A_0}{8\pi^2}\right)^2 \log^2 \frac{M_X}{M_3} \left(\frac{M_3}{m_t}\right)^4 \\ & \times \left| m_c^2 \lambda \log \frac{m_t^2}{m_c^2} + m_t^2 \log \frac{m_t^2}{m_u^2} A^2 (\rho - i\eta) \lambda^5 \right|^2. \end{aligned} \tag{5.8}$$

Similarly to the process $\mu \rightarrow e\gamma$ we can compute the contribution to $\tau \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$. We get

$$\mathrm{BR}(\tau \longrightarrow e\gamma) \propto \frac{\Gamma(\tau \longrightarrow e\nu\nu)}{\Gamma(\tau)} \frac{\alpha^3}{G_f m_s^8 v_u^4} \tan^2\beta \left(\frac{3m_0 + A_0}{8\pi^2}\right)^2 \hat{L}^2 \left(\frac{M_3}{m_t}\right)^4 \left| m_t^2 \log \frac{m_t^2}{m_u^2} A (\rho - i\eta) \lambda^3 \right|^2, \tag{5.9}$$

where we used a hierarchical structure for the Dirac neutrino masses and introduced the structure of L. We observe that $\mathrm{BR}(\mu \rightarrow e\gamma)$ is suppressed by a factor λ^4 with respect to $\mathrm{BR}(\tau \rightarrow e\gamma)$.

The other τ decay is the least suppressed process that violates the individual lepton number. In fact we have

$$\mathrm{BR}(\tau \longrightarrow \mu\gamma) \propto \frac{\Gamma(\tau \longrightarrow \mu\nu\nu)}{\Gamma(\tau)} \frac{\alpha^3}{G_f m_s^8 v_u^4} \tan^2\beta \left(\frac{3m_0 + A_0}{8\pi^2}\right)^2 \hat{L}^2 \left(\frac{M_3}{m_t}\right)^4 \left| m_t^2 \log \frac{m_t^2}{m_u^2} A \lambda^2 \right|^2. \tag{5.10}$$

We observe that $\mathrm{BR}(\mu \rightarrow e\gamma)$ is in general suppressed by a factor λ^6 with respect to $\mathrm{BR}(\tau \rightarrow \mu\gamma)$, and $\mathrm{BR}(\tau \rightarrow \mu\gamma)$ by a factor λ^2. Our conclusions are equivalent to the one in [14, 15], and also in our analysis it can be a further suppression of the branching ratios if the leading term in (5.5) cancels. We can conclude that in this case, for general values of the SUSY parameters, the expected branching ratios are compatible with the actual experimental data, and will be observable only for high value of the low-energy neutrino masses and for particular point in the SUSY parameter space. However our discussion is more general since in fact we showed that these results do not depend on the form of the correlation matrix V_M.

5.2. Degenerate M_D

If we assume that the eigenvalues of the Dirac Yukawa matrix are degenerate, as computed in Section 4.3, we have two cases depending on the degeneration of m_{low}.

5.2.1. Nondegenerate m_{low}

For nondegenerate m_{low} we have the right-handed neutrinos with the same hierarchy of the low-energy neutrinos, and $V_0^\dagger V_R^\star$ close to V_M. In this case we get

$$\begin{aligned}
\text{BR}(\mu \longrightarrow e\gamma) &\propto \frac{\Gamma(\mu \longrightarrow e\nu\nu)}{\Gamma(\mu)} \frac{\alpha^3}{G_f m_s^8 v_u^4} \tan^2\beta \left(\frac{3m_0 + A_0}{8\pi^2}\right)^2 \\
&\quad \times \left| M_1 M_2 \log \frac{m_\beta}{m_\alpha} \cos\alpha_{12} \cos\alpha_{23} \sin\alpha_{12} \right|^2, \\
\text{BR}(\tau \longrightarrow e\gamma) &\propto \frac{\Gamma(\mu \longrightarrow e\nu\nu)}{\Gamma(\mu)} \frac{\alpha^3}{G_f m_s^8 v_u^4} \tan^2\beta \left(\frac{3m_0 + A_0}{8\pi^2}\right)^2 \\
&\quad \times \left| M_1 M_3 \log \frac{m_\beta}{m_\alpha} \cos\alpha_{12} \sin\alpha_{23} \sin\alpha_{12} \right|^2 \\
\text{BR}(\tau \longrightarrow \mu\gamma) &\propto \frac{\Gamma(\mu \longrightarrow e\nu\nu)}{\Gamma(\mu)} \frac{\alpha^3}{G_f m_s^8 v_u^4} \tan^2\beta \left(\frac{3m_0 + A_0}{8\pi^2}\right)^2 \\
&\quad \times \left| M_2 M_3 \cos\alpha_{23} \sin\alpha_{23} \left(\log \frac{m_\gamma}{m_\alpha} + \sin^2\alpha_{12} \log \frac{m_\beta}{m_\alpha} \right) \right|^2.
\end{aligned} \tag{5.11}$$

The ratios among them become of order one:

$$\begin{aligned}
\frac{\text{BR}(\mu \longrightarrow e\gamma)}{\text{BR}(\tau \longrightarrow e\gamma)} &\simeq \tan^2\alpha_{23} \in [0.5, 1.4], \\
\frac{\text{BR}(\mu \longrightarrow e\gamma)}{\text{BR}(\tau \longrightarrow \mu\gamma)} &\simeq \left| \frac{\cos\alpha_{12} \sin\alpha_{12}}{\sin\alpha_{23}\left(\left(\log(m_\gamma/m_\alpha)/\log(m_\beta/m_\alpha)\right) + \sin^2\alpha_{12}\right)} \right|^2.
\end{aligned} \tag{5.12}$$

We notice that in this case, with respect to the one considered in the previous section, the value of the branching ratio of $\mu \to e\gamma$ is bigger by a factor λ^6. So we obtain that, despite the fact that this case is the most promising to extract information on the structure of V_M, degenerate M_D and nondegenerate m_{low} are excluded by the experimental data for most of the SUSY parameters. Naturally one can fine tune the SUSY parameter and/or the neutrino mass parameters in such a way to escape from our general analysis.

5.2.2. Degenerate m_{low}

If the spectrum of the low-energy neutrino is degenerate, then the mixing matrix $V_0^\dagger V_R^\star$ becomes close to the identity. In this case the branching ratios depend on the common M_D mass and the Cabibbo parameter. By assuming (If this relation does not hold then we are in the case of degenerate M_D and nondegenerate m_{low}.) $M_2^2 - M_1^2 > \lambda^4 M_3^2$ we get

$$\begin{aligned}
\text{BR}(\mu \longrightarrow e\gamma) &\propto \left|\left(M_2^2 - M_1^2\right)\right|^2 \lambda^2 \\
\text{BR}(\tau \longrightarrow e\gamma) &\propto \left|\left((1 - (\rho - i\eta))M_1^2 - M_2^2 + M_3^2(\rho - i\eta)\right)\right|^2 \left(A\lambda^3\right)^2 \\
\text{BR}(\tau \longrightarrow \mu\gamma) &\propto \left|\left(M_3^2 - M_2^2\right)\right|^2 \left(A\lambda^2\right)^2,
\end{aligned} \tag{5.13}$$

and the ratios among them are

$$\mathrm{BR}(\mu \longrightarrow e\gamma) : \mathrm{BR}(\tau \longrightarrow e\gamma) : \mathrm{BR}(\tau \longrightarrow \mu\gamma) = 1 : \lambda^4 : \lambda^2. \tag{5.14}$$

To compare this case with the case of hierarchical M_D of Section 4.2, we observe that here $\mathrm{BR}(\mu\to e\gamma)$ is the largest one, while in the other case it is the smallest one. Moreover the value of the branching ratios here depends on the differences $M_i^2 - M_j^2$ and they are in general smaller than in the other case. For example, if $M_2^2 - M_1^2 \simeq \lambda^4 M_3^2$ and M_i are of order m_t, we obtain that

$$\begin{aligned}
\mathrm{BR}(\mu \longrightarrow e\gamma) &\propto \left(\frac{M_3}{m_t}\right)^4 \left|m_t^4 \lambda^5\right|^2, \\
\mathrm{BR}(\tau \longrightarrow e\gamma) &\propto \left(\frac{M_3}{m_t}\right)^4 \left|m_t^4 \lambda^7\right|^2, \\
\mathrm{BR}(\tau \longrightarrow \mu\gamma) &\propto \left(\frac{M_3}{m_t}\right)^4 \left|m_t^4 \lambda^6\right|^2.
\end{aligned} \tag{5.15}$$

In this case, not only we cannot extract information on the V_M structure, but also we have no hope to observe these branching ratios because they are too small even with respect to the future experimental sensitivities.

6. Conclusions

We analized the consequences of a nontrivial quark-lepton complementarity and a flavor symmetry on $\mathrm{BR}(l_i\to l_j\gamma)$. The nontrivial quark-lepton complementarity, together with the flavor symmetry, states that the correlation matrix V_M, product of the CKM and the PMNS mixing matrix, is related to the diagonalization of the Majorana right-handed and Dirac neutrino mass matrices. In this framework we obtained that $\mathrm{BR}(l_i\to l_j\gamma)$ is related to the CKM mixing matrix and the Dirac neutrino masses.

We have three cases as follows.

(1) Hierarchical Dirac neutrino eigenvalues (very hierarchical right-handed neutrino masses, $V_0^\dagger V_R^\star \simeq I$) where we get the usual ratios:

$$\mathrm{BR}(\mu \longrightarrow e\gamma) : \mathrm{BR}(\tau \longrightarrow e\gamma) : \mathrm{BR}(\tau \longrightarrow \mu\gamma) = \lambda^6 : \lambda^4 : 1 \propto M_3^4 \lambda^4 \widehat{L}. \tag{6.1}$$

This case is the most promising one for a future observation of the branching ratios. However it will not give us any information about the structure of the V_M matrix.

(2) Degenerate Dirac neutrino eigenvalues, with nondegenerate low-energy neutrino masses (the hierarchy of the right-handed neutrino masses is close to the one of the low-energy spectrum, $V_0^\dagger V_R^\star \simeq V_M$) where we get

$$\mathrm{BR}(\mu \longrightarrow e\gamma) = \tan^2\theta_{23}^{V_M} \mathrm{BR}(\tau \longrightarrow e\gamma) = f\left(\theta_{12}^{V_M}, \theta_{23}^{V_M}\right) \mathrm{BR}(\tau \longrightarrow \mu\gamma) \propto M_3^4 \widehat{L} \tag{6.2}$$

with $f(\theta_{12}^{V_M}, \theta_{23}^{V_M})$ of order one. This case is the only one where the structure of V_M plays a fundamental role in the determination of the branching ratios. However it is already excluded for a large part of the SUSY parameters space by the experimental limits.

(3) Degenerate Dirac neutrino eigenvalues and low-energy neutrino spectrum (right-handed neutrinos close to degenerate, $V_0^\dagger V_R^\star \simeq I$) where we have

$$\mathrm{BR}(\mu \longrightarrow e\gamma) : \mathrm{BR}(\tau \longrightarrow e\gamma) : \mathrm{BR}(\tau \longrightarrow \mu\gamma) = 1 : \lambda^4 : \lambda^2 \propto M_3^4 \lambda^{10} \widehat{L}. \tag{6.3}$$

In this case the branching ratios are too small even with respect to the future experimental sensitivities.

Acknowledgments

The authors would like to thank João Pulido for useful discussions about neutrino physics, and J.C. Romão for enlightening discussion about flavor violating processes in supersymmetry. We acknowledge the MEC-INFN Grant, and the Fundação para a Ciência e a Tecnologia for the Grant SFRH/BPD/25019/2005.

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