

## Research Article

# Thermodynamics of Charged AdS Black Holes in Rainbow Gravity

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In this paper, the thermodynamic property of charged AdS black holes is studied in rainbow gravity. By the Heisenberg Uncertainty Principle and the modified dispersion relation, we obtain deformed temperature. Moreover, in rainbow gravity we calculate the heat capacity in a fixed charge and discuss the thermal stability. We also obtain a similar behaviour with the liquid-gas system in extending phase space (including  $P$  and  $r$ ) and study its critical behavior with the pressure given by the cosmological constant and with a fixed black hole charge  $Q$ . Furthermore, we study the Gibbs function and find its characteristic swallow tail behavior, which indicates the phase transition. We also find that there is a special value about the mass of test particle which would lead the black hole to zero temperature and a diverging heat capacity with a fixed charge.

## 1. Introduction

It is known Lorentz symmetry which is one of most important symmetries in nature; however, some researches indicate that the Lorentz symmetry might be violated in the ultraviolet limit [1–5]. Since the standard energy-momentum dispersion relation relates to the Lorentz symmetry, the deformation of Lorentz symmetry would lead to the modification of energy-momentum dispersion relation. In fact, some calculations in loop quantum gravity have showed the dispersion relations may be deformed. Meanwhile, based on the deformed energy-momentum dispersion relation the double special relativity has arisen [6, 7]. In this theory, in addition to the velocity of light being the maximum velocity attainable there is another constant for maximum energy scale in nature which is the Planck energy  $E_p$ . It gives different picture for the special relativity in microcosmic physics. The theory has been generalized to curved spacetime by Joao Magueijo and Lee Smolin, called gravity's rainbow [8]. In their theory, the geometry of spacetime depends on the energy of the test particle and observers of different energy would see different geometry of spacetime. Hence, a family of energy-dependent metrics named rainbow metrics will describe the geometry of spacetime, which is different from

general gravity theory. Based on the nonlinear of Lorentz transformation, the energy-momentum dispersion relation can be rewritten as

$$E^2 f^2 \left( \frac{E}{E_p} \right) - p^2 g^2 \left( \frac{E}{E_p} \right) = m^2, \quad (1)$$

where  $E_p$  is the Planck energy. The rainbow functions  $f(E/E_p)$  and  $g(E/E_p)$  are required to satisfy

$$\begin{aligned} \lim_{E/E_p \rightarrow 0} f \left( \frac{E}{E_p} \right) &= 1, \\ \lim_{E/E_p \rightarrow 0} g \left( \frac{E}{E_p} \right) &= 1. \end{aligned} \quad (2)$$

In this case, the deformed energy-momentum dispersion relation equation (1) will go back to classical one when the energy of the test particle is much lower than  $E_p$ . Due to this energy-dependent modification to the dispersion relation, the metric  $h(E)$  in gravity's rainbow could be rewritten as [9]

$$h(E) = \eta^{ab} e_a(E) \otimes e_b(E), \quad (3)$$

where the energy dependence of the frame fields is

$$\begin{aligned} e_0(E) &= \frac{1}{f(E/E_p)} \tilde{e}_0, \\ e_i(E) &= \frac{1}{g(E/E_p)} \tilde{e}_i, \end{aligned} \quad (4)$$

and here the tilde quantities refer to the energy-independent frame fields. This leads to a one-parameter Einstein equation

$$\begin{aligned} G_{\mu\nu} \left( \frac{E}{E_p} \right) + \Lambda \left( \frac{E}{E_p} \right) g_{\mu\nu} \left( \frac{E}{E_p} \right) \\ = 8\pi G \left( \frac{E}{E_p} \right) T_{\mu\nu} \left( \frac{E}{E_p} \right), \end{aligned} \quad (5)$$

where  $G_{\mu\nu}(E/E_p)$  and  $T_{\mu\nu}(E/E_p)$  are energy-dependent Einstein tensor and energy-momentum tensor and  $\Lambda(E/E_p)$  and  $G(E/E_p)$  are energy-dependent cosmological constant and Newton constant. Generally, many forms of rainbow functions have been discussed in literatures; in this paper we will mainly employ the following rainbow functions:

$$\begin{aligned} f \left( \frac{E}{E_p} \right) &= 1, \\ g \left( \frac{E}{E_p} \right) &= \sqrt{1 - \eta \left( \frac{E}{E_p} \right)^n}, \end{aligned} \quad (6)$$

which has been widely used in [10–18].

Recently, Schwarzschild black holes, Schwarzschild AdS black holes, and Reissner-Nordstrom black holes in rainbow gravity [19–21] have been studied. Ahmed Farag Alia, Mir Faizald, and Mohammed M. Khalile [15] studied the deformed temperature about charged AdS black holes in rainbow gravity based on Heisenberg Uncertainty Principle (HUP),  $E = \Delta p \sim 1/r_+$ . In this paper, we study the thermodynamical property about the charged AdS black holes in rainbow gravity based on the usual HUP,  $p = \Delta p \sim 1/r_+$ . Moreover, we study how the mass of test particle influences thermodynamical property for charged AdS black holes.

The paper is organized as follows. In the next section, by using the HUP and the modified dispersion relation, we obtain deformed temperature, and we also calculate heat capacity with a fixed charge and discuss the thermal stability. In Section 3, we find the charged AdS black holes have similar behaviour with the liquid-gas system with the pressure given by the cosmological constant while we treat the black holes charge  $Q$  as a fixed external parameter, not a thermodynamic variable. We also calculate the Gibbs free energy and find characteristic swallow tail behavior. Finally, the conclusion and discussion will be offered in Section 4.

## 2. The Thermal Stability

In rainbow gravity the line element of the modified charged AdS black holes can be described as [15]

$$ds^2 = -\frac{N}{f^2} dt^2 + \frac{1}{Ng^2} dr^2 + \frac{r^2}{g^2} d\Omega^2, \quad (7)$$

where

$$N = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} + \frac{r^2}{l^2}. \quad (8)$$

Generally,  $-3/l^2 = \Lambda$  which is cosmological constant. Because all energy dependence in the energy-independent coordinates must be in the rainbow functions  $f$  and  $g$ ,  $N$  is independent on the energy of test particle [8]. In gravity's rainbow, the deformed temperature related to the standard temperature  $T_0$  was [15]

$$T = -\frac{1}{4\pi r} \lim_{r \rightarrow r_+} \sqrt{\frac{-g^{11} (g^{00})'}{g^{00} g^{00}}} = \frac{g(E/E_p)}{f(E/E_p)} T_0, \quad (9)$$

where  $r_+$  is horizon radius.

In gravity's rainbow, although the metric depends on the energy of test particle, the usual HUP can be still used [19]. For simplicity we take  $n = 2$  in the following discussion, by combining (1) with (6) we can get

$$g = \sqrt{1 - \eta G_0 m^2} \sqrt{\frac{r_+^2}{r_+^2 + \eta G_0}}, \quad (10)$$

where  $G_0 = 1/E_p^2$ ,  $m$  is the mass of test particle, and  $\eta$  is a constant parameter.

Generally, the standard temperature was given by [22]

$$T_0 = \frac{1}{4\pi} \left( \frac{1}{r_+} + \frac{3r_+}{l^2} - \frac{Q^2}{r_+^3} \right). \quad (11)$$

When using (6) and (10), we can get the temperature of charged AdS black holes in rainbow gravity

$$T = gT_0 = \frac{1}{4\pi k} \sqrt{\frac{r_+^2}{r_+^2 + \eta G_0}} \left( \frac{1}{r_+} + \frac{3r_+}{l^2} - \frac{Q^2}{r_+^3} \right), \quad (12)$$

where  $k = 1/\sqrt{1 - \eta G_0 m^2}$ . It is easy to find  $T = T_0$  when  $\eta = 0$ . Equation (12) shows that there are two solutions when  $T = 0$ , one corresponds to extreme black hole and the other to  $m^2 = 1/\eta G$ . The second solution indicates the temperature of black holes completely depends on the mass of test particle when the black holes keep with fixed mass, charge, and anti-de Sitter radius. The bigger the mass of test particle is, the smaller the temperature of black holes is. When  $m^2 = 1/\eta G$ , the temperature keeps zero. Generally, due to gravity's rainbow, a minimum radius with respect to the black hole is given and is related to a radius of black hole remnant when the temperature tends to zero [15]. However, our paper shows all black holes can keep zero temperature when the test particle mass approaches a value, such as  $m^2 = 1/\eta G$ . But due to  $m \ll M_p$  in general condition, it may be difficult to test the phenomenon with zero temperature about black holes.

In general, the thermal stability can be determined by the heat capacity, which is also used to the systems of black holes [17, 23–25]. In other words, the positive heat capacity corresponds to a stable state and the negative heat capacity corresponds to unstable state. In following discussions, we

will focus on the heat capacity to discuss the stability of black holes. When  $N = 0$ , the mass of charged AdS black holes can be calculated as

$$M = \frac{1}{2} \left( r_+ + \frac{Q^2}{r_+} + \frac{r_+^3}{l^2} \right). \quad (13)$$

Based on the first law  $dM = TdS$  with the deformed temperature [20], the modified entropy can be computed

$$S = \int \frac{dM}{T} \quad (14)$$

$$= \pi k r_+ \sqrt{r_+^2 + \eta G_0} + \pi k \eta G_0 \ln \left( r_+ + \sqrt{r_+^2 + \eta G_0} \right).$$

Note that the next leading order is logarithmic as  $S \approx \pi r_+^2 + (1/2)\pi\eta G_0 \ln(4r_+^2)$ , which is similar to the quantum correction in [26–31]. With  $A = 4\pi r_+^2$  we can get  $S \approx A/4 + (1/2)\pi\eta G_0 \ln(A/\pi)$ . We can find the result is in agreement with the standard entropy  $S = A/4$  when  $\eta = 0$ , which is standard condition.

The heat capacity with a fixed charge can be calculated as

$$C_Q = T \frac{dS}{dT} = \left( \frac{\partial M / \partial r_+}{\partial T / \partial r_+} \right) \quad (15)$$

$$= 2\pi k \frac{(-Q^2 l^2 r_+^2 + l^2 r_+^4 + 3r_+^6)(r_+^2 + \eta G_0)^{3/2}}{3r_+^7 + (6\eta G_0 - l^2)r_+^5 + 3Q^2 l^2 r_+^3 + 2\eta G_0 Q^2 l^2 r_+},$$

which shows that  $C_Q$  reduces to standard condition [22] with  $\eta = 0$ . Obviously, the heat capacity is diverging when  $m^2 = 1/\eta G$ . Generally, when the temperature vanishes, the heat capacity also tends to zero. However, our paper shows a different and anomalous phenomenon. Fortunately, the phenomenon is just an observation effect; the result gives us a way to test the theory of rainbow gravity. Some of the conditions above indicate that the mass of test particle does not influence the forms of temperature, entropy, and heat capacity but only changes their amplitudes.

The numerical methods indicate there are three situations corresponding to zero, one, and two diverging points of heat capacity respectively, which have been described in Figures 1, 2, and 3. Figure 1 shows a continuous phase and does not appear phase transition with  $l < l_c$ . In Figure 2, there are one diverging point and two stable phases for  $C_Q > 0$  with  $l = l_c$ , phase 1 and phase 2, which individually represent a phase of large black hole (LBH) and a small black hole (SBH). In Figure 3, one can find there are three phases and two diverging points with  $l > l_c$ . Phase 1 experiences a continuous process from a unstable phase  $C_Q < 0$  to a stable phase  $C_Q > 0$ ; phase 2 is a pure unstable phase with  $C_Q < 0$ ; phase 3 is a stable phase with  $C_Q > 0$ . It is easy to see phase 1 represents the phase of SBH and phase 3 represents the phase of LBH. However, there is a special unstable phase 2 between phase 1 and phase 3. This indicates when the system evolves from phase 3 to phase 1, the system must experience a medium unstable state which could be explained as an exotic quark-gluon plasma with negative heat capacity [32, 33].

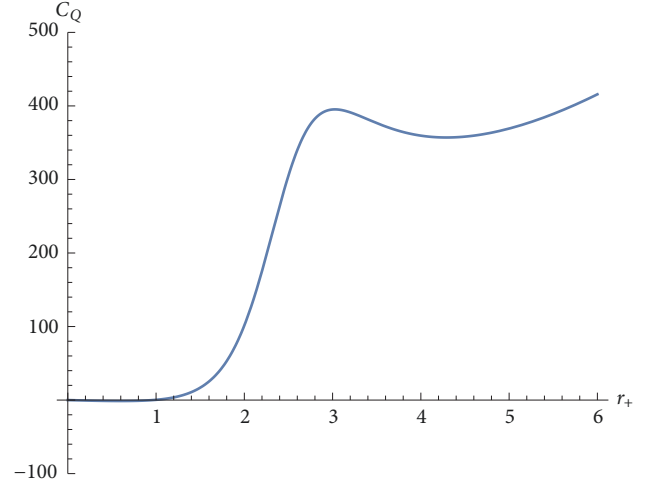


FIGURE 1:  $C_Q - r_+$  diagram of charged AdS black holes in the rainbow gravity. It corresponds to  $l = 6$ . We have set  $Q = 1, \eta = 1, m = 0$ .

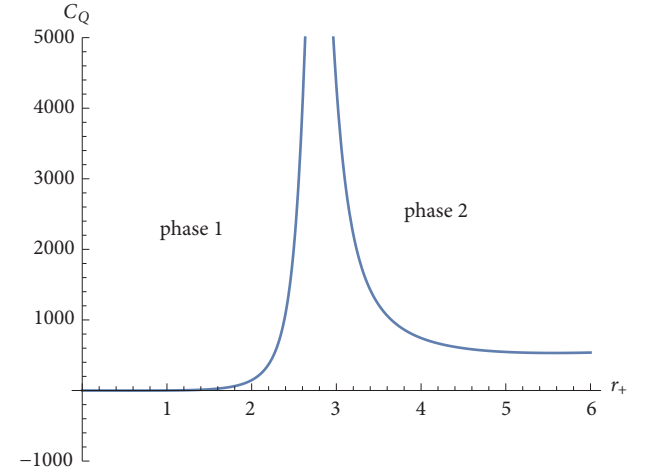


FIGURE 2:  $C_Q - r_+$  diagram of charged AdS black holes in the rainbow gravity. It corresponds to  $l = 7.05$ . We have set  $Q = 1, \eta = 1, m = 0$ .

### 3. The Phase Transition of Charged AdS Black Holes in Extending Phase Space

Surprisingly, although rainbow functions modify the  $\Lambda(E/E_p)$  term, they do not affect thermodynamical pressure related to the cosmological constant [17, 34]. So we can take the following relation:

$$P = -\frac{\Lambda(0)}{8\pi} = \frac{3}{8\pi l^2}. \quad (16)$$

Since David Kubiznak and Robert B. Mann have showed the critical behaviour of charged AdS black holes and completed the analogy of this system with the liquid-gas system [22], in what follows we will study whether the critical behavior of the charged AdS black holes system in rainbow

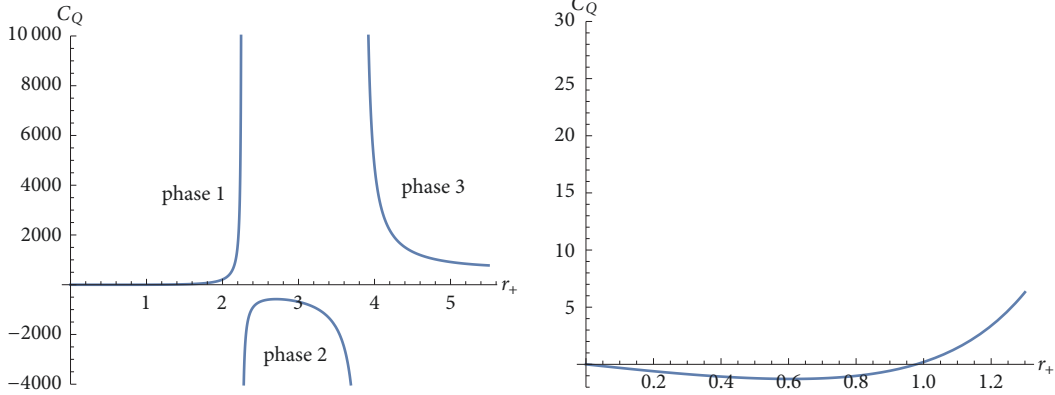


FIGURE 3:  $C_Q - r_+$  diagram of charged AdS black holes in the rainbow gravity. It corresponds to  $l = 8$ , right one corresponds to an extending part of near  $r_+ = 1$ . We have set  $\eta = 1, Q = 1, m = 0$ .

gravity is kept. Using (12) and (16) in extended phase space, we can get

$$P = \frac{k}{2} \sqrt{\frac{r_+^2 + \eta G_0}{r_+^4}} T - \frac{1}{8\pi} \frac{1}{r_+^2} + \frac{1}{8\pi} \frac{Q^2}{r_+^4}. \quad (17)$$

Similarly with [17], the critical point is obtained from

$$\begin{aligned} \frac{\partial P}{\partial r_+} &= 0, \\ \frac{\partial^2 P}{\partial r_+^2} &= 0, \end{aligned} \quad (18)$$

which leads to

$$r_c = \sqrt{\frac{2^{4/3} \eta G_0 Q^2 + 2^{4/3} Q^4 + 2Q^2 (x+y)^{1/3} + 2^{2/3} (x+y)^{2/3}}{(x+y)^{1/3}}}, \quad (19)$$

$$T_c = \frac{1}{2\pi k} \frac{r_c^2 - 2Q^2}{r_c^4 + 2\eta G_0 r_c^2} \sqrt{r_c^2 + \eta G_0},$$

$$P_c = \frac{r_c^4 - 3Q^2 r_c^2 - 2\eta G_0 Q^2}{8\pi r_c^4 (r_c^2 + 2\eta G_0)},$$

where  $x = \eta G_0 Q^2 (\eta G_0 + Q^2)$  and  $y = Q^2 (\eta G_0 + Q^2) (\eta G_0 + 2Q^2)$ . We can obtain

$$\frac{P_c r_c}{T_c} = k \frac{r_c^4 - 3Q^2 r_c^2 - 2\eta G_0 Q^2}{4r_c (r_c^2 - 2Q^2) \sqrt{r_c^2 + \eta G_0}}, \quad (20)$$

which shows the critical ratio is deformed due to the existence of rainbow gravity. It is notable that (20) will back to the usual ratio with  $\eta = 0$ . Generally, for charged AdS black holes, the pressure and temperature are demanded as positive real value. From (19), when  $P_c > 0$  and  $T_c > 0$ , we have

$$\begin{aligned} r_c^4 - 3Q^2 r_c^2 - 2\eta G_0 Q^2 &> 0, \\ r_c &> \sqrt{2}Q, \end{aligned} \quad (21)$$

which indicates a restriction between  $Q$  and  $\eta$ . The  $P - r_+$  diagram has been described in Figure 4. From Figure 4, we can find that charged AdS black holes in rainbow gravity have an analogy with the Van-der-Waals system and have a first-order phase transition with  $T < T_c$ . Namely, when considering rainbow gravity with the form of (6), the behavior like Van-der-Waals system can also be obtained.

Based on [14, 35] the black hole mass is identified with the enthalpy, rather than the internal energy, so the Gibbs free energy for fixed charge in the rainbow gravity will be

$$\begin{aligned} G &= H - TS \\ &= \frac{1}{2} \left( r_+ + \frac{Q^2}{r_+} + \frac{r_+^3}{l^2} \right) - kT (\pi r_+ \sqrt{r_+^2 + \eta G_0} \\ &\quad + \pi \eta G_0 \ln (r_+ + \sqrt{r_+^2 + \eta G_0})), \end{aligned} \quad (22)$$

which has been showed in Figure 5. Because the picture of  $G$  demonstrates the characteristic swallow tail behaviour, there is a first-order transition in the system.

## 4. Conclusion

In this paper, we have studied the thermodynamic behavior of charged AdS black holes in rainbow gravity. By the modified dispersion relation and HUP, we got deformed temperature in charged AdS black holes using no-zero mass of test particle. We have discussed the divergence about the heat capacity with a fixed charge. Our result shows that the phase structure has a relationship with AdS radius  $l$ . When  $l = l_c$ , there is only one diverging point about heat capacity; when  $l > l_c$ , we have found there are two diverging points and three phases including two stable phases and one unstable phase. In particular, an analogy between the charged AdS black holes in the rainbow gravity and the liquid-gas system is discussed. We have also showed  $P - r_+$  critical behavior about the charged AdS black holes in the rainbow gravity. The consequence shows there is the Van-der-Waals like behavior in the rainbow gravity when  $\eta$  and  $Q$  coincide with (21). The rainbow functions deform the forms of critical pressure,

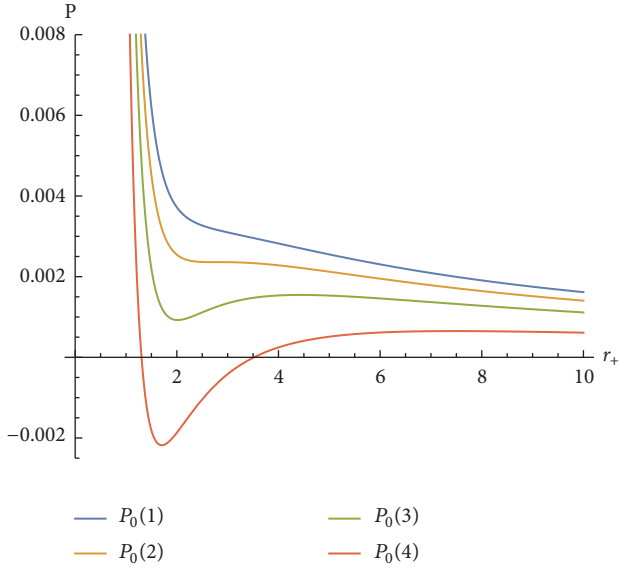


FIGURE 4:  $P - r_+$  diagram of charged AdS black holes in the rainbow gravity. The temperature of isotherms decreases from top to bottom. The  $P_0(1)$  line corresponds to one-phase for  $T > T_c$ . The critical state,  $T_c = 0.0358$ , is denoted by the  $P_0(2)$  line. The lowest two lines correspond to the smaller temperature than the critical temperature. We have set  $Q = 1, \eta = 1, m = 0$ .

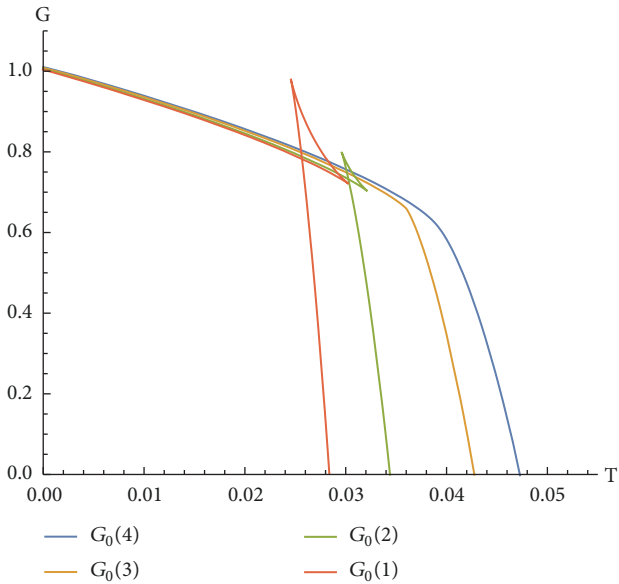


FIGURE 5: Gibbs free energy of charged AdS black holes in rainbow gravity. The blue line  $G_0(3)$  corresponds to the critical pressure  $P_c \approx 0.0024$ , the line  $G_0(4)$  corresponds to pressure  $P > P_c$ , and the others corresponds to pressure  $P < P_c$ . We have set  $Q = 1, \eta = 1, m = 0$ .

temperature, and radius. At last, we have discussed the Gibbs free energy and have obtained characteristic “swallow tail” behaviour which can be the explanation of first-order phase transition.

We find the mass of test particle does not influence the forms of temperature, entropy, and heat capacity but only

changes their amplitudes. Moreover, there is a special value about the mass of test particle encountered  $m^2 = 1/\eta G$ , which would lead to zero temperature and diverging heat capacity for charged AdS black holes in rainbow gravity.

## Data Availability

No data were used to support this study.

## Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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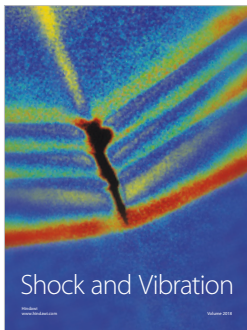
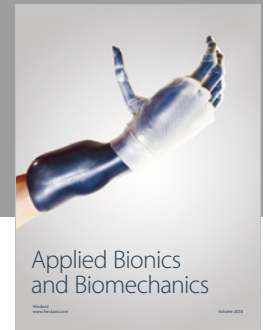
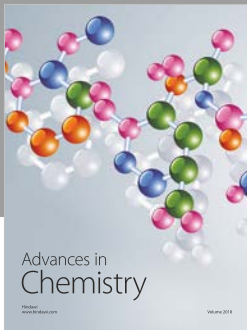
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## References

- [1] R. Iengo, J. G. Russo, and M. Serone, “Renormalization group in Lifshitz-type theories,” *Journal of High Energy Physics*, vol. 2009, no. 11, 2009.
- [2] A. Adams, N. Arkani-Hamed, S. Dubovsky, A. Nicolis, and R. Rattazzi, “Causality, analyticity and an IR obstruction to UV completion,” *Journal of High Energy Physics*, vol. 2006, no. 10, 2006.
- [3] B. M. Gripaios, “Modified gravity via spontaneous symmetry breaking,” *Journal of High Energy Physics*, vol. 2004, no. 10, 2004.
- [4] J. Alfaro, P. González, and Á. Ricardo, “Electroweak standard model with very special relativity,” *Physical Review D*, vol. 91, no. 10, Article ID 105007, 2015.
- [5] H. Belich and K. Bakke, “Geometric quantum phases from Lorentz symmetry breaking effects in the cosmic string space-time,” *Physical Review D: Particles, Fields, Gravitation and Cosmology*, vol. 90, no. 2, 2014.
- [6] J. Magueijo and L. Smolin, “String theories with deformed energy-momentum relations, and a possible nontachyonic bosonic string,” *Physical Review D: Particles, Fields, Gravitation and Cosmology*, vol. 71, Article ID 026010, 2005.
- [7] J. Magueijo and L. Smolin, “Generalized Lorentz invariance with an invariant energy scale,” *Physical Review D: Particles, Fields, Gravitation and Cosmology*, vol. 67, Article ID 044017, 2003.
- [8] J. Magueijo and L. Smolin, “Gravity’s rainbow,” *Classical and Quantum Gravity*, vol. 21, pp. 1725–1736, 2004.
- [9] J.-J. Peng and S.-Q. Wu, “Covariant anomaly and Hawking radiation from the modified black hole in the rainbow gravity theory,” *General Relativity and Gravitation*, vol. 40, no. 12, pp. 2619–2626, 2008.
- [10] G. Amelino-Camelia, J. Ellis, N. E. Mavromatos, and D. V. Nanopoulos, “Distance measurement and wave dispersion in a Liouville-string approach to quantum gravity,” *International Journal of Modern Physics A*, vol. 12, no. 3, pp. 607–623, 1997.
- [11] U. Jacob, F. Mercati, G. Amelino-Camelia, and T. Piran, “Modifications to Lorentz invariant dispersion in relatively boosted frames,” *Physical Review D: Particles, Fields, Gravitation and Cosmology*, vol. 82, no. 8, Article ID 084021, 2010.



- [12] G. Amelino-Camelia, “Quantum-spacetime phenomenology,” *Living Reviews in Relativity*, vol. 16, no. 1, p. 5, 2013.
- [13] A. F. Ali, “Black hole remnant from gravity’s rainbow,” *Physical Review D: Particles, Fields, Gravitation and Cosmology*, vol. 89, no. 10, Article ID 104040, 2014.
- [14] A. F. Ali, M. Faizal, and M. M. Khalil, “Absence of black holes at LHC due to gravity’s rainbow,” *Physics Letters. B. Particle Physics, Nuclear Physics and Cosmology*, vol. 743, pp. 295–300, 2015.
- [15] A. F. Ali, M. Faizal, and M. M. Khalil, “Remnant for all black objects due to gravity’s rainbow,” *Nuclear Physics B*, vol. 894, pp. 341–360, 2015.
- [16] S. H. Hendi and M. Faizal, “Black holes in Gauss-Bonnet gravity’s rainbow,” *Physical Review D: Particles, Fields, Gravitation and Cosmology*, vol. 92, no. 4, Article ID 044027, 2015.
- [17] S. H. Hendi, B. Eslam Panah, and S. Panahiyan, “Three-dimensional dilatonic gravity’s rainbow: exact solutions,” *PTEP. Progress of Theoretical and Experimental Physics*, vol. 76, no. 5, pp. 1–15, 2016.
- [18] S. Gangopadhyay and A. Dutta, “Constraints on rainbow gravity functions from black-hole thermodynamics,” *EPL (Europhysics Letters)*, vol. 115, no. 5, 2016.
- [19] Y. Gim and W. Kim, “Thermodynamic phase transition in the rainbow Schwarzschild black hole,” *Journal of Cosmology and Astroparticle Physics*, vol. 2014, no. 10, 2014.
- [20] Y.-W. Kim, S. K. Kim, and Y.-J. Park, “Thermodynamic stability of modified Schwarzschild–AdS black hole in rainbow gravity,” *The European Physical Journal C*, vol. 76, no. 10, p. 557, 2016.
- [21] C. Liu, “Charged Particle’s Tunneling in a Modified Reissner-Nordstrom Black Hole,” *International Journal of Theoretical Physics*, vol. 53, no. 1, pp. 60–71, 2014.
- [22] D. Kubiznak and R. B. Mann, “P-V criticality of charged AdS black holes,” *Journal of High Energy Physics*, vol. 2012, no. 7, pp. 1–25, 2012.
- [23] G.-M. Deng, J. Fan, X. Li, and Y.-C. Huang, “Thermodynamics and phase transition of charged AdS black holes with a global monopole,” *International Journal of Modern Physics A*, vol. 33, no. 3, Article ID 1850022, 2018.
- [24] G. Deng and Y. Huang, “Q- $\Phi$  criticality and microstructure of charged AdS black holes in  $f(R)$  gravity,” *International Journal of Modern Physics A*, vol. 32, no. 35, Article ID 1750204, 2017.
- [25] S. H. Hendi and M. Momennia, “AdS charged black holes in Einstein–Yang–Mills gravity’s rainbow: Thermal stability and p-v criticality,” *Physics Letters B*, vol. 777, pp. 222–234, 2018.
- [26] D. V. Fursaev, “Temperature and entropy of a quantum black hole and conformal anomaly,” *Physical Review D: Particles, Fields, Gravitation and Cosmology*, vol. 51, no. 10, Article ID R5352, 1995.
- [27] R. K. Kaul and P. Majumdar, “Logarithmic correction to the Bekenstein-Hawking entropy,” *Physical Review Letters*, vol. 84, no. 23, pp. 5255–5257, 2000.
- [28] S. Das, P. Majumdar, and R. K. Bhaduri, “General logarithmic corrections to black-hole entropy,” *Classical and Quantum Gravity*, vol. 19, no. 9, pp. 2355–2367, 2002.
- [29] A. Chatterjee and P. Majumdar, “Universal canonical black hole entropy,” *Physical Review Letters*, vol. 92, no. 14, Article ID 141301, 2004.
- [30] F. J. Wang, Y. X. Gui, and C. R. Ma, “Entropy corrections for Schwarzschild black holes,” *Physics Letters. B. Particle Physics, Nuclear Physics and Cosmology*, vol. 660, no. 3, pp. 144–146, 2008.
- [31] B. Eslam Panah, “Effects of energy dependent spacetime on geometrical thermodynamics and heat engine of black holes: Gravity’s rainbow,” *Physics Letters. B. Particle Physics, Nuclear Physics and Cosmology*, vol. 787, pp. 45–55, 2018.
- [32] P. Burikham and T. Chullaphan, “Comments on holographic star and the dual QGP,” *Journal of High Energy Physics*, vol. 2014, no. 5, 2014.
- [33] P. Burikham and C. Promsiri, “The Mixed Phase of Charged AdS Black Holes,” *Advances in High Energy Physics*, vol. 2016, 2016.
- [34] S. H. Hendi, S. Panahiyan, B. Eslam Panah, M. Faizal, and M. Momennia, “Critical behavior of charged black holes in Gauss-Bonnet gravity’s rainbow,” *Physical Review D: Particles, Fields, Gravitation and Cosmology*, vol. 94, no. 2, Article ID 024028, 2016.
- [35] D. Kastor, S. Ray, and J. Traschen, “Enthalpy and the mechanics of AdS black holes,” *Classical and Quantum Gravity*, vol. 26, no. 19, Article ID 195011, 16 pages, 2009.



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