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Research Article

Closing a Window for Massive Photons

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Working with the assumption of nonzero photon mass and a trajectory that is described by the nongeodesic world line of a spinning top we find, by deriving new astrophysical bounds, that this assumption is in contradiction with current experimental results. This yields the conclusion that such photons have to be exactly massless.

1. Introduction

Although there are good theoretical reasons to believe that the photon mass should be exactly zero, there is no experimental proof of this belief. A long series of very different experiments lead to the current experimental upper bound on the photon mass $m_\gamma < 10^{-18}$ eV. However, even with further improvement of the experimental technology and precision a complete exclusion of a nonzero photon mass by those techniques will never be possible. In order to improve on this situation we will work with the assumption that the photon mass is different from zero $m_\gamma \neq 0$. We will not speculate on the origin of this mass and its underlying theory. In the simplest case, such a photon will be described by a wave equation of the Proca type. For waves describing massless particles with spin, it is well understood, in the eikonal approximation, how the propagating wave can be treated as geometric path that minimizes a Lagrangian described by the length

$$g_{\mu\nu} u^\mu u^\nu = u^2, \tag{1}$$

where $u^\mu = \dot{x}^\mu$. As soon as mass and rotational degrees of free-dom $(m^2, \sigma^{\mu\nu})$ are involved, it is natural to assume that, in the eikonal approximation, the resulting geometric action also involves the invariant functions of the corresponding additional degrees of freedom:

$$\begin{gathered} g_{\mu\nu} g_{\alpha\beta} \sigma^{\alpha\mu} \sigma^{\nu\beta} = \operatorname{tr}\sigma^2, \\ g_{\mu\nu} g_{\alpha\beta} g_{\gamma\delta} u^\mu \sigma^{\nu\alpha} \sigma^{\beta\gamma} u^\delta = u\sigma\sigma u, \\ g_{\mu\nu} g_{\rho\tau} g_{\alpha\beta} g_{\gamma\delta} \sigma^{\delta\mu} \sigma^{\nu\rho} \sigma^{\tau\alpha} \sigma^{\beta\gamma} = \operatorname{tr}\sigma^4. \end{gathered} \tag{2}$$

This generic approach to the geometric action which includes spin and mass was developed and discussed in the context of a spinning top, a point on a world line to which a rotating frame has been attached [1–10]. The resulting equations of motion do not depend on the particular form in which the Lagrangian terms (1, 2) are combined. This formulation and the corresponding equations of motion can also be derived as consistent eikonal limit as quantum field theory in curved space-time. This has been shown independently for spin one half [11, 12], for spin one [13, 14], and for spin three half [15, 16].

Exact solutions to those equations coupled to gravity via the metric $g_{\mu\nu}$ show that spinning tops without mass follow geodesics, while massive spinning tops have different trajectories. Further implications and discussions of the spinning top approach can be found in [17–22].

2. Astrophysical Bound

As master equation for the trajectory of massive particles with spin we use the solution of the spinning top equations in a Schwarzschild background [5, 6, 9, 10]:

$$\frac{d\phi}{dr} = \left(\frac{2\eta + 1}{\eta - 1}\right)\left(\frac{P_\phi}{r^2 P^r}\right), \tag{3}$$

where one has to insert the following definitions:

$$\eta = \frac{J^2 r_0}{2m_\gamma^2 c^2 r^3}, \tag{4}$$

where $J = \hbar$ is the photon spin. For a given angular momentum j, the momenta are

$$\begin{aligned} P_\phi &= \frac{-j + (\pm_\phi)\, EJ/\left(m_\gamma c^2\right)}{1 - \eta}, \\ P_t &= \frac{E - (\pm_t)\, jJr_0/\left(2m_\gamma r^3\right)}{1 - \eta}, \\ P^r &= \pm\left[\frac{P_t^2}{c^2} - \left(\frac{P_\phi^2}{r^2} + m^2c^2\right)\left(1 - \frac{r_0}{r}\right)\right]^{1/2}. \end{aligned} \tag{5}$$

The orientation of this problem is chosen such that the angle $\theta = \pi/2$ is constant along the trajectories, which implies that

$$P_\theta = 0. \tag{6}$$

The solution is such that $\pm_\phi = \pm_t$ and $P_\mu P^\mu = m_\gamma^2 c^2$ are fulfilled.

For small spin corrections and large radii one can approximate

$$\left(\frac{dr}{d\phi}\right)^2 = r^4\left(\frac{1}{j^2} + (\pm_\phi)\frac{2\hbar}{cj^3 m_\gamma}\right) - r^2 + rr_0\left(1 - \frac{(\pm_t)\,\hbar}{cjm_\gamma}\right). \tag{7}$$

In this equation one can easily verify that for spinless case ($\hbar \to 0$), the geodesic trajectory is recovered. One can express the angular momentum j in terms of the minimal radial distance r_m:

$$j = \frac{2cE\hbar m \pm (3r_0 - 2r_m)\, r_m^4}{c\hbar^2 r_0^2 r_m + 4c^3 m^2 (r_0 - r_m)\, r_m^4} + 2\sqrt{r_m (r_m - r_0)} \cdot \frac{\sqrt{E^2\hbar^4 r_0^2 r_m^4 + 4c^8 m^6 (r_0 - r_m)\, r_m^9 + c^6\hbar^2 m^4 r_0 r_m^6 (4r_m - 3r_0) + c^2\hbar^2 r_0 \left(\hbar^4 r_0^3/4 - 4E^2 m^2 r_m^7\right) + c^4 m^2 r_m^3 \left(4E^2 m^2 r_m^7 - \hbar^4 r_0^2 r_m\right)}}{c\hbar^2 r_0^2 r_m + 4c^3 m^2 (r_0 - r_m)\, r_m^4}. \tag{8}$$

Due to the steep behavior of $d\phi/dr$ at the minimal radius r_m one has to use (8), since one cannot rely a priori on the approximate value $j \approx r_m^{3/2}/\sqrt{r_m - r_0}$.

Due to the involved form of (8), the complete angular deviation has to be computed essentially numerically, as it was done here. In order to gain a better intuition of the numerical results one can however do approximations and expansions for small $\hbar$, mc^2/E, and r_0/r_m as follows:

$$\Delta\phi = 2\left(\int_{r_m}^{\infty}\left(\frac{d\phi}{dr}\right)dr\right) - \pi \approx 2\frac{r_0}{r_m}\left(1 + \frac{\hbar}{r_m c m_\gamma}\right), \tag{9}$$

where $\pm_t = -1$ and $\pm_\phi = 1$ were used. One observes that the standard result $2r_0/r_m$ is modified by a spin-dependent correction. In order to be in agreement with the experimentally well confirmed gravitational lensing effect, this dimensionless modification has to be much smaller than one:

$$1 \gg \Delta_J \approx \frac{\hbar}{r_m c m_\gamma}. \tag{10}$$

This modification is inversely proportional to the photon mass m_γ. This inverse proportionality implies that practically no deviations from the usual trajectories are observable for massive standard model particles. Please note that this nonperturbative feature in m_γ is also known from theories of massive gravity where the limit $m_g \to 0$ does not give standard gravity where the graviton was massless right from the start $m_g = 0$. Further similarities to massive gravity have been recently discussed in [23]. It is further interesting to note that Δ_J actually increases the usual angular deflection. However, a deviation from the geodesic bending of light has been excluded to high precision [24]. Thus, relation (10) can be interpreted as an estimate for the numerical lower photon mass limit:

$$m_\gamma \gg \frac{\hbar}{r_m c} \approx 3 \times 10^{-16}\ \mathrm{eV}/c^2. \tag{11}$$

In Figure 1 we compare this estimate to the observed value [25, 26] and to the precise numerical results by using the solar radius $r_m = 6.96 \times 10^8$ m, the solar Schwarzschild radius $r_0 = 2964$ m, and photon energy of $E = 1$ eV.

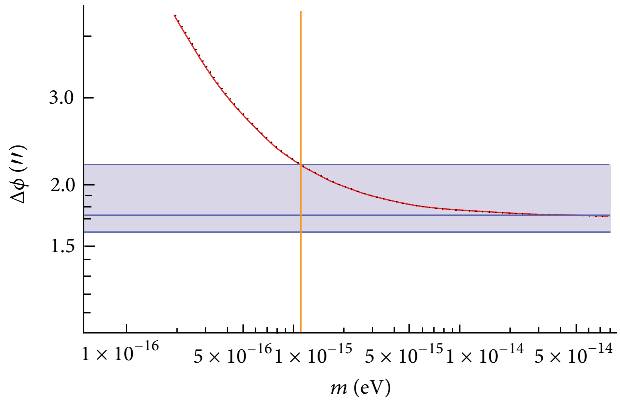

FIGURE 1: Numerical results for the angular deviation $\Delta\phi$ as a function of m_γ. The horizontal lines represent a conservative range for the measured gravitational lensing by the sun [25]. Please note that this range can be largely improved by measurements using radio astronomy [24, 26]. The red line is the numerical value for both configurations of $\pm_\phi = \pm_t$. The black dotted line is the analytic estimate (9). The vertical line is the deduced minimal value for m_γ given in (12).

One finds that the estimated deviation is in very good agreement with the lower numerical result. By using conservative exclusion ranges for $\Delta\phi$ one can read from the figure a more precise limit:

$$m_\gamma > 1.1 \times 10^{-15}\ \mathrm{eV}/c^2. \quad (12)$$

The lower limit (12) can be combined with the upper limit for a photon mass from the Particle Data Group [27]; $m_\gamma < 10^{-18}\ \mathrm{eV}/c^2$. Some of the limits in the PDG tables [27] are however derived by assuming a spinless coupling of the photon to gravity and to matter. But this might not be true. When deriving the limit (12), we found that the gravitational coupling of the massive particles with spin is different from a spinless or massless coupling. Thus, out of the limits in [27] only those can be applied straightforwardly, where the gravitational coupling is not relevant. For example, results from laboratory experiments that do not involve astrophysical or gravitational components [28–34] can be used directly. Also results from other experiments that do involve astrophysical components but where the actual trajectory of the photon does not play any role [35–45] should be applicable, but a sound revision is in order. Nevertheless, the bounds on m_γ from some experiments are not directly applicable because they are not sufficiently general [46–48] or because they make explicit use of the photon trajectory in a gravitational field [34, 49–52] (note that those experimental references are ordered by the strength of their respective bounds on m_γ). The experimental bounds that are directly applicable are $m_\gamma < \{1.2 \times 10^{-18}\ \mathrm{eV}/c^2$ [28, 29], $5.6 \times 10^{-17}\ \mathrm{eV}/c^2$ [30], $1 \times 10^{-14}\ \mathrm{eV}/c^2$ [31], $4.5 \times 10^{-10}\ \mathrm{eV}/c^2$ [32, 33], $1.6 \times 10^{-4}\ \mathrm{eV}/c^2$ [34]$\}$.

Thus, three of the applicable experimental limits [31–34], combined with the limit (12), leave only a window for the photon mass. However, if one refers only to the more stringent upper limits on m_γ [28–30] one finds

$$m_\gamma < \left\{1.2 \times 10^{-18}\ \mathrm{eV}/c^2, 5.6 \times 10^{-17}\ \mathrm{eV}/c^2\right\}, \quad (13)$$

which is in clear contradiction to (12). Therefore, one can conclude that confronting the assumption of a nonzero photon mass with an astrophysical lower bound and the established upper bounds excludes the existence of any photon mass. Thus, the photon, if it is actually correctly described by the model that is discussed throughout this paper, has to be massless right from the start:

$$m_\gamma = 0. \quad (14)$$

At this point, another word of caution is at place: when combining bounds that arise from different descriptions, as it was done here when combining the bounds from a geometrical description (12) with bounds that arise from a wave description (13), one might end up comparing parameters of different models. Even within the upper bounds obtained from wave descriptions of the massive photons there exist problems of generality, which imply that bounds that were observed for one model of m_γ do not apply to other models of m_γ [27].

Thus, despite of the generality of the geometric equations of motion, the criterion of applicability that was used here can be made more precise by explicitly deriving specific geometrical equations as eikonal limit of a particular wave model of photon mass, as it was done in [14, 53]. A useful guiding tool for anticipating those results might be the peculiar noncontinuous behavior in the $m_\gamma \to 0$ limit of the geometric description, as it appears in (10). For example for the wave descriptions of m_γ that use the Proca equations [54], it has been shown that the limit $m_\gamma \to 0$ converges to Maxwell's theory only if one demands $\partial_\mu A^\mu = 0$ [55–60]. On the other hand, the same limit for Proca equations with $\partial_\mu A^\mu \neq 0$ does not lead continuously to Maxwell's theory [61, 62]. The corresponding geometrical descriptions in the eikonal limit are expected to have an analogous behavior in the $m_\gamma \to 0$ limit as their counterpart in the wave description. However, the eikonal limit of Maxwell's theory is described by the usual geodesics. Therefore, one might expect that the limit (12) could only apply to Proca mass models with $\partial_\mu A^\mu \neq 0$ and not to Proca mass models with $\partial_\mu A^\mu = 0$. However, this argument does not disqualify the applicability of the geometric approximation (2) a priori. For example, in [61] it has been shown that although the longitudinal modes of the Proca equation (even with $\partial_\mu A^\mu = 0$) decouple from matter for $m_\gamma \to 0$ in flat space-times, the coupling to gravity of those modes does not vanish in this limit.

3. Discussion and Conclusion

Under the assumption that the trajectory of a photon can be described by the world line of a top with one spin, it was shown that such a photon has to have either zero mass $m_\gamma = 0$ or a minimal nonzero mass $m_\gamma > 1.1 \times 10^{-15}\ \mathrm{eV}/c^2$. This new phenomenological bound is the main result of this paper. If one further imposes an agreement with the observed gravitational light bending by the sun [24–26] one finds that when combining those results with the experimental bounds from laboratory experiments such as $m_\gamma < 5.6 \times 10^{-17}\,\mathrm{eV}/c^2$ [28–30], in this theoretical framework, there is no room for a photon mass different from zero.

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