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Research Article

Topological Gravity on (D, N)-Shift Superspace Formulation

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We reassess the subject of topological gravity by following the shift supersymmetry formalism. The gauge-fixing of the theory goes under the Batalin-Vilkovisky (BV) prescription based on a diagram that contains both ghost and antighost superfields, associated with the supervielbein and the super-Lorentz connection. We extend the formulation of the topological gravity action to an arbitrary number of dimensions of the shift superspace by adopting a formulation based on the gauge-fixing for BF-type models.

1. Introduction

Topological field theories have been introduced by Witten [1] and soon after applied in several areas that describe quantum-mechanical and quantum field-theoretical systems. Over the recent years, they have been applied to study nonperturbative quantum gravity [2–4], the issues of topological phases, spin foam, Loop Quantum Gravity, a topological approach to the cosmological constant, and a number of other relevant applications. One of the basic topics for the construction of topological gravity is the topological Yang-Mills theories, by now fairly well discussed and understood. However, on the other hand, the complexity of the symmetry groups and Lagrangians in topological gravity renders difficult the comparison between the results obtained by using different formalisms. A topological gravity theory may be formulated by gauge-fixing an action that is a topological invariant, which can be achieved by twisting an extended supergravity theory. The latter combines diffeomorphisms and local supersymmetry transformations and exhibits a considerably more complex structure whenever compared with Super-Yang-Mills theories. A good review paper on topological gravity, its corresponding physical observables, and a number of interesting applications may be found in the work of [5].

Ever since its formulation, Chern-Simons theory, treated in a supersymmetric formalism [6–8], has raised particular interest in the framework of gauge theories as a topological geometric model. Its basic geometrical objects in the principal bundle are the dreibein and the Lorentz connection, where the dreibeins form a basis for the tangent space, while the space-time metric appears as the square of the dreibein matrix.

The topological supersymmetrization of Yang-Mills [9] and *BF*-theories by adopting the shift formalism [10–13] provide a viable way to their extension to any superspace dimensions, with the Wess-Zumino supergauge choice [14] used in connection with the Batalin-Vilkovisky prescription. In other words, an alternative construction can be used for the BV diagrams, where the latter no longer accommodate fields but, instead, superfields; they are referred to in the literature as BV superdiagrams.

Thus, it is possible to formulate an immediate extension to an arbitrary superspace dimension [11]. We think it is possible to proceed seemingly for topological gravity [15], that is, to describe this theory by adopting a topological formulation based on supersymmetry (SUSY, from now on): a supersymmetric topological gravity [16]. We accomplished this construction by exploiting the shift supersymmetry formalism and defining the geometric supersymmetric elements of the theory from an extension of the usual elements of the differential Riemannian formulation. For the supersymmetrization, we define these elements in a basis supermanifold $\mathscr{M}$ with a mapping to the Euclidian-flat-space carried out by $D-bein$. This $D-bein$ and the Lorentz connection describe the geometric sector along with the gauge-fixing fields of the theory in an Euclidean flat-space-time and the Grassmann-valued coordinates; the latter are adjoined to the space-time coordinates to constitute the superspace of the theory.

Our starting point consists in rewriting the Chern-Simons action [17] in the $N = 1$-shift supersymmetry formalism of a BF-type model and, next, to carry out its complete gauge-fixing. In the sequel, we shall propose the study of more than a single (simple) SUSY and describe the topological gravity in four space-time dimensions [18]. Consequently, we set up an action for arbitrary superspace dimensions, i.e., with arbitrary $D-$ space-time dimensions and extended N-SUSY.

We shall give a brief presentation of the elements of the space-time theory. In the D–dimensional basis manifold M, we define a metric $g_{\mu\nu}$, with index $\mu, \nu, \ldots = 1, 2, \ldots, D$. $D - bein$, $e^a{}_\mu$, allows the transformation between the basis manifold and the Euclidean flat-space-time whose the flat-metric is δ_{ab}, with the same dimension and index $a, b, \ldots = 1, 2, \ldots, D$. The metric is defined in both spaces

$$g = g_{\mu\nu} dx^\mu \otimes dx^\nu = \delta_{ab} e^a \otimes e^b, \tag{1}$$

such that $e^a = e^a{}_\mu dx^\mu$ represents the basis in the dual $T^*_p M$, of the tangent space $T_p M$, with basis elements given by ∂_μ.

The transformation properties between the basis manifold M and flat space are

$$\begin{aligned} v^\mu &= e^\mu{}_a u^a, \\ u^a &= e^a{}_\mu v^\mu \end{aligned} \tag{2}$$

and

$$\begin{aligned} v_\mu &= e_\mu{}^a u_a, \\ u_a &= e_a{}^\mu v_\mu \end{aligned} \tag{3}$$

where the entry $[e^a{}_\mu]$ has its inverse, $[e^a{}_\mu]^{-1}$.

The connection written on the basis manifold, $\Gamma^\mu{}_{\nu\kappa}$, has its correspondent in the Euclidean flat-space-time, $\omega_\mu{}^a{}_b = \Gamma^\nu{}_{\mu\lambda} e^a{}_\nu e^\lambda{}_b - e^\lambda{}_b \partial_\mu e^a{}_\lambda$, which is a 1-form in the basis manifold, called Lorentz connection. Inversely, we have $\Gamma^\nu{}_{\mu\lambda} = e^\nu{}_a \partial_\mu e^a{}_\lambda + e^\nu{}_a e^b{}_\lambda \omega_\mu{}^a{}_b$.

The elements of the dynamical sector are the curvature, $R = d\omega + \omega \wedge \omega$, and torsion, $T = de + \omega \wedge e$. These entities constitute the geometrical sector and obey the Bianchi identities:

$$\begin{aligned} D_\omega R &= dR + \omega \wedge R = 0, \\ D_\omega T &= dT + \omega \wedge T = 0, \end{aligned} \tag{4}$$

where the covariant derivative with respect to the connection ω is $D_\omega(\cdot) = d(\cdot) + \omega \wedge (\cdot)$ (another notation used for form covariant derivative is $D_\omega(\cdot) = d(\cdot) + [\omega, (\cdot)]$). In the sequel, we shall build up the extension of the Cartan-Maurer equations to a superspace formulation.

The paper is outlined as follows: in Section 2, we present a general formulation of gravity as a super-BF model. This section is split into five subsections to render the presentation clearer. Section 3 presents the so-called super-BF model and an associated on-shell solution. An explicit construction in the $D = 3, N = 1$-case is worked out to render manifest the whole idea of the method we have followed. Finally, we cast our concluding comments in Section 4.

2. Generalized Gravity as a Super-*BF* Model

2.1. Preliminary Definitions. The supermanifold we shall work with consists of the basis manifold with the addition of the Grassmann-valued coordinates. The supercoordinates are parameterized as $z^M = (x^\mu, \theta^I)$ defined in a superchart of the basis supermanifold (we stress that the space-time dimension is D, while N is an internal label associated with the number of supersymmetries of the model; $N = 1$ corresponds to a simple SUSY, and $N > 1$ stands for an N-extended SUSY) $\mathscr{M}$, where θ^I is the Grassmannian coordinates [10–12, 19–21] (the Grassmann Variables of the topological description may be found in [10–12], as well as the notations therein. For example, for $N = 2$, Levi-Civita pseudotensor ϵ^{IJ} is the antisymmetric metric, $\epsilon^{12} = +1 = -\epsilon_{12}$, where $\epsilon^{IJ}\epsilon_{JK} = \delta^I{}_K$ and a field or variable transforms as $\varphi^I = \epsilon^{IJ}\varphi_J$ and $\varphi_I = \epsilon_{IJ}\varphi^J$. A scalar product is invariant under $SU(2)$ such that $(\theta)^2 = (1/2)\theta^I\theta_I = (1/2)\epsilon_{IJ}\theta^I\theta^J$. The derivative is defined as $\partial_I = \partial/\partial\theta^I$ and $\partial^I = \partial/\partial\theta_I$ and applied to a Grassmann coordinate which gives $\partial_I\theta_J = -\epsilon_{IJ}$. Finally, the Berezin Integral is defined by $\int \theta^I = \partial_I$). In the Euclidean flat space-time, the coordinates are represented by $z^A = (x^a, \theta^I)$.

The superderivatives as superforms are given by $\hat{d} = dx^\mu\partial_\mu + d\theta^I\partial_I$. An arbitrary superfield, $F(x, \theta^I)$, is defined by the action of the transformation generated by the derivatives with respect to the Grassmann coordinates [10, 11, 14], known as the shift operator, and given according to what follows:

$$Q_I F(x, \theta) = \partial_I F(x, \theta), \quad I = 1, \ldots, N, \tag{5}$$

where we assign fermionic supersymmetry numbers as follows: $[\theta^I] = -1$ and $[Q_I] = +1$.

In analogy to the Riemannian geometry, we set up a superspace formalism [5] to treat the fundamental elements of the description. In the SUSY formalism, we define the $D - bein$ 1-(super)form as

$$\hat{E}^a = E^a_\mu(x, \theta)\, dx^\mu + E^a_I(x, \theta)\, d\theta^I \tag{6}$$

and the 1-(super)form as

$$\hat{\Omega}^{ab} = \Omega^{ab}_\mu(x, \theta)\, dx^\mu + \Omega^{ab}_I(x, \theta)\, d\theta^I. \tag{7}$$

In both cases, their components are form superfields (the same being true in all our definitions that involve superforms: superform components are superfields).

Then, the dynamical objects of the formalism, the curvature 2-superform and torsion 1-superform, are defined by means of the Cartan algebra associated with the shift SUSY. The (super)curvature as 2-superform reads as

$$\begin{aligned}\widehat{R}^{ab} &= \widehat{d}\widehat{\Omega}^{ab} + \widehat{\Omega}^{a}{}_{c}\widehat{\Omega}^{cb} \\ &= \mathbf{R}^{ab}{}_{\mu\nu}dx^{\nu}dx^{\mu} + \mathbf{R}^{ab}_{I\,\mu}dx^{\mu}d\theta^{I} + \mathbf{R}^{ab}_{IJ}d\theta^{I}d\theta^{J},\end{aligned} \tag{8}$$

where $\mathbf{R}^{ab}$ accommodates the genuine 2-form Riemann tensor that we simply write as

$$R^{ab} = R^{ab}{}_{\mu\nu}dx^{\nu}dx^{\mu}. \tag{9}$$

Also, the supercovariant derivative is given by

$$\widehat{D}_{\widehat{\Omega}}(\cdot) = \widehat{d}(\cdot) + \left[\widehat{\Omega},(\cdot)\right] \tag{10}$$

$$= D_{\Omega}(\cdot) + d\theta^{I}D_{I}(\cdot) \tag{11}$$

$$\begin{aligned}&= dx^{\mu}\left(\partial_{\mu}(\cdot) + \left[\Omega_{\mu},(\cdot)\right]\right) \\ &\quad + d\theta^{I}\left(\partial_{I}(\cdot) + \left[\Omega_{I},(\cdot)\right]\right)\end{aligned} \tag{12}$$

and $(D_{\Omega})_{\mu} = D_{\mu}$, which is the 1-form covariant superfield derivative with the Ω-connection. This yields the (super)components of the curvature superfield:

$$\begin{aligned}\mathbf{R}^{ab} &= \left(D_{\Omega}\Omega\right)^{ab}, \\ \mathbf{R}^{ab}_{I} &= d\Omega^{ab}_{I} + \partial_{I}\Omega^{ab} + \left[\Omega^{ac},\Omega^{cb}_{I}\right], \\ \mathbf{R}^{ab}_{IJ} &= \left(D_{I}\Omega_{I}\right)^{ab},\end{aligned} \tag{13}$$

and we can also write curvature (9) as $R = D_{\omega}\omega$.

2.2. Reassessing $D = 3, N = 1$ and $D = 4, N = 1$ Topological Gravities. After the papers [6, 7] have appeared, a whole line of works has approached supergravity and its topological version with the aim of accomplishing the description based on a maximally extended SUSY. The challenge is to conveniently formulate it as a gauges theory, in view of the huge number of degrees of freedom accommodated in all superfields. In the formalism of shift SUSY, or topological supersymmetric formalism, it is understood that (simple) $N = 1$ superspace is considered [10, 22]. We wish, in the present contribution, to formulate, with the help of the shift SUSY [5, 10, 11, 13, 14], gravity for any dimensionality, D, and for a general number of SUSY generators, N. We shall adopt the Batalin-Vilkovisky (BV) prescription [23] combined with the Blau-Thompson minimal action gauge-fixing [12, 24], which fix the Lagrange multipliers associated with the gauge conditions (we propose the following notation for the superfield charges: ${}^{s}\Omega^{g}_{p}$, where s stands for the SUSY number, g denotes the ghost number, and p indicates the form degree). Finally, we need to add the Fadeev-Popov gauge-fixing action along with the BF-model term, forming the full invariant action.

Our starting point is the gravity formulation for $N = 1$-SUSY [5, 6, 25–28]; for that, we define the 1-form superfields by means of the following expansion $\widehat{E}^{a} = E^{a}_{\mu}(x,\theta)dx^{\mu} + E^{a}_{\theta}(x,\theta)d\theta$, as having an odd statistics, such that the component superfields readily follow:

$$E^{a} = e^{a} + \theta\psi^{a}, \tag{14}$$

and

$$E^{a}_{\theta} = \chi^{a} + \theta\phi^{a}. \tag{15}$$

The superform $\widehat{E}^{a}$ has odd statistics, because e^{a} should be odd – this is a 1-form with SUSY number 1 – with its component $e^{a}{}_{\mu}$ being a bosonic field. To build up the model in $D = 3$ space-time dimensions, we define the superconnection as

$$\widehat{\Omega}^{ab} = \Omega^{ab}{}_{\mu}(x,\theta)\,dx^{\mu} + \Omega^{ab}_{\theta}(x,\theta)\,d\theta, \tag{16}$$

where the components 1-form superfields are

$$\begin{aligned}\Omega^{ab} &= \omega^{ab} + \theta\varpi^{ab}, \\ \Omega^{ab}_{\theta} &= \lambda^{ab} + \theta\lambda^{ab}_{\theta}.\end{aligned} \tag{17}$$

The (super)components of the supercurvature can be expanded from expression (13).

The invariant (topological) gravity action in the dimension we are now considering is the Chern-Simons model [1, 10, 12] in the corresponding shift superspace; therefore, here, to ensure the right field content, we need to change E^{a} given above (which would be compatible with a BF-type action) and rewrite it as $E^{a} = \psi^{a} + \theta e^{a}$, such that the integrand contains basically the expression for the gravitational Chern-Simons action:

$$\begin{aligned}\int Q\left[\varepsilon_{abc}E^{a}\mathbf{R}^{bc}\right] &= \int \varepsilon_{abc}\left[e^{a}R^{bc} - \psi^{a}\left(D_{\omega}\varpi\right)^{bc}\right] \\ &= \int d^{3}x\varepsilon_{abc}\varepsilon^{\mu\nu\kappa}\left\{e^{a}{}_{\mu}R^{bc}{}_{\nu\kappa} - \psi^{a}{}_{\mu}\left(D_{\nu}\varpi_{\kappa}\right)^{bc}\right\},\end{aligned} \tag{18}$$

where $R^{ab} = d\omega^{ab} + \omega^{ac}\omega^{cb} = (D_{\omega}\omega)^{ab}$, $(D_{\omega}\varpi)^{ab} = d\varpi^{ab} + [\omega^{ac},\varpi^{cb}]$, and the superintegral in this formalism is $\int d\theta = Q$. The final result of this supersymmetrization is an action free from the supersymmetric charges. Then, this formulation for $D = 3$ dimensions is not convenient to our construction, because as we have seen above, we had to make a new definition of E^{a}, compared with (14), in order that the action acquire the Chern-Simons form. For that reason, we choose to work with a BF-type formulation, which yields the same expected results without, however, the need of changing the form of the basic elements of the formulation, E^{a} for example, to accommodate the supersymmetrization process.

Now, that the construction of the action for the geometric sector is totally fixed, we need to describe the BV superdiagram [11] that contains the supercomponents of (8), using the Blau-Thompson minimal action procedure [12, 13]. The superdiagram in $D = 3$ dimensions is given as below:

$$\begin{array}{ccccc} & & {}^{0}\mathbf{R}^{ab}_{2} & & \\ & {}^{-1}\overline{H}^{ab}_{1} & & {}^{1}\mathbf{R}^{ab}_{1} & \\ {}^{0}\overline{Z}^{ab}_{0} & & {}^{-2}\overline{W}^{ab}_{0} & & {}^{2}\mathbf{R}^{ab}_{0}. \end{array} \tag{19}$$

Let us recall that this gauge-fixing procedure is a shift gauge-fixing rather than the BRST gauge-fixing (Faddev-Popov), though it has the similar effect on the shift degrees of freedom (shift ghosts elimination [5, 11]. In some cases, this is named BRST gauge-fixing [1, 10]. However, all curvature superfields and Lagrangian multipliers exhibit the same covariant BRST transformation: $s(\cdot) = -[C^{ab}, (\cdot)]$, where C^{ab} is a zero-form superfield ghost associated with Ω^{ab}. According to this diagram, the action can be written as

$$S^{N=1}_{D=3} = \int d\theta \left\{ {}^{-1}\overline{H}^{ab}_1 \left({}^0\mathbf{R}^{ab}_2 \right) + {}^{-2}\overline{W}^{ab}_0 \left(D_\Omega * {}^1\mathbf{R}^{ab}_1 \right) + {}^0\overline{Z}^{ab}_0 \left(D_\Omega * {}^{-1}\overline{H}^{ab}_1 \right) \right\}, \tag{20}$$

For $D = 4$ dimensions, we can reproduce the results of (20), by generalizing the systematization [22], where the BV superdiagram is now given by

$$\begin{array}{ccccccc} & & & {}^0\mathbf{R}^{ab}_2 & & & \\ & & {}^{-1}\overline{H}^{ab}_2 & & {}^1\mathbf{R}^{ab}_1 & & \\ & {}^0\overline{Z}^{ab}_1 & & {}^{-2}\overline{W}^{ab}_0 & & {}^2\mathbf{R}^{ab}_0 & \\ {}^{-1}\overline{U}^{ab}_0 & & & & & & \end{array} \tag{21}$$

In this case, we clearly notice, by means of the Blau-Thompson procedure, that the ghost fields for each Lagrange multiplier are eliminated by a simple redefinition in the action, leading to a minimal gauge-fixing action [12, 13]. Therefore, the associated action with (21) can be written as

$$S^{N=1}_{D=4} = \int d\theta \left\{ {}^{-1}\overline{H}^{ab}_2 \left({}^0\mathbf{R}^{ab}_2 \right) + {}^{-2}\overline{W}^{ab}_0 \left(D_\Omega * {}^1\mathbf{R}^{ab}_1 \right) + {}^0\overline{Z}^{ab}_0 \left(D_\Omega * {}^{-1}\overline{H}^{ab}_2 \right) + {}^{-1}\overline{U}^{ab}_0 \left(D_\Omega * {}^0\overline{Z}_1 \right)^{ab} \right\}. \tag{22}$$

Now, we are ready to build up the supergravity action in $N = 2$ superspace. This task is the subject of our next section.

2.3. The topological gravity in $N = 2, D = 4$. Here, it is possible to write down the topological gravity action, using the same construction as the one in the previous Section. Now, the $D - bein$ 1-form-superfield is given by

$$\begin{aligned} E^a &= e^a + \theta^I \psi^a_I + \frac{1}{2}\theta^2 \rho^a, \\ E^a_I &= \chi^a_I + \theta^J \phi_{JI} + \frac{1}{2}\theta^2 \varphi^a_I. \end{aligned} \tag{23}$$

The connection components if the 1-superform (7) can be read as below:

$$\begin{aligned} \Omega^{ab} &= \omega^{ab} + \theta^I \varpi^{ab}_I + \frac{1}{2}\theta^2 \tau^{ab}, \\ \Omega^{ab}_I &= \lambda^{ab}_I + \theta^J \lambda^{ab}_{JI} + \frac{1}{2}\theta^2 \kappa^{ab}_I. \end{aligned} \tag{24}$$

The action for $D = 3$ space-time dimensions obeys the same gauge-fixing systematization as ion the $N = 1$ case. This superdiagram here can be written as

$$\begin{array}{ccccc} & & {}^0\mathbf{R}^{ab}_2 & & \\ & {}^{-2}\overline{H}^{ab}_1 & & {}^1\mathbf{R}^{ab}_{1\ I} & \\ {}^0\overline{Z}^{ab}_0 & & {}^{-3}\overline{W}^{ab\ I}_0 & & {}^2\mathbf{R}^{ab}_{0\ IJ}, \end{array} \tag{25}$$

For the invariant topological gravity model in $D = 4$ dimensions we propose

$$\begin{array}{cccccc} & & & {}^0\mathbf{R}^{ab}_2 & & \\ & & {}^{-2}\overline{H}^{ab}_2 & & {}^1\mathbf{R}^{ab}_{1\ I} & \\ & {}^0\overline{Z}^{ab}_1 & & {}^{-3}\overline{W}^{ab\ I}_0 & & {}^2\mathbf{R}^{ab}_{0\ IJ}. \\ {}^{-2}\overline{U}^{ab}_0 & & & & & \end{array} \tag{26}$$

Therefore the action associated with this diagram is given as

$$S^{N=2}_{D=4} = \int d^2\theta \left\{ {}^{-2}\overline{H}^{ab}_2 \left({}^0\mathbf{R}^{ab}_2 \right) + {}^{-3}\overline{W}^{ab\ I}_0 \left(D_\Omega * {}^1\mathbf{R}^{ab}_{1\ I} \right) + {}^0\overline{Z}^{ab}_1 \left(D_\Omega * {}^{-2}\overline{H}^{ab}_2 \right) + {}^{-2}\overline{U}_0 \left(D_\Omega * {}^0\overline{Z}^{ab}_1 \right) \right\}, \tag{27}$$

Once we have understood how an action can be written, we shall define the geometric elements for a general superspace dimension and we shall present the action of topological gravity in the sequel.

2.4. The (N, D)*−Superspace.* We generalize the formulation by expanding the action from $N = 2$ to an arbitrary number of SUSYs. We need to define the form superfields for N superspace dimensions. The shift index runs as $I = 1, \ldots, N$, and the Levi-Civita pseudotensor is defined as follows: $\epsilon^{1\ldots N} = 1$. The $D - bein$ (super)component fields of (6) are 1-form superfields and their expansion in component fields is given by

$$E^a = e^a + \theta^I \psi^a_I + \cdots + \frac{1}{N!}\theta^N \psi^a_N, \tag{28}$$

$$E^a_I = \phi^a_I + \theta^J \phi^a_{JI} + \cdots + \frac{1}{N!}\theta^N \varphi^a_I. \tag{29}$$

The component connections of the 2-superform (7) are

$$\Omega^{ab} = \omega^{ab} + \theta^I \omega^{ab}_I + \cdots + \frac{1}{N!}\theta^N \varpi^{ab}, \tag{30}$$

$$\Omega^{ab}_I = \lambda^{ab}_I + \theta^J \lambda^{ab}_{IJ} + \cdots + \frac{1}{N!}\theta^N \varrho^{ab}_I. \tag{31}$$

In $D = 3$ dimensions, the construction is the same for all N, i.e., as (19). In $D = 4$ dimensions, the diagram is also similar to

(21). We can extend the space-time dimension from 4 to any D. We start off the construction with the Levi-Civita symbol, where we assume $\varepsilon^{012\ldots D-1} = 1$. The Euclidian flat-space-time indices are $a_1, a_2, \ldots, a_D$ and they run from 1 to D. The BV superdiagram in the (D, N)−superspace is

$$\begin{array}{ccccccc} & & & & {}^{0}\mathbf{R}_2^{ab} & & \\ & & & {}^{-N}\overline{H}_{D-2}^{ab} & & {}^{1}\mathbf{R}_{1\ I}^{ab} & \\ & & {}^{0}\overline{Z}_{D-3}^{ab} & & {}^{-N-1}\overline{W}_0^{ab\ I} & & {}^{2}\mathbf{R}_{0\ IJ}^{ab} \\ & {}^{-N}\overline{Z}_{D-4}^{ab} & & & & & \\ \swarrow & & & & & & \\ {}^{-s}\overline{Z}_0^{ab} & & & & & & \end{array} \quad (32)$$

with $s = 0$ if D is even and $s = -N$ if D is odd. Therefore, the invariant action in the (D, N)−superspace is

$$\begin{aligned} S_D^N = \int d^N\theta \Big\{ & {}^{-N}\overline{H}_{D-2}^{ab}\, {}^{0}\mathbf{R}_2^{ab} \\ & + {}^{-N-1}\overline{W}_0^{ab\ I} \left(D_\Omega * {}^{1}\mathbf{R}_1^{ab} \right) \\ & + {}^{0}\overline{Z}_{D-3}^{ab} \left(D_\Omega * {}^{-N}\overline{H}_{D-2}^{ab} \right) \\ & + {}^{-N}\overline{Z}_{D-4}^{ab} \left(D_\Omega * {}^{0}\overline{Z}_{D-3}^{ab} \right) + \cdots \\ & + {}^{-s}\overline{Z}_0^{ab} \left(D_\Omega * {}^{-N-s}\overline{Z}_1^{ab} \right) \Big\}, \end{aligned} \quad (33)$$

where each N−power covariant derivative is written as $D_{\Omega_I}^N = \epsilon^{I_1\ldots I_N} D_{I_1} \ldots D_{I_N}$.

This action still presents still spurious degrees of freedom; they are localized in the connection $\Omega_\mu(x, \theta)$; thus, we need to suppress them from the model by fixing the gauge with the help of the BRST ghosts, as we are next going to do.

2.5. The Fadeev-Popov Gauge-Fixing. According to [10, 11], the gauge-fixing of topological gravity consists in writing the Batalin-Vilkovisky diagram [5] and defining the respective ghost for each field. The ghost field and the diffeomorphism transformation fix the principal action [29]. Here, we need to fix only the Lorentz gauge condition. The ghost superfield, for $N = 1$-SUSY, for example, is

$$\widehat{C}^{ab} = C^{ab} = c^{ab} + \theta c_\theta^{ab}. \quad (34)$$

Associated with this ghost, we need to define the antighosts and their Lagrange multipliers as zero-form superfields:

$$\begin{aligned} \overline{C}^{ab} &= \overline{c}^{ab} + \theta \overline{c}_\theta^{ab}, \\ B^{ab} &= b^{ab} + \theta b_\theta^{ab}. \end{aligned} \quad (35)$$

The diffeomorphism-generating ghost field (or, simply, the diffeomorphism ghost field) is a supervector in shift superspace [5], such that

$$\widehat{\Xi} = \Xi^M \partial_M = \Xi^\mu (x, \theta)\, \partial_\mu + \Xi^I (x, \theta)\, \partial_I; \quad (36)$$

its component fields read as follows:

$$\begin{aligned} \Xi^\mu &= \xi^\mu + \theta \xi_\theta^\mu, \\ \Xi^\theta &= \zeta + \theta \zeta_\theta, \end{aligned} \quad (37)$$

and this superform obeys the following properties:

$$\begin{aligned} \widehat{\Xi}^2 &= \frac{1}{2} \left[\widehat{\Xi}, \widehat{\Xi} \right], \\ \left(\widehat{\Xi}^2 \right)^M &= \left(\Xi^N \partial_N \right) \Xi^M, \end{aligned} \quad (38)$$

$$\begin{aligned} \pounds_{\widehat{\Xi}} &= \left[i_{\widehat{\Xi}}, \widehat{d} \right], \\ \left[\pounds_{\widehat{\Xi}_1}, \pounds_{\widehat{\Xi}_2} \right] &= \pounds_{[\widehat{\Xi}_1, \widehat{\Xi}_2]} \end{aligned} \quad (39)$$

where $\pounds_{\widehat{\Xi}}$ is the Lie derivative and i_Ξ stands for the inner product operation [5].

The BRST transformation consists of the ghost and the diffeomorphism transformations. Those transformations on the superforms described in (7) are given by

$$s\widehat{\Omega}^{ab} = \pounds_{\widehat{\Xi}} \widehat{\Omega}^{ab} - \left(\widehat{D}_{\widehat{\Omega}} C \right)^{ab}, \quad (40)$$

$$sC^{ab} = \pounds_{\widehat{\Xi}} C^{ab} - (C * C)^{ab}, \quad (41)$$

$$s\overline{C}^{ab} = \pounds_{\Xi} \overline{C}^{ab} + B^{ab}, \quad (42)$$

$$sB^{ab} = 0, \quad (43)$$

$$s\widehat{\Xi} = \widehat{\Xi}^2 \quad (44)$$

where the BRST operator is nilpotent and $s^2 = 0$ and anticommutes with $\widehat{d}$. From the complete set of the BRST transformations described above, we can write the superdiagram of the Batalin-Vilkovisky to fix the spurious degrees of freedom of the Lorentz connection. The superdiagram associated with the general action (33) will be

$$\begin{array}{ccc} & \Omega^{ab} & \\ \overline{C}^{ab} & & C^{ab}. \end{array} \quad (45)$$

The Fadeev-Popov gauge-fixing super-Lagrangian, invariant under the BRST operator, is

$$\mathscr{L}_{gf} = s \left\{ \overline{C}^{ab} \left(D_\Omega * \Omega \right)^{ab} \right\}, \quad (46)$$

so that, according to the BRST transformation above, we have the action

$$\begin{aligned} S_{gf} = \int d\theta \Big\{ & B^{ab} \left(D_\Omega * \Omega \right)^{ab} - \overline{C}^{ab} \left(D_\Omega * D_\Omega C \right)^{ab} \\ & + \pounds_\Xi \overline{C}^{ab} \left(D_\Omega * \Omega \right)^{ab} + \overline{C}^{ab} \left(D_\Omega * \pounds_\Xi \Omega \right)^{ab} \Big\}, \end{aligned} \quad (47)$$

where "$*$" is the dual star transformation (or star product) of an object in the tangent space to the cotangent space. The action written above for just one Grassmann dimension has the same structure, in superfield form, as the action written in N Grassmannian dimensions; this is so by virtue of the star product operation. Therefore, the invariant gauge-fixed gravity action for any dimension is determined by the sum of (33) and (47); this yields

$$S = S_D^N + S_{gf}. \tag{48}$$

3. The Super-*BF* Model and an On-Shell Solution

The model studied in the previous section is acceptable, because it correctly describes the particular models previously studied; nevertheless, we can formulate a new model whose geometric sector in the action is stable under the BRST invariance of all the other sectors, which we cannot guarantee in the general model of the previous section. Here, we do not follow any longer a Blau-Thompson minimal action procedure. The difference consists basically in considering the total action written as a BRST invariance, considering also the on-shell instanton solutions of the geometric sector. This means that the first Lagrangian multiplier contains the background zero-modes of the degrees of freedom. Therefore, by virtue of the BRST transformation for the first Lagrangian multiplier ${}^{-N}\overline{H}_{D-2}^{ab}$ of the superdiagram (32), the latter gets restricted to the geometric section and the first multipliers according to what follows:

$$\begin{array}{ccccc} & & {}^{0}\mathbf{R}_{2}^{ab} & & \\ {}^{-N}\overline{H}_{D-2}^{ab} & & & {}^{1}\mathbf{R}_{1\,I}^{ab} & \\ & & {}^{-N-1}\overline{W}_{0}^{ab\,I} & & {}^{2}\mathbf{R}_{0\,IJ}^{ab}, \end{array} \tag{49}$$

and the rest of the gauge-fixing diagram splits from the one above; it involves only the fixation of the $\overline{H}^{ab}$ and its antighosts. The BRST transformation of the Lagrangian multiplier is then

$$s\overline{H}^{ab} = -D_{\Omega}\Sigma^{ab} - \left[C^{ac}, \overline{H}^{cb}\right] + £_{\Xi}\overline{H}^{ab}. \tag{50}$$

For the other superfields of the superdiagram (49), the BRST transformation is covariant: $s(\cdot) = -[C, (\cdot)]$. To preserve the nilpotency of the BRST operator s, we take the on-shell solution for the new model as an instanton solution, $\mathbf{R}^{ab} - *\mathbf{R}^{ab} = 0$ in four dimensions, and for other dimensionalities the null curvature solution, $\mathbf{R}^{ab} = 0$. We now write the BV superdiagram associated with this Lagrangian multiplier:

$$\begin{array}{ccccccc} & & & {}^{-N}\overline{H}_{D-2}^{0} & & & \\ & & {}^{0}\overline{\Sigma}_{D-3}^{-1} & & {}^{-N}\Sigma_{D-3}^{1} & & \\ & {}^{-N}\overline{\Sigma}_{D-4}^{0} & & {}^{0}\overline{\Sigma}_{D-4}^{-2} & & {}^{-N}\overline{\Sigma}_{D-4}^{-2} & \\ \swarrow & & & \cdots & & & \searrow \\ {}^{s}\overline{\Sigma}_{0}^{g} & & & \cdots & & & {}^{-N}\overline{\Sigma}_{0}^{D-2}, \end{array} \tag{51}$$

whose transformations for all ghosts and antighosts read as

$$\begin{aligned} s\left({}^{s}\Phi_{D-2-g}^{-(-1)dig(g)}\right) &= -D_{\Omega}\left({}^{s}\Phi_{D-3-g}^{-(-1)dig(g+1)}\right) \\ &\quad - \left[C, {}^{s}\Phi_{D-2-g}^{-(-1)dig(g)}\right] \\ &\quad + £_{\Xi}\left({}^{s}\Phi_{D-2-g}^{-(-1)dig(g)}\right), \end{aligned} \tag{52}$$

where $\Phi = \{{}^{s}\Sigma_{p}^{g}\,{}^{s}\overline{\Sigma}_{p}^{g}\}$. The superdiagram of the BRST Lagrange multipliers of the antighosts ${}^{s}\overline{\Sigma}_{p}^{g}$ is

$$\begin{array}{ccccc} & & {}^{0}\Pi_{D-3}^{0} & & \\ & {}^{-N}\Pi_{D-4}^{1} & & {}^{0}\Pi_{D-4}^{1} & \\ \cdots & & \cdots & & \cdots, \end{array} \tag{53}$$

with

$$\begin{aligned} s\,{}^{s}\overline{\Sigma}_{p}^{g} &= £_{\Xi}\,{}^{s}\overline{\Sigma}_{p}^{g} + {}^{s}\Pi_{p}^{g-1}, \\ s\,{}^{s}\Pi_{p}^{g-1} &= 0. \end{aligned} \tag{54}$$

We define only the antifields $\overline{H}^{*}$, $\overline{W}^{*}$ of the Lagrangian multipliers $\overline{H}$, $\overline{W}$ in the diagram (49) that fix the geometric sector. Those describe the following relation:

$$\begin{aligned} s_{\ell}\int d^{N}\theta\left(\overline{H}^{*}\overline{H} + \overline{W}^{*}\overline{W}\right) &= \left\{\left(\mathcal{N}_{\overline{H}} - \mathcal{N}_{\overline{H}^{*}}\right) + \left(-\mathcal{N}_{\overline{W}} + \mathcal{N}_{\overline{W}^{*}}\right)\right\} S_{Geom} \\ &= \int d^{N}\theta\left[\overline{H}^{ab}\mathbf{R}^{ab} + \overline{W}^{ab\,I}\left(D_{\Omega} * \mathbf{R}_{I}^{ab}\right)\right] \end{aligned} \tag{55}$$

where the geometric sector action is $S_{Geom} = \int d^{N}\theta[\overline{H}\mathbf{R} + \overline{W}^{I}(D_{\Omega} * \mathbf{R}_{I})]$. $\mathcal{N}$ is the counter field operator and the Slavnov-Taylor operator is given by

$$\mathcal{S} = s_{\ell} + \mathcal{O}(\hbar), \tag{56}$$

with the following properties

$$\mathcal{S}_{S}\mathcal{S}(S) = 0, \quad for\ \forall S, \tag{57}$$

$$\mathcal{S}_{S}\mathcal{S}_{S} = 0, \quad if\ \mathcal{S}(S) = 0. \tag{58}$$

And s_{ℓ} is the linearized BRST operator.

The invariant gauge action is determined by the geometric sector action for the superdiagram (49) and the Fadeev-Popov action of the superdiagrams (45), (51), and (53), such that

$$\begin{aligned} S = \int d^N\theta \Big\{ & {}^{-N}\overline{H}^{ab}_{D-2}\mathbf{R}^{ab} + {}^{-N-2}\overline{W}^{ab\ I}_{0} \left(D_\Omega * \mathbf{R}^{ab}_I\right) \\ & + s\Big[{}^{0}\overline{\Sigma}^{-1}_{D-3}\left(D_\Omega * {}^{-N}\overline{H}^{0}_{D-2}\right) \\ & + {}^{0}\overline{\Sigma}^{-2}_{D-4}\left(D_\Omega * {}^{-N}\Sigma^{1}_{D-3}\right) + {}^{-N}\overline{\Sigma}^{0}_{D-4}\left(D_\Omega * {}^{0}\Sigma^{-1}_{D-3}\right) \\ & + \cdots + {}^{s}\Sigma^{g}_{0}\left(D_\Omega * {}^{-s-1}\Sigma^{-g-1}_{0}\right) \\ & + \overline{C}^{ab}\left(D_\Omega * \Omega\right)^{ab}\Big]\Big\}, \end{aligned} \quad (59)$$

where $s = -N$ and $g = 0$ if D is even or $s = 0$ and $g = -1$ if D is odd. Summarizing, the general form of this new model (59) exhibits trivial cohomology for both operators Q and s:

$$S = Qs\left(\mathscr{L}_{Geometric} + \mathscr{L}_{Fadeev-Popov}\right) \quad (60)$$

while in the previous section the total action (48) obeys the scheme

$$S = Q\left(\mathscr{L}_{Geometric} + s\mathscr{L}_{Fadeev-Popov}\right) \quad (61)$$

For the sake of illustrating the gauge-fixing procedure, we can present a simple example in the particular $D = 3$ and $N = 1$ case.

Example. Let us reproduce the topological gravity in the case of three dimensions with $N = 1$-SUSY. The superdiagram is

$$\begin{array}{ccc} & {}^{0}\mathbf{R}^{ab}_{2} & \\ {}^{-1}\overline{H}^{ab}_{1} & & {}^{1}\mathbf{R}^{ab}_{\theta\ 1} \\ & {}^{-2}\overline{W}^{ab}_{0} & \qquad {}^{2}\mathbf{R}^{ab}_{\theta\theta\ 0}. \end{array} \quad (62)$$

Using the definitions (16), (17), the curvature (13), and the Lagrange multiplier superfields are given, component-wise, as follows:

$$\mathbf{R}^{ab} = R^{ab} - \theta\left(D_\omega\varpi\right)^{ab}, \quad (63)$$

$$\mathbf{R}^{ab}_\theta = \varpi^{ab} + \left(D_\omega\lambda\right)^{ab} + \theta\left(\varpi^{ac}\lambda^{cb} - \left(D_\omega\lambda_\theta\right)^{ab}\right), \quad (64)$$

$$\overline{H}^{ab} = h^{ab} + \theta h^{ab}_\theta, \quad (65)$$

$$\overline{W}^{ab} = w^{ab} + \theta w^{ab}_\theta \quad (66)$$

so that all those superfields are covariant under the BRST transformation, $s(\cdot) = -[C, (\cdot)]$. The BRST transformation for the Lagrange multipliers contains the zero-modes of the degrees of freedom given in (50). We define the BRST ghost and antighost as zero-form superfields

$$\Sigma^{ab} = \sigma^{ab} + \theta\sigma^{ab}_\theta, \quad (67)$$

$$\overline{\Sigma}^{ab} = \overline{\sigma}^{ab} + \theta\overline{\sigma}^{ab}_\theta. \quad (68)$$

The BRST transformation for all superfields of the Fadeev-Popov gauge-fixing action is given by

$$s\widehat{\Omega}^{ab} = \pounds_{\widehat{\Xi}}\widehat{\Omega}^{ab} - \left(\widehat{D}_{\widehat{\Omega}}C\right)^{ab}, \quad (69)$$

$$sC^{ab} = \pounds_{\widehat{\Xi}}C^{ab} - \left(C * C\right)^{ab}, \quad (70)$$

$$s\overline{C}^{ab} = \pounds_{\widehat{\Xi}}\overline{C}^{ab} + B^{ab} \quad (71)$$

$$sB^{ab} = 0, \quad (72)$$

and the transformation for the shift antighost sector carries a new term: $(D_\Omega\Sigma)^{ab}$ discussed in (50) such that

$$s\overline{H}^{ab} = \pounds_{\widehat{\Xi}}\overline{H}^{ab} - \left[C^{ac}, \overline{H}^{cb}\right] - \left(D_\Omega\Sigma\right)^{ab} \quad (73)$$

and the other transformation is given by

$$s\Sigma^{ab} = \pounds_{\widehat{\Xi}}\Sigma^{ab} - \left(\Sigma * \Sigma\right)^{ab}, \quad (74)$$

$$s\overline{\Sigma}^{ab} = \pounds_{\widehat{\Xi}}\overline{\Sigma}^{ab} + \Pi^{ab}, \quad (75)$$

$$s\Pi^{ab} = 0, \quad (76)$$

$$s\widehat{\Xi} = \widehat{\Xi}^2. \quad (77)$$

For the superdiagram of the Lagrange multiplier of $\mathbf{R}^{ab}$ adapted for the $N = 1$ case, have

$$\begin{array}{ccc} & \overline{H}^{ab} & \\ \overline{\Sigma}^{ab} & & \Sigma^{ab}, \end{array} \quad (78)$$

Thus, the Fadeev-Popov gauge-fixing super-Lagrangian, together with (45), becomes

$$s\left\{\overline{C}^{ab}\left(D_\Omega * \Omega\right)^{ab} + \overline{\Sigma}^{ab}\left(D_\Omega * \overline{H}\right)^{ab}\right\}. \quad (79)$$

According to (59) and the BRST transformation above, the explicit topological supergravity invariant action takes the form

$$\begin{aligned} S = \int d\theta \Big\{ & \overline{H}^{ab}R^{ab} + \overline{W}^{ab}R^{ab}_\theta + B^{ab}\left(D_\Omega * \Omega\right)^{ab} \\ & - \overline{C}^{ab}\left(D_\Omega * D_\Omega C\right)^{ab} + \Pi^{ab}\left(D_\Omega * \overline{H}\right)^{ab} \\ & - \overline{\Sigma}^{ab}\left(D_\Omega * D_\Omega\Sigma\right)^{ab} + \pounds_{\widehat{\Xi}}\overline{C}^{ab}\left(D_\Omega * \Omega\right)^{ab} \\ & + \overline{C}^{ab}\left(D_\Omega * \pounds_{\widehat{\Xi}}\Omega\right)^{ab} + \pounds_{\widehat{\Xi}}\overline{\Sigma}^{ab}\left(D_\Omega * \overline{H}\right)^{ab} \\ & + \overline{\Sigma}^{ab}\left(D_\Omega * \pounds_{\widehat{\Xi}}\overline{H}\right) - \overline{\Sigma}^{ab}\left(D_\Omega * \left[C, \overline{H}\right]\right)^{ab}\Big\}. \end{aligned} \quad (80)$$

According to the new term of transformation (73), this requires that the on-shell solution for the superfield curvature be null because we work in a 3-dimensional manifold.

A good example is the topological gravity theory for a two-dimensional Riemannian world-sheet manifold, which usually appears coupled to topological sigma-models. The dimension constrains the connection and curvature superforms not to carry the Euclidean vector index; they should be represented as $\widehat{\Omega}$ and $\widehat{R}$. For the BV superdiagram, we may follow the same systematic construction of example contemplated above, but the Lagrange multiplier associated with $\mathbf{R}$ is a zero-form. For that reason, it does not need to have an associated antighost; thus, the gauge-fixing systematic procedure does not change.

4. Concluding Comments

Based on the investigation pursued here, we are able to write down general models over Riemannian manifolds endowed with a metric, both in the topological supersymmetric and in the Witten-type topological formulations, preserving topological invariants like the Euler characteristic, χ, and the correlation functions, while keeping them free from shift spurious degrees of freedom. A good prospective for the application of our results would be the association with twist techniques to reobtain ordinary supergravity theory in the Weyl representation. This attempt, by using topological supergravity, can be implemented for any superspace dimension because there are no symmetry limitations in superspace that prevents us from following this path. Other possibilities of applications that we may point out could be in the framework of Loop Quantum Gravity, Spin foam, Global effects with continuous deformations, and systems with emergent SUSY, like interesting topological materials such as Weyl semimetals and topological insulators. Finally, an open issue which shall be the subject of a forthcoming work is the coupling of the matter sector in the framework of topological gravity. We shall be soon reporting on that.

Conflicts of Interest

The authors state hereby that there are no conflicts of interest regarding the publication of this paper.

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