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Research Article

Entanglement in a QFT Model of Neutrino Oscillations

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Tools of quantum information theory can be exploited to provide a convenient description of the phenomena of particle mixing and flavor oscillations in terms of entanglement, a fundamental quantum resource. We extend such a picture to the domain of quantum field theory where, due to the nontrivial nature of flavor neutrino states, the presence of antiparticles provides additional contributions to flavor entanglement. We use a suitable entanglement measure, the concurrence, that allows extracting the two-mode (flavor) entanglement from the full multimode, multiparticle flavor neutrino states.

1. Introduction

In the last years, many efforts have been dedicated to the investigation of entanglement in the domain of elementary particle physics and quantum field theory [1–11]. The understanding of the role of nonlocal quantum correlations in infinite-dimensional systems of fields and particles, as well as the underlying mechanism governing their aggregation, represents a main goal. For systems composed by identical particles and/or sets of in general distinguishable field modes (either discrete or continuous, finite or infinite), the characterization and quantification of entanglement are achieved unambiguously only by properly taking into account the algebra of observables besides the tensor product structure of the individual state spaces. In the case of quantum fields, a further extension of such a framework is needed to take into account the correlations among distinguishable physical field modes rather than among indistinguishable particles and excitations [12–27].

For instance, the single-particle Bell superposition state $|0,1\rangle + |1,0\rangle$ between any two modes of the electromagnetic field is a well-known example of a maximally (bipartite) entangled quantum state (i.e., with maximal von Neumann entropy of the reduced single-mode density matrices), despite the fact that it involves only one excitation of the field (a single photon) [26]. In this case, the entanglement is between two different field modes with occupation numbers ranging between 0 and 1. Considering single-particle or multiparticle (e.g., multiphoton) states of many modes leads to straightforward generalizations that allow considering the bipartite and multipartite multimode field entanglement of single-particle and multiparticle states. In the same way, it is possible to make precise sense of Bell nonlocality in the context of single-particle, multimode states [28–30].

In this context, it has been recently recognized that the phenomena of particle mixing and flavor oscillations can be understood in terms of quantum entanglement [4, 5]. In particular, a connection has been established among the flavor transition probabilities and the multimode, single-particle entanglement for oscillating neutrinos [5]. Such a connection allows in principle engineering experimental protocols for the transfer of the quantum information encoded in neutrino states to spatially delocalized two-flavor charged lepton states [4, 5]. The above analysis has been carried out in the context of quantum mechanics (QM), using the well-known Pontecorvo formalism for neutrino oscillations. On the other hand, it has been shown that flavor mixing in the context of quantum field theory (QFT) is associated with a highly

nontrivial nature of the vacuum for the mixed fields [31–34]. As a consequence, neutrino states turn out to be multimode, multiparticle states, with a very rich structure of quantum correlations. In [6, 7], we studied the entanglement in such a system by means of entropic measures and found a relation with experimentally measurable quantities, like the variances of the lepton numbers and charges.

In the present work, we further investigate along this direction by adopting an alternative operational viewpoint on the entanglement associated with the system of oscillating neutrinos. Indeed, entanglement is an observable-induced, relative physical quantity [15–22], endowed with a specific operational meaning according to the selected reference quantum observables and quantum subsystems. By assuming the particle-antiparticle species as further quantum modes, we investigate the entanglement content of the neutrino system in the state obtained by tracing out the antiparticle species. Since such a state turns out to be a mixed one, we adopt the concurrence as a measure for the quantification of its entanglement content. Our results are in line with those of [6, 7] and naturally generalize the QM ones presented in [5].

The paper is organized as follows. In Section 2 we review the quantum information tools exploited in the paper and some results corresponding to the QM framework. In Section 3 we investigate the entanglement phenomenology of neutrino mixing and flavor oscillations adopting a QFT framework, and we discuss the nontrivial structure of flavor entanglement that emerges in the QFT framework.

2. Entanglement and Flavor Oscillations: Quantum Mechanics

In this section, we briefly review the background of the present analysis, that is, the formalism developed and the results obtained within the quantum mechanical framework [4, 5]. Flavor mixing of neutrinos for two generations is described by the 2×2 rotation matrix $\mathbf{U}(\theta)$ [35]

$$\mathbf{U}(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}, \tag{1}$$

where θ is the mixing angle. The two-flavor neutrino states are defined as

$$\left|\underline{\nu}^{(f)}\right\rangle = \mathbf{U}(\theta)\left|\underline{\nu}^{(m)}\right\rangle, \tag{2}$$

where $|\underline{\nu}^{(f)}\rangle = (|\nu_e\rangle, |\nu_\mu\rangle)^T$ are the states with definite flavors e, μ and $|\underline{\nu}^{(m)}\rangle = (|\nu_1\rangle, |\nu_2\rangle)^T$ those with definite masses m_1, m_2. Both $|\nu_\alpha\rangle$ $(\alpha = e, \mu)$ and $|\nu_j\rangle$ $(j = 1, 2)$ are orthonormal. By describing the free propagation of the neutrino mass eigenstates with plane waves of the form $|\nu_j(t)\rangle = e^{-i\omega_j t}|\nu_j\rangle$, ω_j denoting the frequency associated with the mass eigenstate $|\nu_j\rangle$, the time evolution of the flavor states is given by

$$\left|\underline{\nu}^{(f)}(t)\right\rangle = \mathbf{U}(t)\left|\underline{\nu}^{(f)}\right\rangle \equiv \mathbf{U}(\theta)\,\mathbf{U}_0(t)\,\mathbf{U}^{-1}(\theta)\left|\underline{\nu}^{(f)}\right\rangle, \tag{3}$$

where $|\underline{\nu}^{(f)}\rangle$ are the flavor states at $t = 0$, and $\mathbf{U}_0(t) = \mathrm{diag}(e^{-i\omega_1 t}, e^{-i\omega_2 t})$. By assuming the neutrino occupation number associated with a given flavor (mode) as reference quantum number, one can establish the following correspondences with two-qubit states:

$$\begin{aligned} \left|\nu_e\right\rangle &\equiv |1\rangle_{\nu_e}|0\rangle_{\nu_\mu} \equiv |10\rangle_{e\mu}, \\ \left|\nu_\mu\right\rangle &\equiv |0\rangle_{\nu_e}|1\rangle_{\nu_\mu} \equiv |01\rangle_{e\mu}, \end{aligned} \tag{4}$$

where $|j\rangle_{\nu_\alpha}$ stands for a j-occupation number state of a neutrino in mode α. Entanglement is thus established among flavor modes, in a single-particle setting. For instance, the free evolution of the electron-neutrino state $|\nu_e(t)\rangle$ can be written in the form

$$\left|\nu_e(t)\right\rangle = \mathbf{U}_{ee}(t)\,|10\rangle_{e\mu} + \mathbf{U}_{e\mu}(t)\,|01\rangle_{e\mu}, \tag{5}$$

where $|\mathbf{U}_{ee}(t)|^2 + |\mathbf{U}_{e\mu}(t)|^2 = 1$ due to normalization. Thus, the time-evolved states $|\underline{\nu}^{(f)}(t)\rangle$ are entangled Bell-like superpositions of the two masses with time-independent coefficients or flavor eigenstates with time-dependent coefficients. It is worth remarking that the entanglement of (5) is in principle experimentally accessible, throughout a scheme for its transfer from single-neutrino states to two-flavor charged lepton states [5].

The entanglement content of the pure two-qubit state equation (5) is quantified by the von Neumann entropy of the reduced density matrix (or any other monotonic function of it), which is the widely accepted measure of the bipartite entanglement of pure states [36]. For other measures of entanglement (bipartite and multipartite systems), see [37–42]. In the instance of mixed states, a particularly convenient measure for two-qubit systems is the concurrence $C(\rho)$ [43, 44]. For the particular case of a pair of qubits, such a measure is closely related to the entanglement of formation $E_F(\rho)$, which is the prototype of the convex-roof-based measures. The entanglement of formation has a simple physical interpretation: it is the minimal amount of entanglement needed for the production of a mixed state described by a given density matrix.

We briefly recall the definition of the concurrence, which will be used in this work. Let ρ be the density operator corresponding to an arbitrary N-qubit state and describing a system partitioned into N parties. The reduced density operator $\rho^{(\alpha;\beta)}$ associated with ρ is defined as

$$\rho^{(\alpha;\beta)} = \mathrm{Tr}_{\gamma \neq \alpha,\beta}\left[\rho\right], \tag{6}$$

where the trace operation is made over all the parties different from α and β. Next, the spin-flipped state $\widetilde{\rho}^{(\alpha;\beta)}$ reads

$$\widetilde{\rho}^{(\alpha;\beta)} = \left(\sigma_y \otimes \sigma_y\right)\rho^{(\alpha;\beta)*}\left(\sigma_y \otimes \sigma_y\right), \tag{7}$$

where the complex conjugate is taken in the standard basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$. Then the concurrence is expressed in terms of square roots λ_i of the eigenvalues of the non-Hermitian matrix $\rho^{(\alpha;\beta)}\widetilde{\rho}^{(\alpha;\beta)}$:

$$C\left(\rho^{(\alpha;\beta)}\right) = \max\left\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\right\}, \tag{8}$$

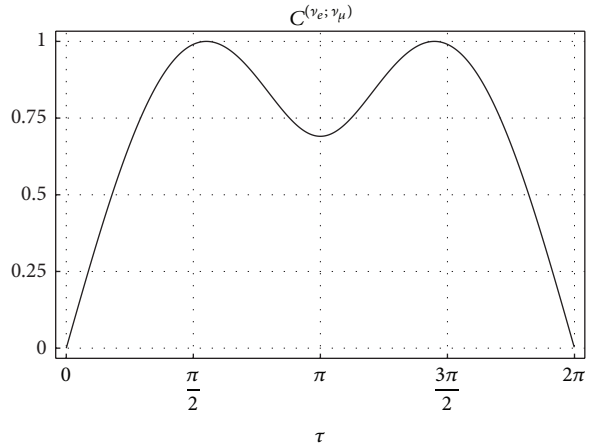

FIGURE 1: QM instance. The concurrence $C^{(\nu_e;\nu_\mu)}$ as function of the scaled time $\tau = (\omega_2 - \omega_1)t$. The mixing angle θ is fixed at the experimental value $\sin^2\theta = 0.314$.

where the λ_is are nonnegative real numbers taken in decreasing order with respect to the index i. The concurrence equation (8) can be easily computed for the pure two-qubit Bell state $|\nu_e(t)\rangle$, that is, (5), with density matrix $\rho_e = |\nu_e(t)\rangle\langle\nu_e(t)|$. The concurrence $C^{(\nu_e;\nu_\mu)} \equiv C(\rho^{(\nu_e;\nu_\mu)})$ writes

$$C^{(\nu_e;\nu_\mu)} = 2\left|\mathbf{U}_{ee}(t)\right|\left|\mathbf{U}_{e\mu}(t)\right|. \tag{9}$$

It is worth noticing that, in the instance of pure states, the concurrence coincides with the square root of the linear entropy. In Figure 1, we show the behavior of $C^{(\nu_e;\nu_\mu)}$ as functions of the scaled, dimensionless time $\tau = (\omega_2 - \omega_1)t$. At time $\tau = 0$, the two flavors are not mixed, the entanglement is zero, and the global state of the system is factorized. For $\tau > 0$, flavor oscillations occur, and the linear entropy exhibits a typical oscillatory behavior; the entanglement is maximal at largest mixing.

3. Entanglement and Flavor Oscillations: Quantum Field Theory

In this section, first we review the essential features of a specific QFT model of particle mixing describing the phenomena of neutrino oscillations [31, 32]. For a general theory of mixing for an arbitrary number of fields see also [45].

Then, by using such a model, we present a generalization of the above analysis to the QFT framework. The neutrino Dirac fields $\nu_e(x)$ and $\nu_\mu(x)$ are defined through the mixing relations

$$\begin{aligned}\nu_e(x) &= \cos\theta\nu_1(x) + \sin\theta\nu_2(x),\\ \nu_\mu(x) &= -\sin\theta\nu_1(x) + \cos\theta\nu_2(x),\end{aligned} \tag{10}$$

where, in standard notation, x stands for the four-vector $x \equiv (t, \mathbf{x})$ and the free fields $\nu_1(x)$ and $\nu_2(x)$ with definite masses m_1 and m_2. The generator of the mixing transformations is given by

$$G_\theta(t) = \exp\left[\theta\int d^3\mathbf{x}\left(\nu_1^\dagger(x)\nu_2(x) - \nu_2^\dagger(x)\nu_1(x)\right)\right] \tag{11}$$

so that

$$\nu_\sigma^\alpha(x) = G_\theta^{-1}(t)\,\nu_i^\alpha(x)\,G_\theta(t), \tag{12}$$

where $(\sigma, i) = (e, 1), (\mu, 2)$, and the superscript $\alpha = 1, \ldots, 4$ denotes the spinorial component. At finite volume, $G_\theta(t)$ is a unitary operator, that is, $G_\theta^{-1}(t) = G_{-\theta}(t) = G_\theta^\dagger(t)$, preserving the canonical anticommutation relations. The generator $G_\theta^{-1}(t)$ maps the Hilbert space for free fields $\mathscr{H}_{1,2}$ to the Hilbert space for mixed fields $\mathscr{H}_{e,\mu}$; that is, $G_\theta^{-1}(t) : \mathscr{H}_{1,2} \mapsto \mathscr{H}_{e,\mu}$. In particular, the flavor vacuum is given by $|0(t)\rangle_{e,\mu} = G_\theta^{-1}(t)\ |0\rangle_{1,2}$ at finite volume V. We denote by $|0\rangle_{e,\mu}$ the flavor vacuum at $t = 0$. It is worth noticing that, in the infinite volume limit, the flavor and the mass vacua are unitarily inequivalent. The free fields $\nu_i(x)$ $(i = 1, 2)$ are given by the following expansions:

$$\nu_i(x) = \frac{1}{\sqrt{V}}\sum_{\mathbf{k},r}\left[u_{\mathbf{k},i}^r\alpha_{\mathbf{k},i}^r(t) + v_{-\mathbf{k},i}^r\beta_{-\mathbf{k},i}^{r\dagger}(t)\right]e^{i\mathbf{k}\cdot\mathbf{x}}, \tag{13}$$

where $\mathbf{k}$ is the momentum vector, $r = 1, 2$ denotes the helicity, $\alpha_{\mathbf{k},i}^r(t) = \alpha_{\mathbf{k},i}^r e^{-i\omega_{k,i}t}$, $\beta_{\mathbf{k},i}^{r\dagger}(t) = \beta_{\mathbf{k},i}^{r\dagger}e^{i\omega_{k,i}t}$, and $\omega_{k,i} = \sqrt{\mathbf{k}^2 + m_i^2}$. The operators $\alpha_{\mathbf{k},i}^r$ and $\beta_{\mathbf{k},i}^r$ are the annihilation operators for the vacuum state $|0\rangle_m \equiv |0\rangle_1 \otimes |0\rangle_2$; that is, $\alpha_{\mathbf{k},i}^r|0\rangle_m = \beta_{\mathbf{k},i}^r|0\rangle_m = 0$. The anticommutation relations are the usual ones; extended details, for example, on the orthonormality and completeness relations are presented in our previous works [31, 32]. By use of $G_\theta(t)$, the flavor fields can be expanded as

$$\nu_\sigma(\mathbf{x}) = \frac{1}{\sqrt{V}}\sum_{\mathbf{k},r}\left[u_{\mathbf{k},i}^r\alpha_{\mathbf{k},\sigma}^r(t) + v_{-\mathbf{k},i}^r\beta_{-\mathbf{k},\sigma}^{r\dagger}(t)\right]e^{i\mathbf{k}\cdot\mathbf{x}}. \tag{14}$$

The flavor annihilation operators are defined as $\alpha_{\mathbf{k},\sigma}^r(t) \equiv G_\theta^{-1}(t)\alpha_{\mathbf{k},i}^r G_\theta(t)$ and $\beta_{\mathbf{k},\sigma}^{r\dagger}(t) \equiv G_\theta^{-1}(t)\beta_{\mathbf{k},i}^{r\dagger}G_\theta(t)$. Without any loss of generality, let us choose the reference frame such that $\mathbf{k} = (0, 0, |\mathbf{k}|)$; we have

$$\begin{aligned}\alpha_{\mathbf{k},e}^r(t) = {}& \cos\theta\alpha_{\mathbf{k},1}^r(t)\\ &+ \sin\theta\left(\left|U_{\mathbf{k}}\right|\alpha_{\mathbf{k},2}^r(t) + \epsilon^r\left|V_{\mathbf{k}}\right|\beta_{-\mathbf{k},2}^{r\dagger}(t)\right),\end{aligned} \tag{15}$$

where $\epsilon^r = (-1)^r$ and

$$\begin{aligned}\left|U_{\mathbf{k}}\right| &\equiv u_{\mathbf{k},i}^{r\dagger}u_{\mathbf{k},j}^r = v_{-\mathbf{k},i}^{r\dagger}v_{-\mathbf{k},j}^r\\ &= \frac{|\mathbf{k}|^2 + (\omega_{k,1} + m_1)(\omega_{k,2} + m_2)}{2\sqrt{\omega_{k,1}\omega_{k,2}(\omega_{k,1} + m_1)(\omega_{k,2} + m_2)}},\\ \left|V_{\mathbf{k}}\right| &\equiv \epsilon^r u_{\mathbf{k},1}^{r\dagger}v_{-\mathbf{k},2}^r = -\epsilon^r u_{\mathbf{k},2}^{r\dagger}v_{-\mathbf{k},1}^r\\ &= \frac{(\omega_{k,1} + m_1) - (\omega_{k,2} + m_2)}{2\sqrt{\omega_{k,1}\omega_{k,2}(\omega_{k,1} + m_1)(\omega_{k,2} + m_2)}}\,|\mathbf{k}|,\end{aligned} \tag{16}$$

with $i, j = 1, 2$, $i \neq j$, and $|U_{\mathbf{k}}|^2 + |V_{\mathbf{k}}|^2 = 1$. The explicit expression for the flavor states $|\nu^r_{\mathbf{k},e}\rangle$ at time $t = 0$ is

$$\begin{aligned}\left|\nu^r_{\mathbf{k},e}\right\rangle &\equiv \alpha^{r\dagger}_{\mathbf{k},e}(0)\,|0\rangle_{e,\mu} \\ &= \left[\cos\theta\alpha^{r\dagger}_{\mathbf{k},1} + \left|U_{\mathbf{k}}\right|\sin\theta\alpha^{r\dagger}_{\mathbf{k},2}\right. \\ &\quad \left. - \epsilon^r\left|V_{\mathbf{k}}\right|\sin\theta\alpha^{r\dagger}_{\mathbf{k},1}\alpha^{r\dagger}_{\mathbf{k},2}\beta^{r\dagger}_{-\mathbf{k},1}\right] \\ &\quad \times G^{-1}_{\theta,\mathbf{k},r}(t)\prod_{\mathbf{p}\neq\mathbf{k}} G^{-1}_{\theta,\mathbf{p},s}(t)\,|0\rangle_{1,2},\end{aligned} \tag{17}$$

where $G_\theta(t) = \prod_{\mathbf{p}}\prod_{s=1}^{2} G_{\theta,\mathbf{p},s}(t)$. In the state (17), a multiparticle component is present, disappearing in the relativistic limit $|\mathbf{k}| \gg \sqrt{m_1 m_2}$: indeed, for large $|\mathbf{k}|$, since one gets $|U_{\mathbf{k}}|^2 \to 1$ and $|V_{\mathbf{k}}|^2 \to 0$, the (quantum-mechanical) Pontecorvo states are recovered. In order to simplify the notation, we omit the superscript r (by fixing $r = 2$) and the subscript $\mathbf{k}$, thus restricting the analysis to the flavor neutrino state $|\nu_e\rangle$ of fixed momentum and helicity. Let us consider again the free evolution of the electron-neutrino state (17):

$$\left|\nu_e(t)\right\rangle = e^{-iH_0 t}\left|\nu_e\right\rangle, \tag{18}$$

where H_0 is the standard QFT free Hamiltonian. Finally, in the Hilbert space $\mathscr{H}_{e,\mu}$, (18) can be written in the form

$$\begin{aligned}\left|\nu_e(t)\right\rangle = &\left[\mathbf{U}_{ee}(t)\,\alpha^\dagger_e + \mathbf{U}_{e\mu}(t)\,\alpha^\dagger_\mu + \mathbf{U}^{e\bar{e}}_{e\mu}(t)\,\alpha^\dagger_e\alpha^\dagger_\mu\beta^\dagger_e\right. \\ &\left. + \mathbf{U}^{\mu\bar{\mu}}_{ee}(t)\,\alpha^\dagger_e\alpha^\dagger_\mu\beta^\dagger_\mu\right]|0\rangle_{e,\mu},\end{aligned} \tag{19}$$

where the time-dependent coefficients are given by

$$\begin{aligned}\mathbf{U}_{ee}(t) &= e^{-i\omega_1 t}\left[\cos^2\theta + \sin^2\theta\left(e^{-i(\omega_2-\omega_1)t}|U|^2\right.\right. \\ &\qquad \left.\left. + e^{-i(\omega_2+\omega_1)t}|V|^2\right)\right], \\ \mathbf{U}_{e\mu}(t) &= e^{-i\omega_1 t}U\cos\theta\sin\theta\left(e^{-i(\omega_2-\omega_1)t} - 1\right), \\ \mathbf{U}^{e\bar{e}}_{e\mu}(t) &= e^{-i\omega_1 t}V\cos\theta\sin\theta\left(1 - e^{-i(\omega_2+\omega_1)t}\right), \\ \mathbf{U}^{\mu\bar{\mu}}_{ee}(t) &= e^{-i\omega_1 t}UV\sin^2\theta\left(e^{-i(\omega_2+\omega_1)t} - e^{-i(\omega_2-\omega_1)t}\right), \\ \left|\mathbf{U}_{ee}(t)\right|^2 &+ \left|\mathbf{U}_{e\mu}(t)\right|^2 + \left|\mathbf{U}^{e\bar{e}}_{e\mu}(t)\right|^2 + \left|\mathbf{U}^{\mu\bar{\mu}}_{ee}(t)\right|^2 = 1.\end{aligned} \tag{20}$$

In the following, in order to conveniently parameterize the neutrino masses m_1 and m_2, momentum $|\mathbf{k}|$, and the evolution time t, we use the real parameters $x = m_2/m_1$, $p = |\mathbf{k}|/\sqrt{m_1 m_2}$, and $\tau = (\omega_2 - \omega_1)t$. Therefore, x represents the ratio between the two masses eigenvalues; p expresses the ratio between the momentum and the masses geometrical mean and corresponds to the relativistic limit for $p \gg 1$.

Evidently, the time-evolved state $|\nu_e(t)\rangle$ in the flavor eigenstates Hilbert space, that is, (19), is a multiparticle entangled state. Analogously with the Pontecorvo states (5), we assume the neutrino occupation number as reference quantum number. However, with respect to (5), we have still two flavors, but we have a further degree of freedom, that is, the neutrino species, that is, particles and antiparticles. Therefore, we obtain multipartite entanglement in a four-qubit state. In the instance $\mathscr{H}_{e,\mu}$, (19) can be written in the form

$$\begin{aligned}\left|\nu_e(t)\right\rangle = &\,\mathbf{U}_{ee}(t)\,|1000\rangle_{e\mu\bar{e}\bar{\mu}} + \mathbf{U}_{e\mu}(t)\,|0100\rangle_{e\mu\bar{e}\bar{\mu}} \\ &+ \mathbf{U}^{e\bar{e}}_{e\mu}(t)\,|1110\rangle_{e\mu\bar{e}\bar{\mu}} + \mathbf{U}^{\mu\bar{\mu}}_{ee}(t)\,|1101\rangle_{e\mu\bar{e}\bar{\mu}},\end{aligned} \tag{21}$$

where $|ijkh\rangle_{e\mu\bar{e}\bar{\mu}}$ denotes the four-qubit vector $|i\rangle_{\nu_e}|j\rangle_{\nu_\mu}|k\rangle_{\bar{\nu}_e}|h\rangle_{\bar{\nu}_\mu}$ with $i, j, k, h = 0, 1$. Let us analyze the entanglement content possessed by a given pair of modes of $|\nu_e\rangle$, that is, (21), by using the concurrence, that is, by (8). Specifically, we compute the quantity $C^{(\alpha;\beta)}$ associated with the two modes $\alpha \neq \beta = \nu_e, \nu_\mu, \bar{\nu}_e, \bar{\nu}_\mu$. We do not report the analytical expressions for the concurrences, as they are long and unwieldy and provide no further significant physical insight.

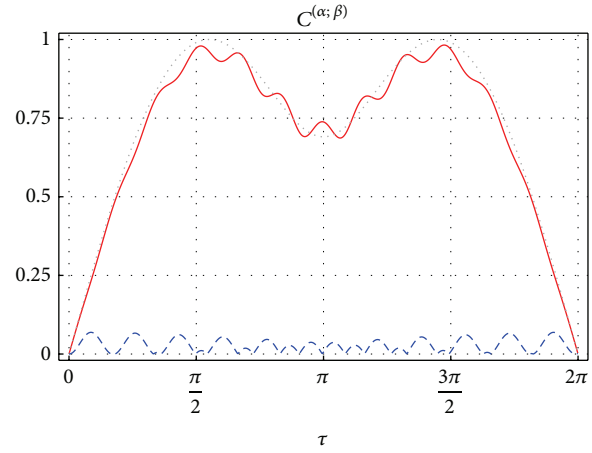

FIGURE 2: QFT instance (color online). The concurrences $C^{(\nu_e;\nu_\mu)}$ (full line) and $C^{(\bar{\nu}_e;\bar{\nu}_\mu)}$ (dashed line) as functions of the scaled time $\tau = (\omega_2 - \omega_1)t$. The mixing angle θ is fixed at the experimental value $\sin^2\theta = 0.314$; the parameters x and p are fixed as $x = 10$ and $p = 5$. The concurrence associated with the quantum-mechanical Pontecorvo states (dotted line), that is, Figure 1, is also reported for comparison.

In Figure 2, we plot the quantities $C^{(\nu_e;\nu_\mu)}$ and $C^{(\bar{\nu}_e;\bar{\nu}_\mu)}$ as functions of the scaled time τ for $x = 10$ and $p = 5$; it is worth noticing that such a choice of the parameters corresponds to the following assumptions: mass m_2 greater than mass m_1 of one order of magnitude and momentum of the same order of magnitude as the masses geometrical mean. We observe that the particle-mode entanglement $(\nu_e; \nu_\mu)$ is predominant; thus, most entanglement is shared between the two particle modes. Notwithstanding, a nonvanishing (although suppressed), nontrivial, oscillating contribution originates from the antiparticle-mode pair $(\bar{\nu}_e, \bar{\nu}_\mu)$.

For completeness, in Figure 3 we also plot the concurrences $C^{(\nu_e;\bar{\nu}_e)}$ and $C^{(\nu_\mu;\bar{\nu}_e)}$, panel (a), and $C^{(\nu_e;\bar{\nu}_\mu)}$ and $C^{(\nu_\mu;\bar{\nu}_\mu)}$, panel (b). Specifically, plots in Figure 3 represent the concurrences associated with the pairs particle-antiparticle $(\nu_\alpha; \bar{\nu}_\beta)$. We observe that there exists strong entanglement content in these pairs; this fact is due to the Bogoliubov contribution in (15). Let us notice that the curves $C^{(\nu_e;\bar{\nu}_\beta)}$ and $C^{(\nu_\mu;\bar{\nu}_\beta)}$ (with

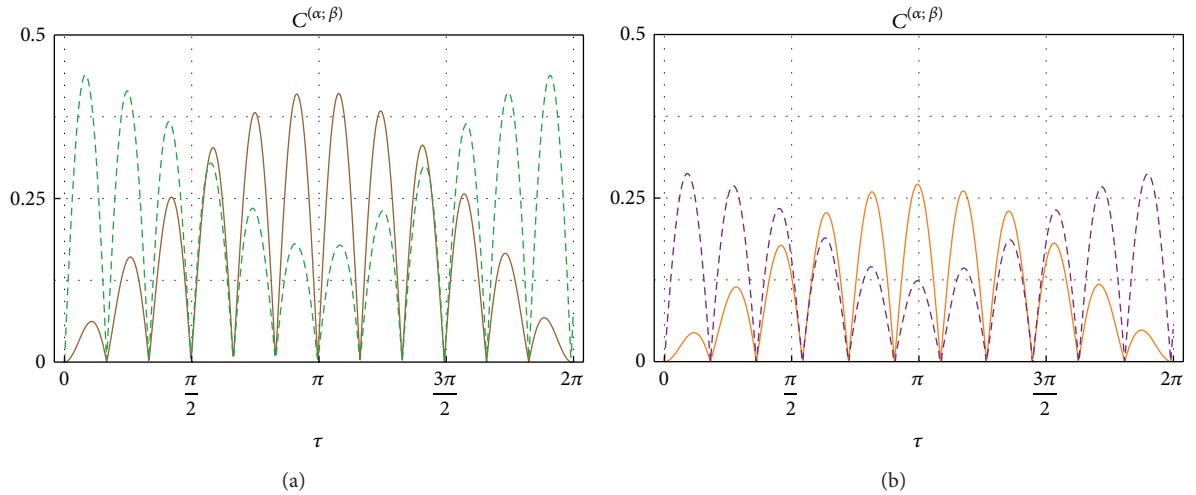

FIGURE 3: QFT instance (color online). In (a) the concurrences $C^{(\nu_e;\bar{\nu}_e)}$ (full line) and $C^{(\nu_\mu;\bar{\nu}_e)}$ (dashed line), in (b) the concurrences $C^{(\nu_e;\bar{\nu}_\mu)}$ (full line) and $C^{(\nu_\mu;\bar{\nu}_\mu)}$ (dashed line), as functions of the scaled time $\tau = (\omega_2 - \omega_1)t$. The mixing angle θ and the parameters x and p are fixed as in Figure 2.

$\beta = e, \mu$) exhibit an opposite behavior; on average, when the former increases (decreases), the latter decreases (increases).

4. Conclusions

In this paper, we analyzed a paradigmatic phenomenon of particle physics, that is, neutrino oscillations, from the point of view of entanglement, one of the fundamental aspects of quantum theory. More specifically, we studied the entanglement associated with a QFT model of neutrino oscillations, generalizing our previous results derived in the context of QM. The two-mode state, obtained by tracing out two modes, is a mixed one, and we characterized the entanglement of such a state by means of the concurrence.

We showed that such a phenomenon, described in a QFT framework, exhibits significantly more complex effects with respect to that found in a QM setting. This procedure is applicable as well to other multiparticle QFT systems, beyond the model considered here.

The present analysis has been carried out for the case of two generations. A further extension can be carried out by considering the case of three flavors, including CP violation, for which the structure of QFT flavor states is considerably more involved [34].

Moreover, the entanglement dynamics has been studied in the time domain; an analogous study can be carried out in the space domain, by considering the spatial distribution of entanglement.

At last, it is also worth remarking that the (exact) results obtained in our paper represent a canonical example of evaluation of entanglement for a relativistic system.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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