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Research Article

Spectroscopy of $z = 0$ Lifshitz Black Hole

Gulnihal Tokgoz and Izzet Sakalli

Physics Department, Eastern Mediterranean University, Famagusta, Northern Cyprus, Mersin 10, Turkey

Correspondence should be addressed to Gulnihal Tokgoz; gulnihal.tokgoz@emu.edu.tr

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We studied the thermodynamics and spectroscopy of a 4-dimensional, $z = 0$ Lifshitz black hole ($Z0$LBH). Using Wald's entropy formula and the Hawking temperature, we derived the quasi-local mass of the $Z0$LBH. Based on the exact solution to the near-horizon Schrödinger-like equation (SLE) of the massive scalar waves, we computed the quasi-normal modes of the $Z0$LBH via employing the adiabatic invariant quantity for the $Z0$LBH. This study shows that the entropy and area spectra of the $Z0$LBH are equally spaced.

1. Introduction

Ever since the publication of the seminal papers of Bekenstein and Hawking [1–3], it has been known that black hole (BH) entropy (S_{BH}) should be quantized in discrete levels as discussed in detail by Bekenstein [4–7]. The proportionality between S_{BH} and BH area ($\mathscr{A}_{BH}$) is justified from the adiabatic invariance [8] properties of area, that is, $S_{BH} = \mathscr{A}_{BH}/4$. Therefore, $\mathscr{A}_{BH}$ should also be quantized in equidistant levels to account for the discrete S_{BH}. Bekenstein [5, 6] proposed that, for the family of Schwarzschild BHs, $\mathscr{A}_{BH}$ should have the following discrete, equidistant spectrum:

$$\mathscr{A}_{BH} = \epsilon \hbar n, \quad n = 0, 1, 2, \ldots, \tag{1}$$

where ϵ is known as the undetermined dimensionless constant. According to (1), the minimum increase in the horizon area becomes $\Delta\mathscr{A}_{\min} = 8\pi\hbar$ for the Schwarzschild BH ($\epsilon = 8\pi$) [9–11]. Following the seminal works of Bekenstein, new methods have been developed to derive the entropy/area spectra of the numerous BHs (see [12] and references therein). Among them, Maggiore's method (MM) [9] fully supports Bekenstein's result (1). In fact, MM [9] was based on Kunstatter's study [13], in which the adiabatic invariant quantity (I_{adb}) is expressed as follows:

$$I_{adb} = \int \frac{dM}{\Delta\omega}, \tag{2}$$

where $\Delta\omega = \omega_{n-1} - \omega_n$ denotes the transition frequency between the successive levels of a BH having mass M. Further, (2) was generalized to the hairy BHs (massive, charged, and rotating ones) as follows (see [14] and references therein):

$$I_{adb} = \int \frac{\mathscr{T}_{\mathscr{H}} dS_{BH}}{\Delta\omega}, \tag{3}$$

where $\mathscr{T}_{\mathscr{H}}$ is the temperature of the BH. Bohr-Sommerfeld quantization rule [15] states that I_{adb} acts as a quantized quantity ($I_{adb} \simeq n\hbar$) when the highly excited modes ($n \longrightarrow \infty$) are considered. In such a case, the imaginary part of the frequency dominates the real part of the frequency ($\omega_I \gg \omega_R$), implying that $\Delta\omega \simeq \Delta\omega_I$. Meanwhile, for the first time, Hod [16, 17] argued that the quasi-normal modes (QNMs) [18, 19] can be used for computing transition frequency. Hod's arguments inspired Maggiore who considered the Schwarzschild BH as a highly damped harmonic oscillator (i.e., $\Delta\omega \simeq \Delta\omega_I$) and managed to rederive Bekenstein's original result (1) using a different method. Today, there are numerous studies in the literature in which MM has been employed for various BHs (see, e.g., [20–25]).

This study mainly explores the entropy/area spectra of a four-dimensional Lifshitz BH [26] possessing a particular dynamical exponent $z = 0$. To analyze the physical features of the $z = 0$ Lifshitz BH ($Z0$LBH) geometry, we first calculate its quasi-local mass M_{QL} [27] and temperature via Wald's entropy [28] and statistical Hawking temperature formula

[29, 30], respectively. QNM calculations of the $Z0$LBH must be performed in order to implement the MM successfully. To this end, we consider the Klein-Gordon equation (KGE) for a massless scalar field in the $Z0$LBH background. Separation of the angular and the radial equations yields a Schrödinger-like wave equation (SLE) [31]. Asymptotic limits of the potential (36) show that the effective potential may diverge beyond the BH horizon for the massive scalar particle; thus, in the far region, the QNMs might not be perceived by the observer. Hence, following a particular method [23, 32–35], we focus our analysis on the near-horizon (NH) region and impose the boundary conditions (45): ingoing waves at the event horizon and no wave at spatial infinity (since at infinity the effective potential of Schrödinger-like wave equation is divergent). After getting NH form of the SLE, we show that the radial equation is reduced to a confluent hypergeometric (CH) differential equation [36]. Performing some manipulations on the NH solution and using the pole structure of the Gamma functions [36], we show how one finds out the QNMs as in [32, 33, 37–40]. The imaginary part of the QNMs is used in (3), and the quantum spectra of entropy and area of the $Z0$LBH are obtained.

The following statements elaborate on the organization of this study. In Section 2, we briefly introduce the $Z0$LBH metric. In addition, we present the derivation of M_{QL} of the $Z0$LBH based on Wald's entropy formula. Section 3 is devoted to the separation of the KGE and finding the effective potential $V_{eff}(r)$. Next, we solve the NH SLE and show how QNMs are calculated. Then, we compute the entropy/area spectra of the $Z0$LBH. Finally, we draw our conclusions in Section 4 (throughout this work, the geometrized unit system is used: $G = c = k_B = 1$ and $\ell_\rho^2 = \hbar$).

2. $Z0$LBH Spacetime

In this section, we introduce the four-dimensional Lifshitz spacetime and its special case, that is, $Z0$LBH [26]. Conformal gravity (CG) covers gravity theories that are invariant under Weyl transformations. CG, which is adapted to static and asymptotically Lifshitz BH solutions, has received intensive attention from the researchers studying condensed matter and quantum field theories [41]. The Lifshitz BHs are invariant under anisotropic scale and characterize the gravitational dual of strange metals [42].

The action of the Einstein-Weyl gravity [26] is given by

$$\mathcal{S} = \frac{1}{2\tilde{\kappa}^2}\int\sqrt{-g}d^4x\left(R - 2\Lambda + \frac{1}{2}\alpha\left|Weyl\right|^2\right), \qquad (4)$$

where $\tilde{\kappa}^2 = 8\pi G$, $|Weyl|^2 = R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} - 2R^{\mu\nu}R_{\mu\nu} + (1/3)R^2$, and $\alpha = (z^2 + 2z + 3)/4z(z-4)$ (constant), which diverges ($\alpha = \infty$) with $z = 0$ and/or $z = 4$. The Lifshitz BH solutions exist in the CG theory for both $z = 4$ and $z = 0$ [26, 43]; however, when $z = 3$ and $z = 4$, Lifshitz BHs appear in the Horava–Lifshitz gravity [26, 44, 45].

Now, we focus on the $Z0$LBH of the CG theory whose metric is given by [26]:

$$ds^2 = -fdt^2 + \frac{4dr^2}{r^2 f} + r^2 d\Omega_{2,k}^2, \qquad (5)$$

where the metric function f is defined by

$$f = 1 + \frac{c}{r^2} + \frac{c^2 - k^2}{3r^4}. \qquad (6)$$

In the above metric, which is conformal to (A)dS (AdS if $c + 2k < 1$ and dS if $c + 2k > 1$) [26], $k = 0$, $k = 1$, and $k = -1$ stand for 2-torus ($d\theta^2 + d\phi^2$), unit 2-sphere ($d\theta^2 + \sin^2\theta d\phi^2$), and unit hyperbolic plane ($d\theta^2 + \sinh^2\theta d\phi^2$), respectively [45]. The metric solution has a curvature singularity at $r = 0$, which becomes naked for $k = 0$. There is an event horizon for $k = \pm 1$ solution expressed as follows [26, 45]:

$$r_h^2 = \frac{1}{6}\left(\sqrt{3\left(4 - c^2\right)} - 3c\right). \qquad (7)$$

Note that the requirement of $r_h^2 \geq 0$ is conditional on this inequality: $-2 \leq c < 1$. When $c = -2$, the solution becomes extremal. Throughout this study, for simplicity, we consider the choice of $c^2 = k^2 = 1$ for which the solution corresponds to a dS BH. Thus, the metric function f becomes

$$f = 1 - \frac{r_h^2}{r^2}. \qquad (8)$$

Thus, at spatial infinity, the Ricci and Kretschmann scalars of the $Z0$LBH can be found as follows:

$$R = R_\lambda^\lambda \sim \frac{5\left(c^4 - 2c^2 + 1\right)}{12r^8},$$
$$K = R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} \sim \frac{25\left(c^4 - 2c^2 + 1\right)}{12r^8}. \qquad (9)$$

By performing the surface gravity calculation [1, 2, 30], we obtain

$$\kappa = \frac{1}{2}. \qquad (10)$$

Therefore, the Hawking or BH temperature [30] of the $Z0$LBH reads

$$\mathcal{T}_{\mathcal{H}} = \frac{\kappa}{2\pi} = \frac{1}{4\pi}. \qquad (11)$$

2.1. Mass Computation of Z0LBH via Wald's Entropy Formula. The GR unifies space, time, and gravitation and the gravitational force is represented by the curvature of the spacetime. Energy conservation is a sine qua non in GR as well. Because the metric (5) of $Z0$LBH represents a non-asymptotically-flat geometry, one should consider the quasi-local mass M_{QL} [27], which measures the density of matter/energy of the spacetime. In this section, we shall employ Wald's entropy calculation [29, 30] and derive M_{QL} using Wald's entropy formula. To this end, we follow the study of Eune and Kim [28].

Starting with the time-like Killing vector ξ^μ, which describes the symmetry of time translation in a spacetime, Wald's entropy is expressed by [29, 30] as

$$S_{BH} = \frac{2\pi}{\kappa}\int_\Sigma d^2x\sqrt{h}\beta, \tag{12}$$

where

$$\beta = \epsilon_{\mu\nu}J^{\mu\nu}, \quad \epsilon_{\mu\nu} = \frac{1}{2}\left(n_\mu u_\nu - n_\nu u_\mu\right). \tag{13}$$

Here, κ and h are the surface gravity and the induced metric on a hypersurface Σ of the horizon (here 2-sphere with $\sqrt{h} = r_h^2\sin\theta$), respectively. u_μ is the four-vector velocity defined as the proper velocity of a fiducial observer moving along the orbit of $\xi^\mu = \gamma(\partial/\partial t)$ (where γ is a normalization constant), which must satisfy $g_{\mu\nu}\xi^\mu\xi^\nu = -1$ at spatial infinity. Thus, one can immediately see that $\gamma = 1$. $J^{\mu\nu}$ is called the Noether potential [46, 47], which is given by

$$J^{\mu\nu} = -2\Theta^{\mu\nu\rho\sigma}\left(\nabla_\rho\xi_\sigma\right) + 4\left(\nabla_\rho\Theta^{\mu\nu\rho\sigma}\right)\xi_\sigma, \tag{14}$$

with

$$\Theta^{\mu\nu\rho\sigma} = \frac{1}{32}\left(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}\right). \tag{15}$$

The surface gravity κ can be calculated by [30]

$$\kappa = \lim_{r\longrightarrow r_h}\sqrt{\frac{\xi^\mu\nabla_\mu\xi_\nu\xi^\rho\nabla_\rho\xi^\nu}{-\xi^2}} = \frac{1}{2}. \tag{16}$$

To have an outward unit vector n_μ on Σ, the equality $n_\mu n^\mu = 1$ must also be satisfied. Hence, one can get

$$n_r = \frac{1}{n^r} = \frac{2}{\sqrt{(r^2 - r_h^2)}}. \tag{17}$$

On the contrary, u_μ is the four-vector velocity and is given by

$$u^\mu = \frac{1}{\alpha}\xi^\mu, \tag{18}$$

with

$$\alpha = \sqrt{-\xi^\mu\xi_\mu}. \tag{19}$$

Therefore, the nonzero components of $\epsilon_{\mu\nu}$ are found to be

$$\epsilon_{tr} = -\epsilon_{rt} = -\frac{1}{r}. \tag{20}$$

In sequel, the four-vector velocity reads

$$u^t = \frac{r}{\sqrt{r^2 - r_h^2}}. \tag{21}$$

The nonzero components of $\Theta^{\mu\nu\rho\sigma}$ (15) are obtained as follows:

$$\begin{aligned}
\Theta^{trtr} &= \Theta^{rtrt} = -\Theta^{trrt} = -\Theta^{rttr} = \frac{-r^2}{128\pi},\\
\Theta^{\theta r\theta r} &= \Theta^{r\theta r\theta} = -\Theta^{\theta rr\theta} = -\Theta^{r\theta\theta r} = \frac{r^2 - r_h^2}{128\pi r^2},\\
\Theta^{\phi r\phi r} &= \Theta^{r\phi r\phi} = -\Theta^{\phi rr\phi} = -\Theta^{r\phi\phi r} = \frac{r^2 - r_h^2}{128\pi r^2\sin^2\theta},\\
\Theta^{\theta\phi\theta\phi} &= \Theta^{\phi\theta\phi\theta} = -\Theta^{\theta\phi\phi\theta} = -\Theta^{\phi\theta\theta\phi} = \frac{1}{32\pi r^4\sin^2\theta},\\
\Theta^{t\phi t\phi} &= \Theta^{\phi t\phi t} = -\Theta^{t\phi\phi t} = -\Theta^{\phi tt\phi}\\
&= \frac{-1}{32\pi\left(r^2 - r_h^2\right)\sin^2\theta},\\
\Theta^{\theta t\theta t} &= \Theta^{t\theta t\theta} = -\Theta^{\theta tt\theta} = -\Theta^{t\theta\theta t} = \frac{-1}{32\pi\left(r^2 - r_h^2\right)}.
\end{aligned} \tag{22}$$

One can verify that $\nabla_\rho\Theta^{\mu\nu\rho\sigma} = 0$. The latter result yields that the second term of the Noether potential vanishes. Therefore, we have

$$J^{\mu\nu} = -2\Theta^{\mu\nu\rho\sigma}\left(\nabla_\rho\xi_\sigma\right) = -2\Theta^{\mu\nu\rho\sigma}C_{\rho\sigma}, \tag{23}$$

The nonzero components of $C_{\rho\sigma} = \nabla_\rho\xi_\sigma$ are as follows:

$$C_{tr} = -C_{rt} = -\frac{r_h^2}{r^3}. \tag{24}$$

After substituting those findings into (14), we find the nonzero components of the Noether potential:

$$J^{tr} = -J^{rt} = \frac{-r_h^2}{32\pi}. \tag{25}$$

Thus, from (20) and (25), β is found as

$$\beta = \frac{r_h^2}{16r^2\pi}, \tag{26}$$

and, in sequel computing the entropy through the integral formulation (12), we obtain

$$\mathcal{S} = \pi r_h^2 = \frac{\mathcal{A}_h}{4}. \tag{27}$$

The above result is fully consistent with the Bekenstein-Hawking entropy. The quasi-local mass M_{QL} can be derived from this entropy by integrating the first law of thermodynamics $dM_{QL} = \mathcal{T}_{\mathcal{H}}dS_{BH}$. After some manipulation, one easily finds the following result:

$$M_{QL} = \frac{r_h^2}{4}. \tag{28}$$

which matches with the quasi-local mass computation of Brown and York [27].

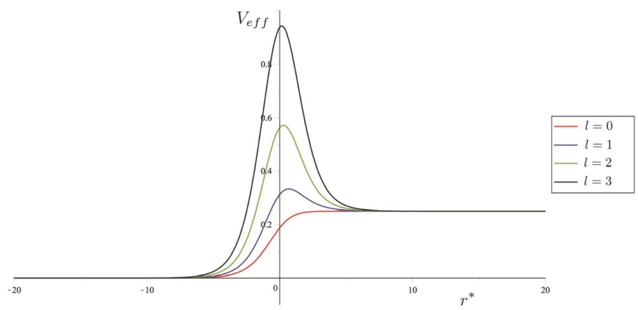

FIGURE 1: Effective potential versus tortoise coordinate graph for various orbital quantum numbers.

3. QNMs and Spectroscopy of $Z0$LBH

In this section, we shall study the QNMs and the entropy of a perturbed Z0LBH via MM [9]. QNMs of a considered BH can be derived by solving the eigenvalue problem of the KGE with the proper boundary conditions. The boundary condition at the horizon implies that there are no outgoing waves at the event horizon (i.e., only ingoing waves carry the QNMs at the event horizon) and the boundary condition at spatial infinity imposes that only outgoing waves are allowed to survive at spatial infinity. The second boundary condition is appropriate for bumpy shape effective potential that dies off at the two ends. Yet, as seen in (36) and shown in Figure 1, the potential never terminates at spatial infinity; instead it diverges for very massive ($m \longrightarrow \infty$) scalar particles. Thus, the potential blocks the waves that come off from the BH and prevents them from reaching spatial infinity. Hence, in this section, we will consider the very massive scalar particles and employ the particular method of [23, 32–35], in which only the QNMs are defined to be those for which one has purely ingoing plane wave at the horizon and no wave at spatial infinity (see (44). Namely, we will find the QNMs of the Z0LBH using their NH boundary condition. For this purpose, we first consider the massive KGE:

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\right)\Psi - m^2\Psi = 0, \tag{29}$$

where Ψ adopts the ansatz for the above wave equation chosen as

$$\Psi = \frac{1}{r}F(r)e^{i\omega t}Y_{lm}(\theta,\phi), \tag{30}$$

in which $F(r)$ is the function of r and $Y_{lm}(\theta,\phi)$ represents the spherical harmonics with the eigenvalues $-l(l+1)$ and m. After performing some straightforward calculations, the radial part of the KGE reduces to a SLE [31]:

$$\left[-\frac{d}{dr^{*2}} + V_{eff}\right]F(r) = \omega^2 F(r), \tag{31}$$

where V_{eff} and r^* are called the effective potential and the tortoise coordinate, respectively. The tortoise coordinate r^* can be found by the following integral:

$$r^* = 2\int\frac{dr}{rf}, \tag{32}$$

which results in

$$r^* = \ln\left(\frac{r^2}{r_h^2} - 1\right). \tag{33}$$

One may check that the limits of r^* admit the following:

$$\lim_{r\longrightarrow r_h} r^* = -\infty,$$
$$\lim_{r\longrightarrow\infty} r^* = \infty. \tag{34}$$

The effective potential seen in (33) is obtained as

$$V_{eff} = f\left\{\frac{l(l+1)}{r^2} + \frac{1}{4}\left[f + r\frac{df}{dr}\right] + m^2\right\}, \tag{35}$$

which admits these limits:

$$\lim_{r\longrightarrow r_h} V_{eff}(r) = 0,$$
$$\lim_{r\longrightarrow\infty} V_{eff}(r) = m^2 + \frac{1}{4}. \tag{36}$$

It is clear that, for any constant m, the potential is finite at infinite radius (see Figure 1). The potential diverges when the scalar particle is very massive. In other words, the waves tend to cease as $m \longrightarrow \infty$.

3.1. Entropy/Area Spectra of Z0LBH. In this section, we shall nudge (perturb) the Z0LBH by the massive scalar fields propagating near the event horizon and read their corresponding QNM frequencies.

We can expand Z0LBH's metric function f to a series around r_h and express it in terms of the surface gravity κ as

$$f = f(r_h) + f'(r_h)(r - r_h) + O\left[(r - r_h)^2\right] \approx 2\kappa y, \tag{37}$$

where $y = r - r_h$ and prime ($'$) denotes the derivative with respect to r. After substituting (37) into (35) and performing Taylor expansion, the NH form of the effective potential is obtained as

$$V = 4\kappa Gy\left[G^2(1 - 2Gy)l(l+1) + \kappa(1 + 2Gy) + m^2\right], \tag{38}$$

with the parameter $G = 1/r_h$. The tortoise coordinate in the NH region becomes $r^* \simeq (1/2\kappa)\ln y$, which enables us to find the NH form of the one-dimensional SLE:

$$\left[-4\kappa^2 y\left(y\frac{d^2}{dy^2} + \frac{d}{dy}\right) + V - \omega^2\right]F(y) = 0. \tag{39}$$

The solutions to the above equation can be expressed in terms of the CH functions of the first and second kinds [36] as follows:

$$F(y) = y^{i\omega/2\kappa}e^{-z/2}\left[C_1 M(a,b,z) + C_2 U(a,b,z)\right], \tag{40}$$

with the parameters

$$a = \frac{\lambda}{\sqrt{\delta}} + \frac{b}{2},$$
$$b = 1 + i\frac{\omega}{\kappa}, \qquad (41)$$
$$z = 2iG\sqrt{\delta}y,$$

where

$$\lambda = \frac{1}{2}\left\{1 + \frac{1}{\kappa}\left[l(l+1)G^2 + m^2\right]\right\},$$
$$\delta = 2\left[l(l+1)\frac{G^2}{\kappa} - 1\right]. \qquad (42)$$

With the aid of the limiting forms of the CH functions [36], one can find the NH limit of solution (40) as

$$F(y) \sim \left[C_1 + C_2\frac{\Gamma(1-b)}{\Gamma(1+a-b)}\right]y^{i\omega/2\kappa} + C_2\frac{\Gamma(b-1)}{\Gamma(a)}y^{-i\omega/2\kappa}. \qquad (43)$$

We can alternatively represent (43) in terms of r^* ($y \simeq e^{2\kappa r^*}$). Thus, the NH ingoing and outgoing waves are distinguished:

$$\Psi \sim \left[C_1 + C_2\frac{\Gamma(1-b)}{\Gamma(1+a-b)}\right]e^{i\omega(t+r^*)} + C_2\frac{\Gamma(b-1)}{\Gamma(a)}e^{i\omega(t-r^*)}. \qquad (44)$$

For QNMs, imposing the boundary conditions that the outgoing waves must vanish at the horizon and no wave at the spatial infinity, that is,

$$F(r^*)_{QNM} \sim \begin{cases} e^{i\omega r^*} & \text{at } r^* \longrightarrow -\infty \\ 0 & \text{at } r^* \longrightarrow \infty, \end{cases} \qquad (45)$$

the solution having coefficient C_2 should be terminated. By using the pole structure of the Gamma function for the denominator of the second term, the outgoing waves vanish for $a = -n$ ($n = 0, 1, 2, ..$). The latter remark yields the frequencies of the QNMs of the Z0LBH as

$$\omega_n = 2\kappa\left[\left(n + \frac{1}{2}\right)i + \frac{\lambda}{\sqrt{\delta}}\right], \qquad (46)$$

where n is known as the overtone quantum number [48]. Accordingly, the transition frequency between two highly excited subsequent states ($\omega_I \gg \omega_R$) is easily obtained as follows:

$$\Delta\omega \approx \Delta\omega_I = 2\kappa = \frac{4\pi\mathcal{T}_{\mathcal{H}}}{\hbar}. \qquad (47)$$

Subsequently, using the adiabatic invariant quantity (3) and Bohr-Sommerfeld quantization rule,

$$I_{adb} = \frac{\hbar}{4\pi}\int\frac{dM}{\mathcal{T}_{\mathcal{H}}} = \frac{\hbar}{4\pi}S_{BH} = \hbar n, \qquad (48)$$

we can read the entropy/area spectra of the Z0LBH as follows:

$$S_{BHn} = \frac{\mathcal{A}_n}{4\hbar} = 4\pi n, \quad \mathcal{A}_n = 16\pi\hbar n. \qquad (49)$$

Therefore, the minimum spacing of the BH area becomes

$$\Delta\mathcal{A}_{\min} = 16\pi\hbar. \qquad (50)$$

Our finding is in agreement with Bekenstein's conjecture [7], and the equispacing of the entropy/area spectra of the Z0LBH supports Kothawala et al.'s hypothesis [21], which states that BHs should have equally spaced area spectrum in Einstein's gravity theory.

4. Conclusions

In this work, the quantum spectra of the Z0LBH were studied using the MM, which is based on the adiabatic invariant quantity (3). After separating the radial and angular parts of the massive KGE on the Z0LBH background, we have found the SLE (31) and its corresponding effective potential (35). We have checked the behaviors of the potential around horizon and at the spatial infinity [see (36)]. In addition, we have depicted the effective potential for different values and have shown that the potential never terminates at the spatial infinity. The SLE associated with the Z0LBH is approximated to a CH differential equation. We have derived QNMs of the Z0LBH using the pole feature of the Gamma functions. The MM is applied for the highly excited modes ($n \longrightarrow \infty$) and the entropy/area spectra are calculated. Both spectra are evenly spaced and independent of the parameters of the BH as expected. On the contrary, we obtained different area equispacing with dimensionless constant $\epsilon = 16\pi$ in comparison to the usual value ($\epsilon = 8\pi$ [9–11]). However, as shown by Hod [17], the spacing between two adjacent levels might be different depending on which method is applied for studying the BH quantization. Besides, our findings are in agreement with both Bekenstein's conjecture [7] and Wei et al. and Kothawala et al.'s studies [21, 49].

In addition, M_{QL} of the Z0LBH is also investigated via Wald's entropy formula (12) by integrating the total energy (mass). The result obtained is in agreement with the BH thermodynamics [50]. Our next target is to study the Dirac QNMs of the 4-dimensional Z0LBH and analyze the spin effect on the area/entropy quantization.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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