Research Article

Confinement Mechanism in the Field Correlator Method

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Confinement in QCD results from special properties of vacuum fluctuations of gluon fields. There are two numerically different scales, characterizing nonperturbative QCD vacuum dynamics: "small" one, corresponding to gluon condensate, critical temperature etc, which is about 0.1–0.3 GeV, and a "large" one, given by inverse confining string width, glueball and gluelump masses, and so forth, which is about 1.5–2.5 GeV. We discuss the origin of this hierarchy in a picture where confinement is ensured by quadratic colorelectric field correlators of the special type. These correlators, on the other hand, can be calculated via gluelump Green's function, whose dynamics is defined by the correlators themselves. In this way one obtains a self-consistent scheme, where string tension can be expressed in terms of Λ_{QCD}.

1. Introduction

Confinement of color is the most important property of quantum chromodynamics (QCD), ensuring stability of matter in the Universe. Attempts to understand physical mechanism of confinement are incessant since advent of constituent quark model and QCD; for reviews, see, for example, [1–4]. Among many suggestions, one can distinguish three major approaches as follows.

(i) Confinement is due to classical field lumps like instantons or dyons.

(ii) Confinement can be understood as a kind of Abelian-like phenomenon, for example, according to seminal suggestion by Hooft [5] and Mandelstam [6] of dual Meissner scenario.

(iii) Confinement results from properties of quantum stochastic ensemble of nonperturbative (*np*) fields filling QCD vacuum.

The first approach has a body of successful phenomenology; see review in [7]. On the other hand, as is well known, instanton gas model lacks confinement. As for the second approach, it has gained much popularity in the lattice community, and studies of various projected objects such as Abelian monopoles and center vortices are going on (see, e.g., [8, 9]). It is worth mentioning that despite tremendous recent progress in lattice calculations, the problem of identifying correct gauge-invariant picture of the QCD vacuum is extremely difficult on the lattice as well as in the continuum. For example, the results of [10, 11]—lowering deconfinement critical temperature with rising magnitude of external Abelian chromomagnetic (not chromoelectric!) field—are not easily incorporated to the conventional dual superconductor picture with space-time independent monopole condensate and presumably suggest the formation of nontrivial inhomogeneous condensates. It seems to be correct to conclude that the exact pattern of the Abelian confinement scenario is not yet fixed.

The development of the third approach in a systematic way started in 1987 [12–15] (see, e.g., [16] as example of earlier investigations concerning stochasticity of QCD vacuum). The nontrivial structure of np vacuum can be described by a set of nonlocal gauge-invariant field strength correlators (FCs). The discussion presented below is in the framework of this scenario.

QCD sum rules [17–19] were suggested as an independent approach to npQCD dynamics, not addressing directly confinement mechanism. The key role is played by gluon condensate G_2, which is defined as np average of the following type:

$$G_2 = \frac{\alpha_s}{\pi} \left\langle F_{\mu\nu}^a(0) F_{\mu\nu}^a(0) \right\rangle. \tag{1.1}$$

In the sum rule framework, it is a universal quantity characterizing QCD vacuum as it is, while it enters power expansion of current-current correlators with channel-dependent coefficients. This is the cornerstone of QCD sum rules ideology.

It is worth stressing from the very beginning that G_2 and other condensates are usually considered as finite physical quantities. Naively in perturbation theory, one would get $G_2 \sim a^{-4}$, where a is a space-time ultraviolet cutoff (e.g., lattice spacing). It is always assumed that this "hard" contribution is somehow subtracted from (1.1) and the remaining finite quantity results from "soft" np fields, in some analogy with Casimir effect where modification of the vacuum by boundaries of typical size L yields nonzero shift of energy-momentum tensor by the amount of order L^{-4}. In QCD, the role of L is played by dynamical scale $\Lambda_{\text{QCD}}^{-1}$.

The idea of np gluon condensate has proved to be very fruitful. However the relation of G_2 to confinement is rather tricky. Let us stress that since there can be no local gauge-invariant order parameter for confinement-deconfinement transition, G_2 is not an order parameter, and, in particular, neither perturbative nor np contributions to this quantity vanish in either phase. (Let us repeat that following [17, 18], we define G_2 in (1.1) as purely np object.) On the other hand, one can show (see details in [3]) that the scale of deconfinement temperature is set by the condensate, or, more precisely, by its electric part: $T_c \sim G_2^{1/4}$. Let us remind the original estimate for the condensate [17, 18] given by $G_2 = 0.012 \, \text{GeV}^4$, with large uncertainties. One clearly sees some tension between this "small" scale and a "large" mass scale in QCD given by, for example, the mass of the lightest 0^{++} glueball, which is about $1.5 \, \text{GeV}$.

The existence of two numerically different np scales in strong interaction physics is a common feature one encounters in many models. As an early example, let us mention

seminal discussion [20] concerning the difference between confinement scale and chiral symmetry breaking scale. Another example is given by instanton model [21] where the mean instanton size (taken to be about 0.3 Fm) is assumed to be smaller than mean instanton radius (about 1 Fm) and this fact is nothing but the "diluteness" of the instanton gas in this model. Dual superconductor picture [8, 9] provides more recent example of this sort: for the QCD vacuum understood as the so-called type-II superconductor, the mass of diagonal field—the "photon" (inverse London penetration depth)—is parametrically larger than the effective Higgs mass. Correspondingly the nonabelian electric field penetrates the vacuum forming Abrikosov fluxes (in dual analogy with the magnetic fields in conventional metal superconductors).

Of immediate interest are recent lattice studies of relevant nonperturbative scales [22]. The authors project each lattice link variable to specific zone in the momentum space bound by some infrared and ultraviolet cuts—the momentum ring—and study dependence of various nonperturbative quantities of physical interest on the location and width of this ring. This analysis demonstrates numerically that the relevant scales are indeed different for different quantities. It is far from obvious, however, what gauge-invariant statements based on these results can be made (since the approach is gauge dependent).

The connections between these models and the approach discussed here were studied in many papers (see reviews in [12–15] and references therein). For example, direct analogy between λ and effective instanton size was analyzed in [23]. The relation between dual superconductor picture and field correlator method was studied in details in [24, 25]. Notice that in both cases the main advantage of field correlator method is its manifest gauge invariance.

The phenomenon of this hierarchy of scales is of prime importance for the stochastic scenario. It was suggested in [12–15]; see also, [26–28] that qualitative physical explanation is the following: *QCD vacuum is filled by short-distance correlated nonperturbative gluon fields.* In other words, besides gluon condensate G_2, there is another important dimensionfull parameter characterizing *np* dynamics of vacuum fields: correlation length λ (also denoted as T_g in some papers). The gauge-invariant dimensionless product $(G_2\lambda^4)^{1/2}$ is about a few percents (see below). This leads to numerous consequences, some of which are discussed in the rest of this paper.

An immediate question to ask is about the origin of this smallness. To answer it is one of the goals of the present paper. The key observation is the following: correlation length λ is nothing but inverse mass of certain gluelump. Gluelumps (for details, we refer the reader to [29, 30]) are rather unusual objects from perturbative field theory point of view. In some sense they are analogous to heavy-light mesons. The latter can be approximated as a bound state of fundamental color charge (light quark) in the static field of fundamental color anticharge (heavy antiquark). In the limit of heavy mass M going to infinity, the difference between the meson mass and M stays constant and the object as a whole is of course color singlet. Analogously, gluelump is a singlet bound state of adjoint color charge "valent gluons" in the static field of "infinitely heavy" adjoint color source. Roughly speaking, gluelump is what replaces gluon propagator in gauge-invariant formulation of the theory. Of course there are no elementary heavy particles with adjoint charge in QCD, and in this sense, gluelump cannot be "detected" as a real particle. Nevertheless, gluelumps happen to be very useful objects in discussions of the most fundamental properties of confining QCD vacuum (see more on that below).

Let us illustrate the appearance of numerical hierarchy of the kind we are discussing on a simple phenomenological example. Consider leading the following ρ-meson Regge

trajectory:

$$J = \alpha'_J M^2(J) + 0.48, \qquad \alpha'_J = 0.89 \, \mathrm{GeV}^{-2}. \tag{1.2}$$

This corresponds to the product

$$\sigma \alpha'_J = \frac{1}{2\pi} = 0.16, \tag{1.3}$$

where $\sigma = 0.18 \, \mathrm{GeV}^2$ is the confining string tension. One can say that the mass squared is a factor "2π" larger than string tension. On the other hand, in stochastic picture one has, parametrically, $\sigma \propto \lambda^2 G_2$. Here the smallness of string tension is controlled by two powers of λ. Thus to get self-consistent picture with the desired hierarchy, roughly speaking, one is to make manifest this "$1/2\pi$" factor in λ^2. This is the essence of the phenomenon discussed in the present paper.

Let us come back to the stochastic picture. Its crucial feature found in [31–33] is that the lowest, quadratic, nonlocal FC, $\langle F_{\mu\nu}(x)F_{\lambda\sigma}(y)\rangle$, describes all np dynamics with very good accuracy. It was also shown that a simple exponential form of quadratic correlators found on the lattice [34–38] allows to calculate all properties of lowest mesons, glueballs, hybrids, and baryons, including Regge trajectories, lepton widths, etc.; see reviews in [2, 27, 28] and references therein. Since the correlation length λ entering these exponents is small, potential relativistic quantum-mechanical picture is applicable and all QCD spectrum is defined mostly by string tension σ (which is an integral characteristic of the nonlocal correlator; see below) and not by its exact profile. This is discussed in details in the next section.

However to establish the confinement mechanism unambiguously, one should be able to calculate vacuum field distributions, that is, field correlators, self-consistently. In the long run, it means that one is to demonstrate that it is essential property of QCD vacuum fields ensemble to be characterized by correlators, which support confinement for temperatures T below some critical value T_c and deconfinement at $T > T_c$.

Attempts to achieve this goal have been undertaken in [39, 40]; however, the resulting chain of equations is too complicated to use in practice.

Another step in this direction is done in [41], where FCs are calculated via gluelump Green's functions and self-consistency of this procedure was demonstrated for the first time. These results were further studied and confirmed in [42].

The main aim of this paper is to present this set of equations as a self-consistent mechanism of confinement and to clarify qualitative details of the FC-gluelump connection. In particular we demonstrate how the equivalence of color-magnetic and colorelectric FCs for $T = 0$ ensures the Gromes relation [43, 44]. We also show, that self-consistency condition for FC as gluelump Green's function allows to connect the mass scale to α_s, and in this way, to express Λ_{QCD} via string tension σ.

Lattice computations play important role as a source of independent knowledge about vacuum field distributions. Recently a consistency check of this picture was done on the lattice [45] by measurement of spin-dependent potentials. The resulting FCs are calculated and compared with gluelump predictions in [46] demonstrating good agreement with analytic results; in particular small vacuum correlation length $\lambda \sim 0.1 \, \mathrm{Fm}$ is shown to correspond to large gluelump mass $M_0 \approx 2 \, \mathrm{GeV}$.

The paper is organized as follows. In the next section, we remind the basic expressions for Green's functions of $q\bar{q}$ and gg systems in terms of Wegner-Wilson loops, and qualitative picture of FCs dynamics is discussed. Section 3 is devoted to the expressions of spin-dependent potentials in terms of FC. We also shortly discuss the lattice computations of FC. In Section 4 FC are expressed in terms of gluelump Green's functions, while in Section 5 the self-consistency of resulting relations is studied. Our conclusions are presented in Section 6.

2. Wegner-Wilson Loop, Field Correlators, and Green's Functions

Since our aim is to study confinement for quarks (both light and heavy) and also for massless gluons, we start with the most general Green's functions for these objects in a proper physical background using Fock-Feynman-Schwinger representation. The formalism is built in such a way that gauge invariance is manifest at all steps. A reader familiar with this technique can skip this section and go directly to Section 3.

First, we consider the case of mesons made of quarks while gluon bound states are discussed below. For quarks, one forms the initial and final state operators

$$\Psi_{\text{in,out}}(x, y) = \psi^\dagger(x)\Gamma_{\text{in,out}}\Phi(x, y)\psi(y), \tag{2.1}$$

where $\psi^\dagger$, ψ are quark operators, Γ is a product of Dirac matrices, that is, 1, γ_μ, γ_5, $(\gamma_\mu\gamma_5)$, ..., and $\Phi(x, y) = P\exp(ig\int_y^x A_\mu dz_\mu)$ is parallel transporter (also known as Schwinger line or phase factor). The meson Green's function can be written (in quenched case) as

$$G_{q\bar{q}}(x, y \mid x', y') = \left\langle \Psi_{\text{out}}^+(x', y')\Psi_{\text{in}}(x, y) \right\rangle_{A,q}$$
$$= \left\langle \Gamma^\dagger\Phi(x', y')S_q(y', y)\Gamma\Phi(y, x)S_q(x, x') \right\rangle_A, \tag{2.2}$$

and the quark Green's function S_q is given in Euclidean space time (see [47, 48] and references therein) as

$$S_q(x, y) = \left(m + \hat{D}\right)^{-1} = \left(m - \hat{D}\right)\int_0^\infty ds(Dz)_{xy}\exp(-K)P_A\exp\left(ig\int_y^x A_\mu dz_\mu\right)P_\sigma(x, y, s), \tag{2.3}$$

where K is kinetic energy term as

$$K = m^2 s + \frac{1}{4}\int_0^s d\tau\left(\frac{dz_\mu(\tau)}{d\tau}\right)^2, \tag{2.4}$$

where m is the pole mass of quark, and quark trajectories $z_\mu(\tau)$ with the end points x and y are being integrated over in $(Dz)_{xy}$.

The factor $P_\sigma(x, y, s)$ in (2.3) is generated by the quark spin (color-magnetic moment) and is equal to

$$P_\sigma(x, y, s) = P_F \exp\left[g \int_0^s \sigma_{\mu\nu} F_{\mu\nu}(z(\tau)) d\tau\right], \tag{2.5}$$

where $\sigma_{\mu\nu} = (1/4i)(\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu)$, and $P_F(P_A)$ in (2.5) and (2.3) are, respectively, ordering operators of matrices $F_{\mu\nu}(A_\mu)$ along the path $z_\mu(\tau)$.

With (2.2) and (2.3), one easily gets

$$G_{q\bar{q}}(x, y; A) = \int_0^\infty ds \int_0^\infty ds' (Dz)_{xy} (Dz')_{xy} e^{-K-K'} \, \mathrm{Tr}\left(\Gamma\left(m - \hat{D}\right) W_\sigma(x, y) \Gamma^\dagger\left(m' - \hat{D}'\right)\right), \tag{2.6}$$

where Tr means trace operation both in Dirac and color indices, while

$$W_\sigma(x, y) = P_A \exp\left(ig \int_{C(x,y)} A_\mu dz_\mu\right) P_\sigma(x, y, s) P'_\sigma(x, y, s'). \tag{2.7}$$

In (2.7) the closed contour $C(x, y)$ is formed by the trajectories of quark $z_\mu(\tau)$ and antiquark $z'_\nu(\tau')$, and the ordering P_A and P_F in P_σ, P'_σ is universal, that is, $W_\sigma(x, y)$ is the Wegner-Wilson loop (W-loop) for spinor particle.

The factors $(m - \hat{D})$ and $(m' - \hat{D}')$ in (2.6) need a special treatment when being averaged over fields. As shown in Appendix 1 of [49], one can use the simple replacement

$$m - \hat{D} \longrightarrow m - i\hat{p}, \quad p_\mu = \frac{1}{2}\left(\frac{dz_\mu}{d\tau}\right)_{\tau=s}. \tag{2.8}$$

The representation (2.6) is exact in the limit $N_c \to \infty$, when internal quark loops can be neglected, and is a functional of gluonic fields A_μ, $F_{\mu\nu}$, which contains both perturbative and *np* contributions, not specified at this level.

The next step is averaging over gluon fields, which yields the physical Green's function $G_{q\bar{q}}$:

$$G_{q\bar{q}}(x, y) = \langle G_{q\bar{q}}(x, y; A)\rangle_A. \tag{2.9}$$

The averaging is done with the usual Euclidean weight $\exp(-\mathrm{Action}/\hbar)$, containing all necessary gauge-fixing and ghost terms.

To proceed, it is convenient to use nonabelian Stokes theorem (see [2] for references and discussion) for the first factor on the r.h.s. in (2.7) (corresponding to the W-loop for scalar particle) and to rewrite it as an integral over the surface S spanned by the contour $C = C(x, y)$:

$$\langle W(C)\rangle = \left\langle \mathrm{Tr}\, P \exp\left(ig \int_C A_\mu(z) dz_\mu\right)\right\rangle = \left\langle \mathrm{Tr}\, \mathcal{P} \exp\left(ig \int_S F(u) ds(u)\right)\right\rangle$$

$$= \mathrm{Tr}\, \mathcal{P}_x \exp \sum_{n=2}^\infty (ig)^n \int_S \langle\langle F(u_1) \cdots F(u_n)\rangle\rangle ds_1 \cdots ds_n = \exp \sum_{n=2}^\infty \Delta^{(n)}[S]. \tag{2.10}$$

Here $F(u_i)ds_i = \Phi(x_0, u_i)F_{\mu\nu}^a(u_i)t^a\Phi(u_i, x_0)ds_{\mu\nu}(u_i)$, and u_i, x_0 are the points on the surface S bound by the contour $C = \partial S$. The double brackets $\langle\langle \cdots \rangle\rangle$ stay for irreducible correlators proportional to the unit matrix in the color space (and therefore, only spacial ordering P_x enters (2.10)). Since (2.10) is gauge-invariant, one can make use of generalized contour gauge [50–52], which is defined by the condition $\Phi(x_0, u_i) \equiv 1$. Notice that throughout the paper we normalize trace over color indices as $\mathrm{Tr}\,\mathbf{1} = 1$ in any given representation.

   Since (2.10) is an identity, the r.h.s. does not depend on the choice of the surface, which is integrated over in $ds_{\mu\nu}(u)$. On the other hand, it is clear that each irreducible n-point correlator $\langle\langle F \cdots F \rangle\rangle$ integrated over S (these are the functions $\Delta^{(n)}[S]$ in (2.10)) depends, in general case, on the choice one has made for S. Therefore it is natural to ask the following question: is there any hierarchy of $\Delta^{(n)}[S]$ as functions of n for a given surface S?

   To get the physical idea behind this question, let us take the limit of small contour C. In this case one has for the np part of the W-loop in fundamental representation the following:

$$-\log \langle W(C) \rangle_{np} \sim G_2 \cdot S^2, \tag{2.11}$$

where S is the minimal area inside the loop C. The np short-distance dynamics is governed by the vacuum gluon condensate G_2; see (1.1). Higher condensates and other possible vacuum averages of local operators, in line with Wilson expansion, have been introduced and phenomenologically estimated in [19]; for a review see [53].

   Suppose now that the size of W-loop is not small, that is, it is larger than some typical dynamical scale $\lambda = \mathcal{O}(1\,\mathrm{GeV})$ to be specified below. Let us now ask the following question: if one still wants to expand the loop formally in terms of local condensates, what would the structure of such expansion be? It is easy to see from (2.10) that there are two subseries in this expansion. The first one is just an expansion in n, that is, in the number of field strength operators. It is just

$$\langle W(C) \rangle = 1 + \Delta^{(2)}[S] + \Delta^{(3)}[S] + \frac{1}{2}\left(\Delta^{(2)}[S]\right)^2 + \cdots. \tag{2.12}$$

The second series is an expansion of each $\Delta^{(n)}[S]$ in terms of local condensates, that is, expansion in powers of derivatives. Taking for simplicity the lowest $n = 2$ term, this expansion reads

$$\Delta^{(2)}[S] = \int_S ds_x \int_S ds_y \langle\langle F(x)F(y) \rangle\rangle$$

$$= \int_S ds_x \int_S ds_y \Big[\langle\langle F(x)F(x) \rangle\rangle + (x-y)_\mu \langle\langle F(x)D_\mu F(x) \rangle\rangle \tag{2.13}$$

$$+ (x-y)_\mu (x-y)_\nu \langle\langle F(x)D_\nu D_\mu F(x) \rangle\rangle / 2 + \cdots \Big].$$

   It is worth mentioning (but usually skipped in QCD sum rules analysis) that the latter series contain ambiguity related to the choice of contours in parallel transporters. The expression (2.13) is written for the simplest choice of straight contour connecting the points x and y (and *not* x and x_0 or y and x_0). This is the choice we adopt in what follows. In general

case each condensate in (2.13) is multiplied by some contour-dependent function of x, y, and x_0. It is legitimate to ask whether this approximation, that is, replacement of

$$\langle \operatorname{Tr} \Phi(x_0, x) F_{\mu\nu}(x) \Phi(x, x_0) \Phi(x_0, y) F_{\rho\sigma}(y) \Phi(y, x_0) \rangle \qquad (2.14)$$

by

$$\langle \operatorname{Tr} F_{\mu\nu}(x) \Phi(x, y) F_{\rho\sigma}(y) \Phi(y, x) \rangle \qquad (2.15)$$

affects our final results. Since the only difference between the above two expressions is profiles of the transporters, this is the question about contour dependence of FCs. We will not discuss this question in details in the present paper and refer an interested reader to [12–15]. Here we only mention that the result (weak dependence on transporters' choice) is very natural in Abelian dominance picture since for Abelian fields the transporters exactly cancel.

Let us stress that both expansions (2.12) and (2.13) are formal since the loop is assumed to be large. Moreover, from dimensional point of view, the terms of these two expansions mix with each other. For example, one has nonzero v.e.v. of two operators of dimension eight:

$$\Delta^{(2)}[S] \longrightarrow \left\langle F_{\mu\nu} D^4 F_{\mu\nu} \right\rangle, \qquad \Delta^{(3)}[S] \longrightarrow \left\langle F_{\mu\sigma} F_{\nu\sigma} D^2 F_{\mu\nu} \right\rangle, \qquad (2.16)$$

with no a priori reason (one could have such reasons if there is some special kinematics in the problem, but this is not the case.) to drop any of them. Moreover, even if one assumes that all these condensates of high orders can be self-consistently defined analogously to G_2, it remains to be seen whether the whole series converges or not.

Here the physical picture behind FC method comes into play. It is assumed (and confirmed a posteriori in several independent ways) that if the surface S is chosen as the minimal one, the dominant contribution to (2.10) results from v.e.v.s of the operators $FD^k F$, and this subseries can be summed up as (2.13). In this language, nonlocal two-point FC can be understood as a representation for the sum of infinite sequence of local terms (2.13).

Physically the dominance of $\langle FD^n F \rangle$ terms (and hence, of two-point FC) corresponds to the fact that the correlation length is small yielding a small expansion parameter $\xi = \bar{F}\lambda^2 \ll 1$ ($\bar{F}$ is average modulus of np vacuum fields), or, in other words, that typical inverse correlation length λ^{-1} characterizing ensemble of QCD vacuum fields is parametrically larger than fields themselves. The dimensionless parameter ξ in terms of condensate is given by $(G_2\lambda^4)^{1/2}$, and it is indeed small: of the order of 0.05 according to lattice estimates. One can say that often repeated statement "QCD vacuum is filled by strong and strongly fluctuating chromoelectric and chromomagnetic fields" is only partly correct, namely, in reality the fields are not *so* strong as those of λ^{-1}. Technically one can say that we sum up the leading subseries in (2.10) in the same spirit as one does in covariant perturbation theory [54] for weak but strongly varying potentials.

Thus, for understanding of confining properties of QCD vacuum dynamics, not only scale of np fields given by G_2 but also another quantity—the vacuum correlation length λ— which defines the nonlocality of gluonic excitations is important. It is worth repeating here that physically λ encodes information about properties of the series (2.13) and has the same theoretical status as that of the corresponding np condensates in the following sense: knowledge of all terms in (2.13) would allow to reconstruct λ. Of course, it is impossible in

practice and one has to use other methods to study large distance asymptotics of FC. On more phenomenologic level, it was discussed in [55–59], while rigorous definition was given in the framework of the FC method (often referred to as Stochastic Vacuum Model.) (see [12–15, 26, 60] for reviews).

It is important to find also the world-line representation for gluon Green's function, which is done in the framework of background perturbation theory (see [61–65] for details). In this way one finds (where a, b are adjoint color indices and dash sign denotes adjoint representation) that

$$G_{\mu\nu}^{ab}(x, y) = \left\{ \int_0^\infty ds (Dz)_{xy} e^{-K} P_a \exp\left(ig \int_y^x \widehat{A}_\mu dz_\mu \right) P_\Sigma(x, y, s) \right\}_{\mu\nu}^{ab}, \tag{2.17}$$

where

$$P_\Sigma(x, y, s) = P_F \exp\left(2ig \int_0^s \widehat{F}_{\lambda\sigma}(z(\tau)) d\tau \right). \tag{2.18}$$

All the above reasoning about asymptotic expansions of the corresponding W-loops is valid here as well.

Thus the central role in the discussed method is played by quadratic (Gaussian) FC of the form

$$D_{\mu\nu,\lambda\sigma} \equiv g^2 \langle \operatorname{Tr} F_{\mu\nu}(x) \Phi(x, y) F_{\lambda\sigma}(y) \Phi(y, x) \rangle, \tag{2.19}$$

where $F_{\mu\nu}$ is the field strength and $\Phi(x, y)$ is the parallel transporter. Correlation lengths λ_i for different channels are defined in terms of asymptotics of (2.19) at large distances: $\exp(-|x - y|/\lambda_i)$. The physical role of λ_i is very important since it distinguishes two regimes: one expects validity of potential-type approach describing the structure of hadrons of spatial size R and at temporal scale T_q for $\lambda_i \ll R, T_q$, while in the opposite case, when $\lambda_i \gg R, T_q$ the description in terms of spatially homogeneous condensates can be applied.

At zero temperature, the $O(4)$ invariance of Euclidean space-time holds and FC (2.19) is represented through two scalar functions $D(z), D_1(z)$ (where $z \equiv x - y$) as follows(all treatment in this paper as well as averaging over vacuum fields is done in the Euclidean space. Notice that only after all Green's functions are computed, analytic continuation to Minkowskii space-time can be accomplished) :

$$D_{\mu\nu,\lambda\sigma}(z) = (\delta_{\mu\lambda}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\lambda})D(z) + \frac{1}{2}\left[\frac{\partial}{\partial z_\mu}(z_\lambda\delta_{\nu\sigma} - z_\sigma\delta_{\nu\lambda}) + \frac{\partial}{\partial z_\nu}(z_\sigma\delta_{\mu\lambda} - z_\lambda\delta_{\mu\sigma}) \right]D_1(z). \tag{2.20}$$

One has to distinguish from the very beginning perturbative and np parts of the correlators $D(z), D_1(z)$. Beginning with the former, one easily finds at tree level that

$$D^{p,0}(z) = 0, \qquad D_1^{p,0}(z) = C_2(f)\frac{4\alpha_s}{\pi}\frac{1}{z^4}, \tag{2.21}$$

where $C_2(f)$ is fundamental Casimir $C_2(f) = (N_c^2 - 1)/2N_c$. At higher orders, situation becomes more complicated. Namely, one has at n-loop order

$$D^{p,n}(z) = D_1^{p,0}(z) \cdot G^{(n)}(z), \qquad D_1^{p,n}(z) = D_1^{p,0}(z) \cdot G_1^{(n)}(z), \qquad (2.22)$$

where the gauge-invariant functions $G^{(n)}(z)$, $G_1^{(n)}(z)$ have the following general structure:

$$G^{(n)}(z), G_1^{(n)}(z) \sim \alpha_s^n \left[c_n (\log \mu z)^n + \cdots \right], \qquad (2.23)$$

where c_n is numerical coefficient and μ is renormalization scale, and we have omitted subleading logarithms and constant terms in the r.h.s.; explicit expressions for the case $n = 1$ can be found in [66, 67].

Naively one could take perturbative functions $D^{p,n}(z)$, $D_1^{p,n}(z)$ written above and use them for computations of W-loop or static potentials in Gaussian approximation. This, however, would be incorrect. The reason is that at any given order in perturbation expansion over α_s one should take into account perturbative terms of the given orders coming from all FCs, and not only from the Gaussian one. For example, at one loop level, the function $D^{p,1}(z)$ is nonzero which naively would correspond to area law at perturbative level. Certainly this cannot be the case since self-consistent renormalization program for W-loops is known not to admit any terms of this sort [68, 69]. Technically in FC language, the correct result is restored by cancelation of contributions proportional to $D^{p,1}(z)$ to all observables by the terms coming from triple FC (see details in [70]).

In view of this general property of cluster expansion, it is more natural just to take relation $D^{p,n}(z) \equiv 0$ as valid at arbitrary n, having in mind that proper number of terms from (2.12) has to be included to get this result. Thus we assume that the following decomposition takes place:

$$D(z) = D^{np}(z), \qquad D_1(z) = D_1^p(z) + D_1^{np}(z), \qquad (2.24)$$

and $D(z)$ has a smooth limit when $z \to 0$. As for $D_1(z)$, its general asymptotic at $z \to 0$ reads as

$$D_1(z) = \frac{c}{z^4} + \frac{a_2}{z^2} + O\left(z^0\right), \qquad (2.25)$$

where c and a_2 weakly (logarithmically) depend on z. Notice that by $D_1(0)$, we always understand np "condensate" part in what follows:

$$G_2 = \frac{6N_c}{\pi^2} \left(D^{np}(0) + D_1^{np}(0) \right). \qquad (2.26)$$

The term $1/z^2$ deserves special consideration. One may argue that at large distances the simple additivity of perturbative and np contributions to $V_{Q\bar{Q}}$ potential is violated [71]. It is interesting whether this additivity holds at small, distances and, in particular, mixed condensate a_2 was suggested in [72] and argued to be phenomenologically desirable.

As we demonstrate below, our approach is self-consistent with $a_2 = 0$, that is, when perturbative-np additivity holds at small distances and there is no dimension-two condensate. However, at intermediate distances, a complicated interrelation occurs, which does not exclude (2.25) being relevant in this regime.

We proceed with general analysis of (2.19). As proved in [12–15], $D(z)$, $D_1(z)$ do not depend on relative orientation of plane $(\mu\nu)$, plane $(\lambda\sigma)$, and vector z_α. This orientation can be of the following 3 types: (a) planes $(\mu\nu)$ and $(\lambda\sigma)$ are perpendicular to the vector z_α; (b) planes $(\mu\nu)$ and $(\lambda\sigma)$ are parallel or intersecting along one direction and z_α lies in one of planes; (c) planes $(\mu\nu)$ and $(\lambda\sigma)$ are perpendicular and z_α lies necessarily in one of them.

It is easy to understand that this classification is $O(4)$ invariant, and therefore, one can assign indices a, b, c to $D_{\mu\nu,\lambda\sigma}$, resulting in general case in three different functions. Physically speaking, case (a) refers to, for example, color magnetic fields F_{ik}, F_{lm} with z_α in the 4th direction, and the corresponding FCs will be denoted as $D_\perp(z)$. Case (b) refers to, for example, the color electric fields $E_i(x)$, $E_k(y)$, connected by the same temporal links along 4th axis, and the corresponding FCs are denoted as $D_\parallel(z)$. Finally, in case (c) only D_1 part survives in (2.20) and it will be given by the subscript EH, D_1^{EH}.

In case of zero temperature, when $O(4)$ invariance holds, these three functions $D_\perp(z)$, $D_\parallel(z)$, and $D_1^{EH}(z)$ are easily expressed via $D(z)$, $D_1(z)$:

$$D_\perp(z) = D(z) + D_1(z),$$

$$D_\parallel(z) = D(z) + D_1(z) + z^2 \frac{\partial D_1(z)}{\partial z^2}, \tag{2.27}$$

$$D_1^{EH}(z) \equiv D_1(z)$$

(note that just $D_\perp$, $D_\parallel$ were measured on the lattice in [34–38]). For nonzero temperature, the correlators of color-electric and color-magnetic fields can be different and in general there are five FCs: $D^E(z)$, $D_1^E(z)$, $D^H(z)$, $D_1^H(z)$, and $D_1^{EH}(z)$. As a result, one obtains the full set of five independent quadratic FCs which can be defined as follows:

$$g^2 \langle \operatorname{Tr} H_i(x)\Phi H_j(y)\Phi^+ \rangle = \delta_{ij}\left(D^H(z) + D_1^H(z) + \mathbf{z}^2 \frac{\partial D_1^H}{\partial z^2} \right) - z_i z_j \frac{\partial D_1^H}{\partial z^2},$$

$$g^2 \langle \operatorname{Tr} E_i(x)\Phi E_j(y)\Phi^+ \rangle = \delta_{ij}\left(D^E(z) + D_1^E(z) + z_4^2 \frac{\partial D_1^E}{\partial z^2} \right) + z_i z_j \frac{\partial D_1^E}{\partial z^2}, \tag{2.28}$$

$$g^2 \langle \operatorname{Tr} H_i(x)\Phi E_j(y)\Phi^+ \rangle = e_{ijk} z_k z_4 \frac{\partial D_1^{EH}}{\partial z^2}.$$

It is interesting that the structure (2.28) survives for nonzero temperature, where $O(4)$ invariance is violated: it tells that (2.28) give the most general form of the FCs. Note that at zero temperature, $D^H = D^E$, $D_1^H = D_1^E$ at coinciding point; the $O(4)$ invariance

requires that colorelectric and colormagnetic condensates coincide: $g^2\langle \mathrm{Tr}\, H_i(x)H_i(x)\rangle =$ $g^2\langle \mathrm{Tr}\, E_j(x)E_j(x)\rangle$; hence,

$$D^H(0) + D_1^H(0) = D^E(0) + D_1^E(0). \tag{2.29}$$

Here we assume that the *np* part of all FCs is finite.

At $z \neq 0$ for zero temperature, FCs can be expressed through $D_\perp(z)$, $D_\parallel(z)$ which do not depend on whether E_i or H_k enters in them, since, for example, $D_\parallel^E$ can be transformed into $D_\parallel^H$ by an action of $O(4)$ group elements (the same for $D_\perp^E$, $D_\perp^H$). From this, one can deduce that both D_1 and D do not depend on subscripts E, H for zero temperature and D_1^{EH} coincides with D_1.

Below we will illustrate this coincidence by concrete calculations of D^E, D^H, and so forth, through the gluelump Green's functions and show that the corresponding correlation lengths satisfy the relations (with obvious notations)

$$\lambda^E = \lambda^H \equiv \lambda; \qquad \lambda_1^E = \lambda_1^H = \lambda_1^{EH} \equiv \lambda_1. \tag{2.30}$$

As shown below, all correlation lengths λ_j appear to be just inverse masses of the corresponding gluelumps $\lambda_j = 1/M_j$.

Having analytic expressions for FCs, one might ask that how to check them versus experimental and lattice data. On experimental side in hadron spectroscopy, one measures masses and transition matrix elements, which are defined by dynamical equation, and the latter can be used of potential type due to smallness of λ_j. In static potential, only integrals over distance enter and the spin-independent static potential can be written as [12–15, 73, 74]

$$V_{Q\overline{Q}}(r) = 2\int_0^r (r-\lambda)d\lambda \int_0^\infty d\nu D^E\left(\sqrt{\lambda^2 + \nu^2}\right) + \int_0^r \lambda d\lambda \int_0^\infty D_1^E\left(\sqrt{\lambda^2 + \nu^2}\right). \tag{2.31}$$

At this point one should define how perturbative and *np* contributions combine in D^E, D_1^E, and this analysis was done in [70]. Making use of (2.21) together with definition of the string tension [12–15], one yeilds

$$\sigma^E = \frac{1}{2}\iint d^2z D^E(z). \tag{2.32}$$

One obtains from (2.31) the standard form of the static potential for $N = 3$ at distances $r \gg \lambda^E$ as

$$V_{Q\overline{Q}}(r) = \sigma^E r - \frac{4\alpha_s(r)}{3r} + \mathcal{O}\left(\frac{\lambda^E}{r}, \alpha_s^2\right). \tag{2.33}$$

As argued below, $\lambda^E \approx 0.1$ Fm, so that the form (2.33) is applicable in the whole range of distances $r > 0.1$ Fm provided that asymptotic freedom is taken into account in $\alpha_s(r)$ in (2.33).

At the smallest values of r, $r \lesssim \lambda^E$, one would obtain a softening of confining term, $\sigma r \rightarrow cr^2$ [12–15], which is not seen in accurate lattice data at $r \gtrsim 0.2$ Fm [75, 76], imposing a stringent limit on the value of λ^E; $\lambda^E \lesssim 0.1$ Fm; see discussion in [76].

3. Spin-Dependent Potentials and FC

We now turn to the spin-dependent interactions to demonstrate that FC can be extracted from them [73, 74, 77–79]. $V_1'(r)$, $V_2'(r)$, $V_3(r)$, $V_4(r)$, plus static term $V_{Q\bar{Q}}'(r)$ contain in integral form all five FCs: $D^E, D_1^E, D^H, D_1^H, D_1^{HE}$, and one can extract the properties of the latter from the spin-dependent potentials. This procedure is used recently for comparison of analytic predictions [46] with the lattice data [45].

To express spin-dependent potentials in terms of FC, one can start with the W-loop (2.7), entering as a kernel in the $q\bar{q}$ Green's function, where quarks $q, \bar{q}$ can be light or heavy. Since both kernels P_σ, P_σ' contain the matrix $\sigma_{\mu\nu}F_{\mu\nu}$, the terms of spin-spin and spin-orbit types appear. As before, we keep Gaussian approximation, and in this case spin-dependent potentials can be computed not only for heavy but also for light quarks. They have the following Eichten-Feinberg form [80]:

$$
V_{SD}^{(\text{diag})}(R) = \left(\frac{\vec{\sigma}_1 \vec{L}_1}{4\mu_1^2} - \frac{\vec{\sigma}_2 \vec{L}_2}{4\mu_2^2} \right) \left(\frac{1}{R} \frac{d\varepsilon}{dR} + \frac{2dV_1(R)}{RdR} \right)
$$
$$
+ \frac{\vec{\sigma}_2 \vec{L}_1 - \vec{\sigma}_1 \vec{L}_2}{2\mu_1\mu_2} \frac{1}{R} \frac{dV_2}{dR} + \frac{\vec{\sigma}_1 \vec{\sigma}_2 V_4(R)}{12\mu_1\mu_2} + \frac{\left(3\vec{\sigma}_1 \vec{R} \vec{\sigma}_2 \vec{R} - \vec{\sigma}_1 \vec{\sigma}_2 R^2 \right) V_3}{12\mu_1\mu_2 R^2}.
\tag{3.1}
$$

Spin-spin interaction appears in W_σ in (2.7) which can be written as

$$
\exp \left\{ -\frac{g^2}{2} \int_0^{s_1} d\tau_1 \int_0^{s_2} d\tau_2 \left\langle \begin{pmatrix} \sigma^{(1)}\mathbf{B} & \sigma^{(1)}\mathbf{E} \\ \sigma^{(1)}\mathbf{E} & \sigma^{(1)}\mathbf{B} \end{pmatrix}_{z_1} \begin{pmatrix} \sigma^{(2)}\mathbf{B} & \sigma^{(2)}\mathbf{E} \\ \sigma^{(2)}\mathbf{E} & \sigma^{(2)}\mathbf{B} \end{pmatrix}_{z_2} \right\rangle \right\},
\tag{3.2}
$$

where $z_{1,2} = z(\tau_{1,2})$, and spin-orbit interaction arises in (2.19) from the products $\langle \sigma_{\mu\nu}F_{\mu\nu}ds_{\rho\lambda}F_{\rho\lambda} \rangle$.

It is clear that the resulting interaction will be of matrix form $(2 \times 2) \times (2 \times 2)$ (not accounting for Pauli matrices). If one keeps only diagonal terms in $\sigma_{\mu\nu}F_{\mu\nu}$ (as the leading terms for large $\mu_i \approx M$) then one can write for the spin-dependent potentials the representation of the Eichten-Feinberg form (3.1).

At this point one should note that the term with $d\varepsilon/dR$ in (3.1) was obtained from the diagonal part of the matrix $(m - \hat{D})\sigma_{\mu\nu}F_{\mu\nu}$, namely, as product $i\sigma_k D_k \cdot \sigma_i E_i$; see [73, 74, 77, 78] for details of derivation, while all other potentials V_i, $i = 1, 2, 3, 4$ are proportional to FCs

$\langle H_i \Phi H_k \Phi \rangle$. One can relate FCs of colorelectric and colormagnetic fields to D^E, D_1^E, D^H, D_1^H defined by (2.28) with the following result:

$$
\begin{aligned}
\frac{1}{R}\frac{dV_1}{dR} &= -\int_{-\infty}^{\infty} dv \int_0^R \frac{d\lambda}{R}\left(1 - \frac{\lambda}{R}\right)D^H(\lambda, v), \\
\frac{1}{R}\frac{dV_2}{dR} &= \int_{-\infty}^{\infty} dv \int_0^R \frac{\lambda d\lambda}{R^2}\left[D^H(\lambda, v) + D_1^H(\lambda, v) + \lambda^2 \frac{\partial D_1^H}{\partial \lambda^2}\right], \\
V_3 &= -\int_{-\infty}^{\infty} dv R^2 \frac{\partial D_1^H(R, v)}{\partial R^2}, \\
V_4 &= \int_{-\infty}^{\infty} dv\left(3D^H(R, v) + 3D_1^H(R, v) + 2R^2 \frac{\partial D_1^H}{\partial R^2}\right), \\
\frac{1}{R}\frac{d\varepsilon(R)}{dR} &= \int_{-\infty}^{\infty} dv \int_0^R \frac{d\lambda}{R}\left[D^E(\lambda, v) + D_1^E(\lambda, v) + \left(\lambda^2 + v^2\right)\frac{\partial D_1^E}{\partial v^2}\right].
\end{aligned}
\tag{3.3}
$$

One can check that Gromes relation [43, 44] acquires the form [79]

$$
V_1'(R) + \varepsilon'(R) - V_2'(R) = \int_{-\infty}^{\infty} dv\left[\int_0^R d\lambda\left(D^E(\lambda, v) - D^H(\lambda, v)\right) + \frac{1}{2}R\left(D_1^E(R) - D_1^H(R)\right)\right].
$$

$$\tag{3.4}$$

For $T = 0$, when $D^E = D^H$, $D_1^E = D_1^H$, the Gromes relations are satisfied identically; however, for $T > 0$ electric and magnetic correlators are certainly different and Gromes relation is violated, as one could tell beforehand; since for $T > 0$, the Euclidean $O(4)$ invariance is violated.

4. Gluelumps and FC

In this section we establish a connection of FC $D(z)$ and $D_1(z)$ with the gluelump Green's functions.

To proceed, one can use the background field formalism [61–65, 81–83], where the notions of valence gluon field a_μ and background field B_μ are introduced, so that total gluonic field A_μ is written as

$$A_\mu = a_\mu + B_\mu. \tag{4.1}$$

The main idea we are going to adopt here is suggested in the second paper of [39, 40]. To be self-contained, let us explain it here in simple form. First, we single out some color index a and fix it at a given number. Then in color components we have, by definition, that

$$A_\mu^c = \delta^{ac} a_\mu^a + (1 - \delta^{ac})B_\mu^c, \tag{4.2}$$

and there is summation neither over a in the first term nor over c in the second. As a result, the integration measure factorizes

$$\int \prod_c \mathfrak{D} A_\mu^c = \int \mathfrak{D} a_\mu^a \int \prod_{c \neq a} \mathfrak{D} B_\mu^c, \qquad (4.3)$$

so one first averages over the fields B_μ^c, and after that one integrates over the fields a_μ^a. The essential point is that the integration over $\mathfrak{D} B_\mu^c$ provides colorless adjoint string attached to the gluon a_μ^a, which keeps the color index "a" unchanged in the course of propagation of gluon a in background made of B. This is another way of saying that gluon field of each particular color enters QCD Lagrangian quadratically. Despite the fact that such separation of the measure is a kind of trick, the formation of adjoint string is the basic physical mechanism behind this background technic, and it is related to the properties of gluon ensemble: even for $N_c = 3$, one has one color degree of freedom a_μ^a and 7 fields B_μ^c; also confining string is a colorless object, and therefore, the singled-out adjoint index "a" can be preserved during interaction process of the valence gluon a_μ^a with the background (B_μ^c). These remarks to be used in what follows make explicit the notions of the valence gluon and background field.

We assume the background Feynman gauge $D_\mu a_\mu = 0$ and assign field transformations as follows:

$$a_\mu \longrightarrow U^+ a_\mu U, \qquad B_\mu \longrightarrow U^+ \left(B_\mu + \frac{i}{g} \partial_\mu \right) U. \qquad (4.4)$$

As a result, the parallel transporter $\Phi(x, y) = P \exp ig \int_y^x B_\mu(z) dz$ keeps its transformation property, and every insertion of a_μ between Φ transforms gauge covariantly. In what follows we will assume that Φ is made entirely of the field B_μ. Note that in the original cluster expansion of the W-loop $W(B + a)$ in FC [81–83]

$$W(B + a) = \left\langle \mathrm{Tr} \, P \exp ig \int_C (B_\mu + a_\mu) dz_\mu \right\rangle, \qquad (4.5)$$

it does not matter whether $(B_\mu + a_\mu)$ or B_μ enters Φ's since those factors cancel in the sum. Thus we assume from the beginning that Φ's are not renormalized, since such renormalization is unphysical anyway according to what have been said above.

As for renormalization having physical meaning, it is known [68, 69] to reduce to the charge renormalization and renormalization of perimeter divergences on the contour C. For static quarks, the latter reduces to the mass renormalization.

In background field formalism, one has an important simplification: the combination gB_μ is renorminvariant [61–65, 68, 69], and so is any expression made of B_μ only. This point is important for comparison of FC with those of the lattice correlators in [45, 46]. Renormalization constants for field strengths Z_b used there account for finite size of plaquettes and they are similar to the lattice tadpole terms.

Now we turn to the analytic calculation of FC in terms of gluelump Green's function. To this end, we insert (4.1) into (2.19) and have for $F_{\mu\nu}(x)$ the following:

$$F_{\mu\nu}(x) = \partial_\mu A_\nu - \partial_\nu A_\mu - ig [A_\mu, A_\nu] = \hat{D}_\mu a_\nu - \hat{D}_\nu a_\mu - ig [a_\mu, a_\nu] + F_{\mu\nu}^{(B)}. \qquad (4.6)$$

Here, the term $F_{\mu\nu}^{(B)}$ contains only the field B_μ^b. It is clear that when one averages over field a_μ^a and sums finally over all color indices a, one actually exploits all the fields with color indices from $F_{\mu\nu}^{(B)}$, so that the term $F_{\mu\nu}^{(B)}$ can be omitted, if summing over all indices a is presumed to be done at the end of calculation.

As a result, $D_{\mu\nu,\lambda\sigma}$ can be written as

$$D_{\mu\nu,\lambda\sigma}(x,y) = D_{\mu\nu,\lambda\sigma}^{(0)} + D_{\mu\nu,\lambda\sigma}^{(1)} + D_{\mu\nu,\lambda\sigma}^{(2)}, \tag{4.7}$$

where the superscripts 0, 1, 2 refer to the power of g coming from the term $ig[a_\mu, a_\nu]$.

We can address now an important question about the relation between FCs and gluelump Green's functions. We begin with 1-gluon gluelump, whose Green's function reads

$$G_{\mu\nu}^{(1g)}(x,y) = \left\langle \mathrm{Tr}_a a_\mu(x)\widehat{\Phi}(x,y)a_\nu(y) \right\rangle. \tag{4.8}$$

According to (4.4), $G_{\mu\nu}^{(1g)}(x,y)$ is a gauge-invariant function.

As shown in [41], the first term in (4.7) is connected to the functions D_1^E, D_1^H and it can be written as follows:

$$D_{\mu\nu,\lambda\sigma}^{(0)}(x,y) = \frac{g^2}{2N_c^2}\left\{ \frac{\partial}{\partial x_\mu}\frac{\partial}{\partial y_\lambda}G_{\nu\sigma}^{(1g)}(x,y) + \mathrm{perm} \right\} + \Delta_{\mu\nu,\lambda\sigma}^{(0)}, \tag{4.9}$$

where $\Delta_{\mu\nu,\lambda\sigma}^{(0)}$ contains contribution of higher FCs, which we systematically discard.

On the other hand, one can find $G_{\mu\nu}^{(1g)}(x,y)$ from the expression [47, 48] written as

$$G_{\mu\nu}^{(1gl)}(x,y) = \mathrm{Tr}_a \int_0^\infty ds (Dz)_{xy} \exp(-K)\left\langle W_{\mu\nu}^F(C_{xy}) \right\rangle, \tag{4.10}$$

where the spin-dependent W-loop is

$$W_{\mu\nu}^F(C_{xy}) = PP_F\left\{ \exp\left(ig \int B_\lambda dz_\lambda \right) \exp F \right\}_{\mu\nu}, \tag{4.11}$$

and the gluon spin factor is $\exp F \equiv \exp(2ig\int_0^s d\tau \widehat{F}_B(z(\tau)))$ with $\widehat{F}_B$ made of the background field B_μ only.

Analogous expression can be constructed for Green's function of 2-gluon gluelump. It is given by the following expression :

$$G_{\mu\nu,\lambda\sigma}^{(2gl)}(x,y) = \left\langle \mathrm{Tr}_a\left(f^{abc}f^{def} a_\mu^a(x)a_\nu^b(x)T^c\widehat{\Phi}(x,y)T^f \times a_\lambda^d(y)a_\sigma^e(y) \right) \right\rangle$$

$$\equiv N_c^2\left(N_c^2 - 1 \right)\left(\delta_{\mu\lambda}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\lambda} \right)G^{(2gl)}(z). \tag{4.12}$$

At small, distances $G^{(2gl)}(x, y)$ is dominated by the perturbative expansion terms

$$G^{(2gl)}(z) \sim \frac{1}{z^4}, \tag{4.13}$$

however, as we already discussed in details, all these perturbative terms are canceled by those from higher FCs (triple, quartic, etc.); therefore, expansion in fact starts with *np* terms of dimension four.

To identify the *np* contribution to $G^{(2gl)}(z)$, we rewrite it as follows:

$$G^{(2gl)}(z) = \int_0^\infty ds_1 \int_0^\infty ds_2 (Dz_1)_{0x} (Dz_2)_{0x} \operatorname{Tr} W_\Sigma(C_1, C_2), \tag{4.14}$$

where the two-gluon gluelump W-loop $W_\Sigma(C_1, C_2)$ is depicted in Figure 6 and can be written as (in the Gaussian approximation)

$$\operatorname{Tr} W_\Sigma(C_1, C_2) = \exp\left\{ -\frac{1}{2} \int_s \int d\pi_{\mu\nu}(u) d\pi_{\lambda\sigma}(v) \left\langle \widehat{F}_{\mu\nu}(u) \widehat{\Phi} \widehat{F}_{\lambda\sigma}(v) \right\rangle \right\}, \tag{4.15}$$

and the total surface S consists of 3 pieces, as shown in Figure 4, as follows:

$$F_{\mu\nu} d\pi_{\mu\nu}(u) = F_{\mu\nu} ds_{\mu\nu}(u) - 2ig d\tau \left(\widehat{F}(u) \right), \tag{4.16}$$

where $(\widehat{F})$ has Lorentz tensor and adjoint color indices F_{ij}^{ab} and lives on gluon trajectories, that is, on the boundaries of S_i. In full analogy with (4.9), we have that

$$D(z) = \frac{g^4 (N_c^2 - 1)}{2} G^{(2gl)}(z). \tag{4.17}$$

The crucial point is that Green's functions (4.9), (4.14) can be calculated in terms of the same FCs $D(z)$, $D_1(z)$. Indeed one has that

$$G^{(w)}(z) = \langle f | \exp(-H_w |z|) | i \rangle, \tag{4.18}$$

where the index w stays for 1-gluon or 2-gluon gluelump Hamiltonians. The latter are expressed via the same FCs $D(z)$, $D_1(z)$ (see [84–86] and references therein):

$$H_w = H_0[\mu] + \Delta H_L[\mu, \nu] + \Delta H_{\text{Coul}}[D_1] + \Delta H_{\text{string}}[D, \nu], \tag{4.19}$$

where the last term $H_{\text{string}}[D, \nu]$ depends on $D(z)$ via the adjoint string tension

$$\sigma_{\text{adj}} = \frac{9}{4} \int_0^\infty d^2 z D(z), \tag{4.20}$$
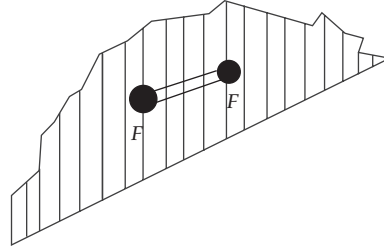
Figure 1

and Hamiltonian depends on einbein fields μ and ν. The term corresponding to perimeter Coulomb-like interaction $\Delta H_{\text{Coul}}[D_1]$ depends on $D_1(z)$ (compare with (2.31), (3.1)).

The self-consistent regimes correspond to different asymptotics of the solutions to these equations. In Coulomb phase of a gauge theory, both H_{1g}, H_{2g} exhibit no mass gap, that is, large-z asymptotic of (4.18) is power like. The function $D(z)$ vanishes in this phase and W-loop obeys perimeter law. In the confinement phase realized in Yang-Mills theory at low temperatures, a typical large-z pattern is given by

$$D(z), D_1(z) \sim \exp\left(-\frac{|z|}{\lambda_i}\right), \tag{4.21}$$

that is, there is a mass gap for both H_{1g}, H_{2g}.

Confining solutions are characterized by W-loops obeying area law. In other words, Hamiltonians expressed in terms of interaction kernels depending on $D(z)$, $D_1(z)$ exhibit mass gap if these kernels are confining. On the other hand, the same mass gap plays a role of inverse correlation length of the vacuum. This should be compared with well-known mean-field technique. In our case the role of mean-field is played by quadratic FC, which develops nontrivial Gaussian term $D(z)$. Notice that the exponential form of its large distance asymptotics $\exp(-|z|/\lambda)$ (and not, let's say, $\exp(-z^2/\lambda^2)$) is dictated by spectral expansion of the corresponding Green's function at large distances.

Full solution of the above equations is a formidable task not addressed by us here. Instead, as a necessary prerequisite, we check below different asymptotic regimes and demonstrate self-consistency of the whole picture.

We begin with small distance region. The np part of contribution to $D_1(z)$ at small distances comes from two possible sources: the area law term (first exponent and the $\exp F$ term in (4.11)). Both contributions are depicted in Figures 1 and 2, respectively. We will disregard the term $\exp F$ in this case, since for the one-gluon gluelump it does not produce hyperfine interaction and only gives rise to the np shift of the gluon mass, which anyhow is eliminated by the renormalization (see the appendix for more details). This is in contrast to the two-gluon gluelump Green's function generating D^E, D^H, where the hyperfine interaction between two gluons is dominating at small distances.

As a result, (4.8) without the $\exp F$ term yields

$$D_1(z) = \frac{4C_2(f)\alpha_s}{\pi}\frac{1}{z^4} + \frac{g^2}{12}G_2. \tag{4.22}$$

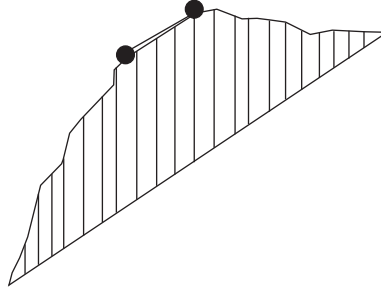It is remarkable that the sign of the np correction is positive.

Figure 2

At large distances, one can use the gluelump Hamiltonian for one-gluon gluelump from [30] to derive the asymptotics [41]

$$D_1(z) = \frac{2C_2(f)\alpha_s M_0^{(1)}\sigma_{\mathrm{adj}}}{|z|}e^{-M_0^{(1)}|z|}, \quad |z|M_0^{(1)} \gg 1, \tag{4.23}$$

where $M_0^{(1)} = (1.2 \div 1.4)\,\mathrm{GeV}$ for $\sigma_f = 0.18\,\mathrm{GeV}^2$ [29, 30].

We now turn to the FC $D(z)$ as was studied in [41]. The relation (4.17) connects $D(z)$ to the two-gluon gluelump Green's function, studied in [30] at large distances. Here we need its small-z behavior and we will write it in the form

$$G^{(2gl)}(z) = G_p^{(2gl)}(z) + G_{np}^{(2gl)}(z), \tag{4.24}$$

where $G_p^{(2gl)}(z)$ contains purely perturbative contributions which are subtracted by higher-order FCs, while $G_{np}^{(2gl)}(z)$ contains *np* and possible perturbative-*np* interference terms. We are interested in the contribution of the FC $\langle FF \rangle$ to $G^{(2gl)}(z)$, when z tends to zero.

One can envisage three types of contributions as follows.

(a) The first one is due to product of surface elements $ds_{\mu\nu}ds_{\lambda\sigma}$, which gives for small surface the factor $\exp(-g^2\langle FF \rangle S_i^2/24N_c)$, similar to the situation discussed for D_1. This is depicted in Figure 4.

(b) The second one is contribution of the type $d\tau_1 d\tau_2 \langle FF \rangle$, where $d\tau$ and $d\tau_2$ belong to different gluon trajectories, 1 and 2, as depicted in in Figure 5. This is the hyperfine gluon-gluon interaction, taken into account in [30] in the course of the gluelump mass calculations (regime of large z); however, in that case mostly the perturbative part of $\langle FF \rangle$ contributes (due to D_1). Here we keep in $\langle FF \rangle$ only the *NP* part and consider the case of small z.

(c) Again the third is the term $d\tau^i d\tau_j$, but now $i = j$. This is actually a part of the gluon selfenergy correction, which should be renormalized to zero, when all (divergent) perturbative contributions are added. As we argued in Appendix of [41], we disregard this contribution here, as well as in the case of $D_1(z)$ (one-gluon gluelump case). Now we treat both contributions (a) and (b).
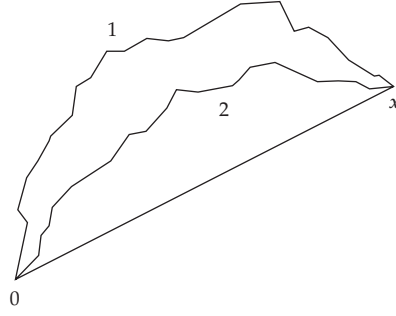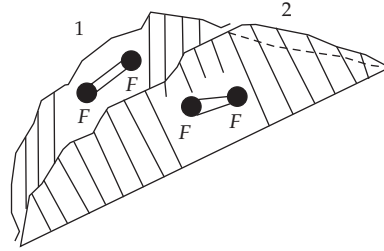
Figure 3

Figure 4

(a) In line with the treatment of D_1, (4.22), one can write the term representing two-gluon gluelump as two nearby one-gluon gluelumps which yield for $D(z)$ the following:

$$\Delta D_a(z) = -\frac{g^4 N_c G_2}{4\pi^2}.$$

(4.25)

(b) In this case one should consider the diagram given in **Figure** 5. which yields the answer (for details see the appendix)

$$G_b^{(2gl)}(z) = \frac{4N_c^2}{N_c^2 - 1} \int \frac{d^4w d^4w' D(w - w')}{(4\pi^2)^4 (w - z)^2 w^2 (w' - z)^2 w'^2},$$

(4.26)

which contributes to $D(z)$ as

$$\Delta D_b(z) = 2N_c^2 g^4 h(z),$$

(4.27)

where at small z, $h(z) \approx (D(\lambda_0)/64\pi^4)\log^2(\lambda_0\sqrt{e}/z)$ and λ_0 is of the order of correlation length λ.
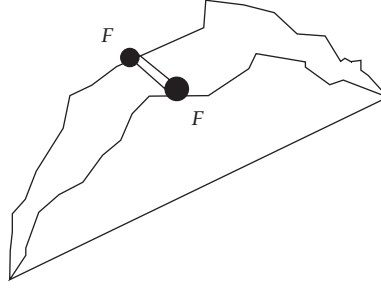
Figure 5

At large distances, one uses the two-gluon gluelump Hamiltonian as in [29] and finds the corresponding spectrum and wave functions; see [29] and Appendix 5 of [41] for details. As a result, one obtains in this approximation

$$D(z) = \frac{g^4(N_c^2-1)}{2}0.1\sigma_f^2 e^{-M_0^{(2)}|z|}, \quad M_0^{(2)}|z| \gg 1, \tag{4.28}$$

where $M_0^{(2)}$ is the lowest two-gluon gluelump mass found in [30] to be about $M_0^{(2)} = (2.5 \div 2.6)\,\text{GeV}$.

We will discuss the resulting properties of $D(x)$ and $D_1(x)$ in the next section.

5. Discussion of Consistency

We start with the check consistency for $D(z)$. As is shown above, $D(z)$ has the following behavior at small z :

$$D(z) \approx -4N_c\alpha_s^2(\mu(z))G_2 + N_c^2\frac{\alpha_s^2(\mu(z))}{2\pi^2}D(\lambda_0)\log^2\left(\frac{\lambda_0\sqrt{e}}{z}\right). \tag{5.1}$$

Since $\alpha_s(\mu(z)) \sim 2\pi/\beta_0 \log(\Lambda z)^{-1}$, the first term is subleading at $z \to 0$ and the last term on the r.h.s. tends to a constant

$$D(0) = \frac{N_c^2}{2\pi^2}D(\lambda_0)\left(\frac{2\pi}{\beta_0}\right)^2. \tag{5.2}$$

From (5.2) one can infer that $D(0) \approx 0.15D(\lambda_0)$ for $N_c = 3$, where $\lambda_0 \gtrsim \lambda^E$. So $D(z)$ is an increasing function of z at small z, $z \lesssim \lambda_0$, and for $z \gg \lambda$, one observes exponential falloff. The qualitative picture illustrating this solution for $D(z)$ is shown in Figure 5.

This pattern may solve qualitatively the contradiction between the values of $D(0)$ estimated from the string tension $D_\sigma(0) \simeq \sigma/\pi\lambda^2 \approx 0.35\,\text{GeV}^4$ and the value obtained in naive way from the gluon condensate $D_{G_2}(0) = (\pi^2/18)G_2 \approx (0.007 \div 0.012)\,\text{GeV}^4$. One can see that $D_\sigma(0) \approx (30 \div 54)D_{G_2}(0)$. This seems to be a reasonable explanation of the mismatch discussed in Section 1.

As shown in [41], the large distance exponential behavior is self-consistent, since (assuming that it persists for all z, while small-z region contributes very little) from the equality $\sigma = \pi\lambda^2 D_\sigma(0)$, comparing with (4.28), one obtains

$$0.1 \cdot 8\pi^2 \alpha_s^2 \left(N_c^2 - 1\right)\sigma_f^2 = \frac{\sigma_f}{\pi\lambda^2}, \tag{5.3}$$

where in $\alpha_s(\mu)$ the scale μ corresponds roughly to the gluelump average momentum (inverse radius) $\mu_0 \approx 1\,\text{GeV}$. Thus (5.3) yields $\alpha_s(\mu_0) \approx 0.4$ which is in reasonable agreement with α_s from other systems [87].

We end this section with discussion of three points as follows.

(1) $D(z)$ and $D_1(z)$ have been obtained here in the leading approximation, when gluelumps of minimal number of gluons contribute 2 for $D(z)$ and 1 for $D_1(z)$. In the higher orders of $O(\alpha_s)$, one has an expansion of the type

$$D(z) = D^{(2gl)}(z) + c_1\alpha_s^3 D^{(1gl)}(z) + c_2\alpha_s^3 D^{(3gl)}(z) + \cdots,$$

$$D_1(z) = D^{(1gl)}(z) + c_1'\alpha_s^3 D^{(2gl)}(z) + \cdots. \tag{5.4}$$

Hence the asymptotic behavior for $D(z)$ will contain exponent of $M_0^{(1)}|z|$ too, but with a small preexponent coefficient.

(2) The behavior of $D(z)$, $D_1^{(np)}(z)$ at small z is defined by NP terms of dimension four, which are condensate G_2 as in (2.11) and the similar term from the expansion of $\exp F$, namely,

$$\langle \exp F \rangle = 1 + 4g^2 \int_0^s d\tau \int_0^{s'} d\tau' \langle F(u(\tau)) F(u(\tau')) \rangle + \cdots, \tag{5.5}$$

therefore, one does not encounter mixed terms like $O(m^2/z^2)$; however, if one assumes that expansion of $\langle F(0)F(z) \rangle$ starts with terms of this sort as it was suggested in [72], then one will have a self-consistent condition for the coefficient in front of this term.

(3) To study the difference between D_1^E, D_1^H at $T \leq T_c$, one should look at (4.9) and compare the situation, when $\mu = \lambda = 4$, $\nu = \sigma = i$ and take $z = x - y$ along the 4th axis (for D_1^E), while for D_1^H one takes $\mu = \lambda = i$, $\nu = \sigma = k$, and the same z. One can see that in both cases one ends up with the one-gluon gluelump Green's function $G_{\mu\nu}$ (4.10) which in the lowest (Gaussian) approximation is the same $G_{\mu\nu} = \delta_{\mu\nu}f(z)$, and $f(z)$ is the standard lowest mass (and lowest angular momentum) Green's function.

Hence $D_1^E = D_1^H$ in this approximation, and for $T = 0$, this is an exact relation, as discussed above. Therefore,

$$\lambda_1 = \lambda_1^E = \lambda_1^H = \frac{1}{M_0^{(1)}}, \quad M_0^{(1)} \approx 1.2 \div 1.4\,\text{GeV}, \ \lambda_1 \cong 0.2 \div 0.15\,\text{Fm}. \tag{5.6}$$

The value $M_0^{(1)}$ in (5.6) is taken from the calculations in [30]. The same is true for D^E, D^H, as it is seen from (4.12), where $G^{(2gl)}$ corresponds to two-gluon subsystem angular momentum $L = 0$ independently of $\mu\nu$, $\lambda\sigma$.

Hence one obtains

$$\lambda \equiv \lambda^E = \lambda^H = \frac{1}{M_0^{(2)}}, \quad \lambda \cong 0.08 \, \text{Fm}, \quad M_0^{(2)} \approx 2.5 \, \text{GeV}, \tag{5.7}$$

where the value $M_0^{(2)} \approx 2.5 \, \text{GeV}$ is taken from [30].

6. Conclusions

We have derived, following the method of [61–65], the expressions for FC $D(z)$, $D_1(z)$ in terms of gluelump Green's functions. This is done in Gaussian approximation. The latter are calculated using Hamiltonian where np dynamics is given by $D^{np}(z)$, $D_1^{np}(z)$. In this way one obtains self-coupled equations for these functions, which allow two types of solutions: (1) $D^{np}(z) = 0$, $D_1^{np}(z) = 0$, that is, no np effects at all; (2) $D^{np}(z)$, $D_1^{np}(z)$ are nonzero and defined by the only scale, which should be given in QCD, for example, string tension σ or Λ_{QCD}. All other quantities are defined in terms of these basic ones. We have checked consistency of self-coupled equations at large and small distances and found that to the order $\mathcal{O}(\alpha_s)$ no mixed perturbative-np terms appear. The function $D_1(z)$ can be represented as a sum of perturbative and np terms, while $D(z)$ contains only np contributions.

We have found a possible way to explain the discrepancy between the average values of field strength taken from G_2 and from σ by showing that $D(z)$ has a local minimum at $z = 0$ and grows at $z \sim \lambda$. Small value of λ and large value of the gluelump mass $M_{gl} = 1/\lambda \approx 2.5 \, \text{GeV}$ explain the lattice data for $\lambda \approx 0.1 \, \text{Fm}$. Thus the present paper argues that relevant degrees of freedom ensuring confinement are gluelumps, described self-consistently in the language of FCs.

Appendix

Nonperturbative Contributions to $D_1(z)$ and $D(z)$ at Small z

We start with $D_1(z)$ expressed via gluelump Green's function in (4.8), (4.10) $G_{\mu\nu}(z) = \delta_{\mu\nu} f(z)$.

The leading NP behavior at small z, proportional to $\langle F(x)F(y) \rangle$, is obtained from the amplitudes, shown in Figures 1 and 2. For the amplitude of Figure 1, one can use the vacuum average of the W-loop (2.11) in the limit of small contours and neglect the factor $\exp F$ in (4.11). One obtains the adjoint loop

$$\langle W(C_{xy}) \rangle = \exp\left(-\frac{g^2 \langle \left(F_{\mu\nu}^a(0)^2 \right) \rangle}{24 N_c} \gamma S^2 \right), \tag{A.1}$$

where $\gamma = C_2(\text{adj})/C_2(f) = 2N_c^2/(N_c^2 - 1)$ (original derivation in [41] referred to the fundamental loop). Proceeding as in Appendix 2 of [41], one obtains the NP correction ΔD_1 as

$$\Delta D_{1a} = C_2(\text{adj})\frac{g^2}{12N_c}G_2 = \frac{g^2}{12}G_2, \tag{A.2}$$

where we have introduced the standard definition [17, 18] $G_2 \equiv (\alpha_s/\pi)\langle F_{\mu\nu}^a(0)F_{\mu\nu}^a(0)\rangle$. One can check that $\Delta D_1 = \mathcal{O}(N_c^0)$.

We turn now to the amplitude of **Figure 2**. The corresponding $f(x)$ for $x \to 0$ can be written as follows:

$$\Delta f_{1b}(x) = \int d^4u\, d^4v\, G(0,u)G(u,v)G(v,x)4\pi^2 N_c^2 G_2, \tag{A.3}$$

where the gluon Green's function $G(x,y)$ for small $|x-y|$ can be replaced by the perturbative part: $G(x,y) \to G_0(x,y) = 1/4\pi^2(x-y)^2$.

Taking into account that $\Delta D_1(x) = -(2g^2/N_c^2)(d/dx^2)\Delta f(x)$, one obtains for $x \to 0$ the following:

$$\Delta D_{1b}(x) = 2g^2 G_2 I(x), \tag{A.4}$$

where we have defined

$$I(x) = -\frac{d}{dx^2}\int \frac{d^4u\, d^4v}{(4\pi^2)^2 u^2(u-v)^2(v-x)^2}. \tag{A.5}$$

The integral $I(0)$ diverges at small v and large u, v. The latter divergence is removed since at large arguments $G(x,y)$ is damped by confining force, keeping the propagating gluon nearby the 4th axis $x_4 = y_4 = 0$ where the static gluonic source resides. One can estimate $I(x \to 0) \sim \mathcal{O}((\ln(\lambda^2/x^2))^a)$. Since G_2 is connected to D, D_1, then

$$G_2 = \frac{3N_c}{\pi^2}\left(D^E(0) + D_1^E(0) + D^H(0) + D_1^H(0)\right), \tag{A.6}$$

and one can see in (A.4) that coefficients of D_1^E on both sides of (A.4) have the same order of magnitude and the same sign, suggesting a self-consistency on this preliminary level. However, as we argued before and in [41], this contribution is actually gluon mass renormalization, which is zero. This is especially clear when $\langle FF\rangle$ in **Figure 2** is replaced by its perturbative part.

We turn now to the most important case of the confining FC $D^E(x)$. The corresponding two-gluon gluelump amplitude is depicted in **Figure 3**, and the *np* contributions to $D^E(x)$ are given in Figures 4 and 5.
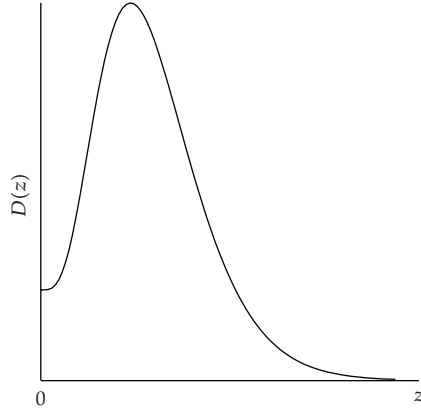
Figure 6

We start with the amplitude of **Figure** 4, which can be represented as a doubled diagram of **Figure** 1, and therefore, the contribution to the two-gluon gluelump Green's function $G^{(2gl)}(x)$ will be

$$G^{(2gl)}(x) = \frac{1}{(4\pi^2 x^2)^2} - \frac{\gamma G_2}{24 N_c (4\pi^2)}. \tag{A.7}$$

Now using relation (2.11), one obtains for the *np* contribution of **Figure** 4

$$\Delta D_a(x) = \frac{g^4 (N_c^2 - 1)}{2} \Delta G^{(2gl)}(x) = -\frac{g^4 N_c}{4\pi^2} G_2. \tag{A.8}$$

Note that this contribution is finite at $x \rightarrow 0$, however, with the negative sign. We now turn to the *np* contribution of **Figure** 5, which can be obtained from (4.12) inserting there the product of operators $\widehat{F}_{fg}(w) d^4 w \widehat{F}_{f'g'}(w') d^4 w'$, where $fg(f'g')$ are adjoint color indices, $\widehat{F}_{fg} = F^a T^a_{fg} = F^a (-i) f_{afg}$. Using the formulas

$$f_{abc} f_{a'bc} = N_c \delta_{aa'}, \qquad f_{abc} f_{ade} f_{bdf} f_{ceg} = \frac{N_c^2}{2} \delta_{fg}, \tag{A.9}$$

one arrives at the expression

$$\Delta D_b(x) = 2 N_c^2 g^4 h(x),$$

$$h(x) = \int d^4 w d^4 w' D(w - w') G(w) G(w') G(w - x) G(w' - x), \tag{A.10}$$

where $G(y)$ is the gluon Green's function in the two-gluon gluelump; the gluon is connected at large distances by two strings to another gluon and to the static gluon source; see **Figure** 5. At small x (we take it along axis 4 for convenience), the integral in $h(x)$ grows when $G(y)$ is

close to the 4th axis and becomes the free gluon $G(y) \rightarrow G_0(y) = 1/4\pi^2 y^2$. As a result, one obtains

$$h(x) \longrightarrow h_0(x) = \int \frac{d^4 w d^4 w'}{(4\pi^2)^4} \frac{D(w - w')}{w^2 w'^2 (w - x)^2 (w' - x)^2}, \tag{A.11}$$

and one should have in mind that the integral for $x \rightarrow 0$ diverges both at small and at large w, w'. The small w, w' region will give the terms $O(\ln(\lambda_0^2/x^2))$, and at large w, w', the integral is protected by the falloff of $G(y)$ at large y due to confinement; therefore, we must imply in the integrals in (A.11) the upper limits for w, w' at some $\lambda_0 \gtrsim 1/\sqrt{\sigma_{\text{adj}}}$. Then introducing for $w(\mathbf{w}, w_4)$ polar coordinates $|\mathbf{w}| = \rho \sin\theta$, $w_4 = \rho \cos\theta$ (and the same for w'), one arrives at the integral

$$\int_0^\pi \frac{\sin^2\theta d\theta}{\rho^2 - 2x\rho\cos\theta + x^2} = \frac{\pi}{2\rho^2} \quad (\rho \geq x), \tag{A.12}$$

or $(\pi/2x^2)$ $(x \geq \rho)$, and finally one has the estimate of contribution to (A.10) from the region of small w, w' $(w|w|, |w'| \lesssim \lambda_0)$ as

$$h(x) \approx \frac{D(\lambda_0)}{4(4\pi^2)^2} \log^2\left(\frac{\lambda_0\sqrt{e}}{x}\right). \tag{A.13}$$

And finally for $D(x)$, one obtains (omitting the perturbative term in (A.7))

$$D(x) = -\frac{g^4 N_c}{4\pi^2} G_2 + \frac{N_c^2}{2\pi^2} \alpha_s^2 D(\lambda_0) \log^2\left(\frac{\lambda_0\sqrt{e}}{x}\right). \tag{A.14}$$

Note that the last term on the r.h.s. dominates at small x and compensates the decrease of the α_s^2 term since

$$\alpha_s^2(x)\ln^2\left(\frac{\lambda_0\sqrt{e}}{x}\right) \cong \left(\frac{4\pi \ln(\lambda_0\sqrt{e}/x)}{\beta_0 \ln(1/x^2\Lambda^2)}\right) \longrightarrow \left(\frac{4\pi}{\beta_0}\right)^2 \quad (x \longrightarrow 0). \tag{A.15}$$

Hence,

$$D(x < \lambda_0) \approx \frac{N_c^2}{2\pi^2}\left(\frac{4\pi}{\beta_0}\right)^2 D(\lambda_0) < D(\lambda_0). \tag{A.16}$$
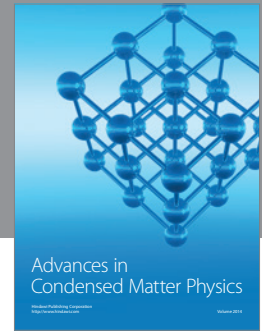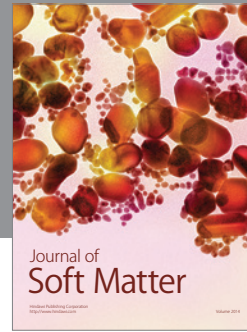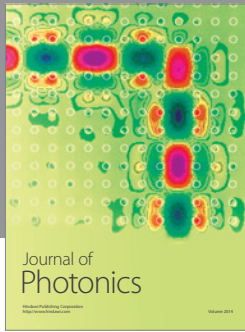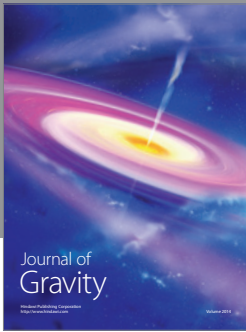
Acknowledgments

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