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Research Article

Bulk-Brane Matching in Bianchi-Types Brane World

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We discuss a comprehensive description of the geometry of the brane-world cosmologies, and present bulk and brane structure and matching between brane and bulk metrics. It is clear that the possibility of the matching condition is not always obvious and therefore it requires a separate analysis. In this study we have shown, under the assumption of consideration of the anisotropic metric except Kasner-AdS like, matching procedure is not achieved for Bianchi-types bulk metrics. Examples of this result are presented by the illustrations of the Bianchi-types II and V bulk metrics.

1. Introduction

Randall and Sundrum (RS) made an intriguing alternative suggestion in which we reside in a universe, 3+1-dimensional surface (the "brane"), of more than four noncompact dimensions. They examine a brane in a space of higher dimension, which is called bulk and it is a slice of anti de Sitter spacetime (AdS) [1]. In these models, five-dimensional Einstein field equations,

$$^{5}G_{I,J} = \kappa_5^2 T_{I,J}, \tag{1.1}$$

where $^{5}G_{I,J}$ is the five-dimensional Einstein tensor, κ_5^2 is the five-dimensional coupling constant, and $T_{I,J}$ is the energy-momentum tensor. It can be written as,

$$T_{I,J} = -\Lambda g_{I,J} + S_{I,J}\delta(y), \qquad S_{I,J} = -\lambda g_{I,J} + \tau_{I,J}, \tag{1.2}$$

where, $g_{I,J}$, λ, and $\tau_{I,J}$ are the metric, tension, and energy-momentum tensors of the brane, respectively.

The effective four-dimensional gravitational equations on the brane take the form [2, 3]:

$$^{4}G_{i,j} = -\Lambda_4 g_{i,j} + \kappa_4^2 \tau_{i,j} + \kappa_5^2 \pi_{i,j} - E_{i,j}, \tag{1.3}$$

where,

$$\Lambda_4 = \frac{1}{2}\kappa_5^2\left(\Lambda + \frac{1}{6}\kappa_5^2\lambda^2\right), \quad \kappa_4^2 = \frac{\kappa_5^2}{6}\lambda,$$
$$\pi_{i,j} = -\frac{1}{4}\tau_{ac}\tau_b^c + \frac{1}{12}\tau\tau_{i,j} + \frac{1}{8}g_{i,j}\tau_{cd}\tau^{cd} - \frac{1}{24}g_{i,j}\tau^2, \tag{1.4}$$

where $\pi_{i,j}$ is the local quadratic energy-momentum correction and $E_{i,j}$ is nonlocal effect from the free bulk gravitational field. Thus, it is not possible to fully understand brane solutions without explicitly knowing the bulk solution.

In the literature, if we take bulk metrics as AdS-like and brane metrics as FRW-like, we make for an exact solution of (1.1), in the isotropic brane-world cosmology [4–9]. For instance, in FRW brane world, the bulk is Schwarzchild AdS and $E_{i,j}$ reduce to simple Coulomb term that gives a dark radiation term on the brane [10–12].

In anisotropic brane-world scenarios, the suitable bulk and brane metrics matching each other were first discovered by frolov [13]. It is clear that Kasner-type brane-world model can be viewed as the generalization of an isotropic model. The five-dimensional Kasner anti de Sitter metric described by

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\sigma_3^2, \tag{1.5}$$

where σ_3^2 is 3-dimentional spatial metric varying with time

$$d\sigma_3^2 = t^{2p_1(t)}dx^2 + t^{2p_2(t)}dy^2 + t^{2p_3(t)}dz^2. \tag{1.6}$$

Here the exponents must satisfy the familiar Kasner restrictions,

$$p_1 + p_2 + p_3 = 1 = p_2^2 + p_2^2 + p_3^2. \tag{1.7}$$

Thus, the brane also has a tension and matter-density given, respectively, as

$$\sigma = \pm\frac{6}{\kappa_5^2 l}, \quad \rho = 0, \tag{1.8}$$

which is the the Randall-Sundrum like fine tuning between the brane tension and the bulk cosmological constant. Since the brane does not include matter, that is to say, it becomes a vacuum, it makes for a poor cosmological model. But the important point here is that it introduces anisotropy into the brane world models.

Some authors have analyzed anisotropic brane worlds including matter content [14, 15]. Particularly, dynamical systems techniques are used by Campos and Sopuerta to look into homogeneous and anisotropic Bianchi-type branes [16]. For a summary of dynamical systems in the context of cosmology, including Bianchi-type cosmologies, refer to [17]. However, in the these early studies many assumptions were made about the Weyl term, $E_{i,j}$, due to the absence of an exact anisotropic bulk solution. This was addressed in [18] for the FLRW and Bianchi I case and shortly after Campos et al. [19] found a family of exact, anisotropic solution to the five-dimensional field equation. Therefore they were able to be explicitly see the relationship between the bulk Weyl curvature and the anisotropy on the brane. They found that it is not possible to have a perfect fluid or scalar field compatible with the anisotropic brane since the junction condition requires anisotropic stress on the brane. Fabbri et al. found more exact bulk solutions and agreed that an anisotropic brane cannot support a perfect fluid in the case where the bulk is static [20]. Harko and Mak investigated Bianchi-type brane-world behaviour near the singularity and at late times and found that they tend to isotropize for certain matter content [21]. Also, they found general solution of the field equations for Bianchi-type I and V in the brane [22].

Up to now, no complete solution for the brane and bulk metrics have been found for cosmological Bianchi brane worlds. The key difficulty is to find anisotropic generalization of AdS that can incorporate anisotropy on a cosmological brane, and that is necessarily nonconformally flat.

In this study we have shown, under the assumption of consideration of the anisotropic metric except Kasner-AdS-like, matching procedure is not achieved for Bianchi-types metrics [23]. Examples of this result are shown by the illustrations of the Bianchi-types II and V metrics. Throughout this paper we will use the following notation: latin letters denote coordinate indices in the bulk spacetime $(I, J, K, \ldots = 0, 1, 2, 3, 4)$ and in the brane $(i, j, k, = 0, 1, 2, 3)$, and also tilde $(\tilde{})$ and upper "5" mean 5-dimensional quantities.

2. Bianchi-Type II and V Space Time

2.1. Bianchi-Type II

We consider the 5-dimensional metric:

$$ds^2 = -e^{\nu(\tilde{t},\tilde{w})} d\tilde{t}^2 + \gamma_{ij} w^i w^j + e^{\mu(\tilde{t},\tilde{w})} d\tilde{w}^2, \tag{2.1}$$

where the 3-dimensional spatial part of the metric can be expressed in diagonal form as

$$\gamma_{ij} = \operatorname{diag}\left(e^{\alpha}, e^{\beta}, e^{\gamma}\right). \tag{2.2}$$

We assume that the metric coefficients $\alpha, \beta, \gamma, \nu$, and μ depend on both $\tilde{t}$ and $\tilde{w}$.

The one-form w^i have the relationship

$$dw^i = \frac{1}{2} C^i_{jk} w^j w^k, \tag{2.3}$$

where, the C^i_{jk} are the structure constants corresponding to the particular Bianchi-type. In the case of type-II, the nonzero structure constants are

$$C^1_{23} = -C^1_{32} = 1. \tag{2.4}$$

The exact solution of the 5-dimentional Einstein Equation for vacuum case was obtained by Halpern [24]. Following his paper, the metric coefficients can be expressed in the following manner:

$$\alpha = 2a_1\tilde{t} + 2a_2\tilde{w},$$
$$\beta = 2b_1\tilde{t} + 2b_2\tilde{w},$$
$$\gamma = 2c_1\tilde{t} + 2c_2\tilde{w},$$
$$\mu = \nu = 2d_1\tilde{t} + 2d_2\tilde{w}, \tag{2.5}$$

where

$$a_1 = \frac{-2a_2^2 + 2a_2c_2 + 1 + a_2\sqrt{6}\sqrt{2a_2^2 + 2c_2^2 - 1}}{2\sqrt{4a_2^2 - 1 - 2a_2c_2 + 4c_2^2 + \sqrt{6}\sqrt{(2a_2^2 + 2c_2^2 - 1)(a_2 - c_2)^2}}},$$

$$b_1 = \frac{-6a_2^2 + 4a_2c_2 - 10c_2^2 + 2 + (a_2 - 3c_2)\sqrt{6}\sqrt{2a_2^2 + 2c_2^2 - 1}}{2\sqrt{4a_2^2 - 1 - 2a_2c_2 + 4c_2^2 + \sqrt{6}\sqrt{(2a_2^2 + 2c_2^2 - 1)(a_2 - c_2)^2}}},$$

$$c_1 = \frac{-2a_2c_2 - 1 + 2c_2^2 + 2 + c_2\sqrt{6}\sqrt{2a_2^2 + 2c_2^2 - 1}}{2\sqrt{4a_2^2 - 1 - 2a_2c_2 + 4c_2^2 + \sqrt{6}\sqrt{(2a_2^2 + 2c_2^2 - 1)(a_2 - c_2)^2}}}, \tag{2.6}$$

$$d_1 = -\frac{2a_2^2 + 4c_2^2 + c_2\sqrt{6}\sqrt{2a_2^2 + 2c_2^2 - 1}}{\sqrt{4a_2^2 - 1 - 2a_2c_2 + 4c_2^2 + \sqrt{6}\sqrt{(2a_2^2 + 2c_2^2 - 1)(a_2 - c_2)^2}}},$$

$$b_2 = -2c_2 - \frac{\sqrt{6}}{2}\sqrt{2a_2^2 + 2c_2^2 - 1},$$

$$d_2 = -a_2 - c_2 - \frac{\sqrt{6}}{2}\sqrt{2a_2^2 + 2c_2^2 - 1}.$$

This set of solutions is purely exponential in character, with monotonic behavior similar to the Kasner (type I) solution. Note, however, that the relationship amongst these exponents is more complex than in the Kasner case.

2.2. Bianchi-Type V

We now consider 5-dimentional Bianchi-type V spatial geometry. We write the metric in the same manner as (2.1) with the nonzero structure constant of the Lie algebra of one-forms equal to

$$C^1_{13} = -C^1_{31} = 1, \qquad C^2_{23} = -C^2_{32} = 1. \tag{2.7}$$

Exact solution of the 5-dimentional Einstein equation was obtained by Halpern [24]. Following his paper, the metric coefficients can be expressed in the following manner:

$$\begin{aligned}
\alpha &= 2a_1 t + \sqrt{2a_2} w, \\
\beta &= 2\ln\left(\frac{1}{2}\sqrt{-4a_1^2 + 2a_2^2}\right) - 2a_1 t - \sqrt{2} a_2 w, \\
\gamma &= 2a_2 t - 2\ln\left(\frac{1}{2}\sqrt{-4a_1^2 + 2a_2^2}\right) + 2a_1\sqrt{2} w, \\
\mu &= \nu = 2a_2 t + 2a_1\sqrt{2} w,
\end{aligned} \tag{2.8}$$

where a_1 and a_2 are independent parameters with $a_2^2 > a_1^2$ to ensure that all scale factors are real.

3. Brane in Anisotropic Bulk

In this section, we consider what will happen if the 3-brane is embedded in the Bianchi types II and V derived above. Following [13, 25], we describe some useful identities for suitable embedding,

$$\begin{aligned}
\tilde{t} &= T(\tau) \longrightarrow d\tilde{t} = \frac{dT}{d\tau} d\tau = \dot{T} d\tau, \\
\tilde{x}^i &= x^i \longrightarrow d\tilde{x}^i = dx^i, \\
\tilde{w} &= \widetilde{W}(\tau) \longrightarrow d\tilde{w} = \frac{d\widetilde{W}}{d\tau} d\tau = \dot{W} d\tau,
\end{aligned} \tag{3.1}$$

where, $\dot{}$ represents derivative with respect to τ.

For generalization of (2.1), we can take its components

$$e^{\nu} = M\left(\tilde{t}, \tilde{w}\right), \qquad e^{\mu} = N\left(\tilde{t}, \tilde{w}\right). \tag{3.2}$$

Then, 5-dimentional metric takes the form

$$ds^2 = -M\left(\tilde{t}, \tilde{w}\right) d\tilde{t}^2 + \gamma_{ij} w^i w^j + N\left(\tilde{t}, \tilde{w}\right) d\tilde{w}^2, \tag{3.3}$$

Induced metric on the brane is

$$ds^2 = -\left(M\dot{T}^2 - N\dot{W}^2\right) d\tau^2 + e^{\alpha(T(\tau),\widetilde{W}(\tau))} dx^2 + e^{\beta(T(\tau),\widetilde{W}(\tau))} dy^2 + e^{\gamma(T(\tau),\widetilde{W}(\tau))} dz^2. \tag{3.4}$$

If we chose

$$M\dot{T}^2 - N\dot{W}^2 = 1 \longrightarrow \dot{T} = +\sqrt{\frac{1 + N\dot{T}^2}{M}} \tag{3.5}$$

and get proper time, we can write the local frame

$$e_i^l = \frac{\partial \tilde{x}^l}{\partial x^i} \longrightarrow e_\tau^{\tilde{t}} = \frac{\partial \tilde{t}}{\partial \tau} = \dot{T}, \qquad e_\tau^{\tilde{x}} = e_\tau^{\tilde{y}} = e_\tau^{\tilde{z}} = 0, \qquad e_\tau^{\tilde{w}} = \frac{\partial \widetilde{W}}{\partial \tau} = \dot{W}, \tag{3.6}$$

or

$$\begin{gathered} e_\tau^l = \left(\dot{T}, 0, 0, 0, \dot{W}\right), \qquad e_{\tau l} = \left(-M\dot{T}, 0, 0, 0, N\dot{W}\right), \\ e_x^l = (0,1,0,0,0), \qquad e_y^l = (0,0,1,0,0), \qquad e_z^l = (0,0,0,1,0). \end{gathered} \tag{3.7}$$

It is not difficult to show these equations implying that the timelike vector is given by

$$u^2 = e_\tau^l e_{\tau l} = -M\dot{T}^2 + N\dot{W}^2 = -1. \tag{3.8}$$

Also, using $n^l e_{\tau l} = 0$ and $n^l n_l = 1$, where, n is normal vector, then we obtain some useful relations,

$$\begin{gathered} n^l e_{xl} = n^l e_{yl} = n^l e_{zl} = 0 \longrightarrow n^1 = n^2 = n^3 = 0, \\ n^l e_{\tau l} = 0 \longrightarrow -M\dot{T}n^0 + N\dot{T}n^4 = 0, \\ n^l e_l = 1 \longrightarrow -M\left(n^0\right)^2 + N\left(n^4\right)^2 = 1. \end{gathered} \tag{3.9}$$

Finally, we find that unit normal vector to the brane is

$$n^0 = \epsilon\sqrt{\frac{N}{M}}\dot{W}, \qquad n^4 = \epsilon\sqrt{\frac{M}{N}}\dot{T}, \tag{3.10}$$

or

$$n^l = \left(\epsilon\sqrt{\frac{N}{M}}\dot{W}, 0, 0, 0, \epsilon\sqrt{\frac{M}{N}}\dot{T}\right), \qquad n_l = \left(-\epsilon\sqrt{MN}\dot{W}, 0, 0, 0, \epsilon\sqrt{MN}\dot{T}\right), \tag{3.11}$$

where, $\epsilon = \pm 1$.

Now,

$$\begin{gathered} K_{ij} = e^l_{(i} e^J_{j)} {}^5\widetilde{\nabla}_l n_j, \\ {}^5\widetilde{\nabla}_l n_j = \widetilde{\partial}_l n_j - {}^5\widetilde{\Gamma}^K_{IJ} n_K. \end{gathered} \tag{3.12}$$

After defining (3.12), we obtain useful form of extrinsic curvature tensor for the brane embedded in the spacetime defined as

$$K_{ij} = \left[e_i^I e_j^J n^L \partial_L {}^5\tilde{g}_{IJ} + {}^5\tilde{g}_{IJ}\left(e_i^I \partial_j n^J + e_j^I \partial_i n^J\right)\right] \tag{3.13}$$

has the following nonvanishing components

$$K_{\tau\tau} = -\epsilon \frac{2N^{*}\dot{W}\dot{T} + (M'\dot{T}^{2}) + (N'\dot{W}^{2}) + (2N\ddot{W})}{2\sqrt{MN}\dot{T}},$$
$$K_{xx} = \epsilon \frac{N\dot{W}\alpha^{*} + M\dot{T}\alpha'}{2\sqrt{MN}} e^{\alpha},$$
$$K_{yy} = \epsilon \frac{N\dot{W}\beta^{*} + M\dot{T}\beta'}{2\sqrt{MN}} e^{\beta},$$
$$K_{zz} = \epsilon \frac{N\dot{W}\gamma^{*} + M\dot{T}\gamma'}{2\sqrt{MN}} e^{\gamma}, \tag{3.14}$$

where we use overdots to represent partial derivatives with respect to τ, asterisks to represent partial derivatives with respect to $\tilde{t}$, and overcommas to represent partial derivatives with respect to $\tilde{w}$.

The Israel's junction condition is given by

$$K_{IJ} = -\frac{\tilde{k}_{5}^{2}}{2}\left(S_{IJ} - \frac{1}{3}Sh_{IJ}\right), \tag{3.15}$$

where S_{IJ} is energy-momentum tensor of the brane and h_{IJ} is induced metric on the brane. S_{IJ}, and its trace S are defined as

$$S_{IJ} = \mu u_{I}u_{J} + (p - \sigma)h_{IJ},$$
$$S = -\mu + 4p - 4\sigma, \tag{3.16}$$

where μ is brane matter-energi density, p is matter pressure, σ is brane tension, and u_I is four vector. The metric inhered by the brane and other hypersurfaces of the foliation is the first fundamental form,

$$h_{IJ} =^{5} \tilde{g}_{IJ} - n_{I}n_{J}, \tag{3.17}$$

and its components are

$$h_{\tau\tau} = -M^{2}\dot{T}^{2}, \qquad h_{xx} = e^{\alpha}, \qquad h_{yy} = e^{\beta}, \qquad h_{zz} = e^{\gamma}. \tag{3.18}$$

Using (3.16) and (3.18), we obtain energy-momentum tensor components

$$S_{\tau\tau} = M^{2}\dot{T}(\mu - p + \sigma),$$
$$S_{xx} = e^{\alpha}(p - \sigma),$$
$$S_{yy} = e^{\beta}(p - \sigma),$$
$$S_{zz} = e^{\gamma}(p - \sigma). \tag{3.19}$$

Finally, if substituting the last equations into (3.15), one gets the brane equation of motions in the bulk

$$\epsilon\frac{2N^*\dot{W}\dot{T}+M'\dot{T}^2+N'\dot{W}^2+2N\ddot{W}}{2\sqrt{MN}\dot{T}}=\frac{\tilde{k}_5^2}{2}M^2\dot{T}(\mu-p+\sigma), \tag{3.20}$$

$$\epsilon\frac{N\dot{W}\alpha^*+M\dot{T}\alpha'}{2\sqrt{MN}}=-\frac{\tilde{k}_5^2}{2}(p-\sigma), \tag{3.21}$$

$$\epsilon\frac{N\dot{W}\beta^*+M\dot{T}\beta'}{2\sqrt{MN}}=-\frac{\tilde{k}_5^2}{2}(p-\sigma), \tag{3.22}$$

$$\epsilon\frac{N\dot{W}\gamma^*+M\dot{T}\gamma'}{2\sqrt{MN}}=-\frac{\tilde{k}_5^2}{2}(p-\sigma). \tag{3.23}$$

Because the anisotropic bulk coefficients α, β, γ, N, and M depend on both $\tilde{t}$ and $\tilde{w}$ differently, the only way to satisfy (3.20) to (3.23) simultaneously without introducing anisotropic matter content on the brane, is to have the anisotropic term vanish in these equations. This is succeeded when $\dot{W}=0\rightarrow W=\text{constant}$, that is, when brane is not moving. Then we obtain $\dot{T}=1/M$. Finally these equations are reduced, respectively, in the following:

$$\frac{M'}{2M^2\sqrt{N}}=\frac{\tilde{k}_5^2}{6}(2\mu+p-\sigma), \tag{3.24}$$

$$\frac{\alpha'}{2\sqrt{N}}=-\frac{\tilde{k}_5^2}{2}(p-\sigma), \tag{3.25}$$

$$\frac{\beta'}{2\sqrt{N}}=-\frac{\tilde{k}_5^2}{2}(p-\sigma), \tag{3.26}$$

$$\frac{\gamma'}{2\sqrt{N}}=-\frac{\tilde{k}_5^2}{2}(p-\sigma). \tag{3.27}$$

From last tree equations, we infer that

$$\alpha'=\beta'=\gamma'. \tag{3.28}$$

Case I (for the Bianchi-type II). If we compare (3.28) with (2.5), we infer that $a_2=b_2=c_2=\pm i/2$, that is, complex values for the metric coefficients.

Case II (for the Bianchi-type V). If we compare (3.28) with (2.8), we infer that $-a_2=a_1$ which is contrary to $a_2^2>a_1^2$ ensuring all scale factors are real.

4. Discussion

Up till now, studies related to the isotropic brane-world models, because of the existence of suitable selections of bulk and brane metrics, there exists many models in the literature such as propose solutions to the current cosmological problems. Brane cosmology with anisotropy has not been clearly understood yet. Apart from Frolov's Kasner-AdS model, there are no additional anisotropic brane-world models which contain bulk-brane matching. The simplest generalizations of FRW brane worlds are Bianchi brane worlds. In this study, by using Frolov's method, after having obtained the equations of motion of brane, we investigated the bulk-brane matching of the Bianchi-type II and Bianchi-type V models whose exact bulk solutions are known. In the result, we have found that the coefficients of bulk and brane metrics are not matching each other since they are imaginary.

Just to finish we would like to mention some current and future work in the line of the present one. First, since Bianchi-types cosmology has large anisotropy, it would be interesting to suppose the matter on the brane possess some anisotropy, then take into account other Bianchi-type bulk solutions. In this sense, a good starting point would be to consider scenarios like those introduced in [20].

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