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Research Article

CP Violation for $B^0 \rightarrow \rho^0(\omega)\rho^0(\omega) \rightarrow \pi^+\pi^-\pi^+\pi^-$ in QCD Factorization

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In the QCD factorization (QCDF) approach we study the direct CP violation in $\overline{B}^0 \rightarrow \rho^0(\omega)\rho^0(\omega) \rightarrow \pi^+\pi^-\pi^+\pi^-$ via the ρ-ω mixing mechanism. We find that the CP violation can be enhanced by double ρ-ω mixing when the masses of the $\pi^+\pi^-$ pairs are in the vicinity of the ω resonance, and the maximum CP violation can reach 28%. We also compare the results from the naive factorization and the QCD factorization.

1. Introduction

CP violation is an extensive research topic in recent years. In standard model (SM), CP violation is related to the weak complex phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix [1, 2]. In the past few years more attention has been focused on the decays of B meson system both theoretically and experimentally. Recently, the large CP violation was found by the LHCb Collaboration in the three-body decay channels of $B^\pm \rightarrow \pi^\pm\pi^+\pi^-$ and $B^\pm \rightarrow K^\pm\pi^+\pi^-$ [3, 4]. Hence, the theoretical mechanism for the three- or four-body decays becomes more and more interesting. In this paper, we focus on the interference from intermediate ρ and ω mesons in the four-body decay.

It is known that the naive factorization [5, 6], the QCD factorization (QCDF) [7–9], the perturbative QCD (PQCD) [10–12], and the soft-collinear effective theory (SCET) [13, 14] are the most extensive approaches for calculating the hadronic matrix elements. These factorization approaches present different methods for dealing with the hadronic matrix elements in the leading power of $1/m_b$ (m_b is the b-quark mass). Direct CP violation occurs through the interference of two amplitudes with different weak phases and strong phases. The weak phase difference is directly determined by the CKM matrix elements, while the strong phase is usually difficult to control from a theoretical approach. The B meson decay amplitude involves the hadronic matrix elements whose computation is not trivial. Different methods may present different strong phases. Meanwhile, we can also obtain a large strong phase difference by some phenomenological mechanism. ρ-ω mixing has been used for this purpose in the past few years [15–25]. In this paper, we will investigate the CP violation via double ρ-ω mixing in the QCDF approach.

In the QCDF approach, at the rest frame of the heavy B meson, B meson can decay into two light mesons with large momenta. In the heavy-quark limit, QCD corrections can be calculated for the nonleptonic two-body B meson decays. The decay amplitude can be obtained at the next-to-leading power in α_s and the leading power in Λ_{QCD}/m_b. In the QCD factorization, there is cancellation of the scale and renormalizaion scheme dependence between the Wilson coefficients and the hadronic matrix elements. However, this does not happen in the naive factorization. The hadronic matrix elements can be expressed in terms of form factors and meson light-cone distribution amplitudes including strong interaction corrections.

The remainder of this paper is organized as follows. In Section 2 we present the form of the effective Hamiltonian. In Section 3 we give the calculating formalism of CP violation from ρ-ω mixing in $B^0 \rightarrow \rho^0(\omega)\rho^0(\omega) \rightarrow \pi^+\pi^-\pi^+\pi^-$. Input parameters are presented in Section 4. We present the numerical rusults in Section 5. Summary and discussion are included in Section 6.

2. The Effective Hamiltonian

With the operator product expansion, the effective weak Hamiltonian can be written as [26]:

$$\mathscr{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}}\left[\sum_{p=u,c}\sum_{q=d,s} V_{pb}V_{pq}^* \times\left(c_1 O_1^p + c_2 O_2^p + \sum_{i=3}^{10} c_i O_i + c_{7\gamma}O_{7\gamma} + c_{8g}O_{8g}\right)\right] + H.c., \quad (1)$$

where G_F represents the Fermi constant, c_i ($i = 1,\ldots,10,7\gamma,8g$) are the Wilson coefficients, and V_{pb}, V_{pq} are the CKM matrix elements. The operators O_i have the following forms:

$$\begin{aligned}
O_1^p &= \bar{p}\gamma_\mu(1-\gamma_5)b\bar{q}\gamma^\mu(1-\gamma_5)p, \\
O_2^p &= \bar{p}_\alpha\gamma_\mu(1-\gamma_5)b_\beta\bar{q}_\beta\gamma^\mu(1-\gamma_5)p_\alpha, \\
O_3 &= \bar{q}\gamma_\mu(1-\gamma_5)b\sum_{q'}\bar{q}'\gamma^\mu(1-\gamma_5)q', \\
O_4 &= \bar{q}_\alpha\gamma_\mu(1-\gamma_5)b_\beta\sum_{q'}\bar{q}'_\beta\gamma^\mu(1-\gamma_5)q'_\alpha, \\
O_5 &= \bar{q}\gamma_\mu(1-\gamma_5)b\sum_{q'}\bar{q}'\gamma^\mu(1+\gamma_5)q', \\
O_6 &= \bar{q}_\alpha\gamma_\mu(1-\gamma_5)b_\beta\sum_{q'}\bar{q}'_\beta\gamma^\mu(1+\gamma_5)q'_\alpha, \\
O_7 &= \frac{3}{2}\bar{q}\gamma_\mu(1-\gamma_5)b\sum_{q'}e_{q'}\bar{q}'\gamma^\mu(1+\gamma_5)q', \\
O_8 &= \frac{3}{2}\bar{q}_\alpha\gamma_\mu(1-\gamma_5)b_\beta\sum_{q'}e_{q'}\bar{q}'_\beta\gamma^\mu(1+\gamma_5)q'_\alpha, \\
O_9 &= \frac{3}{2}\bar{q}\gamma_\mu(1-\gamma_5)b\sum_{q'}e_{q'}\bar{q}'\gamma^\mu(1-\gamma_5)q', \\
O_{10} &= \frac{3}{2}\bar{q}_\alpha\gamma_\mu(1-\gamma_5)b_\beta\sum_{q'}e_{q'}\bar{q}'_\beta\gamma^\mu(1-\gamma_5)q'_\alpha, \\
O_{7\gamma} &= \frac{-e}{8\pi^2}m_b\bar{s}\sigma_{\mu\nu}(1+\gamma_5)F^{\mu\nu}b, \\
O_{8g} &= \frac{-g_s}{8\pi^2}m_b\bar{s}\sigma_{\mu\nu}(1+\gamma_5)G^{\mu\nu}b,
\end{aligned} \quad (2)$$

where α and β are color indices, O_1^p and O_2^p are the tree operators, O_3–O_6 are QCD penguin operators which are isosinglets, and O_7–O_{10} arise from electroweak penguin operators which have both isospin 0 and 1 components. $O_{7\gamma}$ and O_{8g} are the electromagnetic and chromomagnetic dipole operators, $e_{q'}$ are the electric charges of the quarks, and $q' = u,d,s,c,b$ is implied.

For the decay channel $B^0 \rightarrow \rho^0(\omega)\rho^0(\omega)$, neglecting power corrections of order Λ_{QCD}/m_b, the transition matrix element of an operator O_i in the weak effective Hamiltonian is given by [8, 9]:

$$\begin{aligned}
\left\langle V_1V_2\left|O_i\right|\bar{B}\right\rangle &= \sum_j F_j^{B\rightarrow V_1}\left(m_{V_2}^2\right)\int_0^1 du\, T_{ij}^I(u)\,\Phi_{V_2}(u) \\
&\quad + \left(V_1 \longleftrightarrow V_2\right) + \int_0^1 d\xi\, du\, dv T_i^{II}(\xi,u,v)\,\Phi_B \\
&\quad \times(\xi)\,\Phi_{V_1}(v)\,\Phi_{V_2}(u).
\end{aligned} \quad (3)$$

Here $F_j^{B\rightarrow V_{1,2}}(m_{V_{2,1}}^2)$ denotes $B \rightarrow V_{1,2}$ ($V_{1,2}$ represent ρ^0 and ω mesons) form factor, and $\Phi_V(u)$ is the light-cone distribution amplitude for the quark-antiquark Fock state of mesons ρ^0 and ω. $T_{ij}^I(u)$ and $T_i^{II}(\xi,u,v)$ are hard-scattering functions, which are perturbatively calculable. The hard-scattering kernels and light-cone distribution amplitudes (LCDA) depend on the factorization scale and the renormalization scheme. $m_{V_{1,2}}$ denote the ρ^0 and ω masses, respectively.

We match the effective weak Hamiltonian onto a transition operator, the matrix element is given by ($\lambda_p^{(D)} = V_{pb}V_{pD}^*$ with $D = d$) [8, 9]:

$$\left\langle V_1V_2\left|\mathscr{H}_{\text{eff}}\right|\bar{B}\right\rangle = \sum_{p=u,c}\lambda_p^{(D)}\left\langle V_1V_2\left|\mathscr{T}_A^{p,h} + \mathscr{T}_B^{p,h}\right|\bar{B}\right\rangle, \quad (4)$$

where $\mathscr{T}_A^{p,h}$ denotes the contribution from vertex correction, penguin amplitude, and spectator scattering in terms of the operators $a_i^{p,h}$; $\mathscr{T}_B^{p,h}$ refers to annihilation terms contribution by operators $b_i^{p,h}$. h is the helicity of the final state.

The flavor operators a_i^p are defined in [8, 9] as follows:

$$\begin{aligned}
a_i^{p,h}(V_1V_2) &= \left(c_i + \frac{c_{i\pm1}}{N_c}\right)N_i^h(V_2) \\
&\quad + \frac{c_{i\pm1}}{N_c}\frac{C_F\alpha_s}{4\pi}\left[V_i^h(V_2) + \frac{4\pi^2}{N_c}H_i^h(V_1V_2)\right] \\
&\quad + P_i^{p,h}(V_2),
\end{aligned} \quad (5)$$

where N_c is the number of colors, the upper (lower) signs apply when i is odd (even), and $C_F = (N_c^2-1)/2N_c$. It is understood that the superscript "p" is to be omitted for $i = 1,2$. The quantities $V_i^h(V_2)$ account for one-loop vertex corrections, $H_i^h(V_1V_2)$ for hard spectator interactions, and $P_i^{p,h}(V_2)$ for penguin contractions. $N_i^h(V_2)$ is given by

$$N_i^h(V_2) = \begin{cases} 0; & i = 6,8, \\ 1; & \text{all other cases.} \end{cases} \quad (6)$$

The coefficients of the flavor operators $\alpha_i^{p,h}$ can be expressed in terms of the coefficients $a_i^{p,h}$. We will present the form in the following section. Using the unitarity relation

$$\lambda_u^{(D)} + \lambda_c^{(D)} + \lambda_t^{(D)} = 0, \tag{7}$$

we can get

$$\begin{aligned}\sum_{p=u,c} \lambda_p^{(D)} \mathcal{T}_A^{p,h} = {} & \sum_{p=u,c} \lambda_p^{(D)} \left[\delta_{pu}\alpha_1 (V_1V_2) A([\overline{q}_s u][\overline{u}D]) \right. \\ & \left. + \delta_{pu}\alpha_2 (V_1V_2) A([\overline{q}_s D][\overline{u}u])\right] \\ & + \lambda_u^{(D)} \left[(\alpha_4^u (V_1V_2) - \alpha_4^c (V_1V_2)) \right. \\ & \times \sum_q A([\overline{q}_s q][\overline{q}D]) \\ & + \left(\alpha_{4,\mathrm{EW}}^u (V_1V_2) - \alpha_{4,\mathrm{EW}}^c (V_1V_2)\right) \\ & \left. \times \sum_q \frac{3}{2} e_q A([\overline{q}_s q][\overline{q}D]) \right] \\ & - \lambda_t^{(D)} \left[\alpha_3^c (V_1V_2) \sum_q A([\overline{q}_s D][\overline{q}q]) \right. \\ & + \alpha_4^c (V_1V_2) \sum_q A([\overline{q}_s q][\overline{q}D]) \\ & + \alpha_{3,\mathrm{EW}}^c (V_1V_2) \\ & \times \sum_q \frac{3}{2} e_q A([\overline{q}_s D][\overline{q}q]) \\ & + \alpha_{4,\mathrm{EW}}^c (V_1V_2) \\ & \left. \times \sum_q \frac{3}{2} e_q A([\overline{q}_s q][\overline{q}D]) \right],\end{aligned} \tag{8}$$

where the sums extend over $q = u, d, s$ and $\overline{q}_s$ denotes the spectator antiquark.

Next we need to change the annihilation part into the following form [8, 9]:

$$\begin{aligned}\sum_{p=u,c} \lambda_p^{(D)} \mathcal{T}_B^{p,h} = {} & \sum_{p=u,c} \lambda_p^{(D)} \\ & \times \left[\delta_{pu} b_1 (V_1V_2) \sum_{q'} B\left([\overline{u}q'][\overline{q}'u][\overline{D}b]\right) \right. \\ & + \delta_{pu} b_2 (V_1V_2) \\ & \left. \times \sum_{q'} B\left([\overline{u}q'][\overline{q}'D][\overline{u}b]\right) \right] \\ & - \lambda_t^{(D)} \left[b_3 (V_1V_2) \sum_{q,q'} B\left([\overline{q}q'][\overline{q}'D][\overline{q}b]\right) \right. \\ & + b_4 (V_1V_2) \sum_{q,q'} B\left([\overline{q}q'][\overline{q}'q][\overline{D}b]\right) \\ & + b_{3,\mathrm{EW}} (V_1V_2) \\ & \times \sum_{q,q'} \frac{3}{2} e_q B\left([\overline{q}q'][\overline{q}'D][\overline{q}b]\right) \\ & + b_{4,\mathrm{EW}} (V_1V_2) \\ & \left. \times \sum_{q,q'} \frac{3}{2} e_q B\left([\overline{q}q'][\overline{q}'q][\overline{D}b]\right) \right],\end{aligned} \tag{9}$$

where $b_i^{p,h}$, $b_{i,\mathrm{EW}}^{p,h}$, and B will be given in the following section.

3. CP Violation in $B^0 \rightarrow \rho^0(\omega)\rho^0(\omega) \rightarrow \pi^+\pi^-\pi^+\pi^-$

3.1. Formalism. The $B \rightarrow V_1(\epsilon_1, P_1)V_2(\epsilon_2, P_2)$ ($\epsilon_1(P_1)$ and $\epsilon_2(P_2)$ are the polarization vectors (momenta) of V_1 and V_2, resp.); decay rate is written as

$$\Gamma = \frac{G_F^2 P_c}{64\pi m_B^2} \sum_\sigma A^{(\sigma)+} A^{(\sigma)}, \tag{10}$$

where P_c refers to the c.m. momentum. $A^{(\sigma)}$ is the helicity amplitude for each helicity of the final state. The decay amplitude, A, can be decomposed into three components T_0, T_+, and T_- according to the helicity of the final state. With the helicity summation, we can get

$$\sum_\sigma A^{(\sigma)+} A^{(\sigma)} = \left|T_0\right|^2 + \left|T_+\right|^2 + \left|T_-\right|^2. \tag{11}$$

In the vector meson dominance model [27], the photon propagator is dressed by coupling to vector mesons. Based on the same mechanism, ρ-ω mixing was proposed [28, 29]. The formalism for CP violation in the decay of a bottom hadron, $\overline{B}$, will be reviewed in the following. The amplitude for $\overline{B} \rightarrow V\pi^+\pi^-$, A, can be written as

$$A = \left\langle \pi^+\pi^- V \left| H^T \right| \overline{B} \right\rangle + \left\langle \pi^+\pi^- V \left| H^P \right| \overline{B} \right\rangle, \tag{12}$$

where H^T and H^P are the Hamiltonians for the tree and penguin operators, respectively. We define the relative magnitude and phases between these two contributions as follows:

$$A = \left\langle \pi^+\pi^- V \left| H^T \right| \overline{B} \right\rangle \left[1 + r e^{i\delta} e^{i\phi}\right], \tag{13}$$

where δ and ϕ are strong and weak phase differences, respectively. The weak phase difference ϕ arises from the appropriate combination of the CKM matrix elements:

$\phi = \arg[(V_{tb}V_{td}^*)/(V_{ub}V_{ud}^*)]$. The parameter r is the absolute value of the ratio of tree and penguin amplitudes:

$$r = \left| \frac{\langle \pi^+\pi^- V \left| H^P \right| \overline{B} \rangle}{\langle \pi^+\pi^- V \left| H^T \right| \overline{B} \rangle} \right|. \quad (14)$$

The amplitude for $B \to \overline{V}\pi^+\pi^-$ is

$$\overline{A} = \langle \pi^+\pi^-\overline{V} \left| H^T \right| B \rangle + \langle \pi^+\pi^-\overline{V} \left| H^P \right| B \rangle. \quad (15)$$

Then, the CP violating asymmetry, A_{CP}, can be written as

$$A_{\text{CP}} = \frac{|A|^2 - \left|\overline{A}\right|^2}{|A|^2 + \left|\overline{A}\right|^2} = \frac{-2\left(T_0^2 r_0 \sin\delta_0 + T_+^2 r_+ \sin\delta_+ + T_-^2 r_- \sin\delta_-\right)\sin\phi}{\sum_{i=0+-} T_i^2\left(1 + r_i^2 + 2r_i\cos\delta_i\cos\phi\right)}, \quad (16)$$

where

$$|A|^2 = \sum_\sigma A^{(\sigma)+}A^{(\sigma)} = \left|T_0\right|^2 + \left|T_+\right|^2 + \left|T_-\right|^2 \quad (17)$$

and T_i $(i = 0, +, -)$ represent the tree-level helicity amplitudes. We can see explicitly from (16) that both weak and strong phase differences are needed to produce CP violation. ρ-ω mixing has the dual advantages that the strong phase difference is large and well-known [15, 16]. In this scenario one has

$$\langle \pi^+\pi^-\pi^+\pi^- \left| H^T \right| \overline{B} \rangle = \frac{2g_\rho^2}{s_\rho^2 s_\omega}\widetilde{\Pi}_{\rho\omega}\left(t_\omega + t_\omega^a\right) + \frac{g_\rho^2}{s_\rho^2}\left(t_\rho + t_\rho^a\right), \quad (18)$$

$$\langle \pi^+\pi^-\pi^+\pi^- \left| H^P \right| \overline{B} \rangle = \frac{2g_\rho^2}{s_\rho^2 s_\omega}\widetilde{\Pi}_{\rho\omega}\left(p_\omega + p_\omega^a\right) + \frac{g_\rho^2}{s_\rho^2}\left(p_\rho + p_\rho^a\right), \quad (19)$$

where t_V ($V = \rho$ or ω) is the tree amplitude and p_V is the penguin amplitude for producing a vector meson, V. t_V^a ($V = \rho$ or ω) is the tree annihilation amplitude and p_V^a is the penguin annihilation amplitude. g_ρ is the coupling for $\rho^0 \to \pi^+\pi^-$, $\widetilde{\Pi}_{\rho\omega}$ is the effective ρ-ω mixing amplitude, and s_V is from the inverse propagator of the vector meson V:

$$s_V = s - m_V^2 + im_V\Gamma_V, \quad (20)$$

with $\sqrt{s}$ being the invariant mass of the $\pi^+\pi^-$ pair. The direct $\omega \to \pi^+\pi^-$ is effectively absorbed into $\widetilde{\Pi}_{\rho\omega}$, leading to the explicit s dependence of $\widetilde{\Pi}_{\rho\omega}$ [30, 31]. Making the expansion $\widetilde{\Pi}_{\rho\omega}(s) = \widetilde{\Pi}_{\rho\omega}(m_\omega^2) + (s - m_\omega^2)\widetilde{\Pi}'_{\rho\omega}(m_\omega^2)$, the ρ-ω mixing parameters were determined in the fit of Gardner and O'Connell [32]: $\text{Re}\,\widetilde{\Pi}_{\rho\omega}(m_\omega^2) = -3500 \pm 300\,\text{MeV}^2$, $\text{Im}\,\widetilde{\Pi}_{\rho\omega}(m_\omega^2) = -300 \pm 300\,\text{MeV}^2$, and $\widetilde{\Pi}'_{\rho\omega}(m_\omega^2) = 0.03 \pm 0.04$. In practice, the effect of the derivative term is negligible. From (16) and (18), one has

$$re^{i\delta}e^{i\phi} = \frac{2\widetilde{\Pi}_{\rho\omega}\left(p_\omega + p_\omega^a\right) + s_\omega\left(p_\rho + p_\rho^a\right)}{2\widetilde{\Pi}_{\rho\omega}\left(t_\omega + t_\omega^a\right) + s_\omega\left(t_\rho + t_\rho^a\right)}. \quad (21)$$

Defining

$$\frac{t_\omega + t_\omega^a}{t_\rho + t_\rho^a} = \alpha e^{i\delta_\alpha}, \quad (22)$$

$$\frac{p_\rho + p_\rho^a}{p_\omega + p_\omega^a} = \beta e^{i\delta_\beta}, \quad (23)$$

$$\frac{p_\omega + p_\omega^a}{t_\rho + t_\rho^a} = r' e^{i(\delta_q + \phi)}, \quad (24)$$

where δ_α, δ_β, and δ_q are strong phases, one finds the following expression from (21):

$$re^{i\delta} = r' e^{i\delta_q}\frac{2\widetilde{\Pi}_{\rho\omega} + \beta e^{i\delta^\beta}s_\omega}{s_\omega + 2\widetilde{\Pi}_{\rho\omega}\alpha e^{i\delta_\alpha}}. \quad (25)$$

$\alpha e^{i\delta_\alpha}$, $\beta e^{i\delta_\beta}$, and $re^{i\delta}$ will be calculated in the QCD factorization approach in the next section. With (25), we can obtain $r\sin\delta$ and $r\cos\delta$. In order to get the CP violating asymmetry, A_{CP}, in (16), $\sin\phi$ and $\cos\phi$ are needed. ϕ is determined by the CKM matrix elements. In the Wolfenstein parametrization [33, 34], one has

$$\sin\phi = \frac{\eta}{\sqrt{\left[\rho(1-\rho) - \eta^2\right]^2 + \eta^2}}, \qquad \cos\phi = \frac{\rho(1-\rho) - \eta^2}{\sqrt{\left[\rho(1-\rho) - \eta^2\right]^2 + \eta^2}}. \quad (26)$$

3.2. The Calculation Details. The nonfactorizable corrections are included in the coefficients a_i which contain vertex corrections and hard spectator interactions and b_i which contain annihilation contributions.

In the QCD factorization approach, α_i associated with the coefficient a_i can be written as follows (helicity indices are neglected) [8, 9]:

$$\begin{aligned} \alpha_1 &= a_1, \\ \alpha_2 &= a_2, \\ \alpha_3^p &= a_3^p + a_5^p, \\ \alpha_{3,\text{EW}}^p &= a_9^p + a_7^p, \\ \alpha_4^p &= a_4^p - r_\chi^{V_2}a_6^p, \\ \alpha_{4,\text{EW}}^p &= a_{10}^p - r_\chi^{V_2}a_8^p, \end{aligned} \quad (27)$$

where we have used the following notation:

$$r_\chi^V \equiv \frac{2m_V}{m_b}\frac{f_V^\perp}{f_V} \quad (28)$$

with $f_V^{\perp}$ and f_V referring to the transverse decay constant and decay constant of the vector meson, respectively.

The flavor operators $a_i^{p,h}$ include short-distance nonfactorizable corrections such as vertex corrections, hard spectator interactions, and Penguin terms. These contribution and annihilation part are given by [8, 9].

3.3. The Calculation of CP Violation. In order to obtain the CP violation of $\overline{B} \rightarrow \rho^0(\omega)\rho^0(\omega) \rightarrow \pi^+\pi^-\pi^+\pi^-$ in (16), we calculate the amplitudes t_ρ, t_ρ^a, t_ω, t_ω^a, p_ρ, p_ρ^a, p_ω, and p_ω^a in (18) and (19) in the QCDF approach, which are tree-level and penguin-level amplitudes. The decay amplitudes for the process $\overline{B} \rightarrow \rho^0\rho^0(\omega)$ are in the QCD factorization as follows:

$$\begin{aligned}
A_{\overline{B}\rightarrow\rho^0\rho^0} &= A_{\rho^0\rho^0}\left(\alpha_4^p - \delta_{pu}\alpha_2 - \frac{1}{2}\alpha_{4,\mathrm{EW}}^p - \frac{3}{2}\alpha_{3,\mathrm{EW}}^p + \beta_3^p \right.\\
&\quad \left. -\frac{1}{2}\beta_{3,\mathrm{EW}}^p + \delta_{pu}\beta_1 + 2\beta_4^p + \frac{1}{2}\beta_{4,\mathrm{EW}}^p\right),\\
-2A_{\overline{B}\rightarrow\rho^0\omega} &= A_{\rho^0\omega}\left(\delta_{pu}\alpha_2 - \delta_{pu}\beta_1 + 2\alpha_3^p + \alpha_4^p + \frac{1}{2}\alpha_{3,\mathrm{EW}}^p \right.\\
&\quad \left. -\frac{1}{2}\alpha_{4,\mathrm{EW}}^p + \beta_3^p - \frac{1}{2}\beta_{3,\mathrm{EW}}^p - \frac{3}{2}\beta_{4,\mathrm{EW}}^p\right)\\
&\quad + A_{\omega\rho^0}\left(-\delta_{pu}\alpha_2 - \delta_{pu}\beta_1 + \alpha_4^p - \frac{3}{2}\alpha_{3,\mathrm{EW}}^p \right.\\
&\quad \left. -\frac{1}{2}\alpha_{4,\mathrm{EW}}^p + \beta_3^p - \frac{1}{2}\beta_{3,\mathrm{EW}}^p - \frac{3}{2}\beta_{4,\mathrm{EW}}^p\right),
\end{aligned} \tag{29}$$

where

$$A_{V_1V_2} = i\frac{G_F}{\sqrt{2}}\left\langle V_1\left|(\overline{q}b)_{V-A}\right|B\right\rangle\left\langle V_2\left|(\overline{q}q)_V\right|0\right\rangle. \tag{30}$$

From (22), one can get

$$\alpha e^{i\delta_\alpha} = \frac{t_\omega + t_\omega^a}{t_\rho + t_\rho^a} = \frac{Q_2}{Q_1}, \tag{31}$$

where

$$\begin{aligned}
Q_1 &= t_\rho + t_\rho^a\\
&= A_{\rho^0\rho^0}\left[\alpha_4^{u,h} - \alpha_4^{c,h} - \delta_{pu}\alpha_2 \right.\\
&\quad \left. -\frac{1}{2}\left(\alpha_{4,\mathrm{EW}}^{u,h} - \alpha_{4,\mathrm{EW}}^{c,h}\right) + \delta_{pu}\beta_1\right],\\
Q_2 &= t_\omega + t_\omega^a\\
&= -\frac{1}{2}A_{\rho^0\omega^0}\left[\delta_{pu}\alpha_2 - \delta_{pu}\beta_1 + \alpha_4^{u,h} \right.\\
&\quad \left. -\alpha_4^{c,h} - \frac{1}{2}\left(\alpha_{4,\mathrm{EW}}^{u,h} - \alpha_{4,\mathrm{EW}}^{c,h}\right)\right]\\
&\quad -\frac{1}{2}A_{\omega^0\rho^0}\left[-\delta_{pu}y\alpha_2 - \delta_{pu}\beta_1 + \alpha_4^{u,h} \right.\\
&\quad \left. -\alpha_4^{c,h} - \frac{1}{2}\left(\alpha_{4,\mathrm{EW}}^{u,h} - \alpha_{4,\mathrm{EW}}^{c,h}\right)\right].
\end{aligned} \tag{32}$$

In a similar way, with the aid of the Fierz identities, we can evaluate the penguin operator contributions p_ρ and p_ω. From (23), we have

$$\beta e^{i\delta_\beta} = \frac{p_\rho + p_\rho^a}{p_\omega + p_\omega^a} = \frac{Q_3}{Q_4}, \tag{33}$$

where

$$\begin{aligned}
Q_3 &= p_\rho + p_\rho^a\\
&= A_{\rho^0\rho^0}\left(\left[\left(-\frac{1}{2}\right)\alpha_{4,\mathrm{EW}}^{c,h} - \left(\frac{3}{2}\right)\alpha_{3,\mathrm{EW}}^{c,h}\right] \right.\\
&\quad \left. +\beta_3^p - \frac{1}{2}\beta_{3,\mathrm{EW}}^p + 2\beta_4^p + \frac{1}{2}\beta_{4,\mathrm{EW}}^p\right),\\
Q_4 &= p_\omega + p_\omega^a\\
&= -\frac{1}{2}A_{\rho^0\omega}\left(\left[2\alpha_3^c + \alpha_4^c\right] + \frac{1}{2}\alpha_{3,\mathrm{EW}}^c \right.\\
&\quad \left. -\frac{1}{2}\alpha_{4,\mathrm{EW}}^c + \beta_3^p - \frac{1}{2}\beta_{3,\mathrm{EW}}^p - \frac{3}{2}\beta_{4,\mathrm{EW}}^p\right)\\
&\quad -\frac{1}{2}A_{\omega\rho^0}\left(\alpha_4^c - \frac{3}{2}\alpha_{3,\mathrm{EW}}^c - \frac{1}{2}\alpha_{4,\mathrm{EW}}^c + \beta_3^p \right.\\
&\quad \left. -\frac{1}{2}\beta_{3,\mathrm{EW}}^p - \frac{3}{2}\beta_{4,\mathrm{EW}}^p\right).
\end{aligned} \tag{34}$$

Form (24), we have

$$\begin{aligned}
r' e^{i(\delta_q+\phi)} &= \frac{p_\omega + p_\omega^a}{t_\rho + t_\rho^a} = \frac{Q_4}{Q_1},\\
r' e^{i\delta_q} &= \frac{Q_4}{Q_1}\left|\frac{V_{tb}V_{td}^*}{V_{ub}V_{ud}^*}\right|,
\end{aligned} \tag{35}$$

where

$$\left|\frac{V_{tb}V_{td}^*}{V_{ub}V_{ud}^*}\right| = \frac{\sqrt{(1-\rho)^2 + \eta^2}}{(1-\lambda^2/2)\left(\sqrt{\rho^2+\eta^2}\right)}. \tag{36}$$

4. Input Parameters

In the numerical calculations, we should input distribution amplitudes and the CKM matrix elements in the Wolfenstein parametrization. For the CKM matrix elements, which are determined from experiments, we use the results in [35]:

$$\begin{aligned}
\overline{\rho} &= 0.132^{+0.022}_{-0.014}, \qquad \overline{\eta} = 0.341 \pm 0.013,\\
\lambda &= 0.2253 \pm 0.0007, \qquad A = 0.808^{+0.022}_{-0.015},
\end{aligned} \tag{37}$$

where

$$\overline{\rho} = \rho\left(1 - \frac{\lambda^2}{2}\right), \qquad \overline{\eta} = \eta\left(1 - \frac{\lambda^2}{2}\right). \tag{38}$$

The general expressions of the helicity-dependent amplitudes can be simplified by considering the asymptotic distribution amplitudes for Φ_V, Φ_v:

$$\Phi_{\parallel}^{V}(u) = 6u\bar{u}, \qquad \Phi_{v}(u) = 3(2u-1),$$
$$\Phi_{\perp}^{V}(u) = 6u\bar{u}, \qquad \Phi_{+}^{V} = \int_{u}^{1} dv \frac{\Phi_{\parallel}^{V}(v)}{v}, \tag{39}$$
$$\Phi_{-}^{M} = \int_{0}^{u} dv \frac{\Phi_{\parallel}^{V}(v)}{v}.$$

Power corrections in QCDF always involve endpoint divergences which produce some uncertainties. The endpoint divergence $X \equiv \int_0^1 dx/\bar{x}$ in the annihilation and hard spectator scattering diagrams is parameterized as

$$X_A = \ln\left(\frac{m_B}{\Lambda_h}\right)\left(1+\rho_A e^{i\phi_A}\right),$$
$$X_H = \ln\left(\frac{m_B}{\Lambda_h}\right)\left(1+\rho_H e^{i\phi_H}\right), \tag{40}$$

with the unknown real parameters $\rho_{A,H}$ and $\phi_{A,H}$ [8, 9]. For simplicity, we will assume that X_A^h and X_H^h are helicity-independent: $X_A^- = X_A^+ = X_A^0$ and $X_H^- = X_H^+ = X_H^0$.

5. Numerical Results

In the numerical results, we find that for the decay channel we are considering the CP violation can be enhanced via ρ-ω mixing when the invariant mass of $\pi^+\pi^-$ is in the vicinity of the ω resonance. The uncertainties of the CKM matrix elements mainly come from ρ and η. In our numerical results, we let ρ and η vary between the limiting values. We find that the results are not sensitive to the values of ρ and η. Hence, the numerical results are shown in Figures 1, 2, and 3 with the central parameter values of CKM matrix elements. From the numerical results, it is found that there is a maximum CP violating parameter value, A_{CP}^{max}, when the masses of the $\pi^+\pi^-$ pairs are in the vicinity of the ω resonance. In Figure 1, one can find that the maximum CP violating parameter reaches 28% in the case of $(\rho_{central}, \eta_{central})$.

From (16) one can find that the CP violating parameter is related to $\sin\delta$ and r. In Figure 2, we show the plot of $\sin\delta_0$ ($\sin\delta_-$ and $\sin\delta_+$) as a function of $\sqrt{s}$. We can see that the ρ-ω mixing mechanism produces a large $\sin\delta_0$ ($\sin\delta_-$ and $\sin\delta_+$) at the ω resonance. As can be seen from Figure 2, the plots vary sharply in the cases of $\sin\delta_0$ and $\sin\delta_-$. Meanwhile, $\sin\delta_+$ changes weakly compared with the $\sin\delta_0$ and $\sin\delta_-$. It can be seen from Figure 3 that r_0 and r_- change more rapidly than r_+ when the masses of the $\pi^+\pi^-$ pairs are in the vicinity of the ω resonance. The helicity amplitudes mainly come from the contributions of T_0 and T_-. ρ-ω mixing presents a large strong phase which gives a small effect on the amplitudes. Hence, $\sin\delta_+$ associated with T_+ and r is not sensitive to $\sqrt{s}$.

In paper [23], we studied the enhanced CP violation for the decay channel $\overline{B}^0 \to \pi^+\pi^-\pi^+\pi^-$ in the naive factorization. Since nonfactorizable contribution cannot be calculated

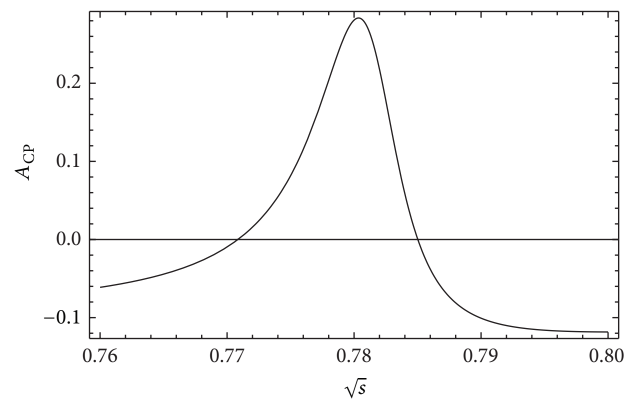

Figure 1: Plot of A_{CP} as a function of $\sqrt{s}$ corresponding to central parameter values of CKM matrix elements for $\overline{B}^0 \to \rho^0(\omega)\rho^0(\omega) \to \pi^+\pi^-\pi^+\pi^-$.

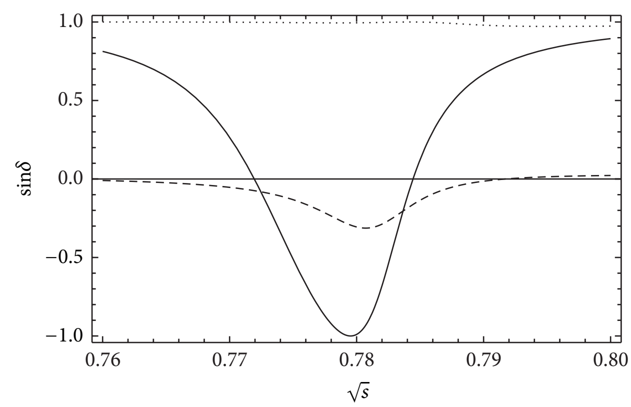

Figure 2: Plot of $\sin\delta$ as a function of $\sqrt{s}$ corresponding to central parameter values of CKM matrix elements for $\overline{B}^0 \to \rho^0(\omega)\rho^0(\omega) \to \pi^+\pi^-\pi^+\pi^-$. The solid (dashed and dotted) line corresponds to $\sin\delta_0$ ($\sin\delta_-$ and $\sin\delta_+$), respectively.

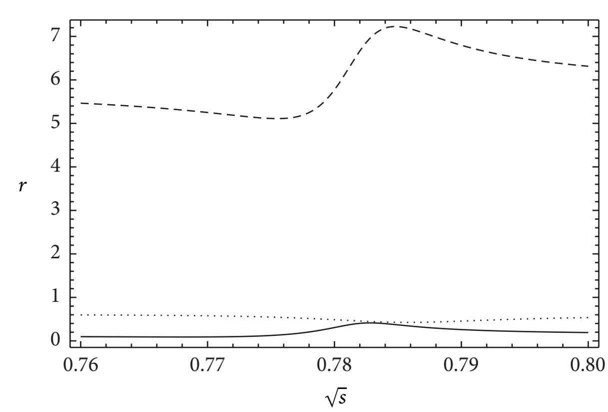

Figure 3: Plot of r as a function of $\sqrt{s}$ corresponding to central parameter values of CKM matrix elements for $\overline{B}^0 \to \rho^0(\omega)\rho^0(\omega) \to \pi^+\pi^-\pi^+\pi^-$. The solid (dashed and dotted) line corresponds to r_0 (r_- and r_+), respectively.

in the naive factorization, N_c was treated as an effective parameter. We found that the CP violating asymmetry was large and ranges from −82% to −98% via the ρ-ω mixing mechanism strongly depending on the value N_c when the invariant mass of the $\pi^+\pi^-$ pair is in the vicinity of the ω resonance. However, the maximum CP violation can only reach 28% via double ρ-ω mixing in the QCD factorization. The naive factorization scheme has been shown to be the leading order result in the framework of QCD factorization when the radiative QCD corrections $O(\alpha_s(m_b))$ and the order $O(1/m_b)$ effects are neglected. The QCD factorization can evaluate systematically corrections to the results from the naive factorization. The distinction between the naive factorization and the QCDF mainly comes from the strong phases of the QCD corrections. In the calculating process, we find that the annihilation contributions in QCDF which introduce the unknown parameters are small. Hence, the uncertainties of the results from the QCDF become small.

6. Summary and Conclusions

In this paper, we studied the CP violation for the decay process $\overline{B}^0 \rightarrow \pi^+\pi^-\pi^+\pi^-$ due to the interference of ρ-ω mixing in the QCDF approach. This process induces two ρ-ω interferences. It was found that the CP violation can be enhanced at the region of ρ-ω resonance. As a result, the maximum CP violation could reach 28%. ρ-ω mixing is small due to the isospin violation. However, the mixing can produce a large strong phase, δ, in (21). This is because when the invariant masses of the $\pi^+\pi^-$ pairs are in the vicinity of ω, $s_\omega \sim im_\omega\Gamma_\omega$, and it becomes comparable with $\widetilde{\Pi}_{\rho\omega}$ in (21). In other words, ρ-ω mixing becomes important in the vicinity of ω. This is the reason why we can see large CP violation in the vicinity of ω. Beyond the ρ-ω interference region, the noticeable values of CP violation are caused by the strong phases provided by the Wilson coefficients.

The LHC experiments are designed with the center-of-mass energy 14 TeV and the luminosity $L = 10^{34}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$. The heavy quark physics is one of the main topics of LHC experiments. In particular, LHCb detector is designed to make precise studies on CP asymmetries and rare decays of b-hadron systems. Recently, the LHCb Collaboration found clear evidence for direct CP violation in some three-body decay channels in charmless decays of B meson. Large CP violation is observed in $B^+ \rightarrow K^+K^-\pi^+$ and $B^\pm \rightarrow \pi^\pm\pi^+\pi^-$ in the regions $m^2_{\pi^+\pi^-\mathrm{low}} < 0.4\,\mathrm{GeV}^2$ and $m^2_{\pi^+\pi^-\mathrm{low}} > 15\,\mathrm{GeV}^2$ [3]. LHCb experiment may collect data in the region of the invariant masses of $\pi^+\pi^-$ associated with the ω resonance for detecting our prediction of CP violation.

In our calculations there are some uncertainties. The QCD factorization scheme provides a framework in which we can evaluate systematically corrections to the results obtained in the naive factorization scheme. However, when we take into account the nonfactorizable and chirally enhanced hard-scattering spectator and annihilation contributions which appear at orders $O(\alpha_s(m_b))$ and $O(1/m_b)$, respectively, the involvement of the twist-3 hadronic distribution amplitudes leads to logarithmical divergence coming from the endpoint integrals. This brings large uncertainties in the predictions of the CP violating asymmetries in the QCD factorization scheme. Furthermore, in addition to the model dependence appearing in the factorized hadronic matrix elements just as in the naive factorization scheme, we cannot avoid the model dependence and process dependence of the hard-scattering spectator and annihilation contributions due to their dependence on the hadronic distribution amplitudes and dependence on different processes. Such dependence will also appear if one tries to include other $1/m_b$ corrections and even higher order corrections. This leads to uncertainity of our results.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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