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Research Article

Mass Mixing Effect and Oblique Radiative Corrections in Extended $SU(2)_R \times SU(2)_L \times U(1)$ Effective Theory

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We analyze the properties of electroweak chiral effective Lagrangian with an extended $SU(2)_R$ gauge group. Right-handed $SU(2)_R$ gauge bosons affect electroweak observables by mixing with electroweak gauge bosons $W_{L,\mu}$ and B_μ. We discuss all possible mass mixing terms and calculate the exact physical mass eigenvalues by diagonalization of mixing matrix without any approximate assumptions. The contributions to oblique radiative corrections parameters STU from $SU(2)_R$ fields are also presented.

1. Introduction

Although the standard model (SM) has been checked very successfully by more and more high energy physics experiments, the as yet undiscovered Higgs, introduced as a basic scaler field in SM, remains as the only unknown component of the electroweak symmetry-breaking mechanism (EWSBM) unknown. That situation has prompted many extensions to SM [1–3]. A new $SU(1)_R$ group, associated with an additional triplet of gauge bosons $W'^{\pm}$ and Z', is often considered for different reasons as an extension to the gauge symmetry [4–6]. This extension often appears in superstring-inspired models as well as GUT models [7]. The non-Abelian $SU(2)_R$ contains sufficient complexity to incorporate interesting issues related to spontaneous parity violation (SPV) and precise electroweak observables, although remains simple enough that phenomenology can be subjected to analysis. $SU(2)_R$ gauge bosons can improve unitarity of not only WW but also WZ scattering processes and delay the breaking scale of unitarity.

Many left-right symmetry models with symmetry group $SU(2)_R \times SU(2)_L \times U(1)$ have been used in studying EWSBM. The common feature of these models is the existence of multi-Higgs bosons that then raises phenomenological issues related to multi-Higgs structure dependencies. To obtain an universal physical analysis, we adopt the nonlinear realization of the chiral Lagrangian to describe extended $SU(2)_R$ electroweak gauge models given the symmetry breaking pattern $SU(2)_R \times SU(2)_L \times U(1) \rightarrow U(1)_{\rm em}$. This chiral Lagrangian has already been written down in [8]. The model is a generalization of the conventional linearly realized models with multi-Higgs. Within the extended non-Abelian chiral effective Lagrangian, multi-Higgs effects are parameterized by a set of coefficients that describes all possible interactions among the gauge bosons and provides a model-independent platform to investigate interesting physics [8].

In the paper, we focus on mass mixing effects in left-right chiral effective Lagrangian. Mass mixings are main focus in the contribution of the right-handed gauge bosons to electroweak observables at low-energy scales. The $SU(2)_R$ gauge triplet can be regarded as a copy of the $SU(2)_L$ gauge triplet of SM, but with heavier masses. Right-handed charged gauge bosons $W_R^{\pm}$ can mix with left-handed $W_L^{\pm}$, and physical mass eigenstates of $W'^{\pm}$ and $W^{\pm}$ are eigenvalues of the charged mass matrix. Similarly, W_R^3 takes part in $W_R^3 - W_L^3 - B$ three-body mixing to form physical massive neutral bosons Z', Z, and a massless photon. The nonlinearly realized chiral effective Lagrangian provides us with all possible mass-mixing channels that are allowed by left-right symmetry. Calculating these mixings, we obtain a complete mass mixing contribution to the electroweak observables and a largest parameter space for new physics. Oblique radiative corrections of $SU(2)_R$ bosons can be obtained from the mass mixing rotation matrix, which indicates shifts to the SM with new physics.

The paper is organized as follows. Section 2 reviews $SU(2)_R \times SU(2)_L \times U(1)$ effective theory with all possible mass mixing terms in the gauge eigenstates basis. Section 3 presents calculations of the charged and neutral mass eigenvalues to obtain physical boson masses estimates. We improved our diagonalization calculation program for the neutral bosons sector in our paper [8] to yield a set of exact solutions for the rotation matrix and the mass eigenvalues without making any approximating assumptions. Oblique radiative corrections coming from the nonstandard mass mixing beyond SM are studied in Section 4. Furthermore, two kinds of special cases are considered corresponding to condition $M_{W_R} \gg M_{W_L}$ case and left-right symmetry. Finally, we give a short summary in Section 5.

2. Left-Right Symmetry Effective Lagrangian

Let $W^a_{R,\mu}$, $W^a_{L,\mu}$, B_μ be electroweak gauge fields ($a = 1, 2, 3$) corresponding to the gauge group $SU(2)_R$, $SU(2)_L$, and $U(1)$, respectively, and $U_{L,R}$ be the two by two unitary unimodular matrices corresponding to left- and right-handed Goldstone boson fields. Under $SU(2)_R \otimes SU(2)_L \otimes U(1)$ transformations, the gauge boson fields transform as

$$\begin{aligned} \frac{\tau^a}{2} W^a_{i,\mu} &\longrightarrow R_i \frac{\tau^a}{2} W^a_{i,\mu}(x) R_i^{\dagger} - \frac{i}{g_i} R_i \partial_\mu R_i^{\dagger}, \\ B_\mu &\longrightarrow B_\mu - \frac{1}{g} \partial_\mu \theta^0, \\ U_i &\longrightarrow R_i U_i R_0^{\dagger} \end{aligned} \tag{2.1}$$

with $R_0 = e^{(i/2)\tau^3\theta^0(x)}$ and $R_i = e^{(i/2)\tau^a\theta_i^a(x)}$ for $i = R, L$. The covariant derivative of the Goldstone fields takes the form

$$\begin{aligned} D_\mu U_R &= \partial_\mu U_R + ig_R\frac{\tau^a}{2}W^a_{R,\mu}U_R - igU_L\frac{\tau_3}{2}B_\mu, \\ D_\mu U_L &= \partial_\mu U_L + ig_L\frac{\tau^a}{2}W^a_{L,\mu}U_L - igU_L\frac{\tau_3}{2}B_\mu. \end{aligned} \tag{2.2}$$

For convenience in present discussion, we will discard conventional EWCL $SU(2)$ covariant building blocks [9–13] and introduce $U(1)$ invariant building blocks (for $i = L, R$)

$$\begin{aligned} X_i^\mu &= U_i^\dagger (D^\mu U_i), \\ \overline{W}_{i,\mu\nu} &= U_i^\dagger g_i W_{i,\mu\nu} U_i, \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu. \end{aligned} \tag{2.3}$$

Here,

$$W_{i,\mu\nu} = W^a_{i,\mu\nu}\frac{\tau^a}{2} = \partial_\mu W^a_{i,\nu}\frac{\tau^a}{2} - \partial_\nu W^a_{i,\mu}\frac{\tau^a}{2} + ig_i\left[W^a_{i,\mu}\frac{\tau^a}{2}, W^b_{i,\nu}\frac{\tau^b}{2}\right]. \tag{2.4}$$

With the help of these building blocks, we can write a leading-order chiral Lagrangian as

$$\begin{aligned} \mathcal{L}_M = &-\frac{1}{4}f_L^2\left\langle X_{L,\mu}X_L^\mu\right\rangle - \frac{1}{4}f_R^2\left\langle X_{R,\mu}X_R^\mu\right\rangle + \frac{1}{2}\tilde{\kappa} f_L f_R\left\langle X_L^\mu X_R^\mu\right\rangle \\ &+ \frac{1}{4}\beta_{L,1}f_L^2\left\langle \tau^3 X_{L,\mu}\right\rangle^2 + \frac{1}{4}\beta_{R,1}f_L^2\left\langle \tau^3 X_{R,\mu}\right\rangle^2 + \frac{1}{4}\tilde{\beta}_1 f_L f_R\left\langle \tau^3 X_{L,\mu}\right\rangle\left\langle \tau^3 X_R^\mu\right\rangle. \end{aligned} \tag{2.5}$$

Here, $\langle\ \rangle$ stands for the trace in flavor space. f_L and f_R are the scales for spontaneous symmetry breaking in the electroweak sector and parity, respectively. The coefficient $\beta_{L,R,1}$ generates extra mass for the left-handed (right-handed) third component in breaking the $SU(2)_{L,R}$ isospin symmetry. The coefficient κ parameterizes the mixing between the left- and right-handed gauge bosons whereas the coefficient $\tilde{\beta}_1$ controls the mixing between left-handed W_L^3 and right-handed W_R^3.

The neutral current interactions are

$$-\mathcal{L}_{\text{NC}} = W^3_{R\mu}J_R^\mu + W^3_{L\mu}J_L^\mu + B_\mu J_0^\mu \tag{2.6}$$

whereas the charged current interactions are

$$-\mathcal{L}_{\text{CC}} = W^+_{R\mu}J_R^{-,\mu} + W^+_{L\mu}J_L^{-,\mu} + \text{h.c.} \tag{2.7}$$

Here,

$$J_{L,R}^{\pm,\mu} = \frac{g_{L,R}}{\sqrt{2}} \overline{\Psi}_{L,R} \tau^{\pm} \gamma^{\mu} \Psi_{L,R}. \tag{2.8}$$

The kinetic part has the simple form

$$\mathcal{L}_K = -\frac{1}{4} W_{L,\mu\nu}^{a} W_L^{\mu\nu,a} - \frac{1}{4} W_{R,\mu\nu}^{a} W_R^{\mu\nu,a} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + i \overline{\Psi}_i \gamma_{\mu} D_{\mu} \Psi_i. \tag{2.9}$$

Adding Yukawa terms

$$\mathcal{L}_Y = \overline{\Psi}_L U_L M U_R^{\dagger} \Psi_R + \text{h.c.}, \tag{2.10}$$

the total Lagrangian is the sum of all the above terms

$$\mathcal{L} = \mathcal{L}_M + \mathcal{L}_K + \mathcal{L}_{\text{NC}} + \mathcal{L}_{\text{CC}} + \mathcal{L}_Y. \tag{2.11}$$

3. Diagonalization and Mass Eigenstates

In this section, we calculate the mass eigenvalues of the left-right symmetry effective Lagrangian by rotating the mass mixing matrix from the gauge basis to the mass basis.

3.1. Charged Gauge Bosons

Taking the unitary gauge $U_L = U_R = 1$, the charged gauge boson mass terms can be expressed as

$$\begin{aligned} \mathcal{L}_{\text{CM}} = {} & \frac{1}{4} f_L^2 g_L^2 W_{L,\mu}^{+} W_{L,\mu}^{-} + \frac{1}{4} f_R^2 g_R^2 W_{R,\mu}^{+} W_{R,\mu}^{-} \\ & - \frac{1}{4} \tilde{\kappa} f_L f_R g_L g_R \left(W_{L,\mu}^{+} W_{R,\mu}^{-} + W_{R,\mu}^{+} W_{L,\mu}^{-} \right). \end{aligned} \tag{3.1}$$

Here, we have used charged boson definitions $W_{i,\mu}^{1} = (W_{i,\mu}^{+} + W_{i,\mu}^{-})/\sqrt{2}$ and $W_{i,\mu}^{2} = i(W_{i,\mu}^{+} - W_{i,\mu}^{-})/\sqrt{2}$ for $i = L, R$.

We make an orthogonal rotation V for $W_L^{\pm}$ and $W_R^{\pm}$

$$\begin{pmatrix} W_R^{\pm} \\ W_L^{\pm} \end{pmatrix} = \begin{pmatrix} \cos\xi & \sin\xi \\ -\sin\xi & \cos\xi \end{pmatrix} \begin{pmatrix} W'^{\pm} \\ W^{\pm} \end{pmatrix} \equiv V \begin{pmatrix} W'^{\pm} \\ W^{\pm} \end{pmatrix} \tag{3.2}$$

to eliminate the cross-terms involving W_L and W_R in (3.1) to keep the kinetic term diagonal. The mixing angle ξ is expressed as

$$\tan 2\xi = \frac{2\tilde{\kappa} f_L f_R g_L g_R}{f_L^2 g_L^2 - f_R^2 g_R^2}. \tag{3.3}$$

After this rotation, the charged boson mass-squared matrix for the charged bosons becomes

$$V^T \mathcal{M}_C V = \mathrm{diag}\left(M_{W'}^2, M_W^2\right), \tag{3.4}$$

and the heavy and light charged boson masses are

$$\begin{aligned}
M_{W'}^2 &= \frac{1}{8}\left[f_L^2 g_L^2 + f_R^2 g_R^2 + \sqrt{\left(f_L^2 g_L^2 - f_R^2 g_R^2\right)^2 + 4\tilde{\kappa}^2 f_L^2 f_R^2 g_L^2 g_R^2}\right] \\
&\simeq \frac{1}{4} f_R^2 g_R^2 \left\{1 + \tilde{\kappa}^2 \frac{f_L^2 g_L^2}{f_R^2 g_R^2 - f_L^2 g_L^2}\right\}, \\
M_W^2 &= \frac{1}{8}\left[f_L^2 g_L^2 + f_R^2 g_R^2 - \sqrt{\left(f_L^2 g_L^2 - f_R^2 g_R^2\right)^2 + 4\tilde{\kappa}^2 f_L^2 f_R^2 g_L^2 g_R^2}\right] \\
&\simeq \frac{1}{4} f_L^2 g_L^2 \left\{1 - \tilde{\kappa}^2 \frac{f_R^2 g_R^2}{f_R^2 g_R^2 - f_L^2 g_L^2}\right\}.
\end{aligned} \tag{3.5}$$

We notice that the charged boson mixing angle ξ is controlled by the coefficient $\tilde{\kappa}$. $W - W'$ mixing causes W couplings to the right-handed fermion with $g_R^W = g_L \sin\xi/\sqrt{2}$. g_R^W can yield the contributions to $b \to s\gamma$ (see paper [14]) and must be restrained so that $g_R^W/g_L^W < 4 \times 10^{-3}$, which requires $\xi < 4 \times 10^{-3}$.

3.2. *Neutral Gauge Bosons*

Now, let us discuss the neutral boson sector. The neutral mass terms in our chiral Lagrangian (2.5) can be readily separated out

$$\begin{aligned}
\mathcal{L}_{M_n} = {} & \frac{1}{8}(1 - 2\beta_{L,1}) f_L^2 \left(g_L W_{L,\mu}^3 - g B_\mu\right)^2 + \frac{1}{8}(1 - 2\beta_{R,1}) f_R^2 \left(g_R W_{R,\mu}^3 - g B_\mu\right)^2 \\
& - \frac{1}{4}\left(\tilde{\kappa} + \tilde{\beta}_1\right) f_L f_R \left(g_L W_{L,\mu}^3 - g B_\mu\right)\left(g_R W_R^{3,\mu} - g B_\mu\right).
\end{aligned} \tag{3.6}$$

It can be written in matrix form

$$\mathcal{L}_{M_n} = \frac{1}{2} \mathcal{G}_\mu^T \mathcal{M}_n \mathcal{G}^\mu \tag{3.7}$$

with neutral gauge bosons $\mathcal{G}_\mu \equiv (W_{R,\mu}, W_{\mathrm{L},\mu}, B_\mu)$ and mass-squared matrix

$$\mathcal{M}_n \equiv \begin{pmatrix}
\dfrac{f_R^2 g_R^2}{4} & -\dfrac{\kappa f_R f_L g_R g_L}{4} & \dfrac{f_R g_R g_0 (\kappa f_L - f_R)}{4} \\
-\dfrac{\kappa f_R f_L g_R g_L}{4} & \dfrac{f_L^2 g_L^2}{4} & \dfrac{f_L g_L g_0 (\kappa f_R - f_L)}{4} \\
\dfrac{f_R g_R g_0 (\kappa f_L - f_R)}{4} & \dfrac{f_L g_L g_0 (\kappa f_R - f_L)}{4} & \dfrac{(f_R^2 + f_L^2 - 2\kappa f_R f_L) g_0^2}{4}
\end{pmatrix}. \tag{3.8}$$

Note that the $\beta_{L,R,1}$ do not appear in the above mass-squared matrix because these can be absorbed by a redefinition of VEV $f_{L,R}$

$$f_{L,R} \longrightarrow \frac{f_{L,R}}{\sqrt{1-2\beta_{L,R,1}}}. \tag{3.9}$$

For the sake of convenience, we will retain using the same notation for the redefined VEV $f_{L,R}$ but keep in mind that this redefinition has been made. The new parameter κ in the above formula is a combination of $\widetilde{\kappa}$ and $\widetilde{\beta}_1$, namely, $\kappa = \widetilde{\kappa} + \widetilde{\beta}_1$. Taking into account the VEVs re-definition, we have

$$\kappa = \frac{\widetilde{\kappa}+\widetilde{\beta}_1}{\sqrt{1-2\beta_{L,1}}\sqrt{1-2\beta_{R,1}}}. \tag{3.10}$$

The physical masses of the neutral bosons are the eigenvalues of the matrix $\mathcal{M}_n$. To obtain the diagonalized eigenvalues, we define the mass eigenstates as $\overline{\mathcal{G}}_\mu = (Z', Z_\mu, A_\mu)^T$ which are related to $\mathcal{G}_\mu$ by a special rotation U^{-1}

$$\begin{aligned}
\overline{\mathcal{G}}_\mu &= \begin{pmatrix}
\dfrac{G_4 g_R}{G_1G_4-G_2G_3} & -\dfrac{G_2 g_L}{G_1G_4-G_2G_3} & \dfrac{(G_2-G_4)g_0}{G_1G_4-G_2G_3} \\[2ex]
-\dfrac{G_3 g_R}{G_1G_4-G_2G_3} & \dfrac{G_1 g_L}{G_1G_4-G_2G_3} & \dfrac{(G_3-G_1)g_0}{G_1G_4-G_2G_3} \\[2ex]
\dfrac{g_R}{G_5} & \dfrac{\lambda_1 g_L}{G_5} & \dfrac{\lambda_2 g_0}{G_5}
\end{pmatrix}\mathcal{G}_\mu && (3.11)\\
&\equiv U^{-1}\mathcal{G}_\mu && (3.12)
\end{aligned}$$

with undetermined couplings G_i $(i=1,\ldots,5)$ and parameters λ_i $(i=1,2)$. This complicated rotation is motivated by the following simple relations: the rotation U relates

$$\begin{aligned}
g_R W_{R,\mu} - g_0 B_\mu &= G_1\overline{Z'}_\mu + G_2\overline{Z}_\mu,\\
g_L W_{L,\mu} - g_0 B_\mu &= G_3\overline{Z'}_\mu + G_4\overline{Z}_\mu,\\
g_R W_{R,\mu} + \lambda_1 g_L W_{L,\mu} + \lambda_2 g_0 B_\mu &= G_5\overline{A}_\mu
\end{aligned} \tag{3.13}$$

which diagonalizes the $B-W_L$ and $B-W_R$ mixings automatically while simultaneously keeping the photon massless. To maintain a diagonal kinetic energy matrix, U must satisfy six independent orthogonality conditions

$$UU^T = 1. \tag{3.14}$$

Adding one mass diagonalization condition for the $W_R - W_L$ mass mixing, there are seven independent equations that determine five G_i ($i = 1, \ldots, 5$) and two λ_i ($i = 1, 2$). Solving these equations, we obtain

$$
\begin{aligned}
\lambda_1 &= \frac{g_R^2}{g_L^2}, \\
\lambda_2 &= \frac{g_R^2}{g_0^2}, \\
G_1 &= \frac{(\kappa f_R C - f_L) f_L}{(f_R C - \kappa f_L) f_R} G_3, \\
G_2 &= C G_4, \\
G_3 &= \frac{(f_R C - \kappa f_L) f_R \sqrt{g_0^2 (1-C)^2 + g_R^2 + C^2 g_L^2}}{f_R^2 C^2 + f_L^2 - 2C\kappa f_R f_L}, \\
G_4 &= \frac{1}{f_R^2 C^2 + f_L^2 - 2C\kappa f_R f_L} \Big(f_R^2 f_L^2 \left[g_R^2 + C^2 g_L^2 + g_0^2 (1+C)^2 \right] \kappa^2 \\
&\qquad - 2 f_R f_L \left[f_R^2 C \left(g_R^2 + g_0^2 \right) + f_L^2 C \left(g_L^2 + g_0^2 \right) + g_0^2 \left(f_R^2 C^2 + f_L^2 \right) \right] \kappa \\
&\qquad + g_R^2 f_R^4 C^2 + g_L^2 f_L^4 + g_0^2 \left(f_R^2 C + f_L^2 \right)^2 \Big)^{1/2}, \\
G_5 &= g_R^2 \sqrt{\frac{1}{g_R^2} + \frac{1}{g_L^2} + \frac{1}{g_0^2}}
\end{aligned}
\tag{3.15}
$$

with a real C that satisfies the quadratic equation

$$
\left(\kappa f_R f_L \left(g_L^2 + g_0^2 \right) - f_R^2 g_0^2 \right) C^2 + \left[f_R^2 \left(g_R^2 + g_0^2 \right) - f_L^2 \left(g_L^2 + g_0^2 \right) \right] C + f_L^2 g_0^2 - \kappa f_R f_L \left(g_R^2 + g_0^2 \right) = 0.
\tag{3.16}
$$

The mass eigenvalues of the physical Z' and Z then become

$$
\begin{aligned}
M_{Z'}^2 &= \left(U^T \mathcal{M} U \right)_{1,1}, \\
M_Z^2 &= \left(U^T \mathcal{M} U \right)_{2,2}.
\end{aligned}
\tag{3.17}
$$

Up to now, we have obtained the exact rotation matrix elements without any approximate assumption. The total rotation U in (3.12) can be expressed in terms of (3.11), (3.15), and (3.16).

4. Oblique Radiative Corrections

To clearly see the new physics correction, we can separate a standard electroweak rotation from the total rotation in (3.12)

$$U \equiv U' U_{\text{em}} \tag{4.1}$$

with the standard electroweak rotation

$$U_{\text{em}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\theta & s_\theta \\ 0 & -s_\theta & c_\theta \end{pmatrix}. \tag{4.2}$$

From (3.12) and (4.1), we can calculate the oblique radiative corrections coming from the right-handed gauge bosons in light of Holdom's work [15]

$$\begin{aligned} S &= \frac{4 s_\theta c_\theta}{\alpha} \left\{ \left(s_\theta^2 - c_\theta^2 \right) U'_{32} - 2 c_\theta s_\theta \left(U'_{33} - 1 \right) + 2 c_\theta s_\theta \left(U'_{22} - 1 \right) \right\}, \\ T &= \frac{2}{\alpha} \left\{ \left(U'_{22} - 1 \right) - \Delta M_Z \right\}, \\ U &= -\frac{8 s_\theta^2}{\alpha} \left\{ c_\theta s_\theta U'_{32} + s_\theta^2 \left(U'_{33} - 1 \right) + c_\theta^2 \left(U'_{22} - 1 \right) \right\}, \end{aligned} \tag{4.3}$$

where s_θ and c_θ are the respective sine and cosine of the standard Weinberg angle from SM, and ΔM_Z is the new physical shift in the Z mass $\Delta M_Z = M_Z - M_Z|_{SM}$. Furthermore, we calculate to leading order the results for two special conditions.

4.1. Case 1: $f_R \gg f_L$ *and* $g_R \gg g_{L/0}$

This case corresponds to a $SU(2)_R$ breaking scale that is much higher than the electroweak breaking scale and $M_{W_R} \gg M_{W_L}$. It is easy to calculate the U' rotation from (4.1), (4.2), and (3.15). We only list leading-order terms

$$\begin{aligned} U'_{11} &\simeq 1, \\ U'_{12} &\simeq \frac{c_\theta s_\theta r^3}{2}, \\ U'_{13} &\simeq r, \end{aligned}$$

$$
\begin{aligned}
U'_{21} &\simeq -\frac{\kappa f_R}{f_L}\frac{s_\theta c_\theta}{r}\left(1+\frac{(3-c_\theta^2)r^2}{2}\right),\\
U'_{22} &\simeq 1,\\
U'_{23} &\simeq -\frac{c_\theta s_\theta r^2}{2},\\
U'_{31} &\simeq r+\frac{\kappa f_R}{r f_L}\left(1+\frac{(1-2c_\theta^2)r^2}{2}\right),\\
U'_{32} &\simeq \frac{c_\theta s_\theta r^2}{2},\\
U'_{33} &\simeq 1
\end{aligned}
\tag{4.4}
$$

with coupling ratio $r \equiv g_0/g_R$. Obviously, in the limit of heavy M_{W_R}, $g_R \gg g_{L,0}$, this new physics rotation matrix U' becomes a unitary matrix. Indeed, it is a requirement of the SM structure and a good self-checking condition of our calculation.

From (3.17), we can calculate the gauge boson mass eigenvalues

$$M_{Z'}^2 = \left(U^T \mathcal{M} U\right)_{1,1} \simeq \frac{f_R^2 g_R^2}{4}\left(1+r^2\right)\left(1-\kappa^2\right), \tag{4.5}$$

$$M_Z^2 = \left(U^T \mathcal{M} U\right)_{2,2} \simeq \frac{f_L^2 (g_L^2+g_0^2)}{4}\left\{1+\left(2\frac{\kappa f_L}{f_R}-s_\theta^2\right)r^2\right\}. \tag{4.6}$$

From (4.5), the mass shift can be calculated

$$\Delta M_Z \simeq -\frac{s_\theta^2}{2}r^2. \tag{4.7}$$

Using (4.3), the leading-order terms to the oblique radiative correction parameters are

$$
\begin{aligned}
\alpha S &\simeq s_\theta^2 c_\theta^2\left(1+2s_\theta^2\right)r^2,\\
\alpha T &\simeq s_\theta^2 r^2,\\
\alpha U &\simeq 4 s_\theta^6 r^2.
\end{aligned}
\tag{4.8}
$$

Adopting the new physics constraints $S < 0.11$, $T < 0.14$, $U < 0.16$ [16] and taking $s_\theta^2 = 0.2311$, $\alpha = 1/137$, we can estimate the coupling ratio $r < 0.05$.

4.2. Case 2: $f_R = f_L$ *and* $g_R \gg g_{L/0}$

The conditions correspond to left-right symmetry. $M_{W_R} \gg M_{W_L}$ requires $g_R \gg g_{L/0}$. Hence, the leading-order terms to the matrix elements of U' are

$$\begin{aligned}
U'_{11} &\simeq 1,\\
U'_{12} &\simeq -\frac{r^3 c_\theta\left(c_\theta^2 + c_\theta^2 s_\theta^2 + 1\right)}{s_\theta} + \frac{r c_\theta\left(2 + s_\theta^4\right)}{s_\theta},\\
U'_{13} &\simeq r - \frac{\kappa r\left(2 + s_\theta^4\right)}{2},\\
U'_{21} &\simeq -r^3 s_\theta c_\theta\left(\frac{3}{2} - c_\theta^2\right) + s_\theta c_\theta r^3 \kappa,\\
U'_{22} &\simeq 1\\
U'_{23} &\simeq -s_\theta c_\theta r^2 + \left\{\frac{s_\theta^3 c_\theta}{2} - \frac{r^2 s_\theta^3 c_\theta\left(1 + 3c_\theta^2\right)}{4}\right\}\kappa,\\
U'_{31} &\simeq r + c_\theta^2 s_\theta^3 r^3 \kappa,\\
U'_{32} &\simeq \frac{s_\theta c_\theta r^2}{2} + \left\{\frac{s_\theta^3 c_\theta}{2} - \frac{r^2 c_\theta}{4 s_\theta}\left[4 + 3s_\theta^4\left(1 + c_\theta^2\right)\right]\right\}\kappa,\\
U'_{33} &\simeq 1.
\end{aligned} \tag{4.9}$$

When taking $r \to 0$ and $\kappa \to 0$, matrix U' becomes unitary. The leading order terms for the gauge boson masses are

$$\begin{aligned}
M_{Z'}^2 &= \left(U^T \mathcal{M} U\right)_{1,1} \simeq \frac{f^2 g_R^2}{4}\left(1 + r^2\right),\\
M_Z^2 &= \left(U^T \mathcal{M} U\right)_{2,2} \simeq \frac{f^2\left(g_L^2 + g_0^2\right)}{4}\left(1 - \frac{s_\theta^2 r^2}{2}\right).
\end{aligned} \tag{4.10}$$

The shift in mass of Z is

$$\Delta M_Z \simeq -\frac{s_\theta^2}{4} r^2. \tag{4.11}$$

In this case, the leading-order terms of the oblique radiative correction parameters are

$$\begin{aligned}
\alpha S &\simeq 2r^2 s_\theta^2\left(1-2s_\theta^2\right)c_\theta^2+\left\{6\left(1-2c_\theta^2\right)s_\theta^4+r^2\left(4c_\theta^2-3\right)\left(3s_\theta^2+4\right)\right\}c_\theta^2\kappa,\\
\alpha T &\simeq \frac{s_\theta^2 r^2}{2},\\
\alpha U &\simeq 4s_\theta^6 r^2+2\left\{2s_\theta^2\left(1-3c_\theta^2 s_\theta^2\right)+r^2\left(2c_\theta^2-1\right)\left(3s_\theta^4+4\right)\right\}s_\theta^2\kappa.
\end{aligned} \tag{4.12}$$

From $T < 0.10$, we can estimate coupling ratio $r < 0.09$ implying a lower limit for the Z' mass of about 0.8 TeV.

5. A Short Summary

To summarize, we have reviewed nonlinearly realized electroweak chiral Lagrangian for the gauge group $SU(2)_R \times SU(2)_L \times U(1)$ and diagonalized gauge eigenstates using all possible mass mixing terms to obtain exact mass eigenstates and the rotation matrix. The oblique radiative corrections from right-handed gauge bosons have been estimated to leading order.

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