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Research Article

Inflation in $f(R,\phi)$ Gravity with Exponential Model

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We are taking action of $f(R)$ gravity with a nonminimal coupling to a massive inflaton field. A $f(R,\phi)$ model is chosen which leads to the *scalar-tensor theory* which can be transformed to Einstein frame by conformal transformation. To avoid the vagueness of the frame dependence, we evaluate the exact analytical solutions for inflationary era in Jordan frame and find a condition for graceful exit from inflation. Furthermore, we calculate the perturbed parameters (i.e., number of e-folds, slow-roll parameters, scalar and tensor power spectra, corresponding spectral indices, and tensor to scalar ratio). It is showed that the tensor power spectra lead to blue tilt for this model. The trajectories of the perturbed parameters are plotted to compare the results with recent observations.

1. Introduction

The late-time cosmic acceleration was discovered in 1998 [1, 2], based on the observations of type Ia supernovae (SN Ia) that opened up a new door of research in the field of cosmology. The main ingredient of this acceleration, named as dark energy (DE) [3], has been still a paradigm in spite of tremendous efforts to understand its origin over the last decade [4, 5]. Dark energy is different from ordinary matter in the respect that it has huge negative pressure whose equation of state (EoS) is close to -1. Independent observational data like SN Ia [6, 7], cosmic microwave background radiation (CMBR) [8, 9], and baryon acoustic oscillations (BAO) [10–12] proved that about 70% of the energy density content of the recent universe comprises DE.

Currently, there are various approaches to construct the models for the explanation of the behavior of DE. One way is to modify the dynamical field equations by taking negative pressure in the form of energy momentum tensor $T_{\mu\nu}$. This class of models includes inflation [13], quintessence [14, 15], k-essence [16, 17], and perfect fluid models. The perfect fluid models are solved with the combination of EoS like Chaplygin gas model and the generalization of this model [18, 19]. On the basis of particle physics, there have been several efforts to find the scalar field models for the explanation of DE [20–22]. The alternative way for the construction of DE model is to modify the "Einstein-Hilbert action" to attain the modified gravity theories like $f(R)$ gravity, $f(T)$ (where T is torsion scalar) gravity, $f(R,T)$ and Gauss-Bonnet gravity [23–31].

The compelling research phenomena of "cosmological inflation" are introduced by Guth (1981) [32]. In hot big-bang (HBB) theory, inflation is a notion implemented on a very initial cosmic stage of expansion. The scale of inflation is assumed to be long enough since over and the standard evolution is rebuilt to hold the prominent triumphs, such as CMBR and nucleosynthesis. In spite of all of its successes, there are some puzzles with HBB theory, which generate inflation [33].

The "flatness issue" [34]$\Omega = 1$ on the time scale, where $\Omega = \rho/\rho_c$, ρ_c being critical density. The curvature term, $(aH)^2$ (a, H is the scale factor and the Hubble parameter) in standard big-bang model, decreases with respect to time lead to $\Omega(t)$ varying from unity. However, recent observations suggested that the value of $\Omega(t)$ is closed to unity, thus it must be same in the early-time such that its value is $|\Omega(t_{Pl})| < \mathcal{O}(10^{-64})$ at *Planck time* while during nucleosynthesis $|\Omega(t_{nucleo})| < \mathcal{O}(10^{-16})$. The difference in numeric values suggests that initial conditions should be fine-tuned. An inappropriate choice generates a cosmos, which either soon expands before the formation of structure or quickly collapses.

The "horizon problem [34]" illustrates "why the temperature of CMBR is the same all over the sky?" The exactly same temperature of CMBR in east and west directions is detected through antenna whereas the radiation coming from opposite directions is separated by $28BLY$. It is well known that travel speed of information is always less than the speed of light, hence neither the radiation nor the regions ever have been in thermal contact. Any two cosmic regions could be in thermal equilibrium if and only if they are closed enough to communicate with each other. So, question arises that without any causal connection, how was thermal equilibrium between two regions developed?

Inflationary mechanism is basically introduced to solve the classical shortcomings attached with HBB model. More precisely, during inflationary phase the factor $a(t)$ grows exponentially ($\ddot{a} < 0$) and evolution equation immediately yields $\rho + 3P < 0$; since ρ is a positive quantity, therefore to hold the mentioned inequality, P must be negative (*i.e.*, $P < -\rho/3$). Symmetry breaking is a technique which helps in achieving this negative pressure. The cosmic model with cosmological constant (Λ) satisfying EoS $P = -\rho$ is the usual example of cosmic inflationary model. The quantity ρ_λ decayed into ordinary matter with passage of time, leading to graceful exit from inflation and sustained the HBB model. Unluckily, Λ is known to be very ad hoc mechanism. An outstanding inflationary model should follow a reasonable hypothesis for the origin of Λ and a graceful exit from the phase of inflation [35].

The *phase transition* is a successful mechanism to achieve inflation, especially a dramatic stage in time-line of the universe where universe really alters its properties. In fact, the present cosmos have undergone a chain of phase transitions as its temperature cooled down. Scalar field, an unusual form of matter with negative pressure, is assumed to be responsible for these transitions in cosmic phases. The inflaton decayed at the end of evolutionary phase and inflation terminates, hopefully expanding the universe by a factor of 10^{27} or more. Moreover, modified gravity theories (MGT) [36–39] provide a new way to get inflation. In these MGT, higher-derivative curvature corrections in Einstein's theory lead to early-time acceleration (see [40, 41] for review and [33, 42–49] for applications).

Liddle and Samuel [50] discussed the effects of nonstandard expansion between two cosmic phases, end of inflation, and the current cosmic stage, resulting that the expected number of e-folding (N) can be reformed and significantly increased in some cases. Walliser [51] solved general scalar-tensor theories of gravity and found the differential equations which successfully inflate the universe. Garcia-Bellido and Quiros [52] solved the problem of inflation, based on a general scalar-tensor theory of gravity. They determined a particular class of models with a Brans-Dicke like behavior during inflation. The result converted continuously to general relativity during the radiation and matter-dominated eras. They solved numerical equations of motion and found a subclass of models. Lahiri and Bhattacharya [53] formulated a general mechanism to analyze the linear perturbations during inflation based on the gauge-ready approach. They solved the first order slow-roll equations for scalar and tensor perturbations and obtained the super-horizon solutions for different perturbations after inflation.

Myrzakulov et al. [54] described the inflation with the reference to $f(R,\phi)$-theories and generated a class of models which support early-time acceleration. Sharif and Saleem [55] studied the warm inflation in the framework of locally rotationally symmetric Bianchi type I universe model. They presented the graphical analysis of the perturbed parameters to check the comparability of the considered model with recent data. In Jordan frame, Mathew et al. [56] constructed exact solution with nonminimal coupled action of $f(R)$ gravity to a massive inflaton field. They proved that the solutions were the same as in scalar-tensor theory. They also explained the dynamics of *tensor power spectrum* associated to this model.

Inspiring by the technique used in [56], we build a cosmic inflationary model with a massive inflaton field that has fundamental place in the standard model of particle physics. The *Einstein-Hilbert (EH) action* is considered as a constrained case of a generalized action with higher order curvature invariants; $f(R)$ gravity is the example of such an action [36, 57, 58]. General theory of relativity (GR) cannot be renormalizable, so it is not possible to quantize it conventionally. However, the modified EH action containing higher order curvature terms can be renormalizable [59, 60], due to which $f(R)$ gravity is taken to be an interesting alternative to GR. The associated $f(R)$ field equations are nontrivial due to its fourth order. In addition, these theories do not experience *Oströgradsky instability* [61].

A conformal transformation can be applied to $f(R,\phi)$ action to convert it to EH action with an additional (canonical) inflaton field [62]. The *scalar-tensor gravity theories* suffer from a long-lasting controversy about the choice of physical frame either Einstein or Jordan [63]. Although the two: Jordan frame (original) as well as the Einstein frame are under conformal transformation, it is unclear how the observable quantities are related to the physical quantities computed in the two frames [35, 64]. To get rid of these controversies and the vagueness in the selection of frame, here we take the action without implementing any transformation to frame, any other theory, or variables [63]. Since the EH action does not possess any nonminimal coupling term, so there is no motivation of performing conformal transformation. Generally, we cannot trust in these techniques presented in literature, and we work with a new analytical method developed in [56].

The manuscript is arranged as follows. In Section 2, firstly, we consider a model and obtain the exact analytical solutions in de-Sitter case and secondly we find inflationary solutions numerically with an exit for different initial conditions. In Section 3, we discuss the scalar and tensor power spectra and prove that the solution of Hubble parameter represents a saddle point. The compatibility of the model with recent data is checked through graphical analysis of the perturbed parameters. In the last section, we conclude the results.

2. Model and Background Solution

The $f(R,\phi)$ theory is described by the action given as

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2}f(R,\phi)-\frac{1}{2}\omega g^{ab}\nabla_a\phi\nabla_b\phi-V(\phi)\right], \quad (1)$$

where $V(\phi)$ denotes the effective potential related to inflaton field. We are taking the following $f(R,\phi)$ model

$$f(R,\phi)=\frac{1}{\kappa}\left[R+h(\phi)R^2e^{\alpha R}\right], \quad (2)$$

with a coupling function denoted by $h(\phi)$. By expanding the exponential in terms of Ricci scalar up to first order, we have

$$f(R,\phi)\simeq\frac{1}{\kappa}\left[R+h(\phi)\left(R^2+\alpha R^3\right)\right]. \quad (3)$$

In Jordan frame, the corresponding $f(R,\phi)$ field equations are as follows [65]:

$$\Box\phi+\frac{1}{2\omega}\left(\omega_{,\phi}\phi^{;a}_{;a}+f_{,\phi}-2V_{,\phi}\right)=0, \quad (4)$$

$$\begin{aligned}FG^p_q&=\omega\left(\phi^{;p}\phi_{;q}-\frac{1}{2}\delta^p_q\phi^{;c}\phi_{;c}\right)-\frac{1}{2}\delta^p_q\left(RF-f+2V\right)\\&\quad+F^{;p}_{;q}-\delta^p_q\Box F,\end{aligned} \quad (5)$$

where $F=\partial f(R,\phi)/\partial R$. In case of scalar field and modified gravity, the corresponding stress-tensors are defined, respectively, as

$$T^{\phi}_{\mu\nu}=\omega\left(\partial_\mu\phi\partial_\nu\phi-\frac{1}{2}g_{\mu\nu}\partial_c\phi\partial^c\phi\right)-g_{\mu\nu}V(\phi), \quad (6)$$

$$T^{MG}_{\mu\nu}=\frac{1}{2}g_{\mu\nu}\left(f-FR\right)+\nabla_\mu\nabla_\mu F-g_{\mu\nu}\Box F. \quad (7)$$

Now, we will find exact solution analytically in de-Sitter case.

2.1. Background Inflationary Solution. The line element of flat FRW space time is

$$ds^2=-dt^2+a(t)^2\left(dx^2+dy^2+dz^2\right), \quad (8)$$

where $a(t)$ denotes the scale factor. Using FRW space time, we have equation of motion for ϕ (4) and field equations (5) of the form, respectively,

$$-\omega\dot{\phi}^2-6F\left(\dot{H}+H^2\right)+6\dot{F}H+2V-f=0, \quad (9)$$

$$\begin{aligned}&4FH^2+2F\left(\dot{H}+H^2\right)-\omega\dot{\phi}^2-2\ddot{F}-4FH-2V+f\\&=0,\end{aligned} \quad (10)$$

$$\begin{aligned}&2\omega\ddot{\phi}+6\omega\dot{\phi}H-\frac{h_{,\phi}f}{h}+\frac{12h_{,\phi}H^2}{h\kappa}+\frac{6h_{,\phi}\dot{H}}{h\kappa}+2V_{,\phi}\\&=0,\end{aligned} \quad (11)$$

where $h=h(\phi)$ is the coupling function. Here, we are considering the following assumptions

$$\begin{aligned}a(t)&=a_0e^{H_Dt},\\ \phi&=\phi_0e^{-nH_Dt},\end{aligned} \quad (12)$$

where n, ϕ_0, and H_D are constants. Substituting (12) in (9)-(11) and solving these equations, we get the coupling function of the form

$$h(\phi)=\lambda_0+\lambda_1\phi^2+\lambda_n\phi^{-1/n}, \quad (13)$$

where $\lambda_1=-n\omega\kappa/48H_D{}^2(2n+1)(18\alpha H_D{}^2+1)$ and $\lambda_n=nc_1$. The scalar field potential is

$$V(\phi)=V_0+V_1\phi^2+V_n\phi^{-1/n}, \quad (14)$$

where

$$\begin{aligned}V_0&=\frac{3H_D{}^2\lambda_0}{\kappa},\\ V_1&=24\lambda_1H_D{}^4\left\{-18\alpha H_D{}^2+n^2\left(36\alpha H_D{}^2+2\right)\right.\\&\quad\left.-5n\left(18\alpha H_D{}^2+1\right)\right\},\\ V_n&=-\frac{72\lambda_nH_D{}^4\left(12\alpha H_D{}^2+1\right)}{\kappa}.\end{aligned} \quad (15)$$

Here λ_0 is the constant of integration. It can be seen that this is an exact solution obtained from the background equations. We are mentioning here some important points: first, we have obtained the solution without using conformal transformation. According to the best of our knowledge, in Jordan frame no exact solution exists. Second, in case of exact analytical de-Sitter solution, the inflaton field (12) decreases as time increases. Third, coupling function $h(\phi)$ directly depends on $V(\phi)$.

From (13) and (14), it can be seen that V_0 depends on λ_0, and similarly V_n depends on λ_n and λ_1 is related to V_1. If we choose $\lambda_0=\lambda_n=0$, then it is obvious that V_0 and V_n also vanished.

3. Special Case: $\lambda_0=\lambda_n=0$

Here, we consider $\lambda_0=\lambda_1=0$. The coupling function and potential are reduced to

$$\begin{aligned}h(\phi)&=\lambda_1\phi^2,\\ V(\phi)&=V_1\phi^2.\end{aligned} \quad (16)$$

This leads to conclude the following points. It can be seen that λ_1 is a positive definite implying that $18\alpha H_D{}^2+1<0$ or $\alpha<-1/(18H_D{}^2)$. Since during inflation the parameter H is large, this leads to small negative value of α. From the stress-tensor (6) and (7), we can calculate $\rho+3P$ as

$$\begin{aligned}\rho+3P&\equiv-T^0_0+T^\alpha_\alpha=2\phi_0{}^2H_D{}^2e^{-2nH_Dt}\left[n^2+V1\right.\\&\quad-\lambda_1\left\{72H_D{}^2\left(1+2n+2n^2\right)+648\alpha H_D{}^4\right.\\&\quad\left.\left.\times\left(3+2n+4n^2\right)\right\}\right].\end{aligned} \quad (17)$$

In (17), the first and second terms are related to the canonical inflaton field while the last quantity represents the modifications to the gravity. The numeric value $n < 1/2$ yields the negativity of the third term while it is positive for $n > 1/2$. However, $n \ll 1$ leads to $\rho + 3P < 0$ and for $n \gg 1$, we have $\rho + 3P > 0$. In further analysis, we are taking $n \gg 1$. We can say from the above discussion that either it corresponds to exit for large values of n or by varying the initial condition of $\dot{\phi}$. If $\dot{\phi}_{t=0} \neq \dot{\phi}^{dS}_{t=0}$, then we will check what kind of inflation exists using the relation $\dot{\phi} \propto n$.

4. First Order Scalar and Tensor Model Perturbations

Now we will discuss the scalar/tensor power spectra for considered inflationary model (3). We have used the same notation as used in [66] then it will be easy to compare. For our model, analysis of [66] is not applicable.

4.1. Perturbations. For FRW space time first order perturbations are

$$ds^2 = -(1+2\theta)\,dt^2 - a\left(\beta_{,\alpha} + B_\alpha\right) dt\, dx^\alpha + a^2\left[g^{(3)}_{\alpha\beta}(1-2\psi) + 2\gamma_{,\alpha|\beta} + 2C_{\alpha|\beta} + 2C_{\alpha\beta}\right]. \quad (18)$$

In the above expression, the scalar perturbations are represented by $dt \equiv a d\eta$ and $\theta(x,t)$, $\beta(x,t)$, $\psi(x,t)$, $\gamma(x,t)$. The quantities $B_\alpha(x,t)$ as well as $C_\alpha(x,t)$ denote the trace-free vector perturbation and $C_{\alpha\beta}(x,t)$ presents the trace-free and transverse tensor perturbations. The decomposed form of inflaton field is $\phi(x,t) = \overline{\phi}(t) + \delta\phi(x,t)$.

In Fourier space and scalar perturbation of Newtonian gauge are as follows [65, 66]:

$$-F\psi + F\theta + \delta F = 0, \quad (19)$$

$$-2F\dot{\psi} - 2FH\theta - \dot{F}\theta + \dot{\phi}\delta\phi + \dot{\delta F} - H\delta F = 0, \quad (20)$$

$$6FH\dot{\psi} + 6FH^2\theta + 2F\frac{k^2}{a^2}\psi - \dot{\phi}^2\theta + 3\dot{F}\dot{\psi} + 6\dot{F}H\theta + \dot{\phi}\dot{\delta\phi} - \ddot{\phi}\delta\phi - 3H\dot{\phi}\delta\phi - 3H\dot{\delta F} + 3\dot{H}\delta F + 3H^2\delta F - \frac{k^2}{a^2}\delta F = 0, \quad (21)$$

$$6F\ddot{\psi} + 12F\dot{H}\theta + 6FH\dot{\theta} + 12FH\dot{\psi} + 12FH^2\theta - 2F\frac{k^2}{a^2}\theta + 3\dot{F}\dot{\psi} + 6\dot{F}H\theta + \dot{F}\dot{\theta} + 4\dot{\phi}^2\theta + 6\theta\ddot{F} - 4\dot{\phi}\dot{\delta\phi} - 2\ddot{\phi}\delta\phi - 6H\dot{\phi}\delta\phi - 3\ddot{\delta F} - 3H\dot{\delta F} + 6H^2\delta F - \frac{k^2}{a^2}\delta F = 0, \quad (22)$$

$$\ddot{\delta\phi} + 3H\dot{\delta\phi} - \frac{1}{2}f_{\phi\phi} + V_{\phi\phi}\delta\phi + \frac{k^2}{a^2}\delta\phi - 3\dot{\phi}\dot{\psi} - 6H\dot{\phi}\theta - \dot{\phi}\dot{\theta} - 2\ddot{\phi}\theta + 3F_\phi\ddot{\psi} + 6F_\phi\dot{H}\theta + 3HF_\phi\dot{\theta} + 12F_\phi H\dot{\psi} + 12F_\phi H^2\theta + 2F_\phi\frac{k^2}{a^2}\psi - F_\phi\frac{k^2}{a^2}\theta = 0, \quad (23)$$

$$\delta F - F_\phi\delta\phi + F_R\delta R = 0, \quad (24)$$

where $\delta R = -6\ddot{\psi} - 12\dot{H}\theta - 6H\dot{\theta} - 24H\dot{\psi} - 24H^2\theta - 4(k^2/a^2)\psi + 2(k^2/a^2)\theta$. While the tensor perturbations are as follows [65]:

$$\ddot{C}^\alpha_\beta + \left(\frac{\dot{F}}{F} + 3H\right)\dot{C}^\alpha_\beta + \frac{k^2}{a^2}C^\alpha_\beta = 0. \quad (25)$$

4.2. Scalar Power Spectrum. Here we calculate the equation, which satisfies the 3-curvature perturbation $\mathcal{R}$ and derives the related power spectrum. We are using the technique followed in [67]. The $\mathcal{R}$ in Jordan frame is stated as

$$\mathcal{R} = \psi + \frac{H}{\dot{\phi}}\delta\phi. \quad (26)$$

Equations (19)-(24) are highly ordered and nonlinear, so we use different techniques to calculate the 3-curvature perturbation equation. First, we will use $\theta + \psi = \Theta$ and find the solution of differential equation. Physically, in Einstein frame, Θ represents the Bardeen potential. Using (20)-(22), we have differential equation in Θ as

$$F\ddot{\Theta} + \left(3\dot{F} + FH - \frac{2F\ddot{\phi}}{\dot{\phi}}\right)\dot{\Theta} + \left(\frac{Fk^2}{a^2} + \frac{2\dot{F}\ddot{\phi}}{\dot{\phi}} - \frac{2FH\ddot{\phi}}{\dot{\phi}} - \ddot{F} + \dot{F}H + 4F\dot{H}\right)\Theta + \left(-\dot{\phi}^2 + 3\ddot{F} + 3\dot{F}H - 6F\dot{H} - \frac{6\dot{F}\ddot{\phi}}{\dot{\phi}}\right)\theta = 0. \quad (27)$$

Using background quantities for de-Sitter, assuming $n \ll 1$ and large values of k, we have differential equation in terms of Θ as

$$\ddot{\Theta} + (1-4n)H_D\dot{\Theta} + \frac{k^2}{a^2}\Theta - 4n(1-n){H_D}^2\theta = 0. \quad (28)$$

Applying the small wavelength limit $k/a \gg 1$, the last two terms of the left hand side can be written as

$$\left(\frac{k^2}{a^2} - 4n(1-n){H_D}^2\right)\theta + \frac{k^2}{a^2}\psi \simeq \frac{k^2}{a^2}\Theta, \quad (29)$$

which can further be written as

$$\ddot{\Theta} + (1-4n)H_D\dot{\Theta} + \frac{k^2}{a^2}\Theta = 0. \quad (30)$$

In terms of Θ, $\delta\phi$ is as follows

$$\delta\phi = -\frac{\phi_0 e^{-nH_D t}}{nH_D}\left(\dot{\Theta} + H_D\Theta\right). \quad (31)$$

Rewriting the perturbation equations in terms of $\mathscr{R}$ and using (26) and (30), we have Θ in terms of $\mathscr{R}$ as

$$\Theta = -\left(3a^2\ddot{R} + 12H_D a^2\dot{R} + 2k^2 R\right)\frac{nH_D}{\phi_0}e^{nH_D t}, \quad (32)$$

substituting it into the perturbation equations, we get the differential equation in terms of $\mathscr{R}$:

$$\ddot{\mathscr{R}} + 3H_D\dot{\mathscr{R}} + \frac{k^2}{a^2}\mathscr{R} = 0. \quad (33)$$

It is an important result which shows that higher order differential equation can be reduced to second order. For the power spectrum, we can evaluate solution to the above differential equation for the short wavelength limit and usethe Bunch-Davies vacuum at the initial epoch of inflation as

$$R_< = \frac{H_D}{2a\sqrt{k}}e^{-ik\eta}. \quad (34)$$

In the long wavelength limit, we get $R_> = C$. By matching $R_<$ and $R_>$ at horizon crossing ($|k\eta| = 2\pi$), we get

$$C = \frac{\sqrt{2}H_D\pi}{k^{3/2}}, \quad (35)$$

leading to the following scale-invariant scalar power spectrum

$$\mathscr{P}_R = {H_D}^2. \quad (36)$$

For $n \ll 1$, above analysis is an analytical expression. It is not possible to obtain the semianalytical expression for other cases, which shows the tilted spectrum. Next, we derive the tensor power spectrum without using $n \ll 1$ limit which leads to blue tilt.

4.3. Tensor Power Spectrum. Following [65], the required equation of motion for tensor perturbation of exact de-Sitter solution can be obtained as

$$\ddot{C}^{\alpha}_{\beta} + (3-2n)H_D\dot{C}^{\alpha}_{\beta} + \frac{k^2}{a^2}C^{\alpha}_{\beta} = 0. \quad (37)$$

Defining $C^{\alpha}_{\beta} = v_g/z_g$ and $z_g = ae^{-nH_D t}$, we have

$$v_g'' + \left(k^2 - \frac{z_g''}{z_g}\right) = 0. \quad (38)$$

The above-mentioned differential equation yields a result as a combination of Hankel function:

$$v_g = \sqrt{-\eta}\left(\widetilde{C}_1 H^{(1)}_{3/2-n}(-k\eta) + \widetilde{C}_2 H^{(2)}_{3/2-n}(-k\eta)\right). \quad (39)$$

Fixing Bunch-Davies vacuum to the initial state, we obtained $\widetilde{C}_1 = \sqrt{\pi/4}$; $\widetilde{C}_2 = 0$. Thus tensor perturbation C^{α}_{β} provides

$$v_g = \sqrt{\frac{\pi}{4}}\sqrt{-\eta}H^{(1)}_{3/2-n}(-k\eta). \quad (40)$$

The corresponding power spectrum ($\mathscr{P}_g$) can be evaluated in the following form $\mathscr{P}_g = 8(k^3/2\pi^2)|\mathscr{C}^{\alpha}_{\beta}|^2$ and modified as

$$\mathscr{P}_g = 8\left(\frac{k}{k_*}\right)^{2n}\frac{2^{-2n}}{4\pi^2}{H_D}^2\left(\frac{\Gamma(3/2-n)}{\Gamma(3/2)}\right)^2 e^{2nH_D t_*}. \quad (41)$$

The tensor spectral index is calculated as $n_T = 2n$, implying a blue-tilted spectrum for a decaying inflaton field, i.e., blue tilted for $n > 1/2$ and red tilted for $n < 1/2$.

5. Stability of Inflationary Solution

Now we will discuss the stability of de-Sitter solution and will examine the variations of initial values whether they show inflationary phase, super inflation, or smooth exit. Hence, to show the inflationary solution many initial values exist and a mess of models have been discussed in literature which show the saddle point [68, 69]. The field equations (9)-(11) in terms of the variable $\Delta = \dot{\phi}/\phi$ can be written as

$$\begin{aligned}\dot{\Delta} &= 2V_1 + 2592\alpha\lambda_1\dot{H}H^4 + 1296\alpha\lambda_1\dot{H}^2H^2 \\ &\quad + 1728\alpha\lambda_1H^6 + 216\alpha\lambda_1\dot{H}^3 - 144\lambda_1\dot{H}H^2 \\ &\quad - 144\lambda_1H^4 - 36\lambda_1\dot{H}^2 - 3H\Delta - \Delta^2,\end{aligned} \quad (42)$$

$$\begin{aligned}\ddot{H} &= \frac{1}{36\lambda_1H - 648\alpha\lambda_1H\dot{H} - 1296\alpha\lambda_1H^3}\left[\frac{3H^2}{{\phi_0}^2}\right. \\ &\quad \cdot e^{2nH_D t} - 180\lambda_1\dot{H}H^2 + 972\alpha\lambda_1H^2\dot{H}^2 \\ &\quad - 3024\alpha\lambda_1H^6 + 1296\alpha\lambda_1\dot{H}H^4 + \frac{1}{2}\Delta^2 - 18\lambda_1\dot{H}^2 \\ &\quad + 216\alpha\lambda_1\dot{H}^3 - V_1 - 72\lambda_1H\dot{H}\Delta - 144\lambda_1H^3\Delta \\ &\quad + 648\alpha\lambda_1H\dot{H}^2\Delta + 2592\alpha\lambda_1H^5\Delta \\ &\quad \left. + 2592\alpha\lambda_1H^3\dot{H}\Delta\right].\end{aligned} \quad (43)$$

In terms of Δ, the above-mentioned equations show that the evolution of H and N, etc., does not involve ϕ or $\dot{\phi}$ and only depends on Δ.

A vector v is defined as

$$v = \begin{pmatrix} H \\ \dot{H} \\ \Delta \end{pmatrix}. \quad (44)$$

It is worth noticed that the solution of de-Sitter model ($H = H_D$) behaves as an equilibrium point ($\dot{v}_{eq} = 0$) where

$$\{v\}_{eq} = \begin{pmatrix} H_D \\ 0 \\ -nH_D \end{pmatrix}, \quad (45)$$

and the expression $\dot{v} = f(v)$ is written as

$$\dot{v} = \begin{pmatrix} \dot{H} \\ \ddot{H} \\ \dot{\Delta} \end{pmatrix}. \quad (46)$$

As we have mentioned above, perturbing $v = v_{eq} + \delta v$ and expansion of $f(v)$ for δv around the equilibrium point provide

$$\delta v_i = \left\{\partial_j f_i\right\}_{eq} \delta v_j = J_{ij} \delta v_j, \quad (47)$$

where

$$J_{ij} = \begin{pmatrix} \frac{\partial \dot{H}}{\partial H} & \frac{\partial \dot{H}}{\partial \dot{H}} & \frac{\partial \dot{H}}{\partial \Delta} \\ \frac{\partial \ddot{H}}{\partial H} & \frac{\partial \ddot{H}}{\partial \dot{H}} & \frac{\partial \ddot{H}}{\partial \Delta} \\ \frac{\partial \dot{\Delta}}{\partial H} & \frac{\partial \dot{\Delta}}{\partial \dot{H}} & \frac{\partial \dot{\Delta}}{\partial \Delta} \end{pmatrix}. \quad (48)$$

Let us consider the eigen value λ_i and eigen vector μ_i of the Jacobian. Hence trajectory of phase space is introduced by

$$\delta v_i = \sum_{i=1}^{i=3} c_i \mu_i e^{(\lambda_i t)}. \quad (49)$$

where the values of constants c_i's have to be constraint from initial values of Hi and $\dot{\phi}/\phi$. In our case, we have one real and two complex eigen values which are too lengthy in expression due to which we did not mention them in the paper. For large values of (λ), N is equivalent to

$$N \approx \frac{H_D}{\lambda} \ln \left(\frac{{H_D}^2}{\lambda \left(H_D - H_j \right)} \right). \quad (50)$$

Figure 1 shows the behavior of slow-roll parameter $\epsilon = -\dot{H}/H^2$ versus N for various values of $\dot{\phi}$. The parameter ϵ can be evaluated as follows:

$$\epsilon = \frac{24H^2 \left(18\alpha H^2 + 1\right) \left(2H\lambda_1 \phi\dot{\phi} - 2\lambda_1 \dot{\phi}^2 - 2\lambda_1 \ddot{\phi}\phi\right)}{2H^2 \left\{1 - 12H^2 \lambda_1 \phi^2 \left(36\alpha H^2 + 2\right)\right\}}. \quad (51)$$

Figure 1 is plotted for $\epsilon - N$ trajectories taking some initial values. It can be observed that for $\dot{\phi} < 1.4\phi_D$, the inflationary phase sustained as $\epsilon < 1$ and ϵ attains a constant value less than unity for $\dot{\phi} \geq 1.4\phi_D$, fixing the other parameters as $\omega_0 = -0.005$, $\phi_0 = -0.3$, $n = 200$, $H_D = 4 \times 10^4$, $H_j = 2$, $\alpha = -10^{-7}$. It can be seen that in the space $\dot{\phi} < \phi_D$, the inflationary phase exists without an exit which represents ϵ diverging to $-\infty$ while $\dot{\phi} > \phi_D$ leads to the inflationary era with an exit. The initial condition $\dot{\phi} = \phi_D$ generates $\epsilon = 0$. In this case, the results are obtained for standard number of e-folds, i.e., $N \simeq 50, 60$, which is in good agreement with observational data. Further, it is observed that as the value of $\dot{\phi}/\phi_D$ is directly proportional to N, as increment in initial value produced an increase in N. Hence rate of inflation increases as $\dot{\phi}$ increases.

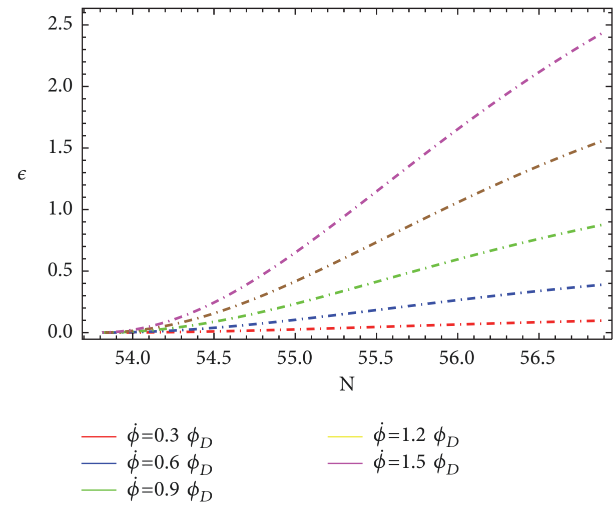

FIGURE 1: Evolution of ϵ versus N for various values of $\dot{\phi}$.

The trajectories are attracted toward its origin with increasing initial values. This shows that divergence of initial constraints form de-Sitter values yields either inflation with graceful exit or super inflation. Hence de-Sitter analytical solution represents saddle point.

Figure 2 is plotted for ϕ versus N (left plot) and versus time t (right plot) for standard number of e-folds. It can be seen that scalar field is decaying with the evolution of time. The trajectories of $\phi - N$ show the same behavior as $\epsilon - N$. The scalar field ϕ is expressed as

$$\phi = \frac{6\kappa\omega H\dot{\phi}}{4\kappa v_1 - 288\lambda_1 - 3456\alpha\lambda_1 H^6}. \quad (52)$$

The scalar spectral index is defined as [70, 71]

$$n_s = 1 - 4\epsilon_1 - 2\epsilon_2 + 2\epsilon_3 - 2\epsilon_4, \quad (53)$$

where

$$\begin{aligned} \epsilon_1 &= -\frac{\dot{H}}{H^2}, \\ \epsilon_2 &= \frac{\ddot{\phi}}{H\dot{\phi}}, \\ \epsilon_3 &= \frac{\dot{F}}{2HF}, \\ \epsilon_4 &= \frac{\dot{E}}{2HE}, \\ E &= \omega F + \frac{3\dot{F}^2}{2\dot{\phi}^2}. \end{aligned} \quad (54)$$

The tensor to scalar ratio is defined as $r = P_g/P_r$. For better understanding of the inflationary model's compatibility with recent data, we have plotted parametric plots (Figure 3) in which scalar spectral index is plotted versus tensor to scalar ratio for $n > 1$. It is observed that for $n = 1.61$, we have

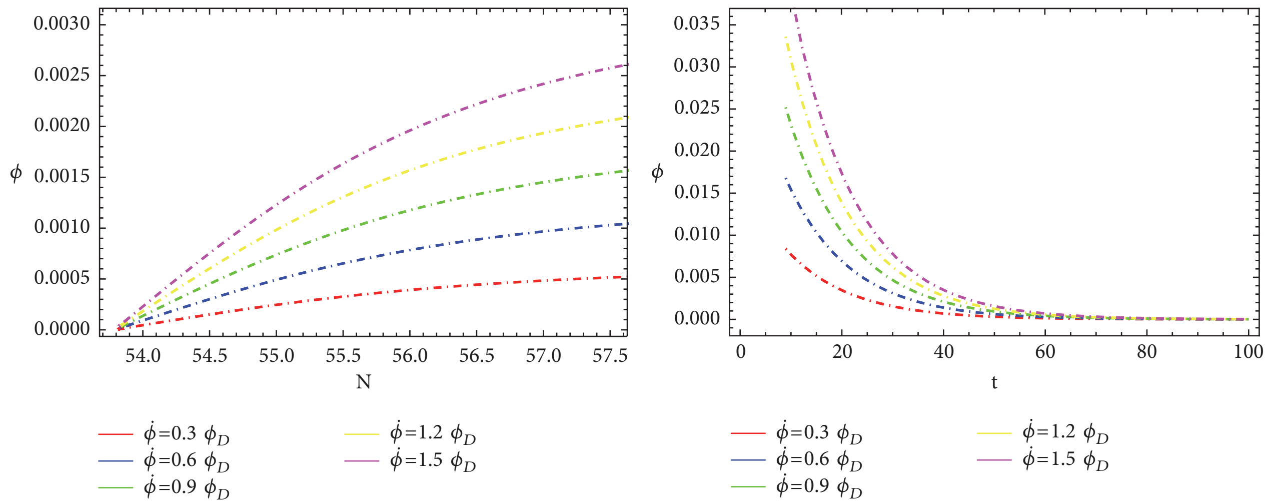

FIGURE 2: Evolution of ϕ versus N for different initial values of $\dot{\phi}$.

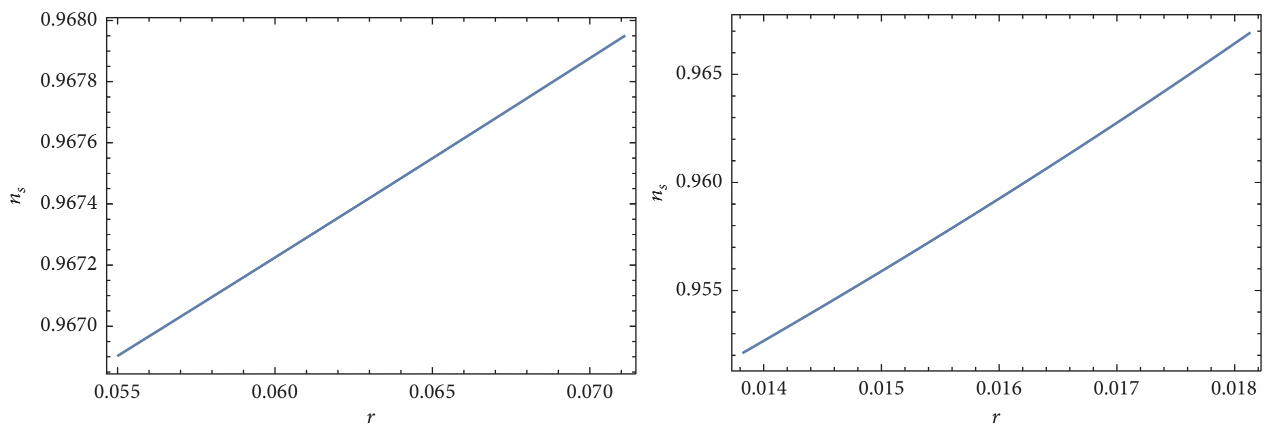

FIGURE 3: Left plot shows the behavior of r versus n_s for $\dot{\phi} = 0.2\phi_D$ with $n = 1.6$ and right plot for $\dot{\phi} = 0.2\phi_D$ with $n = 1.8$.

the standard value of spectral index $n_s = 0.96$ and an upper bound of tensor to scalar ratio is obtained as $r < 0.36$, which is compatible with WMAP7 [72], while for $n = 1.8$, we get $r < 0.11$ in accordance with Plank 2015 [73].

6. Discussion

In this manuscript, we derived the exact inflationary model in Jordan frame for $f(R, \phi)$ model. Even though it is common to study the $f(R)$ models in Einstein frame, we showed explicitly that in the Einstein frame, action contains noncanonical kinetic term. Thus, the advantage of the conformal transformation is negated. Thus, we performed the background and the first order perturbation analysis in the Jordan frame. A nonminimally coupling of massive inflaton field with $f(R)$ action is considered and has ignored the self-interacting inflaton potential. We have evaluated the expression $\rho + 3P$, which led to conclude that for the values $n \ll 1$, $\rho + 3P < 0$ and for $n \gg 1$, we have $\rho + 3P > 0$. In [56], authors also calculated the term $\rho + 3P$ and showed that for $n \gg 1 \Longrightarrow \rho + 3P = 0$ which did not lead to inflation while the inequality $\rho + 3P < 0$ is satisfied for $n \ll 1$.

It is explicitly shown that the model supports inflationary solution with graceful exit and the number of e-folds depends on deviation of initial values from de-Sitter scenario. Further, we have discussed the scalar and tensor perturbations for the chosen $f(R, \phi)$ model. We have used the new analytical method devised in [56] to reduce the higher order scalar perturbation equations to second order in 3-curvature perturbation. We analytically obtained the scalar power spectrum in $n \gg 1$ limit and proved the scale invariance of scalar power spectrum. We obtained the tensor power spectrum for $n > 1/2$ and showed that the spectrum is blue tilted.

To get insight, we plotted the graphs of parameter ϵ and inflaton field for various physical initial conditions of $\dot{\phi}$, i.e., $\dot{\phi} = 0.3\phi_D, 0.6\phi_D, 0.9\phi_D, 1.2\phi_D, 1.5\phi_D$ and $\dot{\phi} = \phi_D$. The $\epsilon - N$ trajectories showed that in the parametric space $\dot{\phi} < 1.4\phi_D$, the inflationary phase sustained as $\epsilon < 1$ and ϵ attains a constant value less than unity for $\dot{\phi} \geq 1.4\phi_D$. It can be seen that in the space $\dot{\phi} < \phi_D$, the inflationary phase exists without an exit which represents ϵ diverging to $-\infty$ while $\dot{\phi} > \phi_D$ leads to the inflationary era with an exit. The initial condition $\dot{\phi} = \phi_D$ generates $\epsilon = 0$. In this case, the results are obtained for standard number of e-folds, i.e., $N \simeq 50, 60$, which is in good agreement with observational data. Further, it is observed that as the value of $\dot{\phi}/\phi_D$ is directly proportional to N, as increment in initial value produced an increase in N. Hence the rate of inflation increases as $\dot{\phi}$ increases. The trajectories are attracted toward its origin with increasing initial values. This shows that change in initial conditions forms value of de-Sitter implying either inflation with an exit or super inflation and proves that the obtained de-Sitter solution behaves as a saddle point. The plot $\phi - N$ represents the decaying behavior with evolution of time. The trajectories of ϵ and ϕ versus N for various initial conditions of $\dot{\phi}$ and $n = 0.01, 0.1$ are also plotted in [56, 67]. These plots show inflationary era and then an exit from inflation for $\dot{\phi} < \phi_D$ (which is opposite to our case) and number of e-folding lies between 80 and 90. Our results are compatible with [56, 67] for standard values of perturbed parameters.

For better understanding of the inflationary model's compatibility with recent data, we have plotted parametric plots (Figure 3) in which scalar spectral index is plotted versus tensor to scalar ratio for $n > 1$. It is observed that for $n = 1.61$, we have the standard value of spectral index $n_s = 0.96$ and an upper bound of tensor to scalar ratio is obtained as $r < 0.36$ which is compatible with WMAP7 [72], while for $n = 1.8$, we get $r < 0.11$ in accordance with Plank 2015 [73].

One of the main points is that the inflationary models in general relativity go to red-tilt [74, 75]. Our discussion and the references [56, 67] led to a conclusion that the modified theories of gravity and general relativity can be distinguished by the fact that, in modified theories, the tensor spectrum is blue tilted. It is worth mentioning here that our results reduced to [67] by choosing $\alpha = 0$.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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