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Research Article

Scattering and Bound States of Duffin-Kemmer-Petiau Particles for q-Parameter Hyperbolic Pöschl-Teller Potential

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The Duffin-Kemmer-Petiau (DKP) equation in the presence of a scalar potential is solved in one spatial dimension for the vector q-parameter Hyperbolic Pöschl-Teller (qHPT) potential. In obtaining complete solutions we used the weak interaction approach and took the scalar and vector potentials in a correlated form. By looking at the asymptotic behaviors of the solutions, we identify the bound and scattering states. We calculate transmission (T) and reflection (R) probability densities and analyze their dependence on the potential shape parameters. Also we investigate the dependence of energy eigenvalues of the bound states on the potential shape parameters.

1. Introduction

In the early 20th century, many scientific studies have been done to explain the structure of nuclei, atoms, and molecules. In the explanation of these physical systems, particle equations describing fermions or bosons were solved for the potentials that vary depending on the physical systems. Also scattering and bound states were examined. Within this context, some potentials used in the previous studies are Morse potential [1], Rosen-Morse potential [2], Yukawa potential [3], Coulomb potential [4], Hylleraas potential [5], Manning-Rosen potential [6], Woods-Saxon potential [7], Hulthen potential [8], Eckart potential [9], Trigonometric Pöschl-Teller potential [10–12], Generalized Pöschl-Teller potential [13–15], and Hyperbolic Pöschl-Teller potential [10, 16–19]. Recently some of these potentials have been introduced in terms of hyperbolic functions whose hyperbolic parts have been extended by a q parameter due to the suggestion of Arai [20]. Of these potentials the one called q-parameter Hyperbolic Pöschl-Teller potential (qHPT) is given by the following form:

$$V(x) = \frac{\lambda(\lambda-1)}{\cosh_q^2(\alpha x)} = \frac{4\lambda(\lambda-1)}{\left(e^{\alpha x}+qe^{-\alpha x}\right)^2}, \qquad (1)$$

where q is the deformation parameter and $q \neq 0$ and λ is the height of the potential barrier and not equal to 0 and 1. Also, α is related to the range of the potential barrier. The graph of the potential against the position is displayed in Figure 1 for different values of the potential parameters. This potential has a feature of shape invariant as obtained from the supersymmetric quantum mechanics. By defining the q deformation parameter energy levels and wavefunctions are altered and, depending on the values that q parameter takes whether $0 < q < 1$ or $q > 1$, the number of energy levels varies. Although the usual PT-type potential provides limited knowledge in describing of atomic interactions, namely, at most two parameters, addition of the q deformation parameter to the potential extends the applications of such kind of potential to the other fields of the physics. PT-type potentials are used for analyzing the bound energies of the Λ-particle in hypernuclei in nuclear physics. Therefore, the qHPT type potential can play an important role in describing the interactions not only in molecular and atomic physics but also in nuclear physics. Moreover, the q-deformed hyperbolic potential is supposed to be used to characterize the curvature in spaces of negative constant curvature [16, 21, 22].

The Schrödinger equation has been solved in one dimension for this potential by using Nikiforov-Uvarov and Path

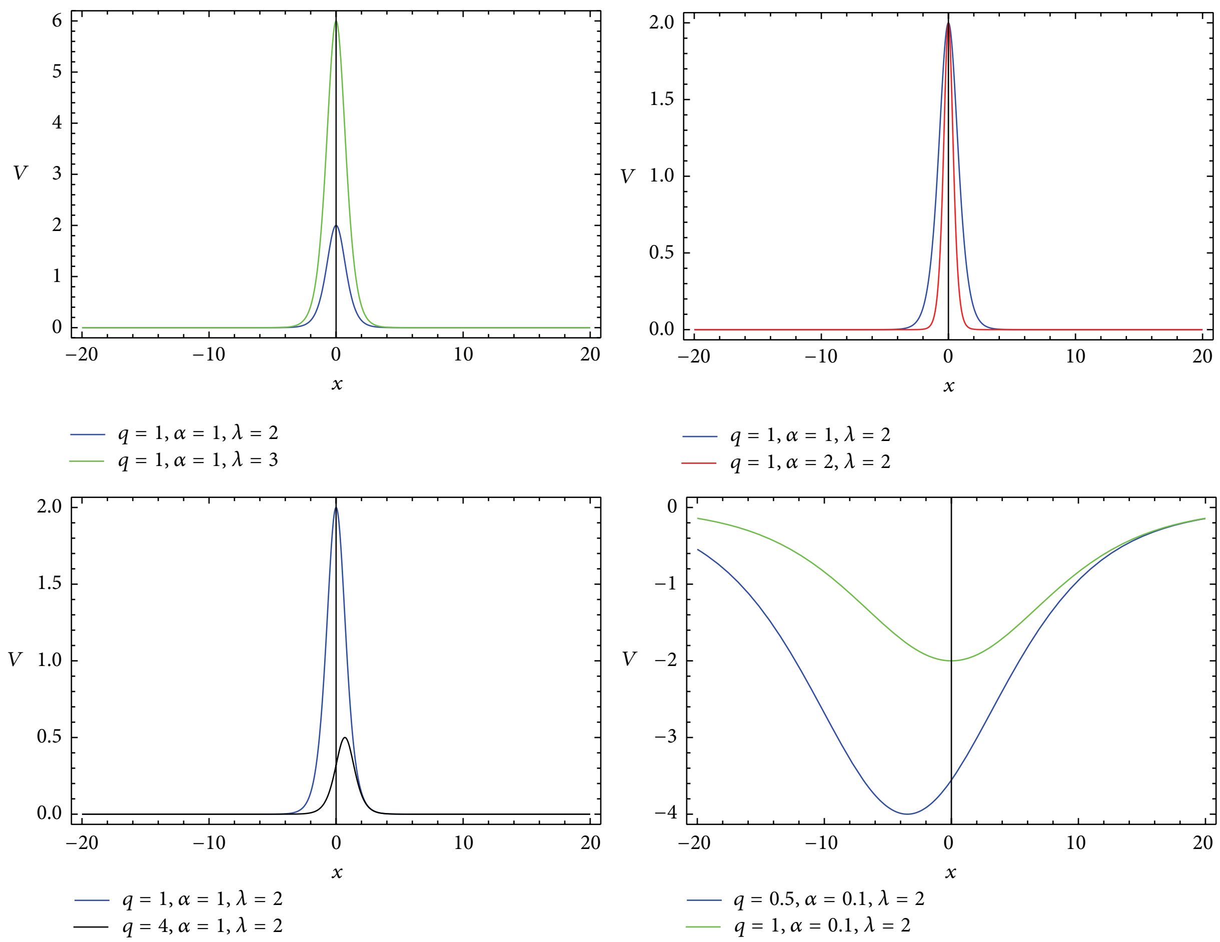

FIGURE 1: The graph of the qHPT potential versus the position for different values of the potential parameters.

integral methods and bound energy states have been obtained [22, 23]. Also the relativistic Klein-Gordon (spin-0) and Dirac (spin-1/2) equations were examined in the existence of the position-dependent mass or scalar potential [24–27]. Usually, in one spatial dimension, position-dependent mass and scalar potential are selected to be equal to the vector potential (external potential). Therefore, the scalar potential can be considered as counterpart of the position-dependent mass which is useful in the cases of quantum dots [28] and electronic properties of semiconductors [29].

Another relativistic equation that defines massive spin-0 (scalar bosons) and spin-1 (vector bosons) particles is called DKP equation. Unlike the Klein-Gordon and Dirac equations, the DKP equation has much more components for wavefunction, namely, it has sixteen components, which makes it difficult to solve exactly. This difficulty makes it harder to study the DKP equation compared to the other relativistic equations.

In contrast to the Klein-Gordon and Dirac equations, the DKP equation has not been studied for the qHPT potential. So, it is the goal of this study to obtain the bound and scattering states of the DKP equation in the presence of a scalar potential for the qHPT potential in one dimension and analyze the transmission (T) and reflection (R) probability densities numerically.

The scheme of this paper is as follows. In Section 2, we solve the DKP equation for the qHPT potential. In Section 3, the transmission (T) and reflection (R) probability densities are calculated by using the left and right solutions. In Section 4, the bound energy states are found for the qHPT potential well. Finally, in Section 5 we discuss the obtained results.

2. Solution of DKP Equation For qHPT Potential

The DKP equation with a scalar potential [30] describing massive spin-0 and spin-1 particles is given by (in natural units $\hbar = c = 1$)

$$\left[i\beta^{\mu}\left(\partial_{\mu} + ieA_{\mu}\right) - \left(m + V_s\right)\right]\Psi_K\left(t, \vec{x}\right) = 0, \quad (2)$$

where A_{μ} is the four-vector potential, V_s is the scalar potential, and $\Psi_K(t, \vec{x})$ is the sixteen components wavefunction of the DKP equation. β^{μ} are the Kemmer matrices which obey the following commutation relation [31, 32]:

$$\beta^{\mu} = \gamma^{\mu} \otimes I + I \otimes \gamma^{\mu}, \tag{3}$$

where γ^{μ} are the Dirac matrices. For (1 + 1)-dimensional case, the Pauli spin matrices σ^{μ} are used instead of the Dirac gamma matrices in (3). In this case the beta matrices become as follows [33]:

$$\beta^{\mu} = \sigma^{\mu} \otimes I + I \otimes \sigma^{\mu}. \tag{4}$$

With the insertion of β^{μ} matrices into (2), we obtain

$$\left[i\left(\sigma^{\mu} \otimes I + I \otimes \sigma^{\mu}\right)\left(\partial_{\mu} + ieA_{\mu}\right) - \left(m + V_s\right)\right]\Psi_K = 0, \tag{5}$$

where the wavefunction is given by

$$\Psi_K^T = \left(\Psi_1 \quad \Psi_0 \quad \Psi_{\tilde{0}} \quad \Psi_2\right). \tag{6}$$

Equation (5) reduces to four coupled differential equations with the choice of $\sigma^{\mu} = (\sigma^z, i\sigma^x)$:

$$\begin{gathered}
\left[2\left(\partial_0 + ieA_0\right) + i\left(m + V_s\right)\right]\Psi_1 + i\left(\partial_1 + ieA_1\right)\left(\Psi_0 + \Psi_{\tilde{0}}\right) = 0 \\
\left(\partial_1 + ieA_1\right)\left(\Psi_1 + \Psi_2\right) + \left(m + V_s\right)\Psi_0 = 0 \\
\left(\partial_1 + ieA_1\right)\left(\Psi_1 + \Psi_2\right) + \left(m + V_s\right)\Psi_{\tilde{0}} = 0 \\
\left[-2\left(\partial_0 + ieA_0\right) + i\left(m + V_s\right)\right]\Psi_2 + i\left(\partial_1 + ieA_1\right)\left(\Psi_0 + \Psi_{\tilde{0}}\right) = 0.
\end{gathered} \tag{7}$$

After some simple algebra, we find a second order differential equation in the form

$$\frac{d^2\left(\chi_1 + \chi_2\right)}{dx^2} - \frac{1}{\left(m + V_s\right)}\frac{dV_s(x)}{dx}\frac{d\left(\chi_1 + \chi_2\right)}{dx} + \left[\left(E - V_v\right)^2 - \left(\frac{m + V_s}{2}\right)^2\right]\left(\chi_1 + \chi_2\right) = 0. \tag{8}$$

The scalar potential we are dealing with has the form $V_s = cf(x)$, where c represents the strength of the weak interaction. It is very small compared with the mass of the particle. Considering this effect the term including the first order derivative can be ignored since

$$\lim_{c \ll m} \frac{c}{m\left(1 + cf(x)/m\right)}\frac{df(x)}{dx}\frac{d\left(\chi_1 + \chi_2\right)}{dx} \longrightarrow 0. \tag{9}$$

Then (8) reduces to

$$\frac{d^2\left(\chi_1 + \chi_2\right)}{dx^2} + \left[\left(E^2 - \widetilde{m}^2\right) - 2\left(E + \widetilde{m}\right)V_v\right]\left(\chi_1 + \chi_2\right) = 0, \tag{10}$$

where $\Psi(t,x) = e^{-iEt}\chi(x)$, $\chi^T = (\chi_1 \quad \chi_0 \quad \chi_{\tilde{0}} \quad \chi_2)$, $\widetilde{m} = m/2$, $eA_0 = V_v = V_s/2$, and $A_1 = 0$.

There are two reasons for choosing the scalar and vector potentials in a correlated form; they are as follows.

(i) Mathematically, with this choice, (8) reduces to a solvable form. In the original form the second order differential equation has a fourth order singular point caused by the external vector qHPT potential. Taking the vector and scalar potential in similar forms removes this difficulty and the resulting equation given in (10) is a solvable form.

(ii) Another reason is that we have no information about the form of potential describing the weak interactions. Our approach to the problem is to take the scalar and vector potentials in a correlated form and to investigate whether we are able to get reasonable quantum mechanical parameters like T and R.

We solve (10) by defining different variables for $x < 0$ and $x > 0$ cases.

For $x < 0$ case, by substituting the qHPT potential given by (1) into (10) and defining the variable

$$y = \left(1 + qe^{-2\alpha x}\right)^{-1}, \tag{11}$$

we obtain the following equation:

$$y(1 - y)\frac{d^2\left(\chi_1 + \chi_2\right)}{dy^2} + (1 - 2y)\frac{d\left(\chi_1 + \chi_2\right)}{dy} + \frac{\left[\varepsilon^2 - \tau y(1 - y)\right]}{y(1 - y)}\left(\chi_1 + \chi_2\right) = 0, \tag{12}$$

where $\varepsilon^2 = (E^2 - \widetilde{m}^2)/4\alpha^2$ and $\tau = 2\lambda(\lambda - 1)(E + \widetilde{m})/q\alpha^2$.

Equation (12) has singularities at $y = 0$ and $y = 1$. Therefore we may suggest the wavefunction in the following form to find the exact solution:

$$\left(\chi_1 + \chi_2\right)(y) = y^{\eta}(1 - y)^{\tilde{\eta}}U(y). \tag{13}$$

By using this definition we obtain the following differential equation:

$$y(1 - y)\frac{d^2U}{dy^2} + \left[(2\eta + 1) - (2\eta + 2\tilde{\eta} + 2)y\right]\frac{dU}{dy} - \left(\eta + \tilde{\eta} + \sigma + \frac{1}{2}\right)\left(\eta + \tilde{\eta} - \sigma + \frac{1}{2}\right)U = 0, \tag{14}$$

where $\eta = \tilde{\eta} = i\varepsilon$ and $\sigma = \sqrt{1/4 - \tau}$. This is the hypergeometric differential equation and its solution is given by [34]

$$U(y) = A\,{}_2F_1\left(2\eta + \sigma + \frac{1}{2}, 2\eta - \sigma + \frac{1}{2}; 1 + 2\eta; y\right) + By^{-2\eta}\,{}_2F_1\left(\sigma + \frac{1}{2}, -\sigma + \frac{1}{2}; 1 - 2\eta; y\right). \tag{15}$$

In this case the left ($x < 0$) side solutions are obtained in the following form:

$$\begin{aligned}(\chi_1+\chi_2)_L(y) &= Ay^{\eta}(1-y)^{\eta}\,{}_2F_1\left(2\eta+\sigma+\frac{1}{2},2\eta-\sigma+\frac{1}{2};1+2\eta;y\right)\\ &\quad+By^{-\eta}(1-y)^{\eta}\,{}_2F_1\left(\sigma+\frac{1}{2},-\sigma+\frac{1}{2};1-2\eta;y\right).\end{aligned}\tag{16}$$

For $x > 0$ case, we define the variable as follows:

$$z=q\left(q+e^{2\alpha x}\right)^{-1}.\tag{17}$$

In this case (10) becomes

$$\begin{aligned}&z(1-z)\frac{d^2(\chi_1+\chi_2)}{dz^2}+(1-2z)\frac{d(\chi_1+\chi_2)}{dz}\\ &\quad+\frac{\left[\varepsilon^2-\tau z(1-z)\right]}{z(1-z)}(\chi_1+\chi_2)=0.\end{aligned}\tag{18}$$

By setting

$$(\chi_1+\chi_2)(z)=z^{\rho}(1-z)^{\tilde{\rho}}f(z)\tag{19}$$

we obtain in the following differential equation:

$$\begin{aligned}&z(1-z)\frac{d^2f}{dz^2}+\left[(2\rho+1)-(2\rho+2\tilde{\rho}+2)z\right]\frac{df}{dz}\\ &\quad-\left(\rho+\tilde{\rho}+\sigma+\frac{1}{2}\right)\left(\rho+\tilde{\rho}-\sigma+\frac{1}{2}\right)f=0,\end{aligned}\tag{20}$$

where $\rho=\tilde{\rho}=i\varepsilon$ and $\sigma=\sqrt{1/4-\tau}$. In this case $\rho=\eta$. The solution of this differential equation is

$$\begin{aligned}f(z)&=D\,{}_2F_1\left(2\eta+\sigma+\frac{1}{2},2\eta-\sigma+\frac{1}{2};1+2\eta;z\right)\\ &\quad+Gz^{-2\eta}\,{}_2F_1\left(\sigma+\frac{1}{2},-\sigma+\frac{1}{2};1-2\eta;z\right)\end{aligned}\tag{21}$$

and the right ($x > 0$) side solutions are obtained in the following form:

$$\begin{aligned}(\chi_1+\chi_2)_R(z) &= Dz^{\eta}(1-z)^{\tilde{\eta}}\,{}_2F_1\left(2\eta+\sigma+\frac{1}{2},2\eta-\sigma+\frac{1}{2};1+2\eta;z\right)\\ &\quad+Gz^{-\eta}(1-z)^{\tilde{\eta}}\,{}_2F_1\left(\sigma+\frac{1}{2},-\sigma+\frac{1}{2};1-2\eta;z\right).\end{aligned}\tag{22}$$

The other components of the Ψ_K wavefunction can be obtained by using (7) and the recurrence formula of the hypergeometric functions [34]:

$$\frac{d\,{}_2F_1(a,b;c;x)}{dx}=\frac{ab}{c}\,{}_2F_1(a+1,b+1;c+1;x).\tag{23}$$

3. Calculation of Transmission and Reflection Probability Densities

Transmission (T) and Reflection (R) probability densities are defined, respectively, in the following form:

$$\begin{aligned}T&=\left|\frac{j_{\text{trans.}}}{j_{\text{inc.}}}\right|,\\ R&=\left|\frac{j_{\text{ref.}}}{j_{\text{inc.}}}\right|,\end{aligned}\tag{24}$$

where $j_{\text{trans.}}$, $j_{\text{ref.}}$, and $j_{\text{inc.}}$ are transmitted, reflected, and incident probability current densities, respectively. Probability of the current density for the DKP equation is given by

$$j^{\mu}=\overline{\Psi}\beta^{\mu}\Psi,\tag{25}$$

where $\overline{\Psi}=\Psi^{\dagger}(\gamma^0\otimes\gamma^0)$. Its explicit form is

$$j=i\left[(\chi_1+\chi_2)^*(\chi_0+\chi_{\bar{0}})-(\chi_0+\chi_{\bar{0}})^*(\chi_1+\chi_2)\right].\tag{26}$$

In order to calculate the transmission (T) and reflection (R) probability densities, we use asymptotic expressions of the wavefunctions.

The incoming wavefunction and its asymptotic behavior as $x\to-\infty$, $y\to 0$, ${}_2F_1(a,b;c;y)\to 1$ are defined as follows, respectively:

$$\begin{aligned}\chi_{\text{inc.}} &= Ay^{\eta}(1-y)^{\eta}\\ &\quad\times\begin{bmatrix}\frac{1}{2}\left[\frac{m+2E}{m+V_s}\right]F_1\\ \left(\frac{-2\alpha}{m+V_s}\right)\left[(\eta(1-y)-\eta y)F_1+y(1-y)F_2\right]\\ \left(\frac{-2\alpha}{m+V_s}\right)\left[(\eta(1-y)-\eta y)F_1+y(1-y)F_2\right]\\ \frac{1}{2}\left[\frac{m-2E+2V_s}{m+V_s}\right]F_1\end{bmatrix}\\ \chi_{\text{inc.}} &\longrightarrow Aq^{-i\varepsilon}e^{2i\alpha\varepsilon x}\times\begin{bmatrix}\frac{1}{2}\left[\frac{m+2E}{m+V_s}\right]\\ \left(\frac{-2\alpha}{m+V_s}\right)i\varepsilon\\ \left(\frac{-2\alpha}{m+V_s}\right)i\varepsilon\\ \frac{1}{2}\left[\frac{m-2E+2V_s}{m+V_s}\right]\end{bmatrix},\end{aligned}\tag{27}$$

where we used

$$F_1 = {}_2F_1\left(2\eta + \sigma + \frac{1}{2}, 2\eta - \sigma + \frac{1}{2}; 1 + 2\eta; y\right)$$
$$F_2 = \frac{(2\eta + \sigma + 1/2)(2\eta - \sigma + 1/2)}{1 + 2\eta} \times {}_2F_1\left(2\eta + \sigma + \frac{3}{2}, 2\eta - \sigma + \frac{3}{2}; 2 + 2\eta; y\right). \quad (28)$$

The reflected wavefunction and its asymptotic behavior as $x \to -\infty, y \to 0, {}_2F_1(a, b; c; y) \to 1$ are defined as follows, respectively:

$$\chi_{\text{ref.}} = By^{-\eta}(1 - y)^{\eta} \times \begin{bmatrix} \frac{1}{2}\left[\frac{m + 2E}{m + V_s}\right] F_3 \\ \left(\frac{-2\alpha}{m + V_s}\right)[(-\eta(1 - y) - \eta y) F_3 + y(1 - y) F_4] \\ \left(\frac{-2\alpha}{m + V_s}\right)[(-\eta(1 - y) - \eta y) F_3 + y(1 - y) F_4] \\ \frac{1}{2}\left[\frac{m - 2E + 2V_s}{m + V_s}\right] F_3 \end{bmatrix}$$

$$\chi_{\text{ref.}} \longrightarrow Bq^{i\varepsilon}e^{-2i\alpha\varepsilon x} \times \begin{bmatrix} \frac{1}{2}\left[\frac{m + 2E}{m + V_s}\right] \\ \left(\frac{2\alpha}{m + V_s}\right) i\varepsilon \\ \left(\frac{2\alpha}{m + V_s}\right) i\varepsilon \\ \frac{1}{2}\left[\frac{m - 2E + 2V_s}{m + V_s}\right] \end{bmatrix}, \quad (29)$$

where we used

$$F_3 = {}_2F_1\left(\sigma + \frac{1}{2}, -\sigma + \frac{1}{2}; 1 - 2\eta; y\right)$$
$$F_4 = \frac{(\sigma + 1/2)(-\sigma + 1/2)}{1 - 2\eta} {}_2F_1\left(\sigma + \frac{3}{2}, -\sigma + \frac{3}{2}; 2 - 2\eta; y\right). \quad (30)$$

The transmitted wavefunction and its asymptotic behavior as $x \to \infty, z \to 0, {}_2F_1(a, b; c; z) \to 1$ are defined as follows, respectively:

$$\chi_{\text{trans.}} = Gz^{-\eta}(1 - z)^{\eta} \times \begin{bmatrix} \frac{1}{2}\left[\frac{m + 2E}{m + V_s}\right] F_5 \\ \left(\frac{2\alpha}{m + V_s}\right)[(-\eta(1 - z) - \eta z) F_5 + z(1 - z) F_6] \\ \left(\frac{2\alpha}{m + V_s}\right)[(-\eta(1 - z) - \eta z) F_5 + z(1 - z) F_6] \\ \frac{1}{2}\left[\frac{m - 2E + 2V_s}{m + V_s}\right] F_5 \end{bmatrix} \quad (31)$$

$$\chi_{\text{trans.}} \longrightarrow Gq^{-i\varepsilon}e^{2i\alpha\varepsilon x} \times \begin{bmatrix} \frac{1}{2}\left[\frac{m + 2E}{m + V_s}\right] \\ \left(\frac{-2\alpha}{m + V_s}\right) i\varepsilon \\ \left(\frac{-2\alpha}{m + V_s}\right) i\varepsilon \\ \frac{1}{2}\left[\frac{m - 2E + 2V_s}{m + V_s}\right] \end{bmatrix}, \quad (32)$$

where we take $D = 0$ to obtain (31) because there is no the reflected wavefunction in the region of the potential and

$$F_5 = {}_2F_1\left(\sigma + \frac{1}{2}, -\sigma + \frac{1}{2}; 1 - 2\eta; z\right)$$
$$F_6 = \frac{(\sigma + 1/2)(-\sigma + 1/2)}{1 - 2\eta} {}_2F_1\left(\sigma + \frac{3}{2}, -\sigma + \frac{3}{2}; 2 - 2\eta; z\right). \quad (33)$$

Transmission (T) and reflection (R) probability densities are found by substituting the obtained asymptotic expressions of wavefunctions into (26):

$$T = \left|\frac{j_{\text{trans.}}}{j_{\text{inc.}}}\right| = \left|\frac{G}{A}\right|^2$$
$$R = \left|\frac{j_{\text{ref.}}}{j_{\text{inc.}}}\right| = \left|\frac{B}{A}\right|^2. \quad (34)$$

In order to correlate the coefficients in (34), we use the continuity condition for the DKP equation that is given by

$$\Psi_{\text{inc.}}(x = 0) + \Psi_{\text{ref.}}(x = 0) = \Psi_{\text{trans.}}(x = 0). \quad (35)$$

By using (35), we obtain

$$\frac{G}{A} = q^{2\eta}(1 + q)^{-2\eta} \times \left\{\frac{\left(S_3\widetilde{F}_3 + S_2\widetilde{F}_4\right)\widetilde{F}_1 - \left(S_1\widetilde{F}_1 + S_2\widetilde{F}_2\right)\widetilde{F}_3}{\left(S_3\widetilde{F}_3 + S_2\widetilde{F}_4\right)\widetilde{F}_5 + \left(S_3\widetilde{F}_5 + S_2\widetilde{F}_6\right)\widetilde{F}_3}\right\}$$

$$\frac{B}{A} = -(1+q)^{-2\eta} \times \left\{ \frac{\left(S_3\widetilde{F}_5 + S_2\widetilde{F}_6\right)\widetilde{F}_1 + \left(S_1\widetilde{F}_1 + S_2\widetilde{F}_2\right)\widetilde{F}_5}{\left(S_3\widetilde{F}_3 + S_2\widetilde{F}_4\right)\widetilde{F}_5 + \left(S_3\widetilde{F}_5 + S_2\widetilde{F}_6\right)\widetilde{F}_3} \right\}, \quad (36)$$

where the constants are given in Table 1.

4. Bound Energy States

To discuss the bound states, the potential must be in the well form. Also, energy values must be smaller than the mass ($|E| < m$). Therefore, the qHPT potential given in (1) is rewritten in the following form:

$$V(x) = -\frac{\lambda(\lambda-1)}{\cosh_q^2(\alpha x)}. \quad (37)$$

Accordingly, τ becomes

$$\tau = -2\frac{\lambda(\lambda-1)(E+\widetilde{m})}{q\alpha^2}, \quad (38)$$

where $\lambda > 1$. In that case the solutions will be in the same form of the once obtained in Section 1, except the above definitions. We will apply the boundary conditions to the solution given in (16) as follows.

(i) First boundary condition: as $x \to -\infty (y \to 0)$, the wavefunction must be equal to zero:

$$(\chi_1 + \chi_2)(x) = Aq^{-\varepsilon}e^{2\alpha\varepsilon x} + Bq^{\varepsilon}e^{-2\alpha\varepsilon x}. \quad (39)$$

In order to satisfy the wavefunction to be continuous, B should be zero. In this case (16) becomes

$$(\chi_1 + \chi_2)(y) = Ay^{\eta}(1-y)^{\eta}\,{}_2F_1\left(2\eta + \sigma + \frac{1}{2}, 2\eta - \sigma + \frac{1}{2}; 1 + 2\eta; y\right), \quad (40)$$

where $\eta = \varepsilon = \sqrt{(\widetilde{m}^2 - E^2)/4\alpha^2}$.

(ii) Second boundary condition: as $x \to \infty (y \to 1)$, the wavefunction must vanish.

As $x \to \infty (y \to 1)$, the hypergeometric functions are defined as follows [34]:

$${}_2F_1(a,b;c;y) = \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)} + \frac{\Gamma(c)\Gamma(-c+a+b)}{\Gamma(a)\Gamma(b)}(1-y)^{c-a-b}. \quad (41)$$

By using (40) and (41), we obtain the following one:

$$(\chi_1 + \chi_2)(x) = Aq^{\varepsilon}e^{-2\alpha\varepsilon x}\frac{\Gamma(1+2\varepsilon)\Gamma(-2\varepsilon)}{\Gamma(1/2-\sigma)\Gamma(1/2+\sigma)} + Aq^{-\varepsilon}e^{2\alpha\varepsilon x}\frac{\Gamma(1+2\varepsilon)\Gamma(2\varepsilon)}{\Gamma(1/2+2\varepsilon+\sigma)\Gamma(1/2+2\varepsilon-\sigma)}, \quad (42)$$

where the factor depending on the variable of the second term of the right side goes to the infinity. Therefore, in order to satisfy this boundary condition, the argument of gamma functions in the denominator of second term should be equal to negative integers:

$$\left(\frac{1}{2} + 2\varepsilon + \sigma\right) = -n \quad \text{or} \quad \left(\frac{1}{2} + 2\varepsilon - \sigma\right) = -n, \quad (43)$$

where n is finite and positive integer. By using one of these equations, the energy relation for bound states is obtained in the following form:

$$E^2 - \widetilde{m}^2 = -\alpha^2\left[n + \frac{1}{2} - \sqrt{\frac{1}{4} + \frac{2\lambda(\lambda-1)(\widetilde{m}+E)}{q\alpha^2}}\right]^2. \quad (44)$$

We give Table 2 in order to evaluate numbers of the energy eigenvalues of the bound states for the qHPT potential against several values of α, q, and λ. From Table 2 we see that numbers of energy eigenvalues depend on the qHPT potential parameters which are α, q, and λ.

5. Conclusion

The main purpose of this study is to determine the solutions of the DKP equation and to obtain the bound and scattering states of the DKP particles for the qHPT potential which have not been done in previous studies. The DKP equation written for the qHPT potential has fourth order singular points. Therefore the equation can not be solved. In order to solve it we make the weak interaction approach and take the scalar and vector potentials in a correlated form. Then, by using the condition that the wavefunction obtained as a solution must be finite as $x \to \mp\infty$ and by analyzing the gamma functions, we find the relation which gives energy eigenvalues for the bound states. As it is seen in Table 2, the number of the bound states (n) depends on the qHPT potential parameters. For example, when evaluating variation of the energy against α for $q = 0.4$, $\lambda = 2$, $m = 1$ it can be seen that the number of the bound states equals 3 for $\alpha = 1.2$. Similarly, when one evaluates variation of the energy according to q for $\alpha = 0.8$, $\lambda = 2$, $m = 1$ one can see that the number of the bound states equals 3 for $q = 1.2$. If one analyzes variation of the energy versus λ which determines depth of the qHPT potential for $\alpha = 0.5$, $q = 0.5$, $m = 1$, one can find that the number of the bound states equals 2 for $\lambda = 1.1$. By adjusting the values of α, q, and λ the number of bound states can be increased or decreased.

The transmission and reflection probability densities are calculated by using asymptotic behavior of the wavefunction and their dependence on the potential shape parameters is analyzed numerically. The plot of energy against the transmission (T) and reflection (R) probability densities is given in Figure 2. We see from this plot that the unitarity condition ($T + R = 1$) is satisfied. The effect of λ, q, and α parameters on the transmission probability density is represented in Figures 3, 4, and 5, respectively. From these figures, it is seen that the transmission probability density decreases as λ increases and it increases with raising q and α parameters.

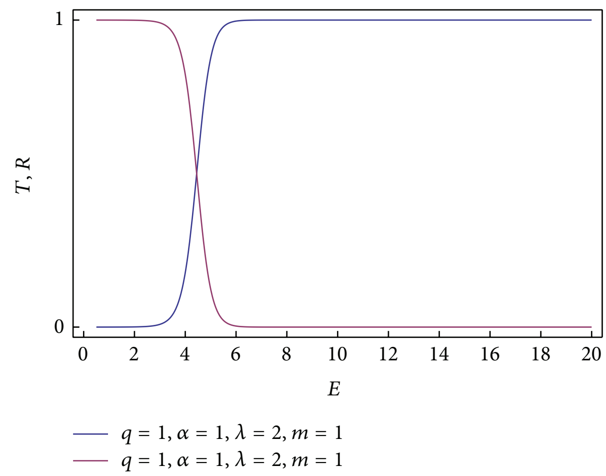

Figure 2: Unitarity condition, $R + T = 1$.

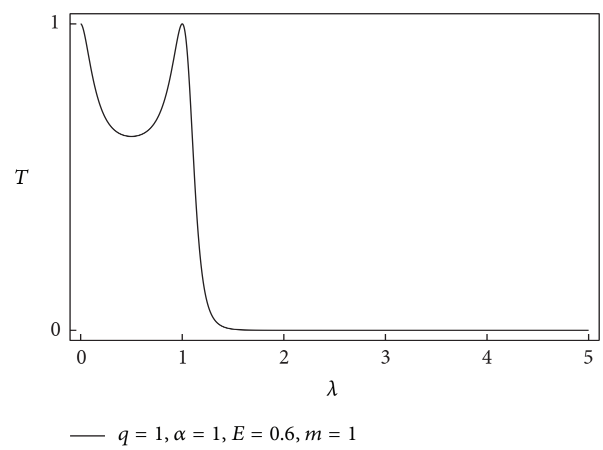

Figure 3: Transmission probability density versus λ parameter.

Table 1: Table for some definitions used in calculations of T and R.

$S_1 = \dfrac{-(1+q)^2(2\alpha\eta)}{[m(1+q)^2 + 8\lambda(\lambda-1)]}\left(\dfrac{q-1}{1+q}\right)$
$S_2 = \dfrac{-(1+q)^2(2\alpha)}{[m(1+q)^2 + 8\lambda(\lambda-1)]}\dfrac{q}{(1+q)^2}$
$S_3 = \dfrac{(1+q)^2(2\alpha\eta)}{[m(1+q)^2 + 8\lambda(\lambda-1)]}$
$\widetilde{F}_1 = {}_2F_1\left(2\eta+\sigma+\dfrac{1}{2}, 2\eta-\sigma+\dfrac{1}{2}; 1+2\eta; \dfrac{1}{1+q}\right)$
$\widetilde{F}_2 = \dfrac{(2\eta+\sigma+1/2)(2\eta-\sigma+1/2)}{1+2\eta}{}_2F_1\left(2\eta+\sigma+\dfrac{3}{2}, 2\eta-\sigma+\dfrac{3}{2}; 2+2\eta; \dfrac{1}{1+q}\right)$
$\widetilde{F}_3 = {}_2F_1\left(\sigma+\dfrac{1}{2}, -\sigma+\dfrac{1}{2}; 1-2\eta; \dfrac{1}{1+q}\right)$
$\widetilde{F}_4 = \dfrac{(\sigma+1/2)(-\sigma+1/2)}{1-2\eta}{}_2F_1\left(\sigma+\dfrac{3}{2}, -\sigma+\dfrac{3}{2}; 2-2\eta; \dfrac{1}{1+q}\right)$
$\widetilde{F}_5 = {}_2F_1\left(\sigma+\dfrac{1}{2}, -\sigma+\dfrac{1}{2}; 1-2\eta; \dfrac{q}{1+q}\right)$
$\widetilde{F}_6 = \dfrac{(\sigma+1/2)(-\sigma+1/2)}{1-2\eta}{}_2F_1\left(\sigma+\dfrac{3}{2}, -\sigma+\dfrac{3}{2}; 2-2\eta; \dfrac{q}{1+q}\right)$

Table 2: Evaluation of numbers of the bound states for the qHPT potential against several values of α, q, and λ.

n	q	$\alpha = 0.8, \lambda = 2, m = 1$ E	α	$q = 0.4, \lambda = 2, m = 1$ E	λ	$\alpha = 0.5, q = 0.5, m = 1$ E
1	0.7	−0.916201	0.7	−0.955329	1.1	−0.904418
	0.8	−0.909189	0.8	−0.941517	1.2	−0.924119
	0.9	−0.902695	0.9	−0.925783	1.3	−0.937065
	1.0	−0.896644	1.0	−0.908094	1.4	−0.946449
	1.1	−0.890978	1.1	−0.888413	1.5	−0.953629
	1.2	−0.885648	1.2	−0.866697	1.6	−0.959318
	1.3	−0.880618	1.3	−0.842897	1.7	−0.963941
2	0.7	−0.726421	0.7	−0.858986	1.1	−0.630982
	0.8	−0.700873	0.8	−0.814433	1.2	−0.730331
	0.9	−0.676677	0.9	−0.76307	1.3	−0.785115
	1.0	−0.653625	1.0	−0.704488	1.4	−0.821553
	1.1	−0.631553	1.1	−0.638169	1.5	−0.848018
	1.2	−0.610328	1.2	−0.563441	1.6	−0.868265
	1.3	−0.589845	1.3	−0.479416	1.7	−0.884302
3	0.7	−0.405568	0.7	−0.708126	1.1	0.226483
	0.8	−0.340312	0.8	−0.612255	1.2	−0.365843
	0.9	−0.275352	0.9	−0.498889	1.3	−0.523306
	1.0	−0.209839	1.0	−0.365023	1.4	−0.615126
	1.1	−0.142822	1.1	−0.205758	1.5	−0.677672
	1.2	−0.0731078	1.2	−0.0119543	1.6	−0.723722
	1.3	0.00100059	1.3	0.238587	1.7	−0.759281

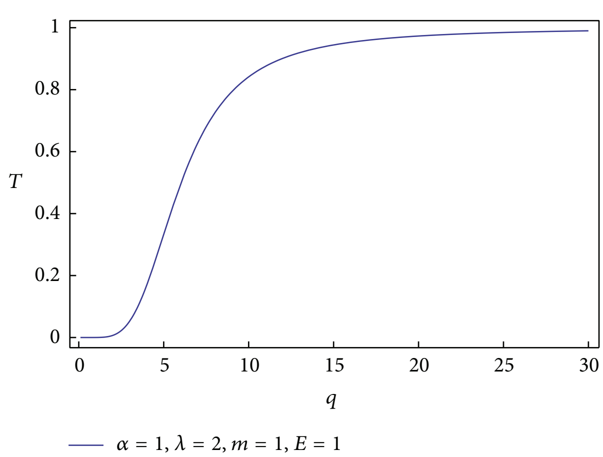

Figure 4: Transmission probability density versus q parameter.

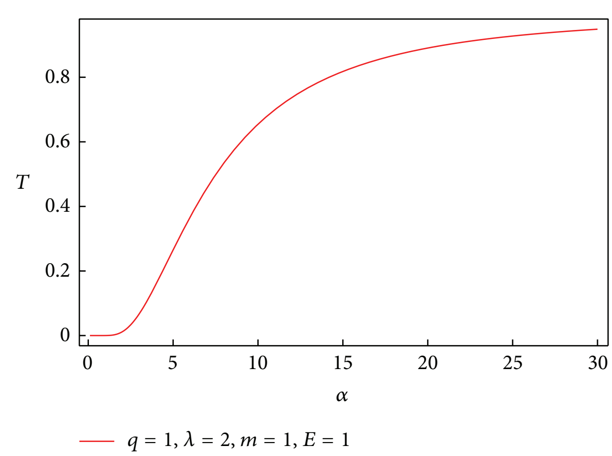

Figure 5: Transmission probability density versus α parameter.

These findings are the expected results, since the increase in the λ also supports the height of the potential, which is shown in Figure 1. Therefore, the transmission probability density decreases. As it can be seen from Figure 1, unlike λ, the height of the qHPT potential decreases due to the increase of q and α parameters. In this case, the transmission probability density increases.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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