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Research Article

Quantum Brownian Representation for the Quantum Field Modes

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When analyzing the particle-like excitations in quantum field theory it is natural to regard the field mode corresponding to the particle momentum as an open quantum system, together with the opposite momentum mode. Provided that the state of the field is stationary, homogeneous, and isotropic, this scalar two-mode system can be equivalently represented in terms of a pair of quantum Brownian oscillators under a Gaussian approximation. In other words, the two-mode system behaves as if it were interacting linearly with some effective environment. In this paper we build the details of the effective linear coupling and the effective environment, and argue that this quantum Brownian representation provides a simple, universal, and nonperturbative characterization of any single particle-like excitation. As immediate applications of the equivalence, we reanalyze the interpretation of the self-energy in terms of decay rates in a general background state and present the master equation for the field mode corresponding to the particle momentum.

1. Introduction and Motivation

Quantum fields can be regarded from the viewpoint of open quantum systems [1–4]. The degrees of freedom of the field which are relevant for the physical problem in question constitute the reduced subsystem and the rest form the environment. If there are several fields in interaction, and the object of interest is a particular field, it is natural to trace over the other environment fields. For instance, in electrodynamics one can study the so-called Euler-Heisenberg effective action for the photons [5], considering the electrons as the environment, or the complementary case, in which the electrons are taken as the system of interest and the photons are integrated out [6–9]. Similarly, in stochastic gravity [10–21] the system of interest is the gravitational field and the matter fields are integrated out. In many other

circumstances it is natural to consider as the reduced system the modes of the quantum field which are below some ultraviolet cutoff, with the ultraviolet modes constituting the environment. This approach has been used, for instance, in studying bubble nucleation [22, 23], analyzing decoherence in field theory [24–26] and in inflationary cosmology [27–30]. As these references illustrate, the open quantum system point of view has often provided new tools and insights to different field theory problems, specially when dealing with states different than the Minkowski vacuum.

The propagation of a particle-like excitation over a given field background is another situation in which there is a natural system-environment separation: the field mode corresponding to the particle momentum is the object of interest, and the rest of modes of the field, along with any other field in interaction, form the environment. In the Minkowski vacuum, the Källén-Lehmann representation of the propagator [5, 31–33] provides a complete description of single particle excitations from the field theory perspective, but there was no equivalent analysis in thermal backgrounds [34–38], or in curved spacetimes [39–42]. The open quantum system approach proves a useful tool when reanalysing the propagation of particle-like excitations in nonvacuum or nonMinkowski backgrounds [43, 44].

Therefore, in this paper we elaborate on the open quantum description of the field mode corresponding to the particle momentum, with the goal of developing useful techniques for the analysis of particle and quasiparticle excitations. We concentrate on the basic methods and tools, and give example applications; the general analysis of the particle-like excitations in nonvacuum or nonflat backgrounds is left for separate publications [43, 44].

We would like to consider the field mode as the system and the remaining modes as the environment. However, it proves difficult to implement this system-environment division directly, since the mode-decomposed field $\phi_{\mathbf{p}}$ is not real, but is a complex quantity obeying the contraint $\phi_{\mathbf{p}} = \phi^*_{-\mathbf{p}}$ and acting on both the Hilbert space sector with momentum $\mathbf{p}$ and the sector with momentum $-\mathbf{p}$. Instead of focusing on a single mode, given that the field naturally links modes with opposite momentum, we will choose as the system of interest any two modes with a given opposite momentum, and as the environment the remaining modes of the field, as well as the modes of any other field in interaction. Namely, given a single scalar field ϕ, the system degrees of freedom are the two modes $\phi_{\mathbf{p}}$ and $\phi_{-\mathbf{p}}$, and the other modes $\phi_{\mathbf{q}}$, with $\mathbf{q} \neq \pm\,\mathbf{p}$, form the environment. A similar system-environment division has been used in [45–47] in an inflationary context.

The main goal of this paper is to show that this two-mode system behaves, under certain assumptions, as if it was an open quantum system interacting linearly with some effective environment, even if the original field interaction is nonlinear. The paradigm of linear open quantum system is the quantum Brownian motion (QBM) model, whose system of interest is a nonrelativistic particle interacting linearly with an infinite bath of harmonic oscillators. This model has had many applications in different contexts, among which one may mention the quantum to classical transition [48, 49], the escape from a potential well [50–53], the Unruh effect [54, 55] or quantum optics [3, 56]. In an influential paper Caldeira and Leggett [57] applied the influence functional model of Feynman and Vernon [58, 59] to the QBM model. The QBM model can be generalised to encompass the general class of linearly interacting open quantum systems [60, 61].

The results of this paper can bee seen as an explicit implementation of the statement by Hu and Matacz [55] that the motion of a Brownian particle can be used to depict the behaviour of a single quantum field mode.

We will allow for stationary and isotropic, but otherwise arbitrary, states for the field (a Gaussian approximation will be also assumed later on). Field theory with arbitrary field states can be studied within the closed time path (CTP), or *in-in*, method, which was originally proposed by Schwinger [62] and Keldysh [63]. The most characteristic feature of the CTP method, in contrast to the conventional *in-out* method, is the doubling of the number of degrees of freedom.

The paper is organised as follows. In Section 2, the relevant system-environment separation is discussed. In Section 3, the central section of this paper, we present and analyse the QBM representation for the field modes, given the structure of the generating functional and the two-point propagators. In Section 4, we illustrate the utility of the QBM representation with two-example applications. First, we rederive the interpretation of the imaginary part of the self-energy. Second, we build and analyze the master equation for the field modes corresponding to the particle momentum. The main body of the paper ends with Section 5, where we summarise and discuss the main results. In order not to break the continuity of the exposition, some details of the analysis are left for the appendices, which also provide background reference material which fixes the notation and makes the paper relatively self-contained. In Appendix A, we briefly present the CTP approach to field theory and apply it to the analysis of the structure of the two-point propagators, and in Appendix B, we introduce the theory of linear open quantum systems, focusing also in the structure of the two-point propagators. The analysis of the propagators done in the appendices is important for the discussion in Section 3.

Throughout the paper we work with a system of natural units with $\hbar = c = 1$, denote quantum mechanical operators with a hat, and use a volume-dependent normalisation in the definition of the field modes (see (2.1) below). The same symbol will be used for a quantity and its Fourier transform whenever there is no danger of confusion.

2. Field Modes Regarded as Open Quantum Systems

Let us now present the relevant system-environment separation. For concreteness, we consider a self-interacting field theory model consisting of a single scalar field ϕ, although results can be straightforwardly extended to any number of fields. The field ϕ can be decomposed in modes according to

$$\phi_{\mathbf{p}} = \frac{1}{\sqrt{V}} \int \mathrm{d}^3\mathbf{x}\, e^{-i\mathbf{p}\cdot\mathbf{x}} \phi(\mathbf{x}), \tag{2.1}$$

where V is the volume of the space, a formally infinite quantity. The factor $V^{-1/2}$ in the definition of $\phi_{\mathbf{p}}$ is chosen so that the propagators verify

$$G_+(t,t';\mathbf{p}) = \int \mathrm{d}^3\mathbf{x} e^{-i\mathbf{p}\cdot\mathbf{x}} \left\langle 0 \left| \hat{\phi}(t,\mathbf{x}) \hat{\phi}(t',\mathbf{0}) \right| 0 \right\rangle = \left\langle 0 \left| \hat{\phi}_{\mathbf{p}}(t) \hat{\phi}_{-\mathbf{p}}(t') \right| 0 \right\rangle. \tag{2.2}$$

As stated in the introduction, given a particular momentum $\mathbf{p} \neq \mathbf{0}$, the system is composed by the two modes $\phi_{\mathbf{p}}$ and $\phi_{-\mathbf{p}}$, and the environment is composed by the other modes of the field, $\phi_{\mathbf{q}}$, with $\mathbf{q} \neq \pm\mathbf{p}$. Should there be other fields in interaction of any arbitrary spin, the modes of these additional fields would also form part of the environment.

The Hilbert space can be decomposed as $\mathscr{H} = \mathscr{H}_{\rm sys} \otimes \mathscr{H}_{\rm env}$, where in turn $\mathscr{H}_{\rm sys} = \mathscr{H}_{\mathbf{k}} \otimes \mathscr{H}_{-\mathbf{k}}$. Notice that this separation does not correspond to the Fock space decomposition. The entire system is in a state $\widehat{\rho}$; the state of the reduced system is $\widehat{\rho}_{\rm s} = {\rm Tr}_{\rm env}\widehat{\rho}$, and the state of the environment is $\widehat{\rho}_{\rm e} = {\rm Tr}_{\rm sys}\widehat{\rho}$. Generally speaking, the state for the entire system is not a factorised product state (i.e., $\widehat{\rho} \neq \widehat{\rho}_{\rm s} \otimes \widehat{\rho}_{\rm e}$).

The action can be decomposed as $S = S_{\rm sys} + S_{\rm count} + S_{\rm env} + S_{\rm int}$, where $S_{\rm sys}$ is the renormalized system action,

$$S_{\rm sys} = \int {\rm d}t \left(\dot{\phi}_{\mathbf{p}} \dot{\phi}_{-\mathbf{p}} - E_{\mathbf{p}}^2 \phi_{\mathbf{p}} \phi_{-\mathbf{p}} \right), \tag{2.3a}$$

$S_{\rm count}$ is the appropriate counterterm action,

$$S_{\rm count} = \int {\rm d}t \left\{ (\mathcal{Z}_{\mathbf{p}} - 1) \dot{\phi}_{\mathbf{p}} \dot{\phi}_{-\mathbf{p}} - \left[\mathcal{Z}_{\mathbf{p}} \left(\mathbf{p}^2 + m_0^2 \right) - E_{\mathbf{p}}^2 \right] \phi_{\mathbf{p}} \phi_{-\mathbf{p}} \right\}, \tag{2.3b}$$

(with m_0 being the bare mass of the field), and $S_{\rm env}$ and $S_{\rm int}$ are the environment and interaction actions, respectively, which depend on the particular field theory model. Notice that we have allowed for an arbitrary rescaling of the field $\phi \to \phi / \mathcal{Z}_{\mathbf{p}}^{1/2}$ and for an arbitrary frequency of the two-mode system $E_{\mathbf{p}}$ (which needs not be necessarily of the form $(\mathbf{p}^2 + m^2)^{1/2}$).

Let us draw our attention on the field rescaling and the frequency renormalisation. Since it is always possible to freely move finite terms from the system to the counterterm action and *vice versa*, both the field rescaling and the frequency renormalisation should be taken into account even if no infinities appeared in the perturbative calculations. A physical criterion needs to be chosen in order to fix the values of these two parameters. In the vacuum, such criterion is provided by the on-shell renormalisation scheme; in nonvacuum situations, it is investigated in [43], and will be briefly discussed in Section 4. Notice that the form of $\mathcal{Z}_{\mathbf{p}}$ and $E_{\mathbf{p}}$ is not necessarily dictated by the Lorentz symmetry: even if the countertems which remove the infinities from the vacuum theory also remove the infinities in general field states, there can be finite Lorentz-breaking contributions. Anyway for the purposes of most of this paper the values of $\mathcal{Z}_{\mathbf{p}}$ and $E_{\mathbf{p}}$ are not relevant and will be left unspecified (some comments will be made in Section 4.2 though).

The system variables $\phi_{\mathbf{p}}$ and $\phi_{-\mathbf{p}}$ are complex quantities verifying $\phi_{-\mathbf{p}}^* = \phi_{\mathbf{p}}$. We can construct real degrees of freedom by introducing the following change of variables:

$$\phi_\Sigma = \frac{1}{\sqrt{2}} (\phi_{\mathbf{p}} + \phi_{-\mathbf{p}}), \qquad \phi_\Delta = \frac{-i}{\sqrt{2}} (\phi_{\mathbf{p}} - \phi_{-\mathbf{p}}). \tag{2.4}$$

In terms of the real variables, the system action can be re-expressed as

$$S_{\rm sys} = \frac{1}{2} \int {\rm d}t \left(\dot{\phi}_\Sigma^2 - E_{\mathbf{p}}^2 \phi_\Sigma^2 + \dot{\phi}_\Delta^2 - E_{\mathbf{p}}^2 \phi_\Delta \right). \tag{2.5}$$

We could alternatively have directly obtained these two real degrees of freedom by working with the sine and cosine Fourier transform [55]. However we prefer to work with the exponential Fourier transform to make manifest the momentum conservation properties.

Most information on the reduced quantum system can be extracted from the set of correlation functions, or equivalently from the CTP generating functional for the reduced system, which can be written as

$$Z[j_{a,\alpha}] = \exp\left[-\frac{1}{2!}\int \mathrm{d}t\,\mathrm{d}t'\, j^a_\alpha(t) j^b_\beta(t) G^{\alpha\beta}_{ab}(t,t') - \frac{1}{4!}\int \mathrm{d}t\,\mathrm{d}t'\,\mathrm{d}t''\,\mathrm{d}t'''\, j^a_\alpha(t) j^b_\beta(t') j^c_\gamma(t'') j^d_\delta(t''') G^{(\mathrm{C})\alpha\beta\gamma\delta}_{abcd}(t,t',t'',t''') + \cdots\right]. \tag{2.6}$$

This expression is somewhat cumbersome and needs some clarification. Latin indices $a, b, c, \ldots$ are CTP indices and take the values 1 and 2, indicating, respectively, the forward and backwards time branches characteristic of the CTP formalism (see Appendix A). Greek indices $\alpha, \beta, \gamma, \ldots$ take the two values $+\mathbf{p}$ and $-\mathbf{p}$ and make reference to the two field modes $\phi_{\mathbf{p}}$ and $\phi_{-\mathbf{p}}$. An Einstein summation convention is used both for Latin and Greek indices. The propagator $G^{\alpha\beta}_{ab}(t,t')$ is the 2-point propagator connecting CTP indices a and b, whose external legs correspond to particles with momenta α and β. When the state is translation-invariant and isotropic, momentum conservation imposes:

$$G^{(+\mathbf{p})(-\mathbf{p})}_{ab}(t,t') = G^{(-\mathbf{p})(+\mathbf{p})}_{ab}(t,t') = G_{ab}(t,t';\mathbf{p}),$$
$$G^{(+\mathbf{p})(+\mathbf{p})}_{ab}(t,t') = G^{(-\mathbf{p})(-\mathbf{p})}_{ab}(t,t') = 0. \tag{2.7}$$

In turn, $G^{(\mathrm{C})\alpha\beta\gamma\delta}_{abcd}(t,t',t'',t''')$ is the connected part of the four-point correlation function having external legs with momenta α, β, γ, and δ. For translation-invariant states momentum conservation implies that only when momentum is balanced (i.e., two incoming and two outgoing external legs) the correlation function is nonvanishing. Terms with a higher number of external legs behave similarly.

The open quantum system is nonlinear, and a systematic treatment of the generating functional can be done by using the tools of nonlinear open quantum systems. We follow a different path in the next section.

3. Quantum Brownian Motion Analogy

Let us reconsider the generating functional (2.6). It depends on the n-point correlation functions, with n being arbitrarily large. However, in many situations one is interested in properties which only depend on the two-point correlation functions. In other cases one is doing a perturbative expansion of the generating functional, and connected higher-order correlation functions are usually also of higher-order in the expansion parameter. Finally there are situations in which one only has access to the two-point correlation functions and expects (or simply hopes) that connected higher-order correlation functions are subdominant. In any of this situations one can be tempted to approximate the generating functional by the following Gaussian expression:

$$Z[j_{a,\alpha}] \approx \exp\left[-\frac{1}{2!}\int \mathrm{d}t\,\mathrm{d}t'\, j^a_\alpha(t) j^b_\beta(t) G^{\alpha\beta}_{ab}(t,t')\right]. \tag{3.1}$$

This Gaussian approximation can be controlled in the framework or the large-N expansion (where N is the number of scalar fields) [64–66]. We should emphasise that it does not necessarily imply any perturbative expansion in the coupling parameter, nor any free field approximation.

The Gaussian approximation of the generating functional can be expanded to

$$Z[j_{a,\mathbf{p}}, j_{a,-\mathbf{p}}] = \exp\left\{-\frac{1}{2}\int \mathrm{d}t\,\mathrm{d}t'\left[j^a_{\mathbf{p}}(t)j^b_{-\mathbf{p}}(t')G_{ab}(t,t') + j^a_{\mathbf{p}}(t')j^b_{-\mathbf{p}}(t)G_{ab}(t,t')\right]\right\}, \tag{3.2}$$

or equivalently, in terms of the real variables ϕ_Σ and ϕ_Δ and their corresponding classical sources,

$$Z[j_{a,\Sigma}, j_{a,\Delta}] = \exp\left\{-\frac{1}{2}\int \mathrm{d}t\,\mathrm{d}t'\left[j^a_{\Sigma}(t)j^b_{\Sigma}(t')G_{ab}(t,t') + j^a_{\Delta}(t)j^b_{\Delta}(t')G_{ab}(t,t')\right]\right\}, \tag{3.3}$$

or

$$Z[j_{a,\Sigma}, j_{a,\Delta}] = Z[j_{a,\Sigma}]Z[j_{a,\Delta}], \qquad Z[j_a] = \exp\left[-\frac{1}{2}\int \mathrm{d}t\,\mathrm{d}t'\, j^a(t)j^b(t')G_{ab}(t,t')\right]. \tag{3.4}$$

Equation (3.4) shows that for translation-invariant states within the Gaussian approximation the reduced two-mode state effectively behaves as two decoupled quantum mechanical degrees of freedom.

The Gaussian approximation implies that the expression of the generating functional of a reduced two-mode in terms of the two-point propagators coincides with that of a QBM model, provided that the system is isotropic and translation invariant: compare (B.27) with (3.4). Moreover, by comparing (B.26) with (A.20), we also realise that the structure of the two-point propagators is identical in both cases. Notice that this latter fact is independent of the Gaussian approximation.

Therefore, we conclude that, assuming homogeneity, isotropy, and stationarity, *there is an equivalent QBM system for every scalar two-mode pair treated under the Gaussian approximation*. In other words, within the Gaussian approximation, every two-mode of a given quantum field theory can be described in terms of a pair of quantum Brownian particles interacting linearly with some effective environment. We must stress that, similarly as the linear interaction does not coincide with the real coupling, the effective environment does not coincide with the real environment. The precise details of the equivalence are summarised in Table 1 for the particular case of the $\lambda\phi^4$ theory. We will see below that both the strength of the linear coupling and the state of the equivalent environment depend on the details of the original environment.

Let us investigate on this correspondence. On the one hand, as is shown in Appendix B, the effect of the environment in the QBM system is fully encoded in two kernels, the dissipation kernel $D(t,t')$—or its closely related counterpart $H(t,t')$; see (B.13b)—and the noise kernel $N(t,t')$. The dynamics of the quantum Brownian particle can be determined once the frequency and the noise and dissipation kernels are known. On the other hand, as shown in Appendix A, in a quantum field theory the two-point correlation functions are fully characterised by the frequency $E_{\mathbf{p}}$ and the retarded and Hadamard self-energies, $\Sigma_{\mathrm{R}}(t,t')$ and

Table 1: Detail of the equivalent linear QBM system for a $\lambda\phi^4$ quantum field theory. The $\lambda\phi^4$ model has been chosen for concreteness, but the correspondence would be analogous for any other field theory model. The symbol (=) indicates that the original and equivalent quantities are indeed identical despite the name change.

	Original system	Equivalent linear QBM
System	Two field modes	Two identical oscillators
System d.o.f.	$\phi_{\mathbf{p}}$, $\phi_{-\mathbf{p}}$	2 copies of q
Environment	Other modes 3-d field	2 1-d field
Env. d.o.f.	$\phi_{\mathbf{q}}$, $\mathbf{q} \neq \pm\mathbf{p}$	φ_p
Frequency	$E_{\mathbf{p}}$	Ω (=)
Coupling	$(\lambda/3!)\sum_{\mathbf{q}\mathbf{q}'} \phi_{\pm\mathbf{p}}\phi_{\mathbf{q}}\phi_{\mathbf{q}'}\phi_{\mp\mathbf{p}-\mathbf{q}-\mathbf{q}'}$	$g^2\sum_p \mathcal{J}(p)\dot{q}\varphi_p$
2-point function	$G_{ab}(t,t';\mathbf{p})$	$G_{ab}(t,t')(=)$

$\Sigma^{(1)}(t,t')$, respectively. By comparing again (B.25) and (B.26) with (A.20) we realise that the precise analogy goes as follows:

$$\begin{aligned} \Omega &= E_{\mathbf{p}}, \\ H(t,t') &= \mathbf{p}^2 + m^2 - E_{\mathbf{p}} + \Sigma_{\mathrm{R}}(t,t';\mathbf{p}), \\ D(t,t') &= -i\,\mathrm{Im}\,\Sigma_{\mathrm{R}}(t,t';\mathbf{p}), \\ N(t,t') &= \frac{i}{2}\Sigma^{(1)}(t,t';\mathbf{p}), \end{aligned} \tag{3.5}$$

where the quantities on the right-hand side correspond to the original field theory system and the quantities on the left-hand side correspond to the equivalent QBM model. The correspondence is valid no matter the renormalisation scheme chosen to fix $\mathfrak{Z}_{\mathbf{p}}$ and $R_{\mathbf{p}}$.

This representation provides thus a first rough interpretation for the retarded and Hadamard self-energies, $\Sigma_{\mathrm{R}}(t,t';\mathbf{p})$ and $\Sigma^{(1)}(t,t';\mathbf{p})$, respectively. The retarded self-energy corresponds to the dissipation kernel, so that it determines the dissipative properties of the system, and it is independent of the state of the equivalent environment (though not independent of the state of the original environment). The Hadamard self-energy corresponds to the noise kernel, and thus it is basically related to fluctuations. Although the description in terms of the noise and dissipation kernels is often sufficient, the equivalent QBM system can be alternatively described in terms of the linear coupling constant to the effective environment, g, the distribution of frequencies of the environment $\mathcal{J}(\omega)$ (see Appendix B), and the occupation number of the modes of the effective environment $n(p) = \mathrm{Tr}[\widehat{\rho}_{\mathrm{e}}^{(\mathrm{eff})}\widehat{a}_p^{\dagger}\widehat{a}_p]$. The product $g^2\mathcal{J}(\omega)$, which determines the coupling strength to the the ω-mode of the effective environment, can be obtained from the imaginary part of the self-energy:

$$\mathrm{Im}\,\Sigma_{\mathrm{R}}(\omega;\mathbf{p}) = iD(\omega) = -\frac{g^2}{2}\omega\mathcal{J}(\omega). \tag{3.6a}$$

This last equation implies that the equivalent coupling depends on the state of the real environment, since the retarded self-energy is state dependent in general. The occupation

numbers of the effective environment $n(p)$ can be reproduced from the Hadamard self-energy:

$$\Sigma^{(1)}(\omega;\mathbf{p}) = -2iN(\omega) = -2ig^2|\omega|\mathcal{J}(\omega)\left[\frac{1}{2} + n(|\omega|)\right]. \tag{3.6b}$$

The knowledge of the occupation numbers fully determines a Gaussian stationary state for the equivalent environment. These results follow from (B.14) and (B.16).

Note that the analogy depends only on the Gaussian approximation, so that it can be extended to all orders in perturbation theory. Notice also that the results derived from the QBM interpretation are exact for all those properties which depend only on the two-point correlation functions; for the properties which depend on higher-order correlation functions, it is a correct approximation depending on the validity of the Gaussian approximation, that is, depending on the relative importance of the connected parts of the correlation functions with respect to the disconnected parts.

4. Example Applications

4.1. Interpretation of the Self-Energy

As a first example application, let us make use of the QBM correspondence to analyse the physical significance of the self-energy in general backgrounds. Our findings will coincide with the result by Weldon [67] (which nowadays is a textbook result [37]) as far as the imaginary part of the self-energy is concerned. However, while Weldon's original analysis was only valid to first-order in perturbation theory, our technique is be valid to all orders. Moreover, our analysis will not be tied to any field theory model. Additionally we will also obtain an interpretation for the other components of the self-energy. In this case the QBM analogy is exact since no four-point correlation functions are involved.

To start we consider the probability that an excitation of energy ω decays into the one-dimensional environment, in the equivalent QBM system. The probability Γ_- that an excitation of the Browian particle with positive energy ω decays into the environment is given by (see, e.g., [37])

$$\Gamma_-(\omega) = \frac{1}{2\omega}\int \frac{\mathrm{d}k}{2\pi 2|k|} 2\pi\delta(\omega - |k|)|\mathcal{M}|^2[1 + n(|k|)], \tag{4.1}$$

where $\mathcal{M}$ is the amplitude of the transition, and $n(|k|)$ is the occupation number of the environment states with energy $|k|$. The factor $1 + n(|k|)$, which is due to the Bose-Einstein statistics, enhances the decay probability to those states which are already occupied. Since the equivalent QBM system is linear, the squared decay amplitude is simply given to first-order in the linear coupling constant g by

$$|\mathcal{M}|^2 = g^2\mathcal{J}(\omega)\omega^2, \tag{4.2}$$

where $\mathcal{J}(\omega)$ is the distribution of frequencies of the effective environment. The factor ω^2 is a consequence of the derivative coupling in the QBM model. The decay probability is therefore

$$\Gamma_{-}(\omega) = \frac{1}{2}g^2\mathcal{J}(\omega)[1 + n(\omega)]. \tag{4.3}$$

Likewise, the probability that an excitation of positive energy ω is created spontaneously from the environment is given by

$$\Gamma_{+}(\omega) = \frac{1}{2\omega}\int \frac{\mathrm{d}k}{2\pi 2|k|} 2\pi\delta(\omega - |k|)|\mathcal{M}|^2 n(|k|) = \frac{1}{2}g^2\mathcal{J}(\omega)n(\omega). \tag{4.4}$$

In the original system, Γ_{-} can be interpreted as the probability that a (possibly off-shell) excitation with energy ω decays into the environment, and Γ_{+} can be interpreted as the probability that an environment spontaneously creates an excitation with energy ω.

Notice that the notion of decay rate in quantum mechanics is meaningful only when the excitations are long-lived, or, to put it differently, when the product $g^2\mathcal{J}(\omega)$ is very small. There is an inherent uncertainty in the concept of decay rate, which can be traced to the time-energy uncertainty principle. Therefore it is sufficient to present results to first-order in g: the inherent uncertainty to the notion of decay rate in quantum mechanics is of the same order as the error done by neglecting higher powers of g. In any case, this does not mean that we are doing any perturbative expansion in the original system: the decay rate can be computed to any desired order in the original perturbative coupling constant (e.g., λ in the case of the $\lambda\phi^4$ theory).

We next analyse the self-energy components in the equivalent QBM mode. Given that $\mathrm{Im}\,\Sigma_{\mathrm{R}}(\omega) = (i/2)[\Sigma^{21}(\omega) - \Sigma^{12}(\omega)]$ (see Appendix A), we start by analyzing $\Sigma^{21}(\omega)$. Applying CTP Feynman rules we get

$$-i\Sigma^{21}(t,t') = -(ig)^2\partial_t\partial_{t'}\int \frac{\mathrm{d}p}{2\pi}\mathcal{J}(p)\,\mathrm{Tr}_{\mathrm{env}}\left[\widehat{\rho}_{\mathrm{e}}\widehat{\varphi}_{\mathrm{I}p}(t)\widehat{\varphi}_{\mathrm{I}(-p)}(t')\right], \tag{4.5}$$

where $\widehat{\varphi}_{\mathrm{I}p}$ is p-mode of the environment field in the interaction picture. We have exploited the fact that first-order perturbation theory yields exact results for the self-energy in linear systems. Introducing two resolutions of the identity in the basis of eigenstates of the environment Hamiltonian is a simple exercise [68] to show that the above equation can be developed to

$$\Sigma^{21}(t,t') = ig^2\partial_t\partial_{t'}\int_0^\infty \mathrm{d}p\,\frac{\mathcal{J}(p)}{p}\sum_n \rho_{p,n}\left[(n+1)e^{-ip(t-t')} + ne^{ip(t-t')}\right], \tag{4.6}$$

where $\rho_{p,n}$ is the diagonal value of the reduced density matrix of the p-mode of the equivalent environment: $\mathrm{Tr}_{q\neq p}\langle n_p|\widehat{\rho}_{\mathrm{e}}|m_p\rangle = \rho_{p,n}\delta_{nm}$. Introducing the Fourier transform we get

$$\Sigma^{21}(\omega) = ig^2\omega^2\int_0^\infty \mathrm{d}p\,\frac{\mathcal{J}(p)}{p}\sum_n \rho_{p,n}\left[(n+1)\delta(\omega - p) + n\delta(\omega + p)\right], \tag{4.7}$$

and restricting to positive energies

$$\Sigma^{21}(\omega) = -ig^2\omega\mathcal{J}(\omega)\sum_n \rho_{\omega,n}(n+1) = ig^2\omega\mathcal{J}(\omega)[1+n(\omega)], \quad \omega > 0, \tag{4.8}$$

where we recall that $n(\omega) = \mathrm{Tr}[\hat{\rho}_{\mathrm{e}}^{(\mathrm{eff})}\hat{a}_\omega^\dagger\hat{a}_\omega] = \sum_n \rho_{\omega,n} n$ is the occupation number of the ω-mode of the environment. By comparing with (4.3), we thus see that i is proportional to the decay rate:

$$\Sigma^{21}(\omega) = 2i\omega\Gamma_-(\omega), \quad \omega > 0. \tag{4.9}$$

Repeating the calculation for $\Sigma^{21}(\omega)$ we similarly find

$$\Sigma^{12}(\omega) = ig^2\omega\mathcal{J}(\omega)\sum_n \rho_{\omega,n} n = ig^2\omega\mathcal{J}(\omega)n(\omega), \quad \omega > 0, \tag{4.10}$$

having the corresponding interpretation in terms of the creation rate

$$\Sigma^{12}(\omega) = 2i\omega\Gamma_+(\omega), \quad \omega > 0. \tag{4.11}$$

When the energies are negative $\Sigma^{21}(\omega)$ and $\Sigma^{12}(\omega)$ exchange roles.

The imaginary part of the retarded self-energy is therefore given by

$$\mathrm{Im}\,\Sigma_{\mathrm{R}}(\omega) = \frac{i}{2}\left[\Sigma^{21}(\omega) - \Sigma^{12}(\omega)\right] = -\frac{1}{2}g^2\omega\mathcal{J}(\omega), \tag{4.12}$$

and can be interpreted as the net decay rate for an excitation of energy ω, that is, decay rate minus creation rate:

$$\mathrm{Im}\,\Sigma_{\mathrm{R}}(\omega) = -\omega[\Gamma_-(\omega) - \Gamma_+(\omega)]. \tag{4.13}$$

We therefore recover Weldon's result [67].

We can additionally get an interpretation for the Hadamard self-energy. It is given by

$$\Sigma^{(1)}(\omega) = -\Sigma^{21}(\omega) - \Sigma^{12}(\omega) = -ig^2|\omega|\mathcal{J}(\omega)[1+2n(|\omega|)], \tag{4.14}$$

and it is proportional to the probability of decay plus the probability of creation,

$$\Sigma^{(1)}(\omega) = -2i|\omega|[\Gamma_-(\omega) + \Gamma_+(\omega)]. \tag{4.15}$$

4.2. Master Equation for Relativistic Quasiparticles

As another example application of the correspondence, let us study the master equation for the second-quantized relativistic quasiparticles. A quasiparticle is an elementary excitation

propagating in some backgroung, characterized by a momentum $\mathbf{p}$, an energy $E_{\mathbf{p}}$, and a decay rate $\gamma_{\mathbf{p}}$ verifying $\gamma_{\mathbf{p}} \ll E_{\mathbf{p}}$ (so that the excitation is long lived). The basic properties characterizing the long-time evolution of the relativistic quasiparticles can be extracted from the analysis of the retarded propagator corresponding to the field mode in a model independent way [43] (differently to the short-time behaviour, which is model dependent). In particular, the real and imaginary parts of the self-energy, when evaluated on-shell, correspond to the physical energy and the decay rate of the quasiparticle, respectively,

$$E_{\mathbf{p}}^2 = m^2 + \mathbf{p}^2 + \text{Re}\ \Sigma_{\text{R}}(E_{\mathbf{p}}, \mathbf{p}), \qquad \gamma_{\mathbf{p}} = -\frac{1}{E_{\mathbf{p}}}\text{Im}\,\Sigma_{\text{R}}(E_{\mathbf{p}}, \mathbf{p}). \tag{4.16}$$

Using the QBM correspondence, these two equations can be equivalently written as:

$$\text{Re}\ H(\Omega) = 0, \qquad \gamma_{\mathbf{p}} = -\frac{i}{\Omega}D(\Omega). \tag{4.17}$$

The value of $E_{\mathbf{p}}$ (or equivalently Ω) is fixed by requiring that $E_{\mathbf{p}}$ represents the physical energy of the quantum mode as would be measured by a particle detector. We refer to [43] (which makes use of the QBM correspondence) for further details on the field theory analysis of the quasiparticle excitations.

Thus, let us study the master equation (B.30) for $q = \{\phi_\Sigma, \phi_\Delta\}$. To this end it will prove useful to express the on-shell values of the noise and dissipation kernels in terms of the energy $\Omega = E_{\mathbf{p}}$ and the decay rate $\gamma = \gamma_{\mathbf{p}}$:

$$D(\Omega) = -H(\Omega) = i\gamma, \qquad N(\Omega) = \gamma\Omega\left(\frac{1}{2} + n\right), \tag{4.18}$$

where $n = n(|\Omega|)$ is the equilibrium occupation number of the mode in question. Notice that since the decay rate is necessarily small, the coupling constant to the effective environment is also small. Therefore the mode can be considered to be weakly coupled to the effective environment.

In the weak coupling regime, the master equation coefficients are given by (B.31), and are usually divergent for short times [60]. These divergences are associated to the fact that the assumption of factorized initial conditions for the inital state is unphysical, as commented in Appendix B. Since we are interested in the long-time behaviour of the quasiparticles, we study the asymptotic value of these coefficients, which is free from the short-time divergences.

It is easy to show that the frequency shift vanishes:

$$\begin{aligned}\delta\Omega^2 &= -2\int_{t_{\text{i}}}^{\infty} D(s, t_{\text{i}}) \cos\Omega(s - t_{\text{i}})\text{d}s \\ &= \frac{1}{2}\,\text{Re}\int_{-\infty}^{\infty} H(s, t_{\text{i}}) e^{i\Omega(s-t_{\text{i}})}\text{d}s = \text{Re}\ H(\Omega) = 0.\end{aligned} \tag{4.19}$$

Taking into account that the dissipation kernel is antisymmetric, the evaluation of the dissipative factor is also straightforward:

$$\begin{aligned}\Gamma &= \frac{1}{\Omega}\int_{t_i}^{\infty} D(s,t)\sin\Omega(s-t_i)\mathrm{d}s \\ &= \frac{1}{2\Omega i}\int_{-\infty}^{\infty} D(s,t)e^{i\Omega(s-t_i)}\mathrm{d}s = \frac{1}{2\Omega i}D(\omega) = \frac{\gamma_{\mathrm{P}}}{2}.\end{aligned} \tag{4.20}$$

The first diffusion factor is also easily computed, this time recalling that the noise kernel is symmetric:

$$\begin{aligned}\Gamma h &= \int_{t_i}^{\infty} N(s,t_i)\cos\Omega(s-t_i)\mathrm{d}s \\ &= \frac{1}{2}\int_{-\infty}^{\infty} N(s,t_i)e^{i\Omega(s-t_i)}\mathrm{d}s = \frac{N(\Omega)}{2} = \Omega\gamma\left(\frac{1}{2}+n\right).\end{aligned} \tag{4.21}$$

The second diffusion factor requires some extra work:

$$\begin{aligned}\Gamma f &= \frac{1}{\Omega}\int_{t_i}^{\infty} N(s,t_i)\sin\Omega(s-t_i)\mathrm{d}s \\ &= \frac{1}{\Omega}\mathrm{Im}\int_{-\infty}^{\infty} N(s,t_i)\theta(s-t_i)e^{i\Omega(s-t_i)}\mathrm{d}s \\ &= \frac{1}{\Omega}\mathrm{Im}\int\frac{\mathrm{d}\omega}{2\pi}\frac{iN(\omega)}{\Omega-\omega+i\epsilon} \\ &= -\frac{1}{\Omega}\mathrm{P}\int\frac{\mathrm{d}\omega}{2\pi}\frac{iD(\omega)\,\mathrm{sign}(\omega)}{\Omega-\omega}[1+2n(|\omega|)].\end{aligned} \tag{4.22}$$

The integrand in the last equality is only signifficantly different from zero when $\omega \sim \Omega$. Therefore as a first approximation we may write

$$\Gamma f \approx -\frac{1}{\Omega}\mathrm{P}\int\frac{\mathrm{d}\omega}{2\pi}\frac{-iD(\omega)}{\Omega-\omega}[1+2n(|\Omega|)] = \frac{1}{2\Omega}\,\mathrm{Re}\;H(\Omega)(1+2n) = 0. \tag{4.23}$$

Thus the master equation can be written as

$$\begin{aligned}i\frac{\partial}{\partial t}\rho_{\mathrm{s}}(q,q',t) = \Bigg[&-\frac{1}{2}\left(\frac{\partial^2}{\partial q^2}-\frac{\partial^2}{\partial q'^2}\right)+\frac{1}{2}\Omega^2\left(q^2-q'^2\right) \\ &-\frac{i}{2}\gamma(q-q')\left(\frac{\partial}{\partial q}-\frac{\partial}{\partial q'}\right)-i\Omega\gamma\left(\frac{1}{2}+n\right)(q-q')^2\Bigg]\rho_{\mathrm{s}}(q,q',t),\end{aligned} \tag{4.24}$$

or, equivalently, in terms of the Wigner function, as

$$\frac{\partial W_{\mathrm{s}}}{\partial t} = -p\frac{\partial W_{\mathrm{s}}}{\partial q} + \Omega^2\frac{\partial W_{\mathrm{s}}}{\partial p} + \gamma\frac{\partial p W_{\mathrm{s}}}{\partial p} + \frac{\Omega\gamma}{4}(1+2n)\frac{\partial^2 W_{\mathrm{s}}}{\partial p^2}. \tag{4.25}$$

In this equation p is the canonical momentum associated to the variable q and has nothing to do with the physical quasiparticle momentum $\mathbf{p}$. This is identical to the master equation found by Caldeira et al. [69], corresponding to weak couplings. Fleming et al. [70] found logarithmically divergent results for the master equation coefficients in the ohmic dissipation case, even in the asymptotic regime and in the weak coupling limit. This divergences are associated to the unphysical ultraviolet behaviour of the ohmic dissipation model. Our approximate treatment overlooks the possible divergences associated to the high energy limit of the noise and dissipation kernel.

The reduced density matrix and the Wigner function are to be interpreted in terms of the field modes, and not directly in terms of particles or quasiparticles themselves. Qualitatively, for Gaussian initial conditions the solution of the master equation can be described in terms of the cumulants of a Gaussian distribution in the following way [48, 70]. The expectation value of the field $\langle q\rangle$ follows the trajectory of a classical underdamped harmonic oscillator ($\ddot{q} + \gamma\dot{q} + \Omega^2 q = 0$), namely, it oscillates and slowly decays towards the origin at a rate $\gamma/2$. However, for single quasiparticle excitations the expectation value of the field is always vanishing. Therefore, it is more appealing to consider the dynamics of the second-order cumulants, and in particular the dynamics of the energy of the mode, $(1/2)\langle\ddot{q} + \Omega^2 q\rangle$. When perturbed by the introduction of a quasiparticle, the energy of the mode slowly decays at a rate γ towards its equilibrium value $\Omega(1/2 + n)$, with n being the original occupation number of the mode [43].

5. Summary and Discussion

In this paper we have explored the open quantum system viewpoint for a pair of field modes of opposite momentum, which are the relevant degrees of freedom for the analysis of particle-like excitations in field theory. The main results have been, on the one hand, showing that in any interacting field theory, assuming homogeneity, stationarity, and Gaussianity, this open quantum system can be equivalently represented by two identical quantum Brownian particles interacting linearly with an effective environment, and, on the other hand, exploring the details of the equivalence, which are expressed in (3.5), (3.6a), and (3.6b) and in Table 1.

The Brownian motion equivalence is based on three simple well-known observations. First, the fact that the structure of the two-point correlation functions, which we re-examined in the appendices, is identical regardless of the nature of the interactions. A byproduct of this fact has been establishing a link between the self-energy and the noise and dissipation kernels, thus connecting the open quantum system and field theory notation and languages. Second, the recognition that a Gaussian truncation leads to a generating functional which coincides with that of a linear theory. Finally, the observation that when assuming homogeneity, stationarity, and isotropy the two-mode pair behaves as two copies of a single degree of freedom.

If the system is Gaussian, stationary, and homogeneous, the equivalence is exact. If the system is nonGaussian, the analogy is also exact as far as two-point correlation functions are concerned, but only approximate for higher-order correlation functions. If the system is

nonstationary or nonhomogeneous, there will be corrections to the results of the order of Lp, where L is the characteristic inhomogeneity time or length scale, and p is the relevant energy or momentum scale. Therefore the analogy is perfectly valid in nonhomogeneous backgrounds as long as we consider modes whose characteristic wavelengths are much smaller than the inhomogeneity scale.

It must be noted that for the equivalence to be useful one still needs the field theoretic computation of the two-point correlation functions (or, equivalently, the self-energy). The correspondence does not help in this calculation but must be regarded as a tool useful for interpreting and analyzing the dynamics of the two-mode system. In this sense, there are three basic characteristics which make the QBM equivalence appealing.

First, the equivalence is *universal*, in the sense that it provides the most general description of the dynamics of the scalar two-mode pair within the Gaussian approximation. The description in terms of a linear open quantum system allows a unified description of many different quantum field theory systems: the details of the quantum field theory are unimportant once the equivalent noise and dissipation kernels are known. Different field theory models can be thus classified in the same equivalence class if they lead to the same QBM equivalent model.

Second, the equivalence provides a *simple* characterization of the interaction, given that an arbitrarily complicated coupling with any number of fields is reduced to a linear interaction with a one-dimensional field. Linearly interacting systems have been thoroughly studied in the literature (see, e.g., references given in the introduction and Appendix B) and exhibit many intersting properties, among which one can cite the fact that they are exactly solvable. Therefore, within the regime of validity of the Gaussian approximation, all the methods developed in the literature for linear quantum systems can be applied to analyse the dynamics of a two-mode pair.

Finally, the equivalence is *nonperturbative*, since it does not rely on any perturbative development in the coupling constant of the original field theory. It can be therefore extended to all orders in perturbation theory and applied to strongly interacting systems.

The QBM correspondence, which has also been implicitly used in [71, 72], proves a useful tool when analyzing the properties of particle-like excitations in general backgrounds from a field theory perspective [43, 44]. The utility of the QBM analogy is already highlighted by the first two immediate applications presented in this paper.

As a first application, we reexplored the well-known result that the imaginary part of the retarded self-energy corresponds to the net decay rate of the particle excitations. This was a rather simple application which however improved the conventional textbook derivation by taking profit of the three properties we have remarked above: in the first place, the universality of the QBM analogy, given that the derivation presented in this paper was not tied to any specific field theory model; in the second place, its simplicity, since the calculation essentially reduced to doing trivial perturbative expansions in the equivalent linear system, and, finally, its nonperturbative character, since the derivation avoided any perturbative expansion in the original system.

As a second application, we presented the relevant master equation for the dynamics of the modes corresponding to the quasiparticle momentum. This was a slighly more involved application, relying on the results of [43] (and also complementing them), which highligted the fact that the linear open quantum system machinery can readly be exported to quantum field theory. The presentation and analysis of the master equation, as done in the paper, must be understood as a first approximation to the problem, which might be sufficiently interesting by itself to deserve further work.

Appendices

A. The Closed Time Path Method and the Two-Point Propagators in Field Theory

In this appendix we give a brief introduction to the closed time path (CTP) method (also called *in-in* method, in contrast to the conventional *in-out* method), stressing those aspects relevant for this paper, and apply it to the analysis of the two-point propagators. We address the reader to [73–77] for further details on the CTP method, and to [68, 78–80] for further details on the structure of the two-point functions. For the purposes of this appendix we will consider a free or an interacting scalar field ϕ, although results also apply for a single quantum mechanical degree of freedom.

The path-ordered generating functional $Z_{\mathcal{C}}[j]$ is defined as

$$Z_{\mathcal{C}}[j] = \mathrm{Tr}\left(\hat{\rho} T_{\mathcal{C}} e^{i\int_{\mathcal{C}} \mathrm{d}t \int \mathrm{d}^3\mathbf{x} \hat{\phi}(x) j(x)}\right), \tag{A.1}$$

where $\hat{\phi}(x)$ is the field operator in the Heisenberg picture, $\mathcal{C}$ is a certain path in the complex t plane, $T_{\mathcal{C}}$ means time ordering along this path, and $j(x)$ is a classical external source. By functional differentiation of the generating functional with respect to ϕ, path-ordered correlation functions can be obtained

$$G_{\mathcal{C}}(x, x') = \mathrm{Tr}[\hat{\rho} T_{\mathcal{C}} \hat{\phi}(x) \hat{\phi}(x')] = -\frac{\delta^2 Z_{\mathcal{C}}}{\delta j(x) \delta j(x')}\bigg|_{j=0}. \tag{A.2}$$

Introducing a complete basis of eigenstates of the field operator in the Heisenberg picture, $\hat{\phi}(t, \mathbf{x})|\phi, t\rangle = \phi(t, \mathbf{x})|\phi, t\rangle$, as a representation of the identity, the generating functional can be expressed as

$$Z_{\mathcal{C}}[j] = \int \tilde{\mathrm{d}}\phi \; \tilde{\mathrm{d}}\phi' \langle \phi, t_{\mathrm{i}} | \hat{\rho} | \phi', t_{\mathrm{i}} \rangle \left\langle \phi', t_{\mathrm{i}} \middle| T_{\mathcal{C}} e^{i\int_{\mathcal{C}} \mathrm{d}t \int \mathrm{d}^3\mathbf{x} \hat{\phi}(x) j(x)} \middle| \phi, t_{\mathrm{i}} \right\rangle. \tag{A.3}$$

The functional measures $\tilde{\mathrm{d}}\phi$ and $\tilde{\mathrm{d}}\phi'$ go over all field configurations of the fields at fixed initial time t. If the path $\mathcal{C}$ begins and ends at the same point t_{i}, then the transition element of the evolution operator can be computed via a path integral:

$$Z_{\mathcal{C}}[j] = \int \tilde{\mathrm{d}}\phi \; \tilde{\mathrm{d}}\phi' \langle \phi, t_{\mathrm{i}} | \hat{\rho} | \phi', t_{\mathrm{i}} \rangle \int_{\varphi(t_{\mathrm{i}}, \mathbf{x}) = \phi(\mathbf{x})}^{\varphi(t_{\mathrm{i}}, \mathbf{x}) = \phi'(\mathbf{x})} \mathfrak{D}\varphi e^{i\int_{\mathcal{C}} \mathrm{d}t \int \mathrm{d}^3\mathbf{x} \{L[\varphi] + \varphi(x) j(x)\}}, \tag{A.4}$$

where $L[\phi]$ is the Lagrangian density of the scalar field.

Let us consider the time path shown in Figure 1. If we define $\varphi_{1,2}(t,\mathbf{x}) = \varphi(t,\mathbf{x})$ and $j_{1,2}(t,\mathbf{x}) = j(t,\mathbf{x})$ for $t \in \mathcal{C}_{1,2}$, then the generating functional can be re-expressed as

$$\begin{aligned} Z[j_1,j_2] = \int \tilde{\mathrm{d}}\phi\ \tilde{\mathrm{d}}\phi' \tilde{\mathrm{d}}\phi''\langle \phi, t_i|\hat{\rho}|\phi', t_i\rangle \\ \times \int_{\varphi_1(t_i,\mathbf{x})=\phi(\mathbf{x})}^{\varphi_1(t_f,\mathbf{x})=\phi''(\mathbf{x})} \mathfrak{D}\varphi_1 e^{i\int \mathrm{d}^4x\{L[\varphi_1]+\varphi_1(x)j_1(x)\}} \\ \times \int_{\varphi_2(t_i,\mathbf{x})=\phi'(\mathbf{x})}^{\varphi_2(t_f,\mathbf{x})=\phi''(\mathbf{x})} \mathfrak{D}\varphi_2 e^{-i\int \mathrm{d}^4x\{L[\varphi_2]+\varphi_2(x)j_2(x)\}}. \end{aligned} \tag{A.5}$$

In the following it will prove useful to use a condensed notation where neither the boundary conditions of the path integral nor the integrals over the initial and final states are explicit. With this simplified notation the above equation becomes

$$Z[j_1,j_2] = \int \mathfrak{D}\phi_1 \mathfrak{D}\phi_2 \langle \phi, t|\hat{\rho}|\phi', t\rangle e^{i\int \mathrm{d}^4x\{L[\varphi_1]-L[\varphi_2]+\varphi_1(x)j_1(x)-\varphi_2(x)j_2(x)\}}. \tag{A.6}$$

An operator representation is also possible:

$$Z[j_1,j_2] = \mathrm{Tr}\left(\hat{\rho}\ \tilde{T} e^{-i\int_{t_i}^{t_f}\mathrm{d}t\int \mathrm{d}^3\mathbf{x}\hat{\phi}(x)j_2(x)} T e^{i\int_{t_i}^{t_f}\mathrm{d}t\int \mathrm{d}^3\mathbf{x}\hat{\phi}(x)j_1(x)}\right). \tag{A.7}$$

By functionally differentiating the generating functional, the Feynman and Dyson propagators and the Whightman functions can be obtained

$$\begin{aligned} G_{11}(x,x') &= G_{\mathrm{F}}(x,x') = \mathrm{Tr}[\hat{\rho}T\hat{\phi}(x)\hat{\phi}(x')] = -\frac{\delta^2 Z}{\delta j_1(x)\delta j_1(x')}\bigg|_{j_1=j_2=0}, \\ G_{21}(x,x') &= G_{+}(x,x') = \mathrm{Tr}[\hat{\rho}\hat{\phi}(x)\hat{\phi}(x')] = \frac{\delta^2 Z}{\delta j_2(x)\delta j_1(x')}\bigg|_{j_1=j_2=0}, \\ G_{12}(x,x') &= G_{-}(x,x') = \mathrm{Tr}[\hat{\rho}\hat{\phi}(x')\hat{\phi}(x)] = \frac{\delta^2 Z}{\delta j_1(x)\delta j_2(x')}\bigg|_{j_1=j_2=0}, \\ G_{22}(x,x') &= G_{\mathrm{D}}(x,x') = \mathrm{Tr}[\hat{\rho}\tilde{T}\hat{\phi}(x)\hat{\phi}(x')] = -\frac{\delta^2 Z}{\delta j_2(x)\delta j_2(x')}\bigg|_{j_1=j_2=0}. \end{aligned} \tag{A.8}$$

These four propagators can be conveniently organised in a 2×2 matrix, the so-called direct matrix,

$$G_{ab}(x,x') = \begin{pmatrix} G_{\mathrm{F}}(x,x') & G_{-}(x,x') \\ G_{+}(x,x') & G_{\mathrm{D}}(x,x') \end{pmatrix}. \tag{A.9}$$

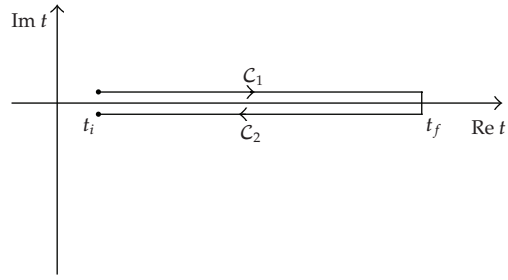

Figure 1: Integration path in the complex-time plane used in the CTP method. The forward and backward lines are infinitesimally close to the real axis.

Lowercase roman indices may acquire the values 1 and 2 are raised and lowered with the "CTP metric" $c_{ab} = \mathrm{diag}(1,-1)$. Higher-order correlation functions can be obtained in a similar way.

We may also consider the Pauli-Jordan or commutator propagator,

$$G(x,x') = \mathrm{Tr}\left(\hat{\rho}\,\left[\hat{\phi}(x),\hat{\phi}(x')\right]\right), \tag{A.10}$$

and the Hadamard or anticonmutator function

$$G^{(1)}(x,x') = \mathrm{Tr}\left(\hat{\rho}\,\left\{\hat{\phi}(x),\hat{\phi}(x')\right\}\right). \tag{A.11}$$

For linear systems (systems whose Heisemberg equations of motion are linear) the Pauli-Jordan propagator is independent of the state and carries information about the system dynamics. Finally, one can also consider the retarded and advanced propagators

$$G_{\mathrm{R}}(x,x') = \theta\left(x^0 - x'^0\right)G(x,x') = \theta\left(x^0 - x'^0\right)\mathrm{Tr}\left(\hat{\rho}\,\left[\hat{\phi}(x),\hat{\phi}(x')\right]\right), \tag{A.12a}$$

$$G_{\mathrm{A}}(x,x') = \theta\left(x'^0 - x^0\right)G(x,x') = \theta\left(x'^0 - x^0\right)\mathrm{Tr}\left(\hat{\rho}\,\left[\hat{\phi}(x),\hat{\phi}(x')\right]\right), \tag{A.12b}$$

which also do not depend on the the state for linear systems. The retarded and advanced propagators and the Hadamard function can be used as an alternative basis to (A.9) in the so-called physical or Keldysh basis.

It is also useful to introduce the correlation functions in momentum space, which are defined as the Fourier transform of the spacetime correlators with respect to the difference variable $\Delta = x - x'$ keeping constant the central point $X = (x + x')/2$:

$$G_{ab}(\omega,\mathbf{p};X) = \int \mathrm{d}^4\Delta e^{i\omega\Delta^0 - i\mathbf{p}\cdot\boldsymbol{\Delta}} G_{ab}\left(X + \frac{\Delta}{2}, X - \frac{\Delta}{2}\right). \tag{A.13}$$

Mixed time-momentum representations of the propagator, $G_{ab}(t,t';\mathbf{p};\mathbf{X})$, can be similarly introduced. For homogeneous and static backgrounds the Fourier transformed propagator does not depend on the mid point X. The canonical example of static and homogeneous

background is the thermal background, in which the state of the field is $\widehat{\rho} = e^{-\beta\widehat{H}}/\mathrm{Tr}(e^{-\beta\widehat{H}})$. Thermal field theory can be thus treated as a particular example of field theory over an arbitrary background. This viewpoint corresponds to the so-called real-time approach to field theory [37, 78, 81].

For interacting theories the self-energy can be introduced similarly to the vacuum case. Interaction theory mixes the two CTP branches and therefore the self-energy has matrix structure and is implicitly defined through the equation

$$G_{ab}(x,x') = G^{(0)}_{ab}(x,x') + \int \mathrm{d}^4y\mathrm{d}^4y' G^{(0)}_{ac}(x,y)\left[-i\Sigma^{cd}(y,y')\right]G_{db}(y',z), \tag{A.14}$$

where $G^{(0)}_{ab}(x,x')$ are the propagators of the free theory, and $G_{ab}(x,x')$ are the propagators of the full interacting theory. Notice that (A.14) is matrix equation relating the four components of the self-energy with the four components of the propagator. Therefore there is no diagonal relation between $G_{11}(x,x')$ and $\Sigma^{11}(y,y')$ as in the vacuum case. The ab component of the self-energy can be computed, similarly to the vacuum case, as the sum of all one-particle irreducible diagrams with amputated external legs that begin and end with type a and type b vertices, respectively. CTP Feynman rules are completed with the prescription of adding one minus sign for every type 2 vertex.

A particularly useful combination is the retarded self-energy, defined as $\Sigma_{\mathrm{R}}(x,x') = \Sigma^{11}(x,x') + \Sigma^{12}(x,x')$. It is related to the retarded propagator through

$$G_{\mathrm{R}}(x,x') = G^{(0)}_{\mathrm{R}}(x,x') + \int \mathrm{d}^4y\,\mathrm{d}^4y'\,G^{(0)}_{\mathrm{R}}(x,y)\left[-i\Sigma_{\mathrm{R}}(y,y')\right]G_{\mathrm{R}}(y',z). \tag{A.15}$$

This equation can be regarded as a consequence of the causality properties of the retarded propagator. A similar relation holds between the advanced propagator $G_{\mathrm{A}}(x,x')$ and the advanced self-energy $\Sigma_{\mathrm{A}}(x,x') = \Sigma^{11}(x,x') + \Sigma^{21}(x,x')$. Another useful combination is the Hadamard self-energy, which is defined as $\Sigma^{(1)}(x,x') = \Sigma^{11}(x,x') + \Sigma^{22}(x,x')$ (or equivalently as $\Sigma^{(1)}(x,x') = -\Sigma^{12}(x,x') - \Sigma^{21}(x,x')$) and which is related to the Hadamard propagator through [68]

$$G^{(1)}(x,x') = -i\int \mathrm{d}^4y\,\mathrm{d}^4y'\,G_{\mathrm{R}}(x,y)\Sigma^{(1)}(y,y')G_{\mathrm{A}}(y',x'). \tag{A.16}$$

All self-energy combinations can be determined from the knowledge of the Hadamard self-energy and the imaginary part of the retarded self-energy. This latter quantity can be obtained from the following cutting rule:

$$\mathrm{Im}\,\Sigma_{\mathrm{R}}(x,y) = \frac{i}{2}\left[\Sigma^{21}(x,y) - \Sigma^{12}(x,y)\right]. \tag{A.17}$$

So far, all expressions in this appendix refer to arbitrary background states $\widehat{\rho}$. For static and homogeneous backgrounds, (A.15) can be solved for the retarded propagator by going to the momentum representation:

$$G_{\mathrm{R}}(\omega,\mathbf{p}) = \frac{-i}{-\omega^2 + m^2 + \mathbf{p}^2 + \Sigma_{\mathrm{R}}(\omega,\mathbf{p}) - p^0 i\epsilon}. \tag{A.18}$$

We have considered that the free propagators of the mode $\mathbf{p}$ are those corresponding to the action (2.3a). Notice that in general the self-energy is a separate function of the energy ω and the 3-momentum $\mathbf{p}$, and not only a function of the scalar p^2, as in the vacuum case. The Hadamard function admits the following expression (which can be derived from (A.16)):

$$\begin{aligned} G^{(1)}(\omega,\mathbf{p}) &= i|G_{\mathrm{R}}(\omega,\mathbf{p})|^2\Sigma^{(1)}(\omega,\mathbf{p}) \\ &= \frac{i\Sigma^{(1)}(\omega,\mathbf{p})}{\left[-\omega^2 + m^2 + \mathbf{p}^2 + \mathrm{Re}\ \Sigma_{\mathrm{R}}(\omega,\mathbf{p})\right]^2 + \left[\mathrm{Im}\,\Sigma_{\mathrm{R}}(\omega,\mathbf{p})\right]^2}. \end{aligned} \tag{A.19}$$

From the retarded propagator and the Hadamard function we can derive

$$\begin{aligned} G_{\mathrm{F}}(\omega,\mathbf{p}) &= \frac{-i\left[-\omega^2 + E_{\mathbf{p}}^2 + \mathrm{Re}\ \Sigma_{\mathrm{R}}(\omega,\mathbf{p})\right] + i\Sigma^{(1)}(\omega,\mathbf{p})/2}{\left[-\omega^2 + m^2 + \mathbf{p}^2 + \mathrm{Re}\ \Sigma_{\mathrm{R}}(\omega,\mathbf{p})\right]^2 + \left[\mathrm{Im}\,\Sigma_{\mathrm{R}}(\omega,\mathbf{p})\right]^2}, \\ G_{\mathrm{D}}(\omega,\mathbf{p}) &= \frac{i\left[-\omega^2 + E_{\mathbf{p}}^2 + \mathrm{Re}\ \Sigma_{\mathrm{R}}(\omega,\mathbf{p})\right] + i\Sigma^{(1)}(\omega,\mathbf{p})/2}{\left[-\omega^2 + m^2 + \mathbf{p}^2 + \mathrm{Re}\ \Sigma_{\mathrm{R}}(\omega,\mathbf{p})\right]^2 + \left[\mathrm{Im}\,\Sigma_{\mathrm{R}}(\omega,\mathbf{p})\right]^2}, \\ G_{-}(\omega,\mathbf{p}) &= \frac{i\Sigma^{(1)}(\omega,\mathbf{p})/2 + \mathrm{Im}\,\Sigma_{\mathrm{R}}(\omega,\mathbf{p})}{\left[-\omega^2 + m^2 + \mathbf{p}^2 + \mathrm{Re}\ \Sigma_{\mathrm{R}}(\omega,\mathbf{p})\right]^2 + \left[\mathrm{Im}\,\Sigma_{\mathrm{R}}(\omega,\mathbf{p})\right]^2}, \\ G_{+}(\omega,\mathbf{p}) &= \frac{i\Sigma^{(1)}(\omega,\mathbf{p})/2 - \mathrm{Im}\,\Sigma_{\mathrm{R}}(\omega,\mathbf{p})}{\left[-\omega^2 + m^2 + \mathbf{p}^2 + \mathrm{Re}\ \Sigma_{\mathrm{R}}(\omega,\mathbf{p})\right]^2 + \left[\mathrm{Im}\,\Sigma_{\mathrm{R}}(\omega,\mathbf{p})\right]^2}. \end{aligned} \tag{A.20}$$

When the field state is not exactly homogeneous, the above expressions are still correct up to order Lp, where L is the relevant inhomogeneity time or length scale.

B. Linear Open Quantum Systems

We present those aspects of the theory of linear open quantum systems relevant in this paper, focusing on the propagators of the Brownian particle. For a more complete presentation check [1–4, 50, 82].

We will consider a quantum Brownian motion (QBM) model: an open quantum system composed of a harmonic oscillator $q(t)$, which will be the subsystem under study, linearly coupled to a free massless field $\varphi(t,x)$, which will act as environment or reservoir.

The action for the full system can be decomposed in the action of the harmonic oscillator, the action of the scalar field and the interaction term as

$$S_{\mathrm{sys}}[q] = \int \mathrm{d}t \left[\frac{1}{2}\dot{q}^2 - \frac{1}{2}\Omega^2 q^2\right], \tag{B.1a}$$

$$S_{\mathrm{env}}[\varphi] = \int \mathrm{d}t\, \mathrm{d}x \left[\frac{1}{2}\left(\partial_t \varphi\right)^2 - \frac{1}{2}\left(\partial_x \varphi\right)^2\right], \tag{B.1b}$$

$$S_{\mathrm{int}}[q, \varphi] = g \int \mathrm{d}t\, \mathrm{d}x\, \delta(x) \dot{q} \varphi, \tag{B.1c}$$

with Ω being the frequency of the harmonic oscillator and g being the coupling constant. The oscillator is taken to have unit mass. A counterterm action including a frequency shift could also be considered.

We use a one-dimensional free field as the environment, following the treatment of [48]. This is equivalent to the alternative representation in which the environment is modelled by a large ensemble of harmonic oscillators [57]. This equivalence can be seen performing a mode decomposition in the interaction term (B.1c)

$$S_{\mathrm{int}}[q, \varphi] = \sqrt{L} \int \mathrm{d}t \frac{\mathrm{d}p}{2\pi} g \dot{q} \varphi_p, \tag{B.2}$$

where $\varphi_p(t)$ is proportional to the spatial Fourier transform of the scalar field

$$\varphi_p(t) = \frac{1}{\sqrt{L}} \int \mathrm{d}x\, e^{-ipx} \varphi(t, x), \tag{B.3}$$

where L is the length of the real axis (formally infinite).

The standard model, also called ohmic model, can be generalised by replacing the delta interaction of (B.1c) by a function $f(x)$. In this case the interaction term is

$$S_{\mathrm{int}}[q, \varphi] = \int \mathrm{d}t\, \mathrm{d}x\, f(x) g \dot{q}(t) \varphi(t, x), \tag{B.4}$$

or equivalently in the Fourier space

$$S_{\mathrm{int}}[q, \varphi] = \sqrt{L} \int \mathrm{d}t \frac{\mathrm{d}p}{2\pi} \tilde{f}(-p) g \dot{q}(t) \varphi_p(t). \tag{B.5}$$

The real even function $\mathcal{J}(p) = \tilde{f}(p)\tilde{f}(-p)$ is called the distribution of frequencies. (In the literature the distribution of frequencies is frequently defined as $\omega\mathcal{J}(\omega)$.) The product $g^2\mathcal{J}(\omega)$ characterises the properties of the coupling with the environment at a given energy ω. The QBM model in this generalised way encompasses the entire class of linearly coupled environments.

When the system and the environment are initially uncorrelated, that is, when the initial density matrix factorises—$\widehat{\rho}(t_{\mathrm{i}}) = \widehat{\rho}_{\mathrm{s}}(t_{\mathrm{i}}) \otimes \widehat{\rho}_{\mathrm{e}}(t_{\mathrm{i}})$, where $\widehat{\rho}_{\mathrm{s}}(t_{\mathrm{i}})$ and $\widehat{\rho}_{\mathrm{e}}(t_{\mathrm{i}})$ mean,

respectively, the density matrix operators of the system and the environment at the initial time—the evolution for the reduced density matrix can be written as

$$\rho_{\mathrm{s}}(q_{\mathrm{f}}, q'_{\mathrm{f}}, t_{\mathrm{f}}) = \int \mathrm{d}q_{\mathrm{i}}\mathrm{d}q'_{\mathrm{i}}\, J(q_{\mathrm{f}}, q'_{\mathrm{f}}, t_{\mathrm{f}}; q_{\mathrm{i}}, q'_{\mathrm{i}}, t_{\mathrm{i}})\rho_{\mathrm{s}}(q_{\mathrm{i}}, q'_{\mathrm{i}}, t_{\mathrm{i}}), \tag{B.6}$$

where the propagator J is defined in a path integral representation by

$$J(q_{\mathrm{f}}, q'_{\mathrm{f}}, t_{\mathrm{f}}; q_{\mathrm{i}}, q'_{\mathrm{i}}, t_{\mathrm{i}}) = \int_{q(t_{\mathrm{i}})=q_{\mathrm{i}}}^{q(t_{\mathrm{f}})=q_{\mathrm{f}}} \mathfrak{D}q \int_{q'(t_{\mathrm{i}})=q'_{\mathrm{i}}}^{q'(t_{\mathrm{f}})=q'_{\mathrm{f}}} \mathfrak{D}q' e^{i(S[q]-S[q']+S_{\mathrm{IF}}[q,q'])}, \tag{B.7}$$

with $S_{\mathrm{IF}}[q,q']$ being the influence action, which is related to the the influence functional introduced by Feynman and Vernon [58, 59] through $F[q,q'] = e^{iS_{\mathrm{IF}}[q,q']}$. In turn, the influence functional can be expressed in the following way:

$$F[q,q'] = \iint \mathfrak{D}\varphi\, \mathfrak{D}\varphi' \rho_{\mathrm{e}}([\varphi_{\mathrm{i}}], [\varphi'_{\mathrm{i}}], t_{\mathrm{i}}) \exp\left[i\left(S[\varphi] - S[\varphi'] + S_{\mathrm{int}}[q,\varphi] - S_{\mathrm{int}}[q',\varphi']\right)\right]. \tag{B.8}$$

The path integral has the boundary conditions $\varphi(x,t_{\mathrm{i}}) = \varphi_{\mathrm{i}}(x)$, $\varphi'(x,t_{\mathrm{i}}) = \varphi'_{\mathrm{i}}(x)$, $\varphi(x,t_{\mathrm{f}}) = \varphi'(x,t_{\mathrm{f}}) = \varphi_{\mathrm{f}}(x)$; there is also an implicit sum over initial and final states, $\varphi_{\mathrm{i}}(x)$, $\varphi'_{\mathrm{i}}(x)$ and $\varphi_{\mathrm{f}}(x)$.

Considering a factorized initial state is a rather unphysical hypothesis that leads to surprising results in many circumstances (see, e.g., [60]). The methods presented in this appendix can be generalized to more natural initial density matrices by the use of the so-called preparation functions [4, 83]. However the preparation function method does not completely solve all the problems because it is based in a sudden change of the density matrix. A more physical approach involves a continuous preparation of the system [84]. In any case, these techniques are increasingly more involved, and we will be only interested in studying the dynamics much after the typical decoherence time. In this case the system and environment have had enough time to interact and become entangled, and the precise form of the initial state becomes unimportant.

When the initial density matrix of the environment $\rho_{\mathrm{e}}([\varphi_{\mathrm{i}}], [\varphi'_{\mathrm{i}}], t_{\mathrm{i}})$ is Gaussian, the path integrals can be exactly performed and one obtains [50, 58, 85]

$$\begin{aligned} S_{\mathrm{IF}}[q,q'] = &-2\int_{t_{\mathrm{i}}}^{t_{\mathrm{f}}} \mathrm{d}t \int_{t_{\mathrm{i}}}^{t} \mathrm{d}t' \dot{\Delta}(t)\mathfrak{D}(t,t')\dot{Q}(t') \\ &+ \frac{i}{2}\int_{t_{\mathrm{i}}}^{t_{\mathrm{f}}} \mathrm{d}t \int_{t_{\mathrm{i}}}^{t_{\mathrm{f}}} \mathrm{d}t' \dot{\Delta}(t)\mathcal{N}(t,t')\dot{\Delta}(t'), \end{aligned} \tag{B.9}$$

where $\Delta(t) = q(t) - q'(t)$ and $Q(t) = [q(t) + q'(t)]/2$.

The kernels can be computed as

$$\begin{aligned}\mathfrak{D}(t,t') &= \frac{ig^2}{2}\int\frac{\mathrm{d}p}{2\pi}\mathcal{J}(p)\mathrm{Tr}\left(\hat{\rho}\left[\hat{\varphi}_{\mathrm{I}(-p)}(t),\hat{\varphi}_{\mathrm{I}p}(t')\right]\right),\\ \mathcal{N}(t,t') &= \frac{g^2}{2}\int\frac{\mathrm{d}p}{2\pi}\mathcal{J}(p)\mathrm{Tr}\left(\hat{\rho}\left\{\hat{\varphi}_{\mathrm{I}(-p)}(t),\hat{\varphi}_{\mathrm{I}p}(t')\right\}\right),\end{aligned} \tag{B.10}$$

where $\hat{\varphi}_{\mathrm{I}}(x,t)$ is the field operator in the interaction picture, and $\hat{\varphi}_{\mathrm{I}p}(t)$ is the p-mode of the same field operator. By integration by parts, the influence action can also be expressed as

$$\begin{aligned}S_{\mathrm{IF}}[q,q'] = &\int_{t_\mathrm{i}}^{t_\mathrm{f}}\mathrm{d}t\int_{t_\mathrm{i}}^{t_\mathrm{f}}\mathrm{d}t'\,\Delta(t)H(t,t')Q(t')\\ &+\frac{i}{2}\int_{t_\mathrm{i}}^{t_\mathrm{f}}\mathrm{d}t\int_{t_\mathrm{i}}^{t_\mathrm{f}}\mathrm{d}t'\,\Delta(t)N(t,t')\Delta(t')\end{aligned} \tag{B.11}$$

or as

$$\begin{aligned}S_{\mathrm{IF}}[q,q'] = &-2\int_{t_\mathrm{i}}^{t_\mathrm{f}}\mathrm{d}t\int_{t_\mathrm{i}}^{t}\mathrm{d}t'\,\Delta(t)D(t,t')Q(t')+\int_{t_i}^{t_f}\mathrm{d}t\,\delta\Omega^2\Delta(t)Q(t)\\ &+\frac{i}{2}\int_{t_\mathrm{i}}^{t_\mathrm{f}}\mathrm{d}t\int_{t_\mathrm{i}}^{t_\mathrm{f}}\mathrm{d}t'\,\Delta(t)N(t,t')\Delta(t'),\end{aligned} \tag{B.12}$$

where the different kernels are defined as

$$H(t,t') = -2\frac{\partial}{\partial t}\frac{\partial}{\partial t'}\left[\theta(t-t')\mathfrak{D}(t,t')\right] \tag{B.13a}$$

$$= -2\theta(t-t')D(t-t')+\delta\tilde{\Omega}^2\delta(t-t'), \tag{B.13b}$$

$$D(t,t') = \frac{\partial}{\partial t}\frac{\partial}{\partial t'}\mathfrak{D}(t,t'), \tag{B.13c}$$

$$N(t,t') = \frac{\partial}{\partial t}\frac{\partial}{\partial t'}\mathcal{N}(t,t'). \tag{B.13d}$$

The kernels $D(t,t')$ and $N(t,t')$ are called, respectively, dissipation and noise kernels. The frequency shift $\delta\tilde{\Omega}^2$ is a formally divergent quantity given by

$$\delta\tilde{\Omega}^2 = 2\lim_{t\to t'}\frac{\partial\mathfrak{D}(t,t')}{\partial t}. \tag{B.13e}$$

The dissipation and noise kernels can be computed following (B.10) and (B.13a)–(B.13e). The value of the dissipation kernel in the frequency space is [68]

$$D(\omega) = \frac{i\omega g^2}{2}\mathcal{J}(\omega). \tag{B.14}$$

The dissipation kernel is closely related to the kernel $H(\omega)$ (see (B.13a) and (B.13b)), which is also state-independent and given by

$$H(\omega) = g^2 \int \frac{\mathrm{d}p}{2\pi} \frac{\omega \mathcal{J}(\omega)}{\omega - \omega' + i\epsilon} + \delta\widetilde{\Omega}^2. \tag{B.15}$$

In contrast, the noise kernel (B.13d) is state dependent. For a general Gaussian stationary environments, characterised by the occupation numbers $n(p) = \mathrm{Tr}(\widehat{\rho}_{\mathrm{e}} \widehat{a}_p^\dagger \widehat{a}_p)$, the noise kernel in Fourier space is given by

$$N(\omega) = g^2 |\omega| \mathcal{J}(\omega) \left[\frac{1}{2} + n(|\omega|) \right]. \tag{B.16}$$

The dissipation and noise kernels are related through

$$N(\omega) = -i \ \mathrm{sign}(\omega) \left[\frac{1}{2} + n(|\omega|) \right] D(\omega). \tag{B.17}$$

For the particular case of an environment in thermal equilibrium at a temperature T the occupation numbers are given by $n(|\omega|) = 1/(e^{|\omega|/T} - 1)$ and the above equation becomes the fluctuation-dissipation theorem:

$$N(\omega) = -i \ \mathrm{sign}(\omega) \coth\left(\frac{|\omega|}{2T} \right) D(\omega). \tag{B.18}$$

By considering an arbitrary distribution of frequencies $\mathcal{J}(\omega)$ and an arbitrary Gaussian state for the environment $\widehat{\rho}_{\mathrm{e}}$ the dissipation and noise kernels may adopt almost any value. In the rest of the appendix we will try to express all results in terms of the dissipation and noise kernels. To this end, it will prove useful to re-express (B.13b) in Fourier space:

$$H(\omega) = -2 \int \frac{\mathrm{d}\omega'}{2\pi} \frac{iD(\omega')}{\omega - \omega' + i\epsilon} + \delta\widetilde{\Omega}^2. \tag{B.19}$$

The kernel $H(\omega)$ can be decomposed in its real and imaginary parts as

$$\begin{aligned} H_{\mathrm{R}}(\omega) &= \mathrm{Re}\ H(\omega) = -2\mathrm{P} \int \frac{\mathrm{d}\omega'}{2\pi} \frac{iD(\omega')}{\omega - \omega'} + \delta\widetilde{\Omega}^2, \\ H_{\mathrm{I}}(\omega) &= \mathrm{Im}\, H(\omega) = iD(\omega). \end{aligned} \tag{B.20}$$

We have used the property $1/(x + i\epsilon) = \mathrm{P}(1/x) - i\pi\delta(x)$. Notice that $H(-\omega) = H^*(\omega) = H_{\mathrm{R}}(\omega) - iH_{\mathrm{I}}(\omega)$. The real and imaginary parts of the kernel in frequency space correspond, respectively, to the even and odd parts in time space. Notice also the Kramers-Kronig relation between the real and imaginary parts of the kernel $H(\omega)$:

$$H_{\mathrm{R}}(\omega) = -2\mathrm{P} \int \frac{\mathrm{d}\omega'}{2\pi} \frac{H_{\mathrm{I}}(\omega')}{\omega - \omega'} + \delta\widetilde{\Omega}^2. \tag{B.21}$$

The frequency shift can be always absorbed in Ω. From now on, and in the main body of the paper, we will assume that such absorption has been carried out.

For a Gaussian environment and asymptotic initial boundary conditions the generating functional can be expressed as [82]

$$Z[j_1, j_2] = e^{(1/2)\int dt_1 dt_2 dt_3 dt_4 j_\Delta(t_1) G_R(t_1,t_2) N(t_2,t_3) j_\Delta(t_4) G_R(t_4,t_3)} e^{-\int dt_1 dt_2 j_\Delta(t_1) G_R(t_1,t_2) j_\Sigma(t_2)}, \tag{B.22}$$

where $j_\Sigma(t) = [j_1(t) + j_2(t)]/2$, $j_\Delta(t) = j_1(t) - j_2(t)$ and $G_R(t,t')$ is the retarded propagator of the kernel

$$L(t,t') = \left(\frac{d^2}{dt^2} + \Omega^2\right)\delta(t-t') + H(t,t'), \tag{B.23}$$

that is, the kernel which verifies

$$\int ds\, G_R(t,s) L(s,t') = -i\delta(t-t'), \quad G_R(t,t') = 0 \quad \text{if } t < t'. \tag{B.24}$$

In the general case there would be a prefactor taking into account the initial conditions for the system. However, since the system has a dissipative dynamics and the initial conditions are given in the remote past initial conditions for the system turn out to be completely irrelevant.

Explicit expressions are most easily obtained in the Fourier space, in which the retarded propagator adopts the form

$$G_R(\omega) = \frac{-i}{L(\omega)} = \frac{-i}{-\omega^2 + \Omega^2 + H(\omega)}. \tag{B.25}$$

It can be checked that $G_R(t,t')$, besides being the retarded propagator of the kernel $L(t,t')$, in the sense of (B.24), is also the retarded propagator of the quantum mechanical system, in the sense of (A.12a).

Differentiating the CTP generating functional, according to (A.8), we obtain the different correlation functions in terms of the noise and dissipation kernels:

$$\begin{aligned}
G_F(\omega) &= \frac{-i[-\omega^2 + \Omega^2 + H_R(\omega)] + N(\omega)}{[-\omega^2 + \Omega^2 + H_R(\omega)]^2 + [H_I(\omega)]^2}, \\
G_D(\omega) &= \frac{i[-\omega^2 + \Omega^2 + H_R(\omega)] + N(\omega)}{[-\omega^2 + \Omega^2 + H_R(\omega)]^2 + [H_I(\omega)]^2}, \\
G_-(\omega) &= \frac{N(\omega) + H_I(\omega)}{[-\omega^2 + \Omega^2 + H_R(\omega)]^2 + [H_I(\omega)]^2}, \\
G_+(\omega) &= \frac{N(\omega) - H_I(\omega)}{[-\omega^2 + \Omega^2 + H_R(\omega)]^2 + [H_I(\omega)]^2}.
\end{aligned} \tag{B.26}$$

The generating functional can be alternatively expressed in terms of these correlation functions as

$$Z[j_1, j_2] = Ne^{-(1/2)\int \mathrm{d}t\mathrm{d}t' j^a(t) G_{ab}(t,t') j^b(t')}. \tag{B.27}$$

If desired, the dynamics of the Brownian oscillator can be analyzed in terms of a Langevin equation

$$\ddot{q}(t) + \Omega^2 q(t) + \int_{t_i}^{\infty} \mathrm{d}t' H(t,t') q(t') = \xi(t), \tag{B.28}$$

where $\xi(t)$ is a stochastic Gaussian field defined by the correlation functions

$$\langle \xi(t) \rangle_\xi = 0, \qquad \langle \xi(t)\xi(t') \rangle_\xi = N(t,t'), \tag{B.29}$$

with $\langle \cdots \rangle_\xi$ meaning stochastic average. The stochastic correlation functions derived from the Langevin equation correspond to a subset of the quantum correlation functions [3, 82].

It is also possible to study the master equation for the reduced density matrix of the system ρ_s, which is given by [60, 82, 86]

$$\begin{aligned} i\frac{\partial}{\partial t}\rho_s(q,q',t) = \Bigg[& -\frac{1}{2}\left(\frac{\partial^2}{\partial q^2} - \frac{\partial^2}{\partial q'^2}\right) + \frac{1}{2}\left[\Omega^2 + \delta\Omega_0^2(t)\right]\left(q^2 - q'^2\right) \\ & - i\Gamma(t)(q-q')\left(\frac{\partial}{\partial q} - \frac{\partial}{\partial q'}\right) - i\Gamma(t)h(t)(q-q')^2 \\ & + \Gamma(t)f(t)(q-q')\left(\frac{\partial}{\partial q} + \frac{\partial}{\partial q'}\right)\Bigg]\rho_s(q,q',t), \end{aligned} \tag{B.30}$$

where $\delta\Omega_0^2(t)$, $\Gamma(t)$, $h(t)$, and $f(t)$ are a frequency shift, a dissipative factor, and two dispersive factors, respectively, which in the weak coupling limit are given by [60]

$$\begin{aligned} \delta\Omega^2(t) &= -2\int_{t_i}^{t} D(s,t_i)\cos\Omega(s-t_i)\ \mathrm{d}s, \\ \Gamma(t) &= \frac{1}{\Omega}\int_{t_i}^{t} D(s,t_i)\sin\Omega(s-t_i)\mathrm{d}s, \\ \Gamma(t)h(t) &= \int_{t_i}^{t} N(s,t_i)\cos\Omega(s-t_i)\mathrm{d}s, \\ \Gamma(t)f(t) &= \frac{1}{\Omega}\int_{t_i}^{t} N(t_i,s)\sin\Omega(s-t_i)\mathrm{d}s. \end{aligned} \tag{B.31}$$

The corresponding expressions valid for arbitrary couplings can be found in [60, 82]. Recall that the time t_i is the time at which the density matrix is assumed to factorize.

Alternatively, it is also possible to introduce the reduced Wigner function

$$W_{\mathrm{s}}(q,p,t) = \frac{1}{2\pi}\int d\Delta e^{i\Delta p}\rho_{\mathrm{s}}\left(q-\frac{\Delta}{2},q+\frac{\Delta}{2},t\right), \tag{B.32}$$

in terms of which the master equation adopts a Fokker-Planck form

$$\begin{aligned}\frac{\partial W_{\mathrm{s}}}{\partial t} &= -p\frac{\partial W_{\mathrm{s}}}{\partial q} + \left[\Omega^2+\delta\Omega_0^2(t)\right]q\frac{\partial W_{\mathrm{s}}}{\partial p} + 2\Gamma(t)\frac{\partial pW_{\mathrm{s}}}{\partial p}\\ &\quad + \Gamma(t)h(t)\frac{\partial^2 W_{\mathrm{s}}}{\partial p^2} + \Gamma(t)f(t)\frac{\partial^2 W_{\mathrm{s}}}{\partial p\partial q}.\end{aligned} \tag{B.33}$$

The Wigner function has some similarities with a classical distribution function, although it cannot be interpreted as a true probability density, because, among other reasons, it can adopt negative values [3].

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