Research Article

Smooth Crossing of $w_\Lambda = -1$ Line in a Single Scalar Field Model

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Smooth double crossing of the phantom divide line $w_\Lambda = -1$ has been found possible with a single minimally coupled scalar field for the most simple form of generalized *k*-essence cosmological model, in the presence of background cold dark matter. Such crossing is a sufficiently late time transient phenomenon and does not have any pathological behaviour.

1. Introduction

The analysis of the three year WMAP data [1–3] and the very recent one of the five-year WMAP data [4] provide no indication of any significant deviations from Gaussianity and adiabaticity of the CMBR power spectrum and therefore suggest that the Universe is spatially flat to within the limits of observational accuracy. Further, the combined analysis of the three-year WMAP data with the supernova Legacy survey (SNLS), in [1], constrains the equation of state w_{de}, corresponding to almost 74% of dark energy present in the currently accelerating Universe, to be very close to that of the cosmological constant value. Moreover, observations appear to favour a dark energy equation of state, $w_{de} < -1$ [5, 6]. The marginalized best fit values of the equation of state parameter are given by $-1.14 \leq w_{de} \leq -0.93$ at 68% confidence level. In case, one considers a flat universe apriori, then the combined data leads to $-1.06 = w_{de} = -0.90$. The five-year WMAP data [4], on the other hand, practically indicates no deviation from ΛCDM model and the combined analysis of WMAP; distance measurement from SN1a and Baryon Acoustic Oscillation (BAO) confirms the presence of 73% of dark energy together with the range of the equation of state parameter $-1.33 \leq w_{de} \leq -0.79$. Thus, it is realized that a viable cosmological model should admit a dynamical equation of state that might have crossed the value $w_\Lambda = -1$, in the recent epoch of cosmological evolution.

So far, it has been administered by Vikman [7] and accepted almost by all [8–22], except perhaps by Andrianov et al. [23], and more recently by Cannata and Kamenshchik [24] that smooth crossing of $w_\Lambda = -1$ line is not possible in minimally coupled theories, even through a generalized k-essence Lagrangian [25–28] in the form $L = (1/2)g(\phi)\dot{\phi}^2 - V(\phi)$. It is not difficult to understand that the standard minimally coupled theory cannot go smoothly over to the phantom [29] domain without violating the stability both at the classical [30] and the quantum mechanical levels [31, 32] (although it has recently been inferred [33] that quantum effects which induce the $w_{de} < -1$ phase are stable in the ϕ^4 model). However, Vikman [7], in particular, argued that transitions from $w_{de} \geq -1$ to $w_{de} < -1$ (or vice versa) of the dark energy described by a general scalar-field Lagrangian $(\rho(\phi), \nabla(\phi))$ are either unstable with respect to the cosmological perturbations or realized on the trajectories of measure zero, even in the presence of k-essence Lagrangian. As a consequence, it has given birth to further complicated models to establish a smooth crossing. Particularly, it requires hybrid model composed of at least two scalar fields [34], onethe quintessence and the other a phantom and is usually dubbed as quintom model [35, 36]. Others, even complicated models like hessence [37, 38], holographic dark energy models [39], nonminimal scalar tensor theories of gravity [40, 41], Gauss-Bonnet gravity [42–46], models with higher-order curvature invariant terms [47, 48] have also been invoked for the purpose. There is yet another mechanism [49, 50], where the crossing is achieved through a dark matter-dark energy interaction in view of exchange of energy between CDM and quintessence field. As a result the Universe appears to be dominated by CDM and a crossing dark energy. However, the same phenomena may be engineered even through the gravitational interaction between dark matter and dark energy.

In the present paper we have been able to show that the so-called phantom divide line corresponding to the state parameter, $w_\Lambda = -1$, can indeed be crossed in a single minimally coupled scalar field model, only by invoking the most simple form of a generalized k-essence Lagrangian [25–28] in the background of baryonic and nonbaryonic cold dark matter. In the model under consideration the state parameter of the dark energy $w_{de} > -1$, in the absence of the background matter, implies that the field is quintessencial in origin. It is only through the gravitational interaction the background matter density pushes the dark energy density down to a minima and then pulls it up to a local maxima along which the phantom divide line is crossed transiently.

The essential feature of the model is a solution of the scale factor in the form $a = a_0 \exp(t^f/n)$, with $0 < f < 1$ and $n > 0$. Such a solution was dubbed as intermediate inflation in the nineties [51–53]. Recently, it has been observed [54] that Gauss-Bonnet interaction in four dimensions with dynamic dilatonic scalar coupling admits such solution leading to late time cosmic acceleration rather than inflation at the very early Universe. Under this consequence, a comprehensive analysis has been carried out [55] with such solution in the context of a generalized k-essence model. It has been observed that it admits scaling solution with a natural exit from it at a later epoch of cosmic evolution, leading to late time acceleration with asymptotic de-Sitter expansion. The corresponding scalar field has also been found to behave as a tracker field [56–61]. Unfortunately, we have not analyzed the behaviour of the state parameter in the intermediate region. In this work, we show that such solution in the presence of background matter leads to late time cosmic acceleration with a transient double crossing of the phantom divide line and the Universe is dominated by the background matter till the second crossing. In Section 3, the stability of the model under linear perturbation has been shown. Thus the model does not appear to develop any pathological features like big-rip or instabilities at the classical and quantum mechanical level, during the cosmological evolution.

2. The Model

As mentioned in the introduction, we start with generalized k-essence [25–28] Lagrangian in the form

$$L = g(\phi)F(X) - V(\phi), \tag{2.1}$$

where $X = (1/2)\partial_\mu \phi \partial^\mu \phi$, which, when coupled to gravity, may be expressed in the following most simplest form:

$$S = \int d^4x \sqrt{-g}\left[\frac{R}{2\kappa^2} - \frac{g(\phi)}{2}\phi_{,\mu}\phi'^\mu - V(\phi) + L_m\right], \tag{2.2}$$

where $F(X)$ appears linearly in X and L_m is the matter Lagrangian. This is the simplest form of an action in which both canonical and noncanonical forms of kinetic energies can be treated and a possible crossing of the phantom divide line may be expatiated. For the spatially flat Robertson-Walker space-time

$$ds^2 = -dt^2 + a^2(t)\left[dr^2 + r^2\left\{d\theta^2 + \sin^2(\theta)d\phi^2\right\}\right], \tag{2.3}$$

the field equations in the units $\kappa^2 = 8\pi G = c = 1$ can be expressed as

$$\dot{H} = -\left[\frac{1}{2}g\dot{\phi}^2 + \frac{\rho_m + p_m}{2}\right], \tag{2.4}$$

$$\dot{H} + 3H^2 = V(\phi) + \frac{\rho_m - p_m}{2}, \tag{2.5}$$

$$g(\ddot{\phi} + 3H\dot{\phi}) + \frac{1}{2}g'\dot{\phi}^2 + V' = 0, \tag{2.6}$$

where the dot and the dash represent derivatives with respect to the time and ϕ, respectively. In the above equations, $H = \dot{a}/a$ is the Hubble parameter, while p_m and ρ_m stand for pressure and the energy density of the background matter. So, altogether, we have got three independent equations, namely, (2.4) through (2.6), corresponding to six variables of the theory, namely, $H, \phi, g(\phi), V(\phi), \rho_m$ and p_m. Therefore, we need three physically reasonable assumptions to obtain complete set of solutions. Our first assumption is to neglect the amount of radiation in the present day Universe, and to consider the background matter to be filled with luminous along with baryonic and nonbaryonic cold dark matter with equation of state $w_m = p_m/\rho_m = 0$. Further, to find a solution viable for crossing the phantom divide line, we present our second assumption, in the form of the following ansatz for the Hubble parameter:

$$H = \frac{f}{nt^{1-f}}, \tag{2.7}$$

with $n > 0$ and $f > 0$. It is clear that $f = 1$ leads to exponential expansion. However, we choose f in between, that is, $0 < f < 1$. Thus the complete set of solutions are

$$a = a_0 \exp\left(\frac{t^f}{n}\right), \qquad \frac{1}{2}g\dot{\phi}^2 = \frac{f(1-f)}{nt^{(2-f)}} - \frac{\rho_m^0}{[2a_0^3 \exp((3/n)t^f)]},$$

$$V = \frac{3f^2}{n^2 t^{2(1-f)}} - \frac{f(1-f)}{nt^{(2-f)}} - \frac{\rho_m^0}{[2a_0^3 \exp((3/n)t^f)]},$$

$$\rho_\phi = \frac{3f^2}{n^2 t^{2(1-f)}} - \frac{\rho_m^0}{[a_0^3 \exp((3/n)t^f)]}, \qquad p_\phi = \frac{2f(1-f)}{nt^{(2-f)}} - \frac{3f^2}{n^2 t^{2(1-f)}},$$

$$\rho_m = \frac{\rho_m^0}{[a_0^3 \exp((3/n)t^f)]},$$

(2.8)

where a_0 and ρ_m^0 are integration constants. The third assumption, which we do not require for the present purpose, expresses the coupling parameter $g(\phi)$ and the potential $V(\phi)$ as functions of ϕ. One can, for example, choose ϕ arbitrarily to find different forms of g and V, which does not affect crossing the phantom divide. We will give one particular form of $g(\phi)$ and $V(\phi)$ at the end. The effective equation of state $w_\phi = p_\phi/\rho_\phi$ corresponding to the scalar field is now expressed as

$$w_\phi = a_0^3 \left(\frac{2nf(1-f) - 3f^2 t^f}{3a_0^3 f^2 t^f - \rho_m^0 n^2 t^{(2-f)} \exp(-(3/n)t^f)} \right).$$

(2.9)

The above form of the state parameter w_ϕ appearing in (2.9) has been found in an earlier work [55], where we just mentioned that it goes over to -1 value asymptotically. Here, our attempt is to analyze its behaviour in the interim region. For this purpose let us express the state parameter w_ϕ as a function of the red-shift parameter. For simplification, we choose $a_0 = 1$, without loss of generality. As a result, the constant ρ_m^0, appearing in (2.8) stands for the amount of matter density present in the Universe at $t = 0$. The red-shift parameter z is defined as

$$1 + z = \frac{a(t_0)}{a(t)} = \exp\left[\frac{1}{n}\left(t_0^f - t^f\right)\right],$$

(2.10)

where $a(t_0)$ is the present value of the scale factor, while $a(t)$ is that value at some arbitrary time t, when the light was emitted from a cosmological source. Thus, w_ϕ can now be expressed as

$$w_\phi = \left(\frac{2nf(1-f) - 3f^2\left[t_o^f - n\ln(1+z)\right]}{3f^2\left[t_o^f - n\ln(1+z)\right] - \rho_m^0 n^2\left[t_o^f - n\ln(1+z)\right]^{(2-f)/f} \exp\left(-(3/n)\left[t_o^f - n\ln(1+z)\right]\right)} \right).$$

(2.11)

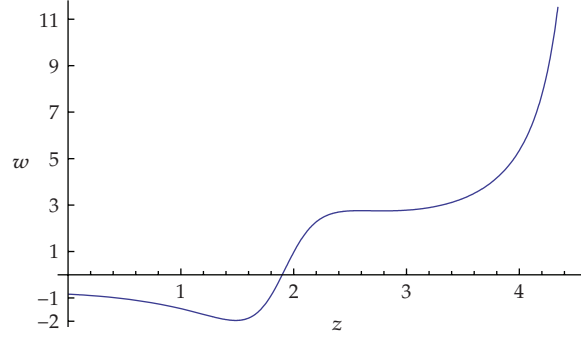**Figure 1:** State parameters $w_\phi(z)$ have been plotted against the red-shift parameter z (with $f = 0.5$, $h = 0.65$, $t_0 = 13\,\mathrm{Gyr}$, $\rho_m^0 \approx 0.63$). Smooth double crossing of the Cosmological constant barrier is observed at sufficiently later epoch, $z \approx 1.8$ from above and $z \approx 0.44$ from below.

For a graphical representation of the state parameter versus the red-shift parameter, we need to select a few parameters of the theory. Firstly, let us choose $f = 0.5$ to find n. The motivation of choosing the value of f in the middle is simply to set a comfortable dimension of time for n^2 and to obtain a reasonably better form of the potential $V(\phi)$. Taking the present value of the Hubble parameter $H_o^{-1} = 9.78/h\,\mathrm{Gyr}$, the age of the Universe $t_0 = 13\,\mathrm{Gyr}$ and with, $h = 0.65$, n can be found from the ansatz (2.7) as $n = 0.5(H_o^{-1}/\sqrt{t_o}) = 2.08$. To estimate the amount of matter density ρ_m^0 present at the time $t = 0$, we take the present value of the matter density parameter $\Omega_{mo} = 0.26$, and so in view of solution (2.8), we have

$$\Omega_{mo} = \frac{\rho_{mo}}{\rho_{co}} = \rho_m^0 \left(\frac{H_o^{-2} \exp\left(-(3/n)t_o^f\right)}{3} \right) = 0.26, \tag{2.12}$$

where ρ_{mo} and ρ_{co} are the present values of the matter density and the critical density, respectively. Thus, $\rho_m^0 \approx 0.63$ and hence, $n^2\rho_m^0 = 2.72$. With these numbers we have plotted the state parameter versus red-shift parameter in Figure 1, which clearly exhibits a smooth double crossing, one from above at $z \approx 1.8, t \approx 2.2$ Gyr and other from below $z \approx 0.44, t \approx 8.2\,\mathrm{Gyr}$ (note that in the present model we have started from the value of the scale factor $a = 1$, at $t = 0$, corresponding to which the red-shift parameter is approximately $z = 4.66$) which is slightly different from that predicted in view of ΛCDM model [62–67]. In Figure 2, the matter density ρ_m (thin line) and the dark energy density ρ_ϕ (thick line) have been plotted against time in Gyr. It demonstrates that initially the Universe was matter dominated. The crossing from above is experienced at the minima and that from below at the local maxima of ρ_ϕ. At the local maxima, ρ_ϕ overtakes ρ_m and the Universe is dominated by dark energy. It should be mentioned at this stage that it is the background CDM that is responsible for the observed transient crossing. The scalar field alone in the absence of the background CDM is not viable of crossing the phantom divide line.
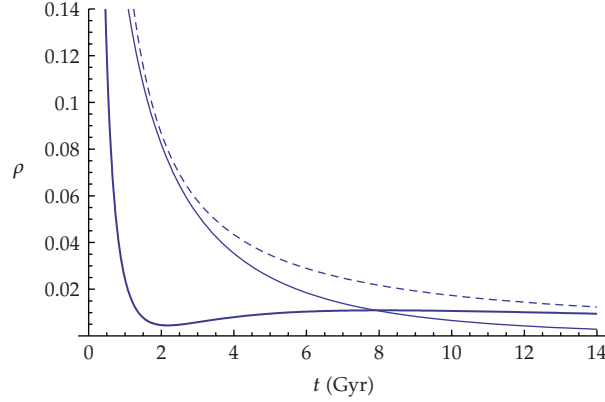
Figure 2: The tracking behaviour of the scalar field is demonstrated. Thick and the thin lines correspond to ρ_ϕ and ρ_m, respectively. The crossing of the phantom divide line occurs at the minima and local maxima (where ρ_ϕ overtakes ρ_m) of the dark energy density. Dashed line corresponds to ρ_ϕ in the absence of background matter. The dark energy is found to remain subdominant till the second crossing.

Now we check how far our model fits with the standard ΛCDM model. In connection with ΛCDM model, the luminosity-redshift relation is

$$H_o dL = (1 + z) \int_0^z \frac{dz}{\sqrt{0.74 + 0.26(1 + z)^3}}, \tag{2.13}$$

while in the present model it is

$$H_o dL = \frac{(1 + z)}{\sqrt{t_0}} \int_0^z \left[\sqrt{t_0} - n \, \ln(1 + z) \right] dz. \tag{2.14}$$

Further, the expression for distance modulus is given by

$$m - M = 5 \log_{10} \left(\frac{dL}{Mpc} \right) + 25 = 5 \log_{10}(DL) + 43, \tag{2.15}$$

where m and M are the apparent and absolute bolometric magnitudes, respectively, and $DL = H_o dL$. In view of (2.13) through (2.15), the distance modulus-redshift graph has been plotted in Figure 3, which shows a perfect fit between the ΛCDM and the present models in agreement with the very recent five-year WMAP data analysis [4].

Surprisingly, as already mentioned, the crossing depicted here does not depend on a particular form of the potential $V(\phi)$ or the coupling parameter $g(\phi)$. One can choose ϕ arbitrarily to find different forms of potential and the coupling parameter without affecting the results. The importance of this fact will be described in the following paragraph. But
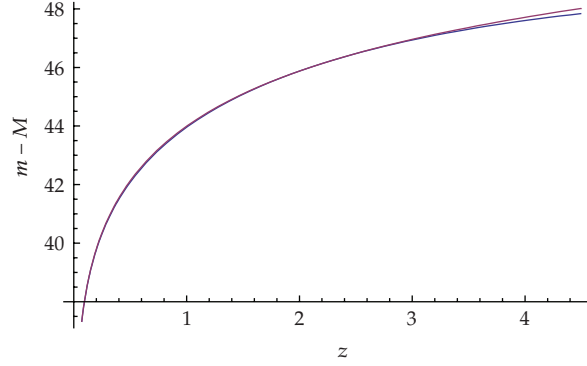
Figure 3: The fit is almost perfect and the models are nearly indistinguishable. Only after $z = 3.5$, the present model deviates slightly by moving up a little bit from the ΛCDM model.
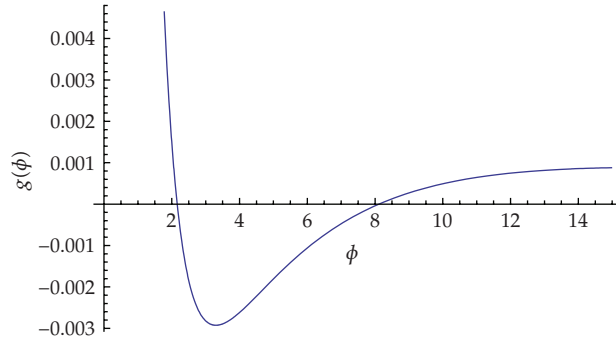

Figure 4: The figure depicts how $g(\phi)$ smoothly phantomizes and dephantomizes the model.

before that, as an example, let us consider a very trivial choice $\phi = t$. Thus, we have in view of (2.8),

$$V = \frac{3f^2}{n^2\phi^{2(1-f)}} - \frac{f(1-f)}{n\phi^{(2-f)}} - \frac{\rho_m{}^{(0)}}{2\left[a_0^3 e^{(3/n)\phi^f}\right]}, \tag{2.16}$$

which has already been demonstrated to be a tracker potential [55], and

$$\frac{1}{2}g(\phi) = \frac{f(1-f)}{n\phi^{(2-f)}} - \frac{\rho_m^0}{2a_0^3 e^{(3/n)\phi^f}}. \tag{2.17}$$

With the above numerics we also present the plots of $g(\phi)$ and $V(\phi)$, in Figures 4 and 5, respectively.

Vikman's argument [7] that the transition across $w_\Lambda = -1$ occurs on trajectories of measure zero has been mentioned in the introduction. On the other hand, Caldwell and Doran [68] argued that there exists a fixed point (ϕ_*, X_*) through which the field must pass to achieve crossing and that for crossing through such a single point, the initial condition must be exceptionally tuned to prevent the slightest deviation from the special trajectory. This means, one has to start from $V = V(\phi)$ and $g = g(\phi)$ and solve (2.6) for several choices of
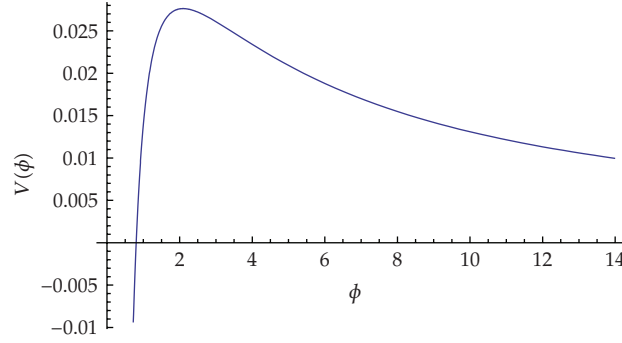
Figure 5: The potential has a maxima at the first crossing and thereafter it is almost flat without any peculiarity during the second crossing.
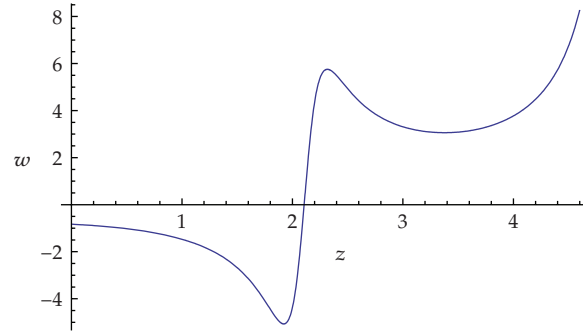

Figure 6: State parameters $w_\phi(z)$ have been plotted against the red-shift parameter z (with $f = 0.5$, $h = 0.68$, $t_0 = 13\,\text{Gyr}$). Smooth double crossing of the Cosmological constant barrier is observed once again for a different initial data $\rho_m^0 \approx 0.77$.

$\phi = \phi(t)$ and $X_* = X_*(t)$ with a pair of initial conditions and show that not only a single precise choice of ϕ and X_* leads to crossing. Following this procedure, it has not been possible to solve (2.6), for the form of the potential $V(\phi)$ and the coupling parameter $g(\phi)$ given by (2.16) and (2.17). Thus the arguments against such crossing could not be tested directly. However, it has been pointed out in the previous paragraph that the transient crossing illustrated here is independent of the choice of $\phi = \phi(t)$, that is, crossing is admissible if one starts from different initial values of ϕ. Thus we can assert that the transient crossing occurs generically for the majority of possible initial conditions of the field and thus is free from the disease of fine tuning. Additionally, we can show that different model parameters admit such transient crossing even for different initial values of ρ_m^0. For this, let us make another plot of the state parameter versus red-shift parameter in Figure 6, taking $f = 0.5$, and $t_0 = 13\,\text{Gyr}$, as before, but with $n = 2$, for which $H_o^{-1} = 14.42\,\text{Gyr}$, which corresponds to $h \approx 0.68$. With these values one can find

$$n^2 \rho_m^0 = 3.09, \tag{2.18}$$

for $\Omega_{mo} = 0.24$ and thus $\rho_m^0 \approx 0.77$ which is quite different from the value $\rho_m^0 \approx 0.63$ obtained previously. The smooth double crossing clearly depicts that it is viable for a wide range of initial data ρ_m^0.

This fact can be illustrated analytically also. It is clear from solutions (2.8) that the kinetic energy term $(1/2)g\dot{\phi}^2$, rather, $g(\phi)$ in particular, vanishes at the crossing. This has also been depicted in Figure 4 under a trivial choice of $\phi = t$. Now, in view of the scalar field (2.6), it is observed that at the crossing, that is, when $g(\phi) = 0$, the term $(1/2)g'\dot{\phi}^2 + V'$ must also vanish. However, this condition is trivially satisfied, as one can see in view of the solutions presented in (2.16) and (2.17) that

$$\frac{1}{2}g'\dot{\phi}^2 + V' = -\frac{6f}{n\phi^{1-f}}g(\phi) = 0. \tag{2.19}$$

In addition, the above condition yields

$$\rho_m^0 = \frac{2f(1-f)}{n}\frac{e^{(3/n)\phi_*^f}}{\phi_*^{2-f}}, \tag{2.20}$$

which makes it apparent that ϕ_*, through which crossing occurs, is different for different initial condition ρ_m^0, and as a result, the crossing does not take place through a particular fixed point (ϕ_*, X_*), rather it occurs through a line. Other way round, it can be said that such crossing is viable for a wide range of initial conditions (ρ_m^0), and so the model does not suffer from the disease of fine tuning.

3. Stability Criteria

Primarily, we note that since there is a dynamical transition of the equation of state from below, so the model avoids big-rip singularity [30] and also prevents undesirable quantum mechanical negative energy graviton and phantom particle production [31, 32]. As a result classically the model is free from future singularity and quantum mechanical stability is guaranteed. Further, the velocity of sound [69–71], $\hat{c}_s^2 = \partial p/\partial \rho = p_{\phi,X}/\rho_{\phi,X}$, is always 1, for the model under consideration. Now the question is, if the model is stable under appropriate perturbation [72, 73].

First, we recall that in the present model, we have two fluids, one is barotropic ($w_m = 0$) and the other, a nonadiabatic scalar field. Now, in the absence of the background matter, $\rho_m = 0$ and $p_m = 0$, the field equations are

$$\rho_\phi^0 = 3H^2, \qquad p_\phi^0 = 2\dot{H} + 3H^2, \tag{3.1}$$

together with (2.6), where, ρ_ϕ^0 and p_ϕ^0 are the energy density and the pressure in the absence of the background, respectively. In view of solutions (2.8) and (2.9) it is clear that in the absence of the background matter, $g\dot{\phi}^2 > 0$ and $w_\phi = -1 + 2n(1-f)/3ft^f > -1$. Thus the scalar field is of quintessence origin and is not viable of crossing the phantom divide line of its own.

Therefore, the corresponding density perturbation equation given by

$$\delta\rho_\phi^0 = (-g'F + V')\delta\phi - gF_{,X}\delta X = \left(\frac{1}{2}g'\dot\phi^2 + V'\right)\delta\phi - g\delta X \qquad (3.2)$$

remains finite. Now, although there is no nongravitational interaction between the two fluids, however they interact strongly under gravitational influence. In Figure 2, the dashed line exhibits the scalar field energy density in the absence of background. It is the background matter that not only reduces the energy density of the scalar field, but also generates a minima through which phantomization and a local maxima through which dephantomization take place. The energy density of the scalar field that has undergone crossing is now given by

$$\rho_\phi = 3H^2 - \rho_m = \rho_\phi^0 - \rho_m. \qquad (3.3)$$

We can now easily see how the problem encountered with energy-density perturbation raised by Caldwell and Doran [68] is cured. The equation for density perturbation is

$$\delta\rho_\phi = \delta\left(\rho_\phi^0 - \rho_m\right) = (-g'F + V')\delta\phi - gF_{,X}\delta X - \delta\rho_m = -\left(\frac{1}{2}g'\dot\phi^2 + V'\right)\delta\phi - g\delta X - \delta\rho_m. \qquad (3.4)$$

We have already demonstrated that if we restrict to $\dot\phi^2 > 0$, then $g(\phi)$ vanishes at the crossing, and the coefficient of $\delta\phi$ also vanishes due to the scalar field equation (2.6). However, $\delta\rho_\phi$ still remains finite at the crossing due to the presence of finite matter density perturbation $\delta\rho_m$. Next, let us look for the pressure perturbation equation [74, 75]

$$\delta p = \hat{c}_s^2 \delta\rho + 3\mathcal{H}(1 + w)\left(\hat{c}_s^2 - c_a^2\right)\rho\frac{\theta}{k^2}, \qquad (3.5)$$

where the symbols have their usual meaning. In expression (3.5) for pressure perturbation, the adiabatic sound velocity c_a^2 diverges at the crossing for one component nonadiabatic fluid and so pressure perturbation also diverges. However, in the present model, the expression for the adiabatic sound velocity is

$$c_a^2 = \frac{\dot{p}_\phi}{\dot{\rho}_\phi} = \frac{\dot{p}_\phi^0}{\dot{\rho}_\phi^0 - \dot{\rho}_m} = \frac{\left[3\mathcal{H}w_\phi(1 + w_\phi) - \dot{w}_\phi\right]\rho_\phi^0}{3\mathcal{H}\left[(1 + w_\phi)\rho_\phi^0 - \rho_m\right]}. \qquad (3.6)$$

Here, dot corresponds to derivative with respect to conformal time and $\mathcal{H}$ is the Hubble parameter in conformal time. It is observed that at the crossing c_a^2 remains finite yielding finite pressure perturbation. So the model is stable under linear perturbations and it appears that everything goes right with it.

4. Summary

First of all it should be admitted that there is nothing crazy in the present model. That the dark energy alone is not viable of crossing the phantom divide line without classical and quantum mechanical pathological behaviour has been proved by several authors and that has not been challenged here. One should also admit that a real cosmic fluid carries baryonic and non-baryonic CDM along with the dark energy and they interact gravitationally. Both Vikman [7] and Caldwell and Doran [68] have studied the behaviour of the k-essence Lagrangian which alone admits crossing in the background of dark matter, while the dark matter in the present model is a quintessence field with $w_\phi > -1$ in the absence of the background. The fact that quintessence field is viable of crossing the barrier in the presence of background matter has already been accounted for [49, 50], where there exists nongravitational interaction between the two. The most interesting feature of the present model is the observation that though there is no non-gravitational interaction between the background matter and the dark energy, yet the background matter pushes the energy density of the dark energy (scalar field) to a minima and a local maxima, around which the transient crossing of the phantom divide line has been experienced. The other interesting feature is that the phenomena of such transient crossing is independent of any particular form of the potential $V(\phi)$ and the coupling parameter $g(\phi)$. Yet another important consequence of the present model, as revealed in Figure 2, is that the matter density had dominant contribution until the second crossing. Finally, the tremendous agreement of the present model with ΛCDM model as shown in Figure 3 supports the cosmological interpretation of the recent five-year WMAP observation [4], that there is practically no deviation from ΛCDM model.
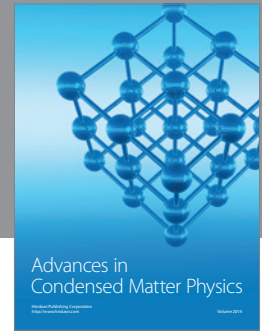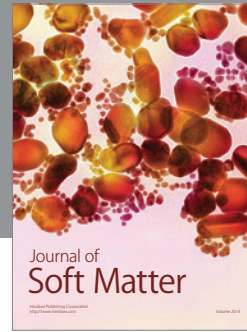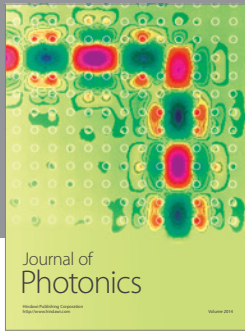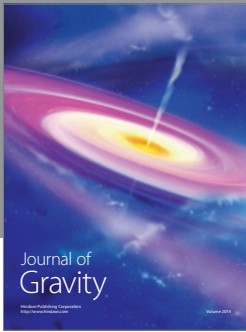
Acknowledgment

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