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Research Article

A New Model for Calculating the Ground and Excited States Masses Spectra of Doubly Heavy Ξ Baryons

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Since the doubly heavy baryons masses are experimentally unknown (except Ξ_{cc}^{+} and Ξ_{cc}^{++}), we present the ground state masses and the positive and negative parity excited state masses of doubly heavy Ξ baryons. For this purpose, we have solved the six-dimensional hyperradial Schrödinger equation analytically for three particles under the hypercentral potential by using the ansatz approach. In this paper, the hypercentral potential is regarded as a combination of the color Coulomb plus linear confining term and the six-dimensional harmonic oscillator potential. We also added the first-order correction and the spin-dependent part contains three types of interaction terms (the spin-spin term, spin-orbit term, and tensor term) to the hypercentral potential. Our obtained masses for the radial excited states and orbital excited states of Ξ_{ccd}, Ξ_{ccu}, Ξ_{bbd}, Ξ_{bbu}, Ξ_{bcd}, and Ξ_{bcu} systems are compared with other theoretical reports, which could be a beneficial tool for the interpretation of experimentally unknown doubly heavy baryons spectrum.

1. Introduction

The doubly heavy baryons have two heavy quarks (c and b) with a light quark (d or u or s). The doubly heavy Ξ baryons family have up or down quarks but Ω family has a light strange quark and their masses spectra have been predicted in the quark model [1]. The SELEX collaboration announced only the experimental mass for the ground state of Ξ_{cc}^{+} baryon and LHCb has determined the ground state of Ξ_{cc}^{++} baryon mass while no triply heavy baryons have been observed yet [2]. Recently experiments and theoretical outcomes have been used in studying the heavy baryons. A lot of new experimental results have been reported by various experimental facilities like CLEO, Belle, BaBar, LHCb, and so forth [3, 4] on ground states and many new excited states of heavy flavor baryons. Bottom baryons are investigated at LHC and Lattice QCD whereas charm baryons are announced at the B-factories [5, 6]. On the other hand, the theoretical works are providing new results for doubly heavy baryons like the Hamiltonian model [7], relativistic quark model [8], the chiral unitary model [9], QCD sum rule [10, 11], and many more. Single- and double- heavy baryons in the constituent quark model were studied by Yoshida et al. They used a model in which there were two exceptions, a color Coulomb term depending on quark masses and an antisymmetric L.S force. They studied the low-lying negative parity states and structures within the framework of a constituent quark model [7]. In [12], the authors calculated the masses of baryons with the quadratic mass relations for ground and orbitally excited states. Wei et al. estimated the masses of singly, doubly, and triply bottom baryons in [13]. Then they studied the linear mass relations and quadratic mass relations.

The light flavor dependence of the singly and doubly charmed states is investigated by Rubio et al. They focused on searching the masses of charmed baryons with positive and negative parity [5]. In [14], the authors used lattice QCD for baryons containing one, two, or three heavy quarks. They applied nonrelativistic QCD for the bottom quarks and relativistic heavy-quark action for the charm quarks. Padmanath et al. determined the ground and excited state spectra of doubly charmed baryons from lattice QCD with dynamical quark fields [15]. The mass of the heavy baryons with two heavy b or c quarks for spin 1/2 in the framework of

QCD sum rules is estimated by Aliev et al. They use the most general form of the interpolating current in its symmetric and antisymmetric forms with respect to the exchange of heavy quarks, to calculate the two point correlation functions describing the baryons under consideration [16]. The authors calculated the masses and residues of the spin 3/2 doubly heavy baryons within the QCD sum rules method. In [17], Eakins et al. ignored all spin-dependent interactions and assume a flavor independent potential, working in the limit where the two heavy quarks are massive enough that their motion can be treated as essentially nonrelativistic, and QCD interactions can be well described by an adiabatic potential [18]. The three-quark problem was solved by Valcarce et al. by means of the Faddeev method in momentum space [19].

The masses of the ground and excited states of the doubly heavy baryons were calculated by Ebert et al. baryons on the basis of the quark-diquark approximation in the framework of the relativistic quark model [20]. In [21], the authors, in the model with the quark-diquark factorization of wave functions, estimated the spectroscopic characteristics of baryons containing two heavy quarks. Albertus et al. used five different quark-quark potentials that include a confining term plus Coulomb and hyperfine terms coming from one–gluon exchange. They solved the three-body problem by means of a variational ansatz made possible by heavy-quark spin symmetry constraints [22].

In this study, we have used the hypercentral constituent quark model (hCQM) with Coulombic-like term plus a linear confining term and the harmonic oscillator potential [23]. We also added the first-order correction and the spin-dependent part to the potential and calculation has been performed by solving six-dimensional hyperradial Schrödinger equations by using the ansatz method. We have obtained the mass spectra of radial excited states up to 5S and orbital excited states for 1P-5P, 1D-4D, and 1F-2F states.

This paper is organized as follows: we briefly present the hypercentral constituent quark model and introduce the interaction potentials between three quarks in doubly heavy baryons in Section 2. In Section 3, we present the exact analytical solution of the hyperradial Schrödinger equation for our proposed potential. In Section 4, our masses spectra results for ground, radial, and orbital excited states of baryon family with six members are given and compared with other predictions. We present the conclusions in Section 5.

2. Theoretical Framework: The HCQM Model and Hypercentral Potential

The hypercentral model has been applied to solve bound states and scattering problems in many various fields of physics. In this model, we consider baryons as three-body systems of constituent quarks. In the center of mass frame, the internal quark motion is described by the Jacobi coordinates (ρ and λ) [37] and the respective reduced masses are given by

$$m_\rho = \frac{2m_1m_2}{m_1+m_2},\qquad m_\lambda = \frac{2m_3\left(m_1^2+m_2^2+m_1m_2\right)}{\left(m_1+m_2\right)\left(m_1+m_2+m_3\right)} \tag{1}$$

Here m_1, m_2, and m_3 are the current quark masses. In order to describe three-quark dynamics, we define hyperradius $x = \sqrt{\rho^2+\lambda^2}$ and hyperangle $\xi = \arctan(\rho/\lambda)$ [38]. In present work, the confining three-body potential is regarded as a combination of three hypercentral interacting potentials. First, the six-dimensional hyper-Coulomb potential $V_{hyc}(x) = \tau/x$, which is attractive for small separations [39–41], while at large separations a hyper-linear term, $V_{con} = \beta x$, gives rise to quark confinement [42], where β corresponds to the string tension of the confinement [43]. Third, the six-dimension harmonic oscillator potential $V_{h.o.} = px^2$, which has a two-body character and turns out to be exactly hypercentral [44], where p is constant. The solution of the hypercentral Schrödinger equation with Coulombic-like term plus a linear confining term potential cannot be obtained analytically [45]; therefore, Giannini et al. used the dynamic symmetry O(7) of the hyper-Coulomb problem to obtain the hyper-Coulomb Hamiltonian and eigenfunctions analytically and they regarded the linear term as a perturbation. Combination of the color Coulomb plus linear confining term and the six-dimensional harmonic oscillator potential has interesting properties since it can be solved analytically, with a good correspondence to physical results. The first-order correction $V^{(1)}(x)$ can be written as [44–47]

$$V^1(x) = -C_F C_A \frac{{\alpha_S}^2}{4x^2} \tag{2}$$

The parameters $C_F = 2/3$ and $C_A = 3$ are the Casimir charges of the fundamental and adjoint representation. The hyper-Coulomb strength $\tau = -(2/3)\alpha_S$, 2/3 is the color factor for the baryon. α_s is the strong running coupling constant, which is written as

$$\alpha_S = \frac{\alpha_S(\mu_0)}{1+\left(\left(33-2n_f\right)/12\pi\right)\alpha_S(\mu_0)\ln\left(\left(m_1+m_2+m_3\right)/\mu_0\right)} \tag{3}$$

The spin-dependent part $V_{SD}(x)$ is given as

$$V_{SD}(x) = V_{SS}(x)\left(\vec{S}_\rho.\vec{S}_\lambda\right) + V_{\gamma S}(x)\left(\vec{\gamma}.\vec{S}\right) + V_T(x)\left[S^2 - \frac{3\left(\vec{S}.\vec{x}\right)\left(\vec{S}.\vec{x}\right)}{x^2}\right] \tag{4}$$

The spin-dependent potential, $V_{SD}(x)$, contains three types of the interaction terms [48], such as the spin-spin term $V_{SS}(x)$, the spin-orbit term $V_{\gamma S}(x)$, and tensor term $V_T(x)$ described as [35]. Here $S = S_\rho + S_\lambda$, where S_ρ and S_λ are the spin vectors associated with the ρ and λ variables, respectively. The coefficient of these spin-dependent terms of the above equation can be written in terms of the vector, $V_V(x) = \tau/x$, and scalar, $V_S(x) = \beta x + px^2$ parts of the static potential as [38]

$$V_{\gamma s} = \frac{1}{2m_\rho m_\lambda x}\left(3\frac{dV_V}{dx} - \frac{dV_S}{dx}\right) \tag{5}$$

Table 1: The quark mass (in GeV) and the fitted values of the parameters used in our calculations.

m_b	m_c	m_d	m_u	α_S	C_F	C_A	β	ω
4.750	1.348	0.35	0.34	0.340	$\frac{2}{3}$	3	0.02	0.11 fm^{-1}

Table 2: The outcomes ground state masses of Ξ are listed with other theoretical predictions (in GeV). Standard devotion of the result is 0.350.

Baryon	Ξ_{ccd} / Ξ_{ccu}		Ξ_{bbd} / Ξ_{bbu}		Ξ_{bcd} / Ξ_{bcu}	
J^P	$\frac{1}{2}^+$	$\frac{3}{2}^+$	$\frac{1}{2}^+$	$\frac{3}{2}^+$	$\frac{1}{2}^+$	$\frac{3}{2}^+$
Our Calc	3.522 / 3.515	3.696 / 3.689	9.716 / 9.711	9.894 / 9.889	6.628 / 6.622	6.688 / 6.682
Ref.[1]	3.520 / 3.511	3.695 / 3.687	10.317 / 10.312	10.340 / 10.335	6.920 / 6.914	6.986 / 6.980
Ref.[24]	3.519					
Ref.[7]	3.685	3.754	10.314			
Ref.[12, 13]	3.520	3.695	10.199	10.316		
Ref.[5]	3.610	3.694				
Ref.[14]	3.610	3.692	10.143	10.178	6.943	6.985
Ref.[25]	3.561	3.642				
Ref.[17]	3.720		9.960		6.720	
Ref.[18]	3.687	3.752	10.322	10.352	7.014	7.064
Ref.[26]	3.676	3.753	10.340	10.367	7.011	7.074
Ref.[27]	3.547	3.719	10.185	10.216	6.904	6.936
Ref.[19]	3.579	3.656	10.189	10.218		
Ref.[20]	3.620	3.727	10.202	10.237	6.933	6.980
Ref.[21]	3.478	3.610	10.093	10.133	6.820	6.900
Ref.[28]	3.627	3.690	10.162	10.184	6.914	
Ref.[29]	3.519	3.620	9.800	9.980	6.650	6.690
Ref.[22]	3.612	3.706	10.197	10.136	6.919	6.986
Ref.[30]	3.510	3.548	10.130	10.144	6.792	6.827
Ref.[31]	3.570	3.610	10.170	10.220		

$$V_T(x) = \frac{1}{6m_\rho m_\lambda}\left(\frac{3d^2V_V}{d^2x} - \frac{1}{x}\frac{dV_V}{dx}\right) \quad (6)$$

$$V_{SS}(x) = \frac{1}{3m_\rho m_\lambda}\nabla^2 V_V \quad (7)$$

In our model, the hypercentral interaction potential is assumed as follows [48]:

$$V(x) = V^{(0)}(x) + \left(\frac{1}{m_\rho} + \frac{1}{m_\lambda}\right)V^{(1)}(x) + V_{SD}(x) \quad (8)$$

where $V^{(0)}(x)$ is given by

$$\begin{aligned} V^{(0)}(x) &= V_{hyc}(x) + V_{con}(x) + V_{h.o.}(x) \\ &= \frac{\tau}{x} + \beta x + px^2 \end{aligned} \quad (9)$$

The baryons masses are determined by the sum of the model quark masses plus kinetic energy, potential energy, and the spin-dependent interaction as $M_B = \sum m_i + \langle H \rangle$ [49]. First, we have solved the hyperradial Schrödinger equation exactly and find eigenvalue under the proposed potential by using the ansatz approach.

3. The Exact Analytical Solution of the Hyperradial Schrödinger Equation under the Hypercentral Potential

The Hamiltonian of three bodies' baryonic system in the hypercentral constituent quark model is expressed as [50]

$$H = \frac{P_\rho^2}{2m} + \frac{P_\lambda^2}{2m} + V(x) \quad (10)$$

and the hyperradial wave function $\psi_{\nu\gamma}(x)$ is determined by the hypercentral Schrödinger equation. The hyperradial Schrödinger equation corresponding to the above Hamiltonian can be written as [51]

$$\begin{aligned} &\left(\frac{d^2}{dx^2} + \frac{5}{x}\frac{d}{dx} - \frac{\gamma(\gamma+4)}{x^2}\right)\psi_{\nu\gamma}(x) \\ &\quad = -2m[E - V(x)]\psi_{\nu\gamma}(x) \end{aligned} \quad (11)$$

where γ is the grand angular quantum number and given by $\gamma = 2n + l_\rho + l_\lambda$, $n = 0, 1, \ldots$; l_ρ and l_λ are the angular momenta associated with the $\vec{\rho}$ and $\vec{\lambda}$ variable and ν denotes the number of nodes of the space three-quark wave function [36]. In (11), m is the reduced mass which is defined as $m =$

TABLE 3: The masses of radial excited states for doubly heavy Ξ baryons (in GeV). Standard devotions of the result are 0.435 and 0.434.

Baryon	State	J^P	Our Calc	Our Calc	[1]	[1]	[7]	[26]	[27]	[19]	[20]	[18]
Ξ_{ccd} and Ξ_{ccu}	2S	$\frac{1}{2}^+$	3.905	3.901	3.925	3.920	4.079	4.029	4.183	3.976	3.910	4.030
	3S		4.185	4.118	4.233	4.159	4.206		4.640		4.154	
	4S		4.430	4.429	4.502	4.501						
	5S		4.653	4.653	4.748	4.748						
	2S	$\frac{3}{2}^+$	3.962	3.958	3.988	3.983	4.114	4.042	4.282	4.025	4.027	4.078
	3S		4.213	4.211	4.264	4.261	4.131		4.719			
	4S		4.446	4.445	4.520	4.519						
	5S		4.663	4.663	4.759	4.759						
Ξ_{bbd} and Ξ_{bbu}	2S	$\frac{1}{2}^+$	9.984	9.981	10.612	10.609	10.571	10.576	10.751	10.482	10.441	10.551
	3S		10.211	10.211	10.862	10.862	10.612		11.170		10.630	
	4S		10.417	10.418	11.088	11.090					10.812	
	5S		10.606	10.610	11.297	11.301						
	2S	$\frac{3}{2}^+$	9.990	9.988	10.619	10.617	10.592	10.578	10.770	10.501	10.482	10.574
	3S		10.205	10.233	10.855	10.866	10.593		11.184		10.673	
	4S		10.418	10.420	11.090	11.092					10.856	
	5S		10.607	10.611	11.298	11.302						
Ξ_{bcd} and Ξ_{bcu}	2S	$\frac{1}{2}^+$	6.922	6.919	7.244	7.240			7.478			7.321
	3S		7.163	7.161	7.509	7.507			7.904			
	4S		7.379	7.377	7.746	7.744						
	5S		7.576	7.581	7.963	7.964						
	2S	$\frac{3}{2}^+$	6.943	6.939	7.267	7.263			7.495			7.353
	3S		7.174	7.171	7.521	7.518			7.917			
	4S		7.384	7.384	7.752	7.752						
	5S		7.580	7.581	7.968	7.969						

$2m_\rho m_\lambda/(m_\rho + m_\lambda)$ [32]. By regarding $\psi_{\nu\gamma}(x) = x^{-5/2}\varphi_{\nu\gamma}$ [20, 35], (11) reduces to the following form:

$$\varphi''_{\nu\gamma}(x) + \left[\varepsilon - r_1 x^2 - r_2 x - \frac{r_3}{x} - \frac{r_4}{x^2} - \frac{r_5}{x^3} + \frac{r_6}{x^5} + r_7 - \frac{(2\gamma+3)(2\gamma+5)}{4x^2}\right]\varphi_{\nu\gamma}(x) = 0 \tag{12}$$

The hyperradial wave function $\varphi_{\nu\gamma}(x)$ is a solution of the reduced Schrödinger equation for each of the three identical particles with the mass m and interacting potential (8), where

$$\begin{aligned} \varepsilon &= 2mE, \\ r_1 &= 2mp, \\ r_2 &= 2m\beta, \\ r_3 &= 2m\tau, \\ r_4 &= 2m\left(\frac{1}{m_\rho} + \frac{1}{m_\lambda}\right)\left(-C_f C_A \frac{\alpha_s^2}{4}\right), \\ r_5 &= 2m\left[\frac{2\tau}{3m_\rho m_\lambda}\left(S_\rho \cdot S_\lambda\right) - \frac{3\tau}{2m_\rho m_\lambda}\left(\vec{\gamma}\cdot\vec{s}\right) + \frac{7\tau}{6m_\rho m_\lambda}s^2\right], \\ r_6 &= 2m\frac{21\tau}{6m_\rho m_\lambda}\left(\vec{s}\cdot\vec{x}\right)\left(\vec{s}\cdot\vec{x}\right), \\ r_7 &= 2m\left(\frac{(\beta+2p)}{2m_\rho m_\lambda}\left(\vec{\gamma}\cdot\vec{s}\right)\right). \end{aligned} \tag{13}$$

We suppose the $\varphi_{\nu\gamma} = h(x)e^{g(x)}$ form for the wave function. Now we make use of the ansatz for $h(x)$ and $g(x)$ [33, 34]:

$$\begin{aligned} h(x) &= \Pi\left(x - a_i^\nu\right) \quad \nu = 1, 2, \ldots, \\ h(x) &= 1 \quad \nu = 0 \\ g(x) &= a\ln x + qx^2 + cx + \frac{d}{x} \end{aligned} \tag{14}$$

Table 4: The masses of orbital excited states for Ξ_{cc} baryon (in GeV). Standard devotions of the result are 0.072 and 0.068.

(a)

State	Our cal Ξ_{cc}^{+}	Our Cal Ξ_{cc}^{++}	[1] Ξ_{cc}^{+}	[1] Ξ_{cc}^{++}	[7]	[26]	[19]	[20]	[12]	[21]	[18]	[5]
$(1^2\ P_{1/2})$	3.851	3.847	3.865	3.861	3.947	3.910	3.880	3.838			4.073	3.892
$(1^2\ P_{3/2})$	3.834	3.830	3.847	3.842	3.949	3.921		3.959	3.786	3.834	4.079	3.989
$(1^4\ P_{1/2})$	3.860	3.856	3.875	3.871								
$(1^4\ P_{3/2})$	3.842	3.838	3.856	3.851								
$(1^4\ P_{5/2})$	3.873	3.872	3.890	3.888	4.163	4.092		4.155	3.949	4.047	4.089	
$(2^2\ P_{1/2})$	4.120	4.101	4.161	4.140	4.135	4.074	4.018	4.085				
$(2^2\ P_{3/2})$	4.104	4.101	4.144	4.140	4.137	4.078	4.197					
$(2^4\ P_{1/2})$	4.127	4.125	4.169	4.167								
$(2^4\ P_{3/2})$	4.111	4.109	4.152	4.149								
$(2^4\ P_{5/2})$	4.140	4.138	4.183	4.181	4.488							
$(3^2\ P_{1/2})$	4.361	4.345	4.426	4.409	4.149							
$(3^2\ P_{3/2})$	4.347	4.345	4.411	4.409	4.159							
$(3^4\ P_{1/2})$	4.367	4.366	4.433	4.432								
$(3^4\ P_{3/2})$	4.354	4.352	4.419	4.417								
$(3^4\ P_{5/2})$	4.336	4.333	4.399	4.396	4.534							
$(4^2\ P_{1/2})$	4.583	4.583	4.671	4.671								
$(4^2\ P_{3/2})$	4.571	4.571	4.658	4.657								
$(4^4\ P_{1/2})$	4.590	4.590	4.678	4.678								
$(4^4\ P_{3/2})$	4.577	4.577	4.664	4.664								
$(4^4\ P_{5/2})$	4.561	4.561	4.646	4.646								
$(5^2\ P_{1/2})$	4.792	4.793	4.901	4.902								
$(5^2\ P_{3/2})$	4.781	4.781	4.889	4.889								
$(5^4\ P_{1/2})$	4.799	4.800	4.908	4.909								
$(5^4\ P_{3/2})$	4.705	4.788	4.895	4.896								
$(5^4\ P_{5/2})$	4.771	4.772	4.878	4.879								
$(1^4\ D_{1/2})$	4.043	4.038	4.077	4.071								
$(1^2\ D_{3/2})$	4.018	4.013	4.049	4.044								
$(1^4\ D_{3/2})$	4.026	4.022	4.058	4.053								
$(1^2\ D_{5/2})$	3.995	3.991	4.024	4.019	4.043	4.115	4.047		4.391	4.034	4.050	4.388
$(1^4\ D_{5/2})$	4.003	4.000	4.033	4.029	4.027	4.052		4.187	4.089	4.393		
$(1^4\ D_{7/2})$	3.975	3.972	4.002	3.998	4.097							
$(2^4\ D_{1/2})$	4.287	4.284	4.345	4.342								
$(2^2\ D_{3/2})$	4.265	4.262	4.321	4.318								
$(2^4\ D_{3/2})$	4.272	4.270	4.329	4.326								
$(2^2\ D_{5/2})$	4.245	4.243	4.299	4.297	4.164	4.091						
$(2^4\ D_{5/2})$	4.252	4.251	4.307	4.305								
$(2^4\ D_{7/2})$	4.228	4.226	4.280	4.278	4.394							

(b)

State	Our Cal Ξ_{cc}^{+}	Our Cal Ξ_{cc}^{++}	[1] Ξ_{cc}^{+}	[1] Ξ_{cc}^{++}	[7]	[32]	[33]	[34]	[35]	[33]	[36]	[5]
$(3^4\ D_{1/2})$	4.511	4.511	4.592	4.592	4.511	4.511	4.592	4.592	4.511	4.511	4.592	4.592
$(3^2\ D_{3/2})$	4.492	4.491	4.571	4.570	4.492	4.491	4.571	4.570	4.492	4.491	4.571	4.570
$(3^4\ D_{3/2})$	4.499	4.499	4.578	4.578								
$(3^2\ D_{5/2})$	4.475	4.474	4.552	4.551	4.348							
$(3^4\ D_{5/2})$	4.481	4.481	4.559	4.558								
$(3^4\ D_{7/2})$	4.460	4.459	4.535	4.534								
$(4^4\ D_{1/2})$	4.723	4.724	4.825	4.826								
$(4^2\ D_{3/2})$	4.706	4.706	4.806	4.806								
$(4^4\ D_{3/2})$	4.711	4.712	4.812	4.813								

(b) Continued.

State	**Our Cal** Ξ_{cc}^{+}	**Our Cal** Ξ_{cc}^{++}	**[1]** Ξ_{cc}^{+}	**[1]** Ξ_{cc}^{++}	**[7]**	**[32]**	**[33]**	**[34]**	**[35]**	**[33]**	**[36]**	**[5]**
$(4^2\ D_{5/2})$	4.690	4.690	4.788	4.788								
$(4^4\ D_{5/2})$	4.696	4.696	4.795	4.795								
$(4^4\ D_{7/2})$	4.675	4.675	4.772	4.772								
$(1^4\ F_{3/2})$	4.198	4.193	4.247	4.242								
$(1^2\ F_{5/2})$	4.169	4.164	4.215	4.210								
$(1^4\ F_{5/2})$	4.142	4.172	4.186	4.219								
$(1^4\ F_{7/2})$	4.150	4.147	4.194	4.191								
$(1^2\ F_{7/2})$	4.178	4.139	4.225	4.182					4.267			
$(1^4\ F_{9/2})$	4.118	4.115	4.159	4.156					4.413			
$(2^4\ F_{3/2})$	4.422	4.425	4.494	4.497								
$(2^2\ F_{5/2})$	4.399	4.399	4.468	4.468								
$(2^4\ F_{5/2})$	4.405	4.406	4.475	4.476								
$(2^4\ F_{7/2})$	4.378	4.382	4.445	4.450								
$(2^2\ F_{7/2})$	4.384	4.376	4.452	4.443								
$(2^4\ F_{9/2})$	4.359	4.355	4.424	4.420								

where a, q, c, and d are positive. From (14), we obtain

$$\varphi''(x) = \left[g''(x) + g'^2(x) + \left(\frac{h''(x) + 2h'(x)\,g'(x)}{h(x)}\right)\right] \cdot \varphi(x) \quad (15)$$

Comparing (12) and (15), it can be found that

$$\left[r_1x^2 + r_2x + \frac{r_3}{x} + \frac{r_4}{x^2} + \frac{r_5}{x^3} - \frac{r_6}{x^5} - r_7 + \frac{(2\gamma+3)(2\gamma+5)}{4x^2} - \varepsilon\right] = \left[g''(x) + g'^2(x) + \frac{h''(x) + 2h'(x)\,g'(x)}{h(x)}\right] \quad (16)$$

By substituting (14) into (16), we obtained the following equation:

$$\begin{aligned} &-\varepsilon + r_1x^2 + r_2x + \frac{r_3}{x} + \frac{r_4}{x^2} + \frac{r_5}{x^3} - \frac{r_6}{x^5} - r_7 + \frac{(2\gamma+3)(2\gamma+5)}{4x^2} \\ &= 4q^2x^2 + 4cqx + \frac{(2ac - 4dq)}{x} + \frac{(a^2 - a - 2cd)}{x^2} + \frac{2d(1-a)}{x^3} + \frac{d^2}{x^4} + (c^2 + 2q + 4ac) \end{aligned} \quad (17)$$

By equating the corresponding powers of x on both sides of (17), we can obtain

$$\begin{aligned} a &= \frac{2\tau}{\beta}\sqrt{\frac{mp}{2}}, \\ c &= \frac{m\beta}{2}\sqrt{\frac{2}{mp}}, \\ q &= \sqrt{\frac{mp}{2}}, \\ \varepsilon &= -\left[\frac{m\beta^2}{2p} + 2\sqrt{\frac{mp}{2}} + \frac{4mp\tau}{\beta} + 2m\left(\frac{(\beta+2p)}{2m_\rho m_\lambda}\left(\vec{\gamma}\cdot\vec{s}\right)\right)\right] \end{aligned} \quad (18)$$

Since $p = m\omega^2/2$, we have $a = 2m\omega/2\beta$, $c = \beta/\omega$, $q = m\omega/2$. The energy eigenvalues for the mode $\nu = 0$ and grand angular momentum γ from (13) and (18) are given as follows:

$$E = -\left[\frac{\beta^2}{2m\omega} + \frac{\omega}{2} + \frac{m\omega^2\tau}{\beta} + \left(\frac{(\beta + m\omega^2)}{2m_\rho m_\lambda}\left(\vec{\gamma}\cdot\vec{s}\right)\right)\right] \quad (19)$$

At last for the best doubly heavy baryons masses (Ξ_{ccd}, Ξ_{ccu}, Ξ_{bbd}, Ξ_{bbu}, Ξ_{bcd}, Ξ_{bcu}) predictions, the values of m_u, m_d, m_c, m_b, α_S, ω, and β (which are listed in Table 1) are selected using genetic algorithm. The cost function of a genetic algorithm is the minimum difference between our calculated baryon mass and the reported baryons mass of other works.

Table 5: The masses of orbital excited states for Ξ_{bb} baryon (in GeV).

State	Our cal Ξ_{bb}^{-}	Our Cal Ξ_{bb}^{0}	[1] Ξ_{bb}^{-}	[1] Ξ_{bb}^{0}	[7]	[26]	[19]	[20]	[12]	[18]	Others
$(1^2\ P_{1/2})$	9.895	9.892	10.514	10.511	10.476	10.493	10.406	10.368		10.691	
$(1^2\ P_{3/2})$	9.890	9.887	10.509	10.506	10.476	10.495		10.408	10.474	10.692	10.390 [31]
$(1^4\ P_{1/2})$	9.897	9.895	10.517	10.514							
$(1^4\ P_{3/2})$	9.893	9.890	10.512	10.509							10.430 [17]
$(1^4\ P_{5/2})$	9.901	9.898	10.521	10.518	10.759				10.588	10.695	
$(2^2\ P_{1/2})$	10.127	10.127	10.77	10.77	10.703	10.710	10612	10.563			
$(2^2\ P_{3/2})$	10.124	10.120	10.766	10.762	10.704	10.713		10.607			
$(2^4\ P_{1/2})$	10.129	10.129	10.772	10.772							
$(2^4\ P_{3/2})$	10.126	10.125	10.768	10.767							
$(2^4\ P_{5/2})$	10.121	10.133	10.763	10.776	10.973	10.713					
$(3^2\ P_{1/2})$	10.337	10.338	11.001	11.002	10.740			10.744			
$(3^2\ P_{3/2})$	10.334	10.335	10.997	10.998	10.742			10.788			
$(3^4\ P_{1/2})$	10.339	10.340	11.003	11.004							
$(3^4\ P_{3/2})$	10.336	10.337	10.999	11.000							
$(3^4\ P_{5/2})$	10.331	10.343	10.994	11.007	11.004						
$(4^2\ P_{1/2})$	10.531	10.534	11.214	11.217				10.900			
$(4^2\ P_{3/2})$	10.527	10.530	11.21	11.213							
$(4^4\ P_{1/2})$	10.533	10.536	11.216	11.219							
$(4^4\ P_{3/2})$	10.529	10.532	11.212	11.215							
$(4^4\ P_{5/2})$	10.526	10.538	11.208	11.222							
$(5^2\ P_{1/2})$	10.712	10.716	11.413	11.418							
$(5^2\ P_{3/2})$	10.709	10.714	11.41	11.415							
$(5^4\ P_{1/2})$	10.714	10.718	11.415	11.420							
$(5^4\ P_{3/2})$	10.711	10.716	11.412	11.417							
$(5^4\ P_{5/2})$	10.706	10.721	11.407	11.423							
$(1^4\ D_{1/2})$	10.043	10.041	10.677	10.675							
$(1^2\ D_{3/2})$	10.037	10.035	10.670	10.668							
$(1^4\ D_{3/2})$	10.038	10.037	10.672	10.670						11.011	
$(1^2\ D_{5/2})$	10.030	10.028	10.663	10.661	10.592	10.676			10.742	11.002	
$(1^4\ D_{5/2})$	10.033	10.031	10.666	10.664							
$(1^4\ D_{7/2})$	10.026	10.024	10.658	10.656		10.608			10.853	11.011	
$(2^4\ D_{1/2})$	10.257	10.257	10.913	10.913							
$(2^2\ D_{3/2})$	10.252	10.252	10.907	10.907							
$(2^4\ D_{3/2})$	10.254	10.254	10.909	10.909							
$(2^2\ D_{5/2})$	10.247	10.247	10.901	10.901		10.712					
$(2^4\ D_{5/2})$	10.248	10.248	10.903	10.903	10.613						
$(2^4\ D_{7/2})$	10.242	10.242	10.896	10.896		11.057					
$(3^4\ D_{1/2})$	10.455	10.457	11.13	11.133			4.592	4.592			
$(3^2\ D_{3/2})$	10.450	10.452	11.125	11.127			4.571	4.570			
$(3^4\ D_{3/2})$	10.451	10.454	11.126	11.129							
$(3^2\ D_{5/2})$	10.446	10.447	11.120	11.122							
$(3^4\ D_{5/2})$	10.447	10.449	11.122	11.124	10.809						
$(3^4\ D_{7/2})$	10.442	10.444	11.116	11.118							
$(4^4\ D_{1/2})$	10.639	10.643	11.333	11.337							
$(4^2\ D_{3/2})$	10.635	10.638	11.328	11.332							
$(4^4\ D_{3/2})$	10.636	10.640	11.330	11.334							
$(4^2\ D_{5/2})$	10.631	10.635	11.324	11.328							

Table 5: Continued.

State	Our cal Ξ_{bb}^{-}	Our Cal Ξ_{bb}^{0}	[1] Ξ_{bb}^{-}	[1] Ξ_{bb}^{0}	[7]	[26]	[19]	[20]	[12]	[18]	Others
$(4^4\ D_{5/2})$	10.632	10.636	11.325	11.33							
$(4^4\ D_{7/2})$	10.627	10.631	11.320	11.324							
$(1^4\ F_{3/2})$	10.173	10.172	10.82	10.819							
$(1^2\ F_{5/2})$	10.166	10.165	10.812	10.811							
$(1^4\ F_{5/2})$	10.158	10.167	10.804	10.813							
$(1^4\ F_{7/2})$	10.167	10.160	10.814	10.806							
$(1^2\ F_{7/2})$	10.160	10.157	10.806	10.803					11.004		
$(1^4\ F_{9/2})$	10.152	10.152	10.797	10.797					11.112		
$(2^4\ F_{3/2})$	10.357	10.376	11.022	11.043							
$(2^2\ F_{5/2})$	10.368	10.369	11.035	11.036							
$(2^4\ F_{5/2})$	10.369	10.371	11.036	11.038							
$(2^4\ F_{7/2})$	10.362	10.365	11.028	11.031							
$(2^2\ F_{7/2})$	10.364	10.363	11.030	11.029							
$(2^4\ F_{9/2})$	10.357	10.357	11.022	11.023							

4. Results and Discussions: Mass Spectrum

The ground and excited states of doubly heavy Ξ baryons are unclear to us experimentally (except Ξ_{cc}^{+} and Ξ_{cc}^{++}). Hence, we have obtained the ground and excited state masses of Ξ_{cc}^{+}, Ξ_{cc}^{++}, Ξ_{bb}^{-}, Ξ_{bb}^{0}, Ξ_{bc}^{0}, and Ξ_{bc}^{+} (see Tables 2, 3, 4, 5, and 6, respectively). These mass spectra are estimated by using the hypercentral potential equation (8) in the hypercentral constituent quark model. We begin with the ground state 1S; the masses are computed for both parities $J^P = (1/2)^+$ and $J^P = (3/2)^+$. Our predicted ground state masses of doubly heavy Ξ baryons are compared with other predictions in Table 2.

We can observe that, in the case of Ξ_{cc} baryon, for 2S states $J^P = (1/2)^+$ and $J^P = (3/2)^+$, our predictions are close to [34] and [1], respectively. Our outcomes for 3S state $J^P = (1/2)^+$ of Ξ_{cc} baryon show 21 MeV (with [7]) and $J^P = (3/2)^+$ shows 51 MeV (with [1]) difference. Analyzing the 2S and 3S states masses for Ξ_{bb} and Ξ_{bc} baryons (with both parities) shows that our masses have a difference in the range of ≈0.5 GeV with [1, 7, 20, 32–34, 36].

To calculate the orbital excited state masses (1P–5P, 1D– 4D, 1F–2F), we have considered all possible isospin splitting and all combinations of total spin S and total angular momentum J. Our outcomes and the comparison of masses with other approaches are also tabulated in Tables 4, 5, and 6.

Our obtained orbital excited masses for Ξ_{cc}, 1P state $J^P = (1/2)^-$ show a difference of 14 MeV (with [1]), 29 MeV (with [33]), 13 MeV (with [34]), and 41 MeV (with [5]), while 1P state $J^P = (3/2)^-$ shows 14 MeV (with [1]), 48 MeV (with [35]), and 0 MeV (with [33]). Our 2P state $J^P = (1/2)^-$ shows a difference of 15 MeV (with [7]), 35 MeV (with [34]), and 41 MeV (with [1]), while 2P state $J^P = (3/2)^-$ shows 26 MeV (with [32]), 33 MeV (with [7]), and 40 MeV (with [1]). Results for 3P states $J^P = (1/2)^-$ and $J^P = (3/2)^-$ show a difference in the range of ≈ 60 MeV with [1]. We can easily observe that our calculated masses for 4P-5P, 1D-3D, and 1F-2F are matched with [1]. Our outcome for 3D state $J^P = (3/2)^+$ is quite equal to the predictions of [7, 32, 33, 35]. For the ground and excited states of doubly heavy baryons (Ξ_{cc}^{+}), the minimum and maximum percentage of relative error values are 0% and 3.53% between our calculations and the masses reported by Shah et al. [1].

For Ξ_{bb} and Ξ_{bc} baryons, the mass difference from our calculations and other references is large.

Comparing our findings with the masses reported by Shah et al. [1], the minimum and maximum percentage of relative error values are 1.2% (0.8%) and 10.317% (6.92%) for the ground and excited states of doubly heavy baryons Ξ_{bb} and Ξ_{bc}, respectively.

5. Conclusion

In this study, we have computed the mass spectra of ground and excited states for doubly heavy Ξ baryons by using a hypercentral constituent quark model. For this goal, we have analytically solved the hyperradial Schrödinger equation for three identical interacting particles under the effective hypercentral potential by using the ansatz method. Our proposed potential is regarded as a combination of the Coulombic-like term plus a linear confining term and the harmonic oscillator potential. We also added the first-order correction and the spin-dependent part to the potential. In our calculations, the u and d quarks have 10 MeV difference mass, so there is a very small mass difference between Ξ_{ccd} and Ξ_{ccu}, Ξ_{bbd} and Ξ_{bbu}, Ξ_{bcd} and Ξ_{bcu}. Our model has succeeded to assign the J^P values to the exited states of doubly heavy baryons (Ξ_{ccd}, Ξ_{ccu}, Ξ_{bbd}, Ξ_{bbu}, Ξ_{bcd}, and Ξ_{bcu}). Comparison of the results with other predictions revealed that they are in agreement and our proposed model can be useful to investigate the doubly heavy baryons states masses. For example, for the ground, radial, and orbital excited states masses of doubly heavy Ξ baryons the minimum and the maximum percentage of relative error values are 0% and 6% between our calculations and the masses reported by Shah et al. [1].

Table 6: The masses of orbital excited states for Ξ_{bc} baryon (in GeV).

State	Our cal Ξ_{bc}^{0}	Our Cal Ξ_{bc}^{+}	[1] Ξ_{bc}^{0}	[1] Ξ_{bc}^{+}	[18]
$(1^2\ P_{1/2})$	6.846	6.842	7.16	7.156	7.390
$(1^2\ P_{3/2})$	6.836	6.831	7.149	7.144	7.394
$(1^4\ P_{1/2})$	6.851	6.847	7.166	7.161	7.399
$(1^4\ P_{3/2})$	6.841	6.837	7.155	7.15	
$(1^4\ P_{5/2})$	6.859	6.856	7.175	7.171	
$(2^2\ P_{1/2})$	7.087	7.084	7.425	7.422	
$(2^2\ P_{3/2})$	7.078	7.075	7.415	7.412	
$(2^4\ P_{1/2})$	7.091	7.088	7.43	7.426	
$(2^4\ P_{3/2})$	7.082	7.079	7.42	7.417	
$(2^4\ P_{5/2})$	7.071	7.095	7.408	7.434	
$(3^2\ P_{1/2})$	7.304	7.302	7.664	7.662	
$(3^2\ P_{3/2})$	7.296	7.295	7.655	7.654	
$(3^4\ P_{1/2})$	7.308	7.306	7.668	7.666	
$(3^4\ P_{3/2})$	7.299	7.299	7.659	7.658	
$(3^4\ P_{5/2})$	7.289	7.312	7.648	7.673	
$(4^2\ P_{1/2})$	7.504	7.623	7.884	8.015	
$(4^2\ P_{3/2})$	7.497	7.498	7.876	7.877	
$(4^4\ P_{1/2})$	7.508	7.508	7.888	7.888	
$(4^4\ P_{3/2})$	7.500	7.500	7.88	7.88	
$(4^4\ P_{5/2})$	7.491	7.514	7.87	7.895	
$(5^2\ P_{1/2})$	7.692	7.693	8.091	8.092	
$(5^2\ P_{3/2})$	7.686	7.687	8.084	8.085	
$(5^4\ P_{1/2})$	7.695	7.697	8.094	8.096	
$(5^4\ P_{3/2})$	7.689	7.689	8.087	8.088	
$(5^4\ P_{5/2})$	7.680	7.681	8.078	8.079	
$(1^4\ D_{1/2})$	7.006	7.004	7.336	7.334	
$(1^2\ D_{3/2})$	6.992	6.989	7.321	7.318	
$(1^4\ D_{3/2})$	6.997	6.980	7.326	7.308	7.324
$(1^2\ D_{5/2})$	6.980	6.977	7.308	7.304	
$(1^4\ D_{5/2})$	6.985	6.969	7.313	7.295	7.309
$(1^4\ D_{7/2})$	6.969	6.953	7.296	7.278	7.292
$(2^4\ D_{1/2})$	7.087	7.227	7.425	7.579	7.579
$(2^2\ D_{3/2})$	7.216	7.214	7.567	7.565	
$(2^4\ D_{3/2})$	7.219	7.219	7.571	7.57	
$(2^2\ D_{5/2})$	7.205	7.203	7.555	7.553	7.538
$(2^4\ D_{5/2})$	7.209	7.208	7.559	7.558	
$(2^4\ D_{7/2})$	7.196	7.195	7.545	7.544	
$(3^4\ D_{1/2})$	7.431	7.431	7.804	7.804	
$(3^2\ D_{3/2})$	7.420	7.420	7.792	7.792	
$(3^4\ D_{3/2})$	7.411	7.424	7.782	7.796	
$(3^2\ D_{5/2})$	7.415	7.410	7.786	7.781	
$(3^4\ D_{5/2})$	7.402	7.414	7.772	7.785	
$(3^4\ D_{7/2})$	7.402	7.402	7.772	7.772	
$(4^4\ D_{1/2})$	7.429	7.504	7.801	7.884	7.797
$(4^2\ D_{3/2})$	7.611	7.613	8.002	8.004	
$(4^4\ D_{3/2})$	7.615	7.617	8.006	8.008	
$(4^2\ D_{5/2})$	7.603	7.604	7.993	7.994	
$(4^4\ D_{5/2})$	7.606	7.608	7.996	7.998	
$(4^4\ D_{7/2})$	7.596	7.597	7.985	7.986	
$(1^4\ F_{3/2})$	7.143	7.141	7.487	7.485	
$(1^2\ F_{5/2})$	7.127	7.125	7.469	7.467	

Table 6: Continued.

State	Our cal Ξ_{bc}^{0}	Our Cal Ξ_{bc}^{+}	[1] Ξ_{bc}^{0}	[1] Ξ_{bc}^{+}	[18]
$(1^4\ F_{5/2})$	7.131	7.129	7.474	7.472	
$(1^4\ F_{7/2})$	7.117	7.114	7.458	7.455	
$(1^2\ F_{7/2})$	7.112	7.109	7.453	7.45	
$(1^4\ F_{9/2})$	7.099	7.097	7.439	7.436	
$(2^4\ F_{3/2})$	7.350	7.350	7.715	7.715	
$(2^2\ F_{5/2})$	7.337	7.336	7.7	7.699	
$(2^4\ F_{5/2})$	7.340	7.339	7.704	7.703	
$(2^4\ F_{7/2})$	7.328	7.327	7.69	7.689	
$(2^2\ F_{7/2})$	7.324	7.323	7.686	7.685	
$(2^4\ F_{9/2})$	7.313	7.311	7.674	7.672	

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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