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Research Article

Neutrino Mass, Coupling Unification, Verifiable Proton Decay, Vacuum Stability, and WIMP Dark Matter in SU(5)

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Nonsupersymmetric minimal SU(5) with Higgs representations 24_H and 5_H and standard fermions in $\overline{5}_F \oplus 10_F$ is well known for its failure in unification of gauge couplings and lack of predicting neutrino masses. Like standard model, it is also affected by the instability of the Higgs scalar potential. We note that extending the Higgs sector by 75_H and 15_H not only leads to the popular type-II seesaw ansatz for neutrino masses with a lower bound on the triplet mass $M_\Delta > 2 \times 10^9$ GeV, but also achieves precision unification of gauge couplings without proliferation of nonstandard light Higgs scalars or fermions near the TeV scale. Consistent with recent LUX-2016 lower bound, the model easily accommodates a singlet scalar WIMP dark matter near the TeV scale which resolves the vacuum stability issue even after inclusion of heavy triplet threshold effect. We estimate proton lifetime predictions for $p \longrightarrow e^+\pi^0$ including uncertainties due to input parameters and threshold effects due to superheavy Higgs scalars and superheavy $X^{\pm 4/3}, Y^{\pm 1/3}$ gauge bosons. The predicted lifetime is noted to be verifiable at Super Kamiokande and Hyper Kamiokande experiments.

1. Introduction

Standard model (SM) of strong and electroweak interactions has been established by numerous experimental tests, yet evidences on neutrino mass [1–5], the phenomena of dark matter [6–25] , and baryon asymmetry of the universe (BAU) [8, 26–29] call for beyond standard model (BSM) physics. It is well known that grand unified theories (GUTs) [30–37] are capable of addressing a number of limitations of the SM effectively. There are interesting theories on neutrino mass generation mechanisms [38–45] based upon various seesaw mechanisms such as type-I, type-II, type-III [46–62], linear [63, 64], and inverse [65–75]. Interesting models for Dirac neutrino mass origin of the neutrino oscillation data have been also proposed [76, 77]. In the absence of experimental evidence of supersymmetry so far, nonsupersymmetric (non-SUSY) GUTs are being extensively exploited by reconciling to the underlying gauge hierarchy problem through fine-tuning [78, 79]. Higher rank GUTs like SO(10) and E_6 can not define a unique symmetry breaking path to the SM gauge theory because of large number of possibilities with one and more intermediate symmetry breakings consistent with electroweak precision data on $\sin^2\theta_W(M_Z)$, $\alpha_S(M_Z)$, and $\alpha(M_Z)$ [80–82]. On the other hand, the rank-4 minimal SU(5) [32] with Higgs representations 5_H and 24_H defines only one unique symmetry breaking path to the standard model

$$SU(5) \longrightarrow SM. \tag{1}$$

Type-I seesaw [46–52] needs nonstandard heavy right-handed neutrino, linear and inverse seesaw both need non-standard fermions and scalars, and type-III seesaw [38–45, 58–62] needs only nonstandard fermionic extension for their implementation. Out of these popular seesaw mechanisms, type-II seesaw mechanism is the one which needs a heavy nonstandard triplet scalar [52–57, 59]. With a second triplet scalar, it is also capable of predicting baryon asymmetry of the universe [57] which is one of the main motivations behind this investigation. This neutrino mass generation mechanism, gauge coupling unification, dark matter, and vacuum stability are the focus of the present work.

Like the minimal SM, with its 15 fermions per generation and the standard Higgs doublet $\phi(2, 1/2, 1)$, the minimal SU(5) with Higgs representations 5_H and 24_H predicts neutrinos to be massless subject to a tiny $\mathcal{O}(10^{-5})$ eV contribution due to nonrenormalizable Planck scale effect which is nearly 4 orders smaller than the requirement of neutrino oscillation data. As the particle spectrum below the GUT symmetry breaking scale is identically equal to the SM spectrum, like SM, the minimal GUT fails to unify gauge couplings [83–87]. Also it predicts instability of the Higgs quartic coupling at mass scales $\mu \geq 5 \times 10^9$ GeV [88–90] after which the coupling continues to be increasingly negative at least up to the unification scale.

A number of interesting models have been suggested for coupling unification by populating the grand desert and for enhancing proton lifetime predictions [60–62, 91–98]. In these models a number of fermion or scalar masses below the GUT scale have been utilised to achieve unification. Interesting possibility of type-III seesaw [60–62] with experimentally verifiable dilepton production [99] at LHC has been also investigated.

The other shortcoming of minimal non-SUSY SU(5) is its inability to predict dark matter which appears to belong to two distinct categories: (i) the weakly interacting massive particle (WIMP) dark matter of bounded mass < 100 TeV and (ii) the decaying dark matter which has been suggested to be a possible source of PeV energy IceCube neutrinos.

In this work we implement a novel mechanism for coupling unification and neutrino masses together. When SU(5) is extended by the addition of its Higgs representations 75_H and 15_H, it achieves two objectives: (i) neutrino mass and mixing generation through type-II seesaw mechanism and (ii) precision gauge coupling unification with experimentally accessible proton lifetime.

But this does not cure the vacuum instability problem persisting in the model as well as the need for WIMP dark matter prediction. Out of these two, as we note in this work, when the dark matter prediction is successfully inducted into the model, the other problem on vacuum stability is automatically resolved.

In contrast to the popular belief on low proton lifetime prediction of the minimal SU(5) [35], we estimate new precise and enhanced predictions of this model including threshold effects [100–108] of heavy particles near the GUT scale. Predicted lifetimes are found to be within the accessible ranges of Superkamiokande and Hyperkamiokande experimental search programmes [109].

This paper is organised in the following manner. In Section 2 we discuss neutrino mass generation mechanism in extended SU(5). Section 3 deals with the problem of gauge coupling unification. In Section 4 we make proton lifetime prediction including possible uncertainties. Embedding WIMP scalar DM in SU(5) is discussed in Section 5 with a brief outline on the current experimental status. Resolution of vacuum stability issue is explained in Section 6. We summarise and conclude in Section 7. Renormalization group equations for gauge and Higgs quartic couplings are discussed in the Appendix.

2. Neutrino Mass Through Type-II Seesaw in SU(5)

As noted in Section 1, in contrast to many possible alternative symmetry breaking paths to SM from non-SUSY SO(10) and E_6 [80–82], SU(5) predicts only one symmetry breaking path which enhances its verifiable predictive capability. Fifteen SM fermions are placed in two different SU(5) representations:

$$\overline{5}_F = \begin{pmatrix} d_1^C \\ d_2^C \\ d_3^C \\ e^- \\ -\nu_e \end{pmatrix}_L,$$
$$10_F = \begin{pmatrix} 0 & u_2^C & -u_3^C & u_1 & d_1 \\ -u_2^C & 0 & u_1^C & u_2 & d_2 \\ u_3^C & -u_1^C & 0 & u_3 & d_3 \\ -u_1 & -u_2 & -u_3 & 0 & e^C \\ -d_1 & -d_2 & -d_3 & -e^C & 0 \end{pmatrix}_L. \quad (2)$$

Lack of RHν in these representations gives vanishing Dirac neutrino mass and vanishing Majorana neutrino mass at renormalizable level. Planck scale induced small Majorana masses can be generated through nonrenormalizable dim.5 interaction

$$-\mathscr{L}_{NR} = \frac{\kappa_{ij}}{M_{\text{Planck}}} \overline{5}_{F_i} \overline{5}_{F_j} 5_H 5_H + h.c, \quad (3)$$

leading to $m_\nu \sim 10^{-5}$ eV which is too low to explain neutrino oscillation data. Mechanism of Dirac neutrino mass generation has been discussed [76, 77] matching the neutrino oscillation data. Using extensions of the minimal GUT type-III seesaw origin of neutrino mass has been discussed where the nonstandard fermionic triplet $\Sigma_F(3, 0, 1)$ mediates the seesaw. This model can be experimentally tested by the production of like-sign dilepton signals at LHC.

Type-II seesaw mechanism for neutrino mass [53–57] does not need any nonstandard fermion but needs only the nonstandard left-handed Higgs scalar triplet $\Delta_L(3, -1, 1)$ with $Y = -2$ which directly couples with the a dilepton pair. It also directly couples to standard Higgs doublet ϕ. As such the standard Higgs VEV can be transmitted as a small induced VEV generating Majorana mass term for the light neutrinos. As this $\Delta_L(3, -1, 1)$ is contained in the symmetric SU(5) scalar representation 15_H, the scalar sector of the minimal GUT needs to include 15_H in addition to 5_H and 24_H.

The Yukawa Lagrangian

$$-\mathscr{L}^{(II)} = l_{L_i}^T C i\tau_2 Y_{ij} \left(\frac{\overrightarrow{\tau}.\overrightarrow{\Delta_L}}{\sqrt{2}} \right)^\dagger l_{L_j} + h.c, \quad (4)$$

combined with the relevant part of the Higgs potential

$$\mathscr{V}_{II} = M_\Delta^2 Tr\left[\Delta_L^\dagger \Delta_L\right] + \mu_\Delta \widetilde{\phi}^\dagger \left(\frac{\overrightarrow{\tau}.\overrightarrow{\Delta_L}}{\sqrt{2}} \right) \phi + h.c. \quad (5)$$

gives rise to the type-II seesaw contribution. In our notation $l_{L_i}^T = (\nu_{L_i}, e_{L_i})$ (i = generation index) and $\phi^T = (\phi^+, \phi^0)$ which are the lepton and scalar doublet of $SU(2)_L$. Here $\widetilde{\phi} = i\tau_2\phi^*$, $\overrightarrow{\tau} = (\tau_1, \tau_2, \tau_3)$ (τ_i are the 2×2 Pauli spin matrices) and, similarly, the scalar triplet Δ_L in the adjoint representation of $SU(2)_L$ is expressed as $\overrightarrow{\Delta_L} = (\Delta_L^1, \Delta_L^2, \Delta_L^3)$. The Majorana type Yukawa coupling Y is a 3×3 matrix in flavor space and C is the charge conjugation matrix. Then

$$\begin{aligned}\left(\frac{\overrightarrow{\tau}.\overrightarrow{\Delta_L}}{\sqrt{2}}\right) &= \frac{1}{\sqrt{2}}\left(\tau_1\Delta_L^1 + \tau_2\Delta_L^2 + \tau_3\Delta_L^3\right) \\ &= \begin{pmatrix} \frac{\Delta^+}{\sqrt{2}} & \Delta^{++} \\ \Delta^0 & -\frac{\Delta^+}{\sqrt{2}} \end{pmatrix}_L \end{aligned} \tag{6}$$

where different components are given by

$$\begin{aligned}\Delta_L^0 &= \frac{1}{\sqrt{2}}\left(\Delta_L^1 + i\Delta_L^2\right), \\ \Delta_L^+ &= \Delta_L^3, \\ \Delta_L^{++} &= \frac{1}{\sqrt{2}}\left(\Delta_L^1 - i\Delta_L^2\right)\end{aligned} \tag{7}$$

A diagrammatic representation for type-II seesaw generation of neutrino mass is shown in Figure 1. From the Feynman diagram shown in Figure 1 the induced VEV of the scalar triplet is

$$v_L = \frac{\mu_\Delta v^2}{2M_\Delta^2}, \tag{8}$$

leading to the type-II seesaw formula

$$m_\nu = 2Yv_L. \tag{9}$$

It is necessary to explain the origin of the $B - L$ breaking scale μ_Δ that occurs in (4), (5), and (8) as well as the Feynman diagram of Figure 1. SU(5) invariance permits the triplet coupling $\mu_\Delta 15_H^\dagger 5_H 5_H$ leading to SM invariant coupling $\mu_\Delta \Delta_L \phi\phi$. Therefore, in one approach, μ_Δ may be treated as explicitly lepton number violating parameter. Alternatively, it is also possible to attribute a spontaneous lepton number violating origin to this parameter. Since the SM model gauge theory has to remain unbroken down to the electroweak scale, the lepton number violating scale can be generated by the VEV of a Higgs scalar that transforms as a singlet under SM. Such a singlet $S_{BL}(1,0,1)$ carrying $B - L = -2$ occurs in the Higgs representation 50_H [36, 110]. The part of the SU(5) invariant potential that generates this scale is

$$V_{BL} = \lambda_5 50_H 15_H 5_H 5_H + h.c, \tag{10}$$

leading to $\mu_\Delta = \lambda_5 < S_{BL} >$. The $U(1)_{B-L}$ symmetric origin of μ_{Δ_L} becomes more transparent if one treats SU(5) as the remnant of $SU(5) \times U(1)_{B-L}$ or higher rank GUTs like SO(10) and E_6. If unification constraint as discussed below is ignored, the order of magnitude of μ_Δ can be anywhere in the range $\mathcal{O}(\mathcal{M}_{\rm W}) - \mathcal{O}(\mathcal{M}_{\rm Planck})$. But as we will find in the subsequent sections, gauge coupling unification in the present SU(5) framework imposes the lower bound $\mu_\Delta \simeq M_\Delta \geq 10^{9.23}$ GeV.

2.1. Type-II Seesaw Fit to the Neutrino Oscillation Data

2.1.1. Neutrino Mass Matrix from Oscillation Data. The effective light neutrino mass matrix (m_ν) is diagonalised by a unitary matrix U (in PMNS parametrisation which is written as $U_{\rm PMNS}$) and yields three mass eigenvalues (m_1, m_2, m_3). The light neutrino mass matrix (m_ν) can be reconstructed as

$$m_\nu = U_{PMNS}\,\mathrm{diag}\left(m_1, m_2, m_3\right) U_{PMNS}^T, \tag{11}$$

where PMNS matrix is parameterised using the PDG convention [111] as

$$U_{\rm PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \mathrm{diag}\left(e^{i\alpha_M/2}, e^{i\beta_M/2}, 1\right) \tag{12}$$

where $s_{ij} = \sin\theta_{ij}, c_{ij} = \cos\theta_{ij}$ with $(i, j = 1, 2, 3)$, δ is the Dirac CP phase, and (α_M, β_M) are Majorana phases.

Here we present our numerical analysis within 3σ and 1σ limits of experimental data. As we do not have any experimental information about Majorana phases, they are varied in the whole 2π interval randomly. From the set of randomly generated values we pick only one set of (α_M, β_M) and use them for our numerical estimations. The procedure adopted here can be repeated to derive corresponding solutions for the Majorana coupling matrix Y for other sets of randomly chosen Majorana phases. Although very recently 3σ and 1σ limits of Dirac CP phase have been announced [1], we prefer to use only their central value as an example. For our present analysis we choose a single set of (α_M, β_M) from a number of sets derived by random sampling and also a single value of δ close to the best fit value. For our analysis all possible

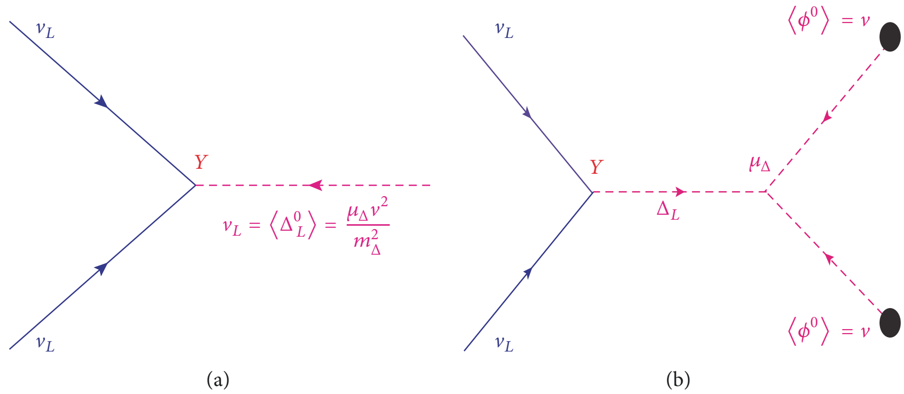


FIGURE 1: Schematic representation of generation of type-II term corresponding to (4) (a) and combination of (5)(b) where dashed line as triplet propagator supplies the damping factor M_Δ^{-2} to the induced vev v_L.

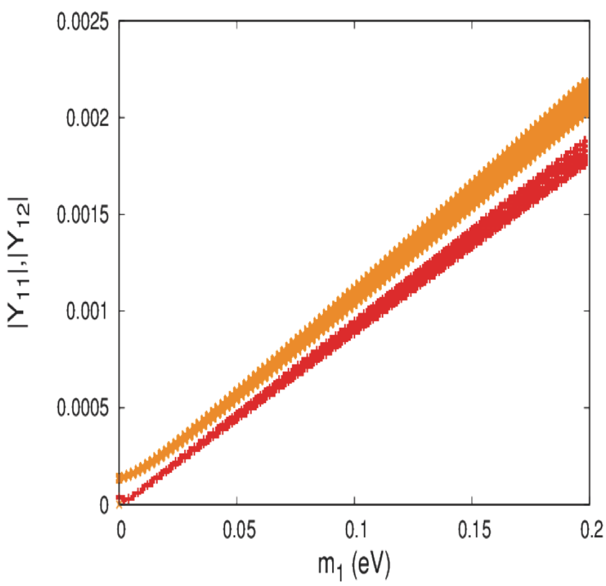
values of the solar and atmospheric mass squared differences and mixing angles have been taken which lie within the 3σ (or 1σ) limit of the oscillation data as determined by recent global analysis [1]. Summary of the global analysis is presented in Table 1. At first we analyze the limits imposed on the neutrino Yukawa couplings by 3σ oscillation constraints taking into account both the mass ordering of light neutrinos, normal ordering (NO), and inverted ordering (IO). In this case we use only one fixed value of the lightest neutrino mass eigenvalue and the other two mass eigenvalues are calculated using the experimental values of the mass squared differences. In this 3σ case we represent the bounds on the elements of Yukawa matrix in a tabular form. Later we proceed to estimate the bounds on the Y matrix elements imposed by 1σ experimental constraints of oscillation observables. In this analysis instead of fixed lightest neutrino mass eigenvalue, we vary it in the range (0–0.2) eV. The other two mass eigenvalues are calculated using 1σ ranges of solar and atmospheric mass squared differences. As already explained we use a single set of randomly chosen (α_M, β_M) and the central value of δ quoted in the Table 1. The variation of Y matrix elements is expressed in terms of their moduli ($|Y_{ij}|$) and the corresponding phases (ϕ_{ij})) with m_1 are shown graphically in Figures 2, 3, and 4 in the NO case. It is clear from the plots that for each single value of m_1 there is a band of allowed values of $|Y_{ij}|$ and ϕ_{ij}. This band signifies the 1σ allowed range of the corresponding matrix element for that single value of m_1. To represent the 1σ bounds in a more transparent manner we produce another set of plots as in Figures 5 and 6 where we show the allowed values of $|Y_{ij}|$ and ϕ_{ij} for a fixed value of m_1. It is to be noted that in this present work graphical representation is done for normally ordered light neutrinos only. Similar kind of exercise can be carried out for inverted mass ordering also.

2.1.2. Majorana Yukawa Coupling for 3σ Bounds of Neutrino Oscillation Data. We now estimate the m_ν matrix for the normally ordered (NO) case. For this purpose we take the mass of the lightest neutrino as $m_1 = 0.00127$ eV. Then using the 3σ ranges of solar and atmospheric mass squared differences for NO case, as mentioned in Table 1, the other two neutrino mass eigenvalues are calculated. Obviously we get a range of values of m_2 and m_3. Plugging in these mass eigenvalues along with all possible combinations and the mixing angles within the 3σ bound in (11) we obtain large number of sets of m_ν matrix. Thus we also get respective bounds on the elements of the m_ν matrix (or equivalently on the Yukawa coupling matrix Y) corresponding to the 3σ oscillation constraints. As mentioned earlier we use single set of randomly chosen Majorana phases while the Dirac CP phase is chosen close to its central value. The effective light neutrino mass matrix m_ν and the coupling matrix Y are connected through the induced VEV v_l which is obtained by assuming the dimensionful coupling $\mu_\Delta \sim M_\Delta$ where $M_\Delta = 10^{12}$ GeV and the electroweak VEV is 246 GeV. With these considerations we estimate the 3σ bound on the elements of Y matrix and present them in Table 2. In the inverted mass ordering the smallest mass eigenvalue is m_3 which is set to be equal to 0.00127 eV. The other two eigenvalues are calculated using the 3σ limit of the solar and atmospheric mass squared differences. In this case also we are able to put a bound on the modulus and phase of the Yukawa coupling matrix following the same procedure as done in the case of NO. The constrained parameters ($|Y_{ij}|, \phi_{ij}$) for inverted mass ordering are given in Table 3.

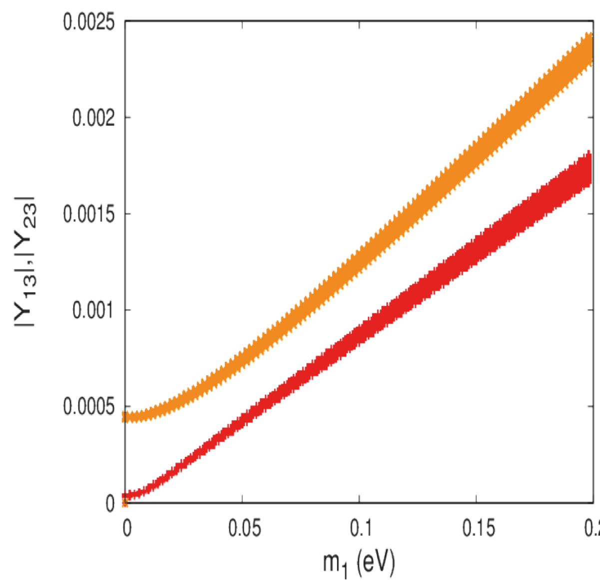
As we have taken a most general complex symmetric structure of the m_ν matrix (or in other words the Yukawa coupling matrix Y) without imposing any kinds specific flavor symmetry, it does not have any definite prediction of the Dirac CP violating phase δ. Any value of δ in the given 3σ range can be accommodated. In this regard few remarks about the present experimental status of the Dirac CP phase are in order. The recent global analysis of oscillation data done in [1] has made it clear that value of the Dirac CP phase $\delta = \pi/2$ is more or less ruled out. In normal mass ordering (NO) $\delta = \pi/2$ is disfavored at more than 4σ confidence level whereas for inverted mass ordering (IO) it is more stringent, where $\delta = \pi/2$ is ruled out at more than 6σ. The best fit value of δ in NO and IO is near 1.2π and 1.5π, respectively. For the sake of simplicity we work with only the best fit values. We have also estimated the highest and lowest values of the CP violating measure, the Jarlskog invariant ($J_{CP} =$

Table 1: Input data from neutrino oscillation experiments [1].

Quantity	best fit values	3σ ranges	2σ ranges	1σ ranges
$\Delta m^2_{21}\ [10^{-5} eV^2]$	7.55	7.05 – 8.14	7.20 – 7.94	7.39 – 7.55
$\lvert\Delta m^2_{31}\rvert\ [10^{-3} eV^2]\ (NO)$	2.50	2.41 – 2.60	2.44 – 2.57	2.47 – 2.53
$\lvert\Delta m^2_{31}\rvert\ [10^{-3} eV^2]\ (IO)$	2.42	2.31 – 2.51	2.34 – 2.47	2.38 – 2.46
$\theta_{12}/^\circ$	34.5	31.5 – 38.0	32.2 – 36.8	33.5 – 35.7
$\theta_{23}/^\circ(NO)$	47.7	41.8 – 50.7	43.1 – 49.8	46 – 48.9
$\theta_{23}/^\circ(IO)$	47.9	42.2 – 50.7	44.5 – 48.9	46.2 – 48.9
$\theta_{13}/^\circ(NO)$	8.45	8 – 8.9	8.2 – 8.8	8.31 – 8.61
$\theta_{13}/^\circ(IO)$	8.53	8.1 – 9	8.3 – 8.8	8.38 – 8.67
$\delta/^\circ(NO)$	218	157 – 349	182 – 315	191 – 256
$\delta/^\circ(IO)$	281	202 – 349	229 – 328	254 – 304



(a)



(b)

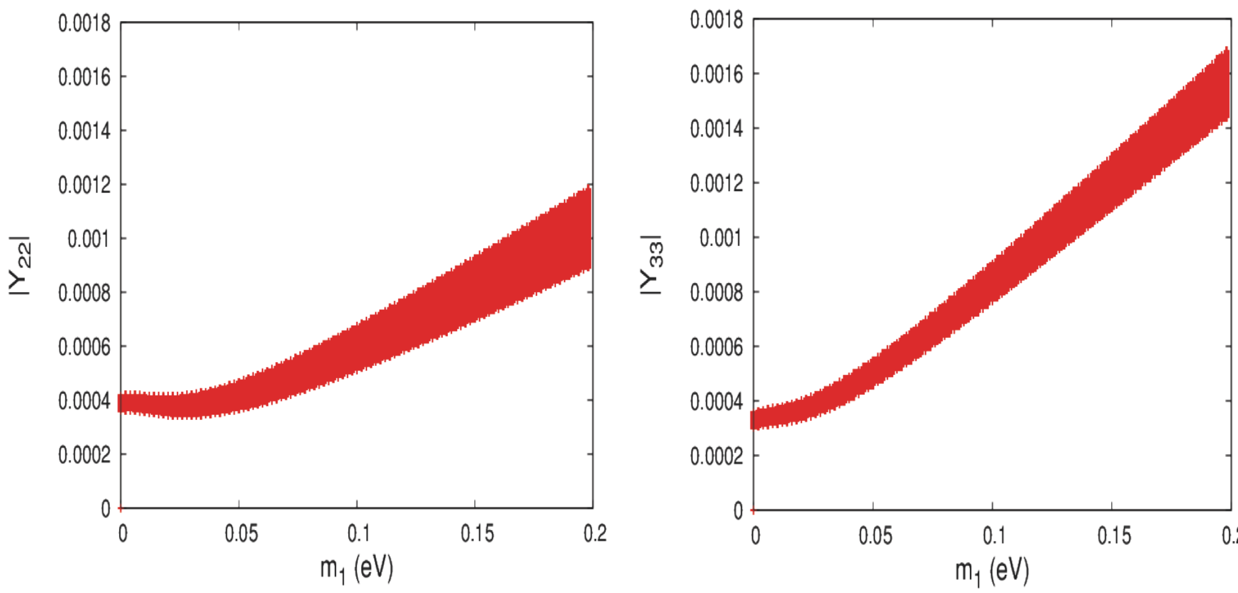
Figure 2: Determination of moduli of Y matrix elements within 1σ uncertainty of oscillation data as a function of lightest neutrino mass eigenvalues m_1. Phase angles used are randomly chosen Majorana phases $\alpha_M = 37.91^\circ$, $\beta_M = 157.91^\circ$ and central value of the Dirac phase $\delta = 216^\circ$. In (a) red and yellow regions denote 1σ allowed values $|Y_{11}|$ and $|Y_{12}|$, respectively. In (b) red patch gives values of $|Y_{13}|$ whereas yellow region denotes the same for $|Y_{23}|$ within the same uncertainty of the oscillation data.

$-s_{12}c_{12}s_{13}c_{13}^2 s_{23}c_{23}\sin\delta$) for both the mass orderings. For NO: $J_{CP} = 0.0175 - 0.0212$, for IO: $J_{CP} = 0.0302 - 0.0365$ when δ is kept fixed at its best fit value whereas all other observables are varying in their respective 3σ ranges.

2.1.3. Majorana Yukawa Coupling for 1σ Bounds of Neutrino Oscillation Data. Here we follow exactly the same methodology as the previous case, however the numerical calculations are done with 1σ ranges of oscillation data instead of 3σ range. Here we are exploring the normally ordered case only. Unlike the previous case the lightest neutrino mass eigenvalue m_1 is not kept fixed; it is varied over a range of $(0 - 0.2)$ eV and the corresponding variations of the modulus and phase of Majorana Yukawa couplings are depicted in Figures 2, 3, and 4. The 1σ allowed ranges of those quantities for a fixed m_1 are also shown in Figures 5 and 6.

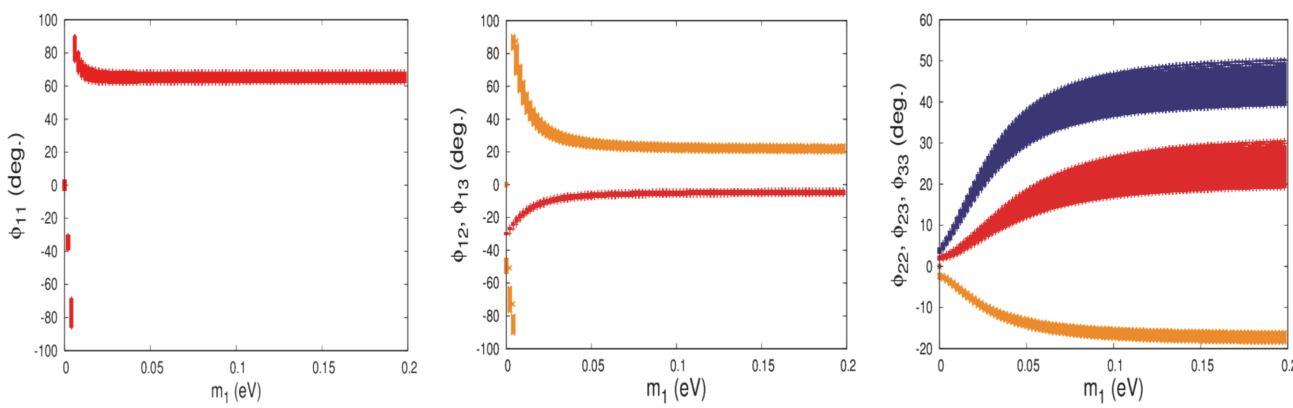
3. Gauge Coupling Unification in the Scalar Extended SU(5)

3.1. Lower Bound on the Scalar Triplet Mass. Exercising utmost economy in populating the grand desert, it was noted that the presence of the scalar component $\kappa(3,0,8) \subset 75_H$ at an intermediate mass $\simeq 10^{10}$ GeV could achieve gauge coupling unification at $M_{GUT} \simeq 10^{15}$ GeV [112] but no neutrino oscillation data was available at that time. Using the most recent electroweak precision data [111, 113, 114], in this work we find that this intermediate scalar mass is now reduced by one order, $M_\kappa = 10^{9.23}$ GeV. Similarly the GUT scale is now determined with high precision including all possible theoretical and experimental uncertainties. Noting the result of this work as discussed in Section 2 that type-II seesaw realisation of neutrino mass needs M_Δ



(a) (b)

FIGURE 3: Determination of moduli of Y matrix elements within 1σ uncertainty of oscillation data as a function of lightest neutrino mass eigenvalues m_1 as shown in (a) for $|Y_{22}|$, and in (b) for $|Y_{33}|$. Phase angles used are the same as in Figure 2.



(a) (b) (c)

FIGURE 4: Determination of phases of Y matrix elements as a function of lightest neutrino mass eigenvalues m_1 within 1σ allowed uncertainty of oscillation data. Phase angles used are the same as in Figure 2. In (a) red region denotes values of ϕ_{11}. In (b) red and yellow regions denote allowed values ϕ_{12} and ϕ_{13}, respectively. In (c) red, yellow, and blue patches give allowed values of ϕ_{22}, ϕ_{23}, and ϕ_{33}, respectively.

substantially lower than the GUT scale leads to the natural apprehension that the presence $\Delta_L(3,-1,1)$ at intermediate mass scale would destroy precision unification achieved by $\kappa(3,0,8)$. This apprehension is logically founded on the basis that nonvanishing contributions to the $SU(2)_L$ and $U(1)_Y$ beta functions would misalign the fine structure constants $\alpha_{2L}(\mu)$ and $\alpha_Y(\mu)$ from the $\kappa(3,0,8)$ realised unification paths substantially for all mass scales $\mu > M_\Delta$.

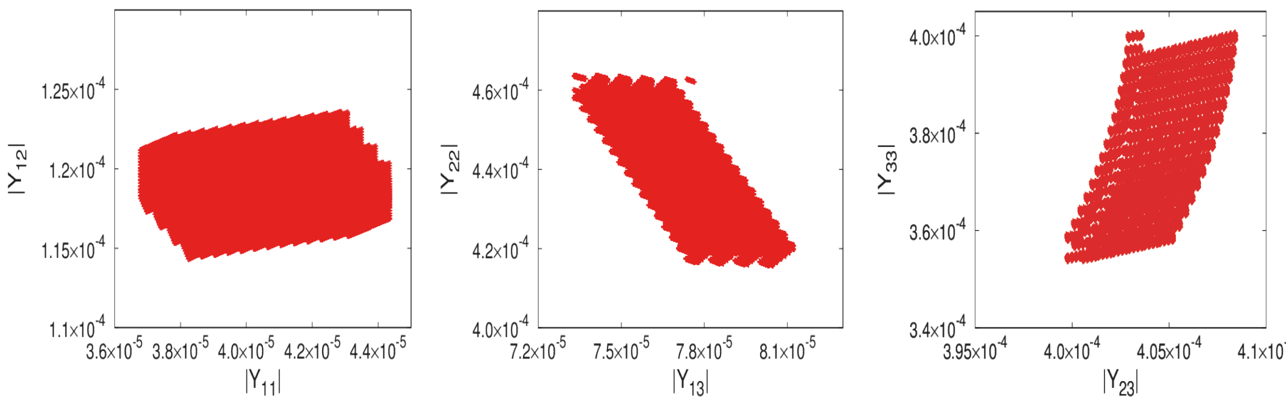
We prevent any such deviation from the $\kappa(3,0,8)$-realisation of precision coupling unification by assuming all the components of $15_H \subset SU(5)$ to have the identical degenerate mass M_Δ which is bounded in the following manner:

$$M_\kappa \le \left(M_\Delta = M_{15_H}\right) \le M_{GUT}. \tag{13}$$

Thus, in order to safeguard precision unification, it is essential that $M_\Delta = M_{15_H} \ge M_\kappa$ in the present scalar extended SU(5) model (the upper limit is due to our observation that type-II seesaw scale is lower than the GUT scale although, strictly speaking, $M_\Delta = M_{15_H} > M_{GUT}$ is possible if type-II seesaw contribution to neutrino mass is ignored).

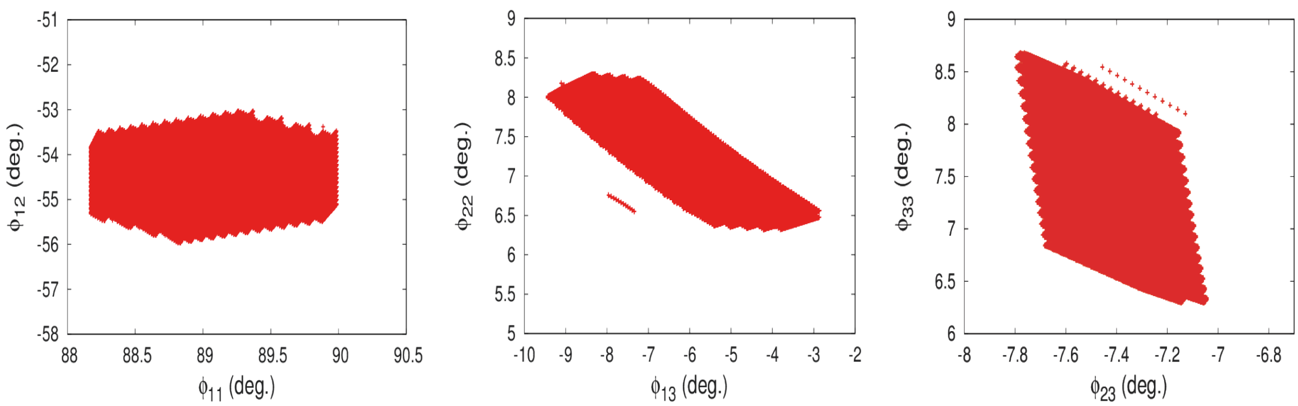
Thus type-II seesaw realisation of neutrino mass and precision unification in SU(5) needs the additional scalar representations 15_H and 75_H.

3.2. RG Solutions to Mass Scales. For realistic unification of gauge couplings we use one loop equations [115]



(a) (b) (c)

Figure 5: Determination of moduli of Y matrix elements for a fixed value of lightest mass eigenvalue $m_1 = 0.00127$ eV. Phase angles used for computation are $\alpha_M = 124.37^\circ$, $\beta_M = 86.27^\circ$ (randomly chosen), and $\delta = 216^\circ$. Neutrino oscillation observables are varied within 1σ range. (a) Variation of $|Y_{11}|$ with $|Y_{12}|$, (b) $|Y_{13}|$ versus $|Y_{22}|$, and (c) $|Y_{23}|$ versus $|Y_{33}|$.



(a) (b) (c)

Figure 6: Determination of phases of Y matrix elements for fixed value of lightest mass eigenvalue $m_1 = 0.001$ eV. Phase angles used for computation are the same as in Figure 5. Neutrino oscillation observables are varied within 1σ range; (a) variation of ϕ_{11} with ϕ_{12}, (b) ϕ_{13} versus ϕ_{22}, and (c) ϕ_{23} versus ϕ_{33}.

supplemented by top-quark threshold effects [83] and two-loop corrections [116]

$$\mu \frac{\partial g_i(\mu)}{\partial \mu} = \frac{a_i}{16\pi^2} g_i^3 + \frac{1}{(16\pi^2)^2} \left[\Sigma_j b_{ij} g_i^3 g_j^2 - \kappa_i y_{\text{top}}^2 \right] \quad (14)$$

In the range of mass scale $\mu = M_Z - M_U$ we include top-quark Yukawa coupling (y_{top}) contribution at the two-loop level with the coefficients in (14) $\kappa_{1Y} = 17/10, \kappa_{2L} = 3/2, \kappa_{3C} = 2$ and the RG evolution equation [83]

$$\mu \frac{\partial y_{top}}{\partial \mu} = \frac{y_{top}}{16\pi^2} \left(\frac{9}{2} y_{top}^2 - \frac{17}{20} g_{1Y}^2 - \frac{9}{4} g_{2L}^2 - 8 g_{3C}^2 \right) \quad (15)$$

The beta function coefficients in three different mass ranges $\mu = M_Z \longrightarrow M_\kappa$, $\mu = M_\sigma - M_\Delta$, and $\mu = M_\Delta - M_U$ are

$\underline{\mu = M_Z \longrightarrow M_\kappa :}$

$$a_Y = \frac{41}{10}, \quad a_{2L} = -\frac{19}{6}, \quad a_{3C} = -7, \quad (16)$$

$\underline{\mu = M_\kappa \longrightarrow M_\Delta :}$

$$a'_Y = \frac{41}{10}, \quad a'_{2L} = -\frac{1}{2}, \quad a'_{3C} = -\frac{11}{2}, \quad (17)$$

Table 2: Numerical values of the moduli ($|Y_{ij}|$) and phases (ϕ_{ij}) ($i, j = 1, 2, 3$) of Yukawa coupling matrix for normally ordered (NO) light neutrino masses corresponding to 3σ global fit of neutrino oscillation data. Lightest neutrino mass eigenvalue is kept fixed at $m_1 = 0.00127$ eV for the sake of simplicity. Randomly chosen Majorana phases $\alpha_M = 74.84^\circ$, $\beta_M = 112.85^\circ$ and the central value of the Dirac phase $\delta = 218^\circ$ have been used.

$\|Y_{11}\|$	$\|Y_{12}\|$	$\|Y_{13}\|$	$\|Y_{22}\|$	$\|Y_{23}\|$	$\|Y_{33}\|$
$(1.74-3.95)\times10^{-5}$	$(1.13-1.44)\times10^{-4}$	$(4.09-6.71)\times10^{-5}$	$(3.20-4.67)\times10^{-4}$	$(4.07-4.35)\times10^{-4}$	$(3.05-4.5)\times10^{-4}$
ϕ_{11} (deg.)	ϕ_{12} (deg.)	ϕ_{13} (deg.)	ϕ_{22} (deg.)	ϕ_{23} (deg.)	ϕ_{33} (deg.)
$(-65.24)-(-61.73)$	$(-48.50)-(-44.22)$	$(-17.48)-8.27$	$4.67-10.6$	$(-6.81)-(-5.34)$	$3.77-10.0$

Table 3: Numerical values of the moduli ($|Y_{ij}|$) and phases (ϕ_{ij}) ($i, j = 1, 2, 3$) of Yukawa coupling matrix Y for invertedly ordered (IO) light neutrino masses corresponding to 3σ global fit of neutrino oscillation data. Lightest neutrino mass eigenvalue is kept fixed at $m_3 = 0.00127$ eV. Phase angles used are the same as in Table 2.

$\|Y_{11}\|$	$\|Y_{12}\|$	$\|Y_{13}\|$	$\|Y_{22}\|$	$\|Y_{23}\|$	$\|Y_{33}\|$
$(4.38-5.3)\times10^{-4}$	$(4.29-5.5)\times10^{-4}$	$(3.55-4.87)\times10^{-4}$	$(8.83-23.5)\times10^{-5}$	$(2.13-2.89)\times10^{-4}$	$(2.84-4.0)\times10^{-4}$
ϕ_{11} (deg.)	ϕ_{12} (deg.)	ϕ_{13} (deg.)	ϕ_{22} (deg.)	ϕ_{23} (deg.)	ϕ_{33} (deg.)
52.96 68.35	$(-6.51)-(-3.16)$	0.5 4.5	$(-60)-(-32.31)$	$(-69.16)-(-51.39)$	$(-78.89)-(-61.92)$

$\underline{\mu = M_\Delta \longrightarrow M_U:}$

$$a''_Y = \frac{79}{15},\quad a''_{2L} = \frac{2}{3},\quad a''_{3C} = -\frac{13}{3}. \tag{18}$$

We have used the most recent electroweak precision data [114]

$$\alpha_S(M_Z) = 0.1182 \pm 0.0005,\quad \sin^2\theta_W(M_Z) = 0.23129 \pm 0.00005,\quad \alpha^{-1}(M_Z) = 127.94 \pm 0.02. \tag{19}$$

Using RGEs and the combinations $1/\alpha(M_Z) - (8/3)1/\alpha_{2L}(M_Z)$ and $1/\alpha(M_Z) - (8/3)1/\alpha_{3C}(M_Z)$, we have derived analytic formulas for the unification scale and intermediate scale(M_κ) treating $SU(2)_L$ triplet scalar scale (M_Δ) constant as

$$\begin{aligned}\ln\frac{M_U^0}{M_Z} &= \frac{2\pi}{187\alpha}\left(7 - \frac{80\alpha}{3\alpha_{3C}} + 8s_W^2\right) + \Delta_U\\ \ln\frac{M_\kappa^0}{M_Z} &= \frac{12\pi}{187\alpha}\left(5 + \frac{23\alpha}{3\alpha_{3C}} - 21s_W^2\right) + \Delta_\kappa\\ \frac{1}{\alpha_{G^0}} &= \frac{3}{8\alpha} + \frac{1}{187\alpha}\left(\frac{347}{8} + \frac{466\alpha}{3\alpha_{3C}} - 271s_W^2\right)\\ &\quad + \Delta_{\alpha_G}\end{aligned} \tag{20}$$

where $s_W^2 = \sin^2\theta_W(M_Z)$ and the first term in (20) represent one loop contributions. The terms Δ_I^i, $i = U, \kappa, \alpha_G$, denote the threshold corrections due to unification scale (M_U), intermediate scale (M_κ), and GUT fine structure constant ($1/\alpha_G$).

Excellent unification of gauge couplings is found for

$$M_U^0 = 10^{15.2+0.0312}\,\text{GeV},\quad M_\kappa^0 = 10^{9.23}\,\text{GeV},\quad \alpha_{G_0}^{-1} = 41.79 \tag{21}$$

where the number 0.0312 in the exponent is due to GUT scale matching of inverse fine structure constant that is present even if all superheavy masses are exactly at $\mu = M_U^0$ [101, 103–105].

3.3. Effects of 15_H *on Unification.* It is well known that when a complete GUT representation is superimposed on an already realised unification pattern in non-SUSY GUTs [117, 118], the GUT scale is unchanged but the inverse fine structure constants change their slopes and deviate from the original paths proportionately so as to increase the unification coupling. As an example in non-SUSY SO(10) [117, 118], at first a precision unification frame has been achieved with the modification of the TeV scale spectrum of the minimal SUSY GUT by taking out the full scalar super partner content of the spinorial super field representation $16 \subset SO(10)$. Then the resulting TeV scale spectrum is [117, 118]

$$\chi\left(2, -\frac{1}{2}, 1\right),$$

$$F_\phi\left(2, \frac{1}{2}, 1\right),$$

$$F_\chi\left(2, -\frac{1}{2}, 1\right),$$
$$F_\sigma(3, 0, 1),$$
$$F_b(1, 0, 1),$$
$$F_c(1, 0, 8) \tag{22}$$

which may be recognised to be the same as the corresponding spectrum in the split-SUSY case supplemented by the additional scalar doublet $\chi(2, -1/2, 1)$. In 3.3 F_i's represent nonstandard fermions. Further adjustment of masses of these particles around TeV scale has been noted to achieve degree of precision coupling unification higher than MSSM [117]. After having thus achieved a precision unification, the full 15_H is superimposed at the type-II seesaw scale M_Δ of the nonsupersymmetrised unification framework. Analogous to MSSM, this model [118] predicts a number of fermions as in 3.3 at the TeV scale which must be verified experimentally at accelerator energies.

In contrast, the present model has only the standard Higgs doublet $\phi(2, 1/2, 1)$ and the WIMP DM scalar singlet $\xi(1, 0, 1)$ near TeV scale as discussed below. Although the TeV scale DM has not been confirmed yet by direct experiments, LUX-16 or Femi-LAT-like experiments may detect it. Moreover, as shown below, the present model ensures vacuum stability through this WIMP dark matter candidate whereas in [118] the vacuum stability and DM issues are yet to be answered. Further, the SM coupling unification scale in [118] being close to the SUSY GUT scale, $M_U \sim 10^{16}$ GeV, predicts proton lifetime nearly 60 times larger than the current experimental limit without threshold effect which is expected to introduce larger uncertainty compared to the present model. It may be more difficult to verify this model by ongoing proton decay experiments. But the present model including such uncertainties is within the experimentally accessible limits. The origin of invariant GUT scale in the presence of 15_H in the present model is due to the invariance of the beta function differences which is $-7/6$ in this model

$$\Delta a_i = \left(a_i' - a_i''\right) = -\left(\frac{7}{6}\right), \quad (i = 1, 2, 3). \tag{23}$$

This results in a change in the inverse GUT coupling constant α_G^{-1} which occurs due to the RG predicted modification

$$\frac{1}{\alpha_G} = \frac{1}{\alpha_{G^0}} - \frac{1}{561\alpha}\left(\frac{229}{2} + \frac{134\alpha}{3\alpha_{3C}} - 350 s_W^2\right) + \frac{7}{12\pi}\ln\left(\frac{M_\Delta}{M_Z}\right). \tag{24}$$

Thus the result of type-II seesaw motivated insertion of 15_H into the $\kappa-$ realised unification framework is to decrease in the inverse GUT fine structure constant while keeping mass scales same as in (21)"

$$\alpha_G^{-1} = 37.765. \tag{25}$$

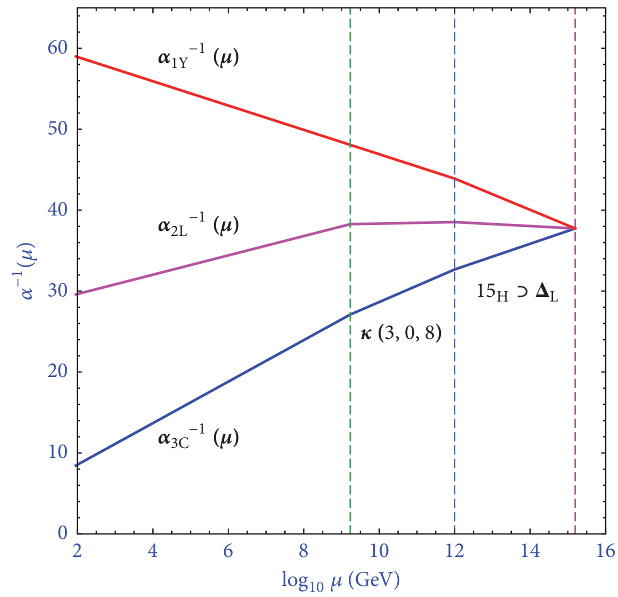


FIGURE 7: Unification of SM gauge couplings in the presence of $\kappa(3, 0, 8)$ at $M_\kappa = 10^{9.23}$ GeV and $15_H \subset SU(5)$ at $M_{15} = 10^{12}$ GeV as discussed in the text. The vertical dashed lines represent the intermediate scale masses and GUT scales.

which is a 9.6% effect. It is essential to take this effect into consideration in the top-down approach for consistency with the precision value of the electromagnetic fine structure constant $\alpha^{-1}(M_Z) = 127.9 \pm 0.01$ [114]. More important is its visible effect on proton lifetime prediction. It is clear from α_G^{-2} dependence in (36) of Section 4, (21), and (25) that the this intermediate type-II seesaw scale has a proton lifetime reduction by 19% that further reduces for lower seesaw scales, $M_\Delta < 10^{12}$ GeV. But the reduction effect decreases as M_Δ increases such that the proton lifetime remains unchanged for the limiting value $M_\Delta = M_{15_H} = M_U$.

In Figure 7 we have shown evolution of inverse fine structure constants of three gauge couplings of SM against mass scales depicting precision unification at $M_U^0 = 10^{15.2}$ GeV.

3.3.1. Implications for Lepton Number and Flavor Violations. It is evident from (13) and (21) that the numerical lower bound on the masses of three members of the triplet in $\Delta_L(3, -1, 1)$ is

$$M_{\Delta^0} \simeq M_{\Delta^-} \simeq M_{\Delta^{--}} \geq 10^{9.23}\text{GeV}. \tag{26}$$

Out of these we have discussed in Section 2 how the mediation of M_{Δ^0} gives type-II seesaw contribution to neutrino masses matching with available neutrino oscillation data at 1σ-3σ levels for all types of hierarchies: NH, IH, and QD. As a result the Higgs-Yukwa interaction and induced VEV of the neutral component of the triplet in the present SU(5) model give similar predictions as in the triplet extended SM based analyses [119] or in the left-right symmetric models and SO(10) with large W_R boson mass [45, 120]. Currently a number of experimental investigations are underway to detect the double beta decay process that would establish Majorana nature of neutrino. The most important difference from such SM based phenomenological analyses is that in the present SU(5) model with type-II seesaw all the parameters of the neutrino oscillation data are theoretically predicted by the seesaw mechanism. Even though $M_{\Delta^0} \geq 10^{9.23}$ GeV, it predicts the double beta decay lifetime close to the observable limit of $\tau_{\beta\beta} \sim 5 \times 10^{25}$ yrs for QD type light neutrino masses $\widehat{m}_i \sim 0.2$ eV. On the other hand for NH type of hierarchy the predicted decay rate is much lower with lifetime $\tau_{\beta\beta} \gg 10^{29}$ yrs. Another theoretical contribution to the double beta decay process is due to the mediation of the doubly charged component Δ^{--} through the physical process $W^-W^- \longrightarrow \Delta^{--} \longrightarrow e^-e^-$ which is negligible because of additional damping of the amplitude caused by the inverse square of its heavy mass $M_{\Delta^{--}} \geq 10^{9.23}$ GeV. The charged component Δ^- also mediates a new loop contributions to lepton flavor violating processes such as $l_\alpha \longrightarrow l_\beta + \gamma$. Again, because of heavy triplet mass the respective contribution to branching ratio turns out to be much smaller than the corresponding prediction with *SM* (supplemented by the oscillation data): $Br.(\mu \longrightarrow e\gamma) < 10^{-53}$ [45, 120]. Similarly the tree level mediation of the LFV process $\mu \longrightarrow ee\bar{e}$ by Δ^{--} is severely damped out compared to the loop mediated $W-$ boson contribution.

3.4. Threshold Effects on the GUT Scale. In the single step breaking model discussed in this work, GUT threshold effects due to superheavy degrees of freedom in different SU(5) representations are expected as major sources of uncertainties on unification scale and proton lifetime prediction. We have estimated the threshold uncertainties following the partially degenerate assumption introduced in [121, 122] which states that the superheavy components belonging to the same GUT representation are degenerate with a single mass scale.

The analytic formulas for GUT threshold effects on the unification scale, intermediate scale, and GUT fine structure constant are

$$\begin{aligned}
\Delta_I^{\kappa} &= \Delta \ln \frac{M_\kappa}{M_Z} = \frac{1}{2244}\left(123\lambda_{2L} - 215\lambda_Y + 92\lambda_{3C}\right),\\
\Delta_I^{U} &= \Delta \ln \frac{M_U}{M_Z} = \frac{5}{3366}\left(3\lambda_{2L} + 13\lambda_Y - 16\lambda_{3C}\right),\\
\Delta_I^{\alpha_G} &= \Delta\left(\frac{1}{\alpha_G}\right)\\
&= \frac{1}{80784\pi}\left(-948\lambda_{2L} - 2425\lambda_Y + 5056\lambda_{3C}\right).
\end{aligned} \tag{27}$$

In (27) $\lambda_i, i = 2L, Y, 3C$ are matching functions due to superheavy scalars (S) and gauge bosons (V) to the three gauge couplings,

$$\alpha_i^{-1}\left(M_U\right) = \alpha_G^{-1} - \frac{\lambda_i\left(M_U\right)}{12\pi}, \tag{28}$$

$$\begin{aligned}
\lambda_i^S\left(M_U\right) &= \sum_j Tr\left(t_{iSj}^2 \widehat{p}_{Sj} \ln \frac{M_j^S}{M_U}\right),\\
\lambda_i^V\left(M_U\right) &= \sum_l Tr\left(t_{iVl}^2\right) - 21\sum_l Tr\left(t_{iVl}^2 \ln \frac{M_l^V}{M_U}\right),
\end{aligned} \tag{29}$$

where t_{iS} and t_{iV} represent the matrix representations due to broken generators of scalars and gauge bosons. The term $\widehat{p}_{Sj}$ denotes the projection operator that removes the Goldstone components from the scalars contributing to spontaneous symmetry breaking.

The decomposition of different SU(5) representations under G_{213} with respect to their superheavy components and values of corresponding matching functions is presented in Table 4.

Using the values of matching function $\lambda^i(M_U)$ from Table 4 in (27) we estimate corrections to different mass scales due to superheavy masses as

$$\begin{aligned}
\Delta \ln \frac{M_\kappa}{M_Z} &= 0.0026738\eta_5 + 0.23262\eta_{24} - 1.24599\eta_{75},\\
\Delta \ln \frac{M_U}{M_Z} &= -0.0160428\eta_5 - 0.0623886\eta_{24}\\
&\quad + 1.142602\eta_{75},\\
\Delta\left(\frac{1}{\alpha_G}\right) &= 0.0160999\eta_5 + 0.0522951\eta_{24}\\
&\quad + 0.0462547\eta_{75}.
\end{aligned} \tag{30}$$

Maximising the uncertainty in M_U leads to

$$\begin{aligned}
\Delta \ln\left(\frac{M_U}{M_Z}\right) &= \pm 0.22103\eta_{SH},\\
\Delta \ln\left(\frac{M_\kappa}{M_Z}\right) &= \pm 1.48128\eta_{SH},\\
\Delta\left(\frac{1}{\alpha_G}\right) &= \pm 0.02214\eta_{SH},
\end{aligned} \tag{31}$$

where $\eta_{SH} = \ln(M_{SH}/M_U)$ and $M_{SH}/M_U = n(1/n)$ with plausible allowed values of real number $n = 1 - 10$.

We also note that the degenerate superheavy gauge bosons contribute a significant correction to unification scale

$$\left(\frac{M_U}{M_U^0}\right)_V = 10^{\pm 0.65508}. \tag{32}$$

Adding all corrections together we obtain

$$M_U = 10^{15.2312 \pm 0.11 \pm 0.221\eta_S \pm 0.655\eta_V}\,\text{GeV}. \tag{33}$$

The first uncertainty (±0.11) represents uncertainty in input parameters given in (19).

Table 4: Superheavy components of SU(5) representations under the SM gauge group G_{213} used to estimate GUT threshold effects.

$SU(5)$representations	G_{213} submultiplet	$(\lambda_{2L}, \lambda_{1Y}, \lambda_{3C})$
5_H	$C_1(1, -1/3, 3)$	$\left(0, \frac{2}{3}, 1\right)$
24_H	$D_1(3, 0, 1)$	$(2, 0, 0)$
	$D_2(1, 0, 8)$	$(0, 0, 3)$
75_H	$E_1(1, 10/3, 3)$	$\left(0, 5, \frac{1}{2}\right)$
	$E_2(2, 5/3, 3)$	$\left(\frac{3}{2}, \frac{5}{2}, 1\right)$
	$E_3(1, -10/3, \overline{3})$	$\left(0, 5, \frac{1}{2}\right)$
	$E_4(2, -5/3, \overline{3})$	$\left(\frac{3}{2}, \frac{5}{2}, 1\right)$
	$E_5(2, -5/3, \overline{6})$	$(3, 5, 5)$
	$E_6(2, 5/3, 6)$	$(3, 5, 5)$
	$E_7(1, 0, 8)$	$(0, 0, 3)$
15_H	$\Delta_L(3, -1, 1)$	$\left(4, \frac{18}{5}, 0\right)$
	$H_2(2, 1/6, 3)$	$\left(3, \frac{1}{5}, 2\right)$
	$H_3(1, 2/3, 6)$	$\left(0, \frac{16}{5}, 5\right)$
24_V	$V_1\left(2, -\frac{5}{6}, 3\right)$	$\left(\frac{3}{4}, \frac{5}{4}, \frac{1}{2}\right)$
	$V_2\left(2, \frac{5}{6}, \overline{3}\right)$	$\left(\frac{3}{4}, \frac{5}{4}, \frac{1}{2}\right)$

4. Proton Lifetime Prediction

Currently the measured value on the lower limit of the proton lifetime for the decay mode $p \longrightarrow e^+\pi^0$ is [109, 123–126]

$$\tau_p^{expt.} \geq 1.6 \times 10^{34} \text{ yrs.} \tag{34}$$

Including strong and electroweak renormalization effects on the d = 6 operator and taking into account quark mixing, chiral symmetry breaking effects, and lattice gauge theory estimations, the decay rates are [37, 127, 128]

$$\Gamma\left(p \longrightarrow e^+\pi^0\right) = \left(\frac{m_p}{64\pi f_\pi^2}\frac{{\alpha_G}^4}{{M_U}^4}\right)\left|A_L\right|^2\left|\overline{\alpha_H}\right|^2\left(1 + D' + F\right)^2 \times R, \tag{35}$$

where $R = [A_{SR}^2 + A_{SL}^2(1 + |V_{ud}|^2)^2]$ for $SU(5)$, $V_{ud} = 0.974$ = the $(1,1)$ element of V_{CKM} for quark mixings, and $A_{SL}(A_{SR})$ is the short-distance renormalization factor in the left (right) sectors. In (35) $A_L = 1.25$ = long distance renormalization factor but $A_{SL} \simeq A_{SR} = 2.542$. These are numerically estimated by evolving the dim.6 operator for proton decay by using the anomalous dimensions of [129] and the beta function coefficients for gauge couplings of this model. In (35) M_U = degenerate mass of superheavy gauge bosons, $\overline{\alpha}_H$ = hadronic matrix elements, m_p =proton mass = 938.3 MeV, f_π = pion decay constant = 139 MeV, and the chiral Lagrangian parameters are $D = 0.81$ and $F = 0.47$. With $\alpha_H = \overline{\alpha_H}(1 + D' + F) = 0.012\text{GeV}^3$ estimated from lattice gauge theory computations [130–132], we obtain $A_R \simeq A_L A_{SL} \simeq A_L A_{SR} \simeq 2.726$ and the expression for the inverse decay rate is

$$\Gamma^{-1}\left(p \longrightarrow e^+\pi^0\right) = \frac{4}{\pi}\frac{f_\pi^2}{m_p}\frac{M_U^4}{\alpha_G^2}\frac{1}{\alpha_H^2 A_R^2}\frac{1}{F_q}, \tag{36}$$

where the GUT fine structure constant $\alpha_G = 0.0263$ and the factor $F_q = (1+(1+|V_{ud}|^2)^2) \simeq 4.8$. This formula has the same form as given in [127] which has been modified here for the SU(5) case.

Using the estimated values of the model parameters, (36) gives

$$\tau_p^{SU(5)} \simeq 10^{33.110 \pm 0.440 \pm 0.884|\eta_S| \pm 2.62|\eta_V|} \text{ yrs.} \tag{37}$$

Numerical estimations on proton lifetime are shown in Table 5 for different splitting factors of superheavy masses.

It is interesting to note that despite three Higgs representations $5_H, 24_H, 75_H$, major contribution to threshold uncertainty in the model is only due to superheavy gauge bosons. When all superheavy gauge boson masses are identically equal to M_U, superheavy scalar mass splitting by a factor 20(1/20) from the GUT scale gives $\eta_S = 1.3(-1.3)$ leading to $[\tau_p]_{max} = 1.80 \times 10^{34}$ yrs. which is consistent with the current experimental bound.

5. Scalar Dark Matter in SU(5)

5.1. Phenomenological and Experimental Constraints. The existence of dark matter (DM) in our galaxy has been established beyond any doubt through its gravitational effects by numerous observations [133]. Hence the hunt for DM has

Table 5: Upper limits on predicted proton lifetime as a function of superheavy scalar (S) and gauge boson(V) mass splittings as defined in the text. The factor $10^{\pm 0.44}$ represents uncertainty due to input parameters.

$\frac{M_S}{M_U}$	$\frac{M_V}{M_U}$	$\tau_P(yrs)$	$\frac{M_S}{M_U}$	$\frac{M_V}{M_U}$	$\tau_P(yrs)$
10	1	$9.77 \times 10^{33\pm 0.44}$	5	5	$3.59 \times 10^{35\pm 0.44}$
10	2	$6.00 \times 10^{34\pm 0.44}$	3	6	$3.68 \times 10^{35\pm 0.44}$
8	3	$1.42 \times 10^{35\pm 0.44}$	1	10	$5.32 \times 10^{35\pm 0.44}$
6	4	$2.35 \times 10^{35\pm 0.44}$	20	1	$1.80 \times 10^{34\pm 0.44}$

been assumed to be of paramount importance for the particle physics community to understand its nature in particular and that of the universe in general. To this end, experiments using a wide range of approaches are being pursued worldwide and giving a large spectrum of interpretations of the DM candidates with masses ranging from a few eV to PeV or even beyond, from axions to wimpzillas and decaying dark matter.

Our motivation in this section is to explore whether SU(5) model can accommodate a scalar singlet ($= \xi$) as a candidate DM which might be instrumental in contributing to the observed relic density or may be detected through ongoing direct or indirect search experiments. The local DM density is observed with some uncertainty to be $0.4\mathrm{GeV/cm}^3$ [134]. Earlier measurements by WMAP [8] and more recent observation by PLANCK satellite [29] indicate 85% of matter content of the Universe to be DM with its relic density

$$\Omega_{\mathrm{dm}} h^2_{\mathrm{Hubble}} = 0.1198 \pm 0.0026 \tag{38}$$

where h_{Hubble} is the Hubble parameter. Various attractive models have been proposed to explain the observed relic density of dark matter and its stability with half-life greater than the age of the universe, $\tau_{DM} > 10^{17}$s. Attempts in this direction include addition of scalar or fermionic dark matter candidates to the RH neutrino (RHν) extended SM. Following the work of Lee-Weinberg [135] and in big-bang cosmology, a weakly interacting massive particle (WIMP) has enjoyed a special status as a DM candidate as it can naturally explain the observed relic density. Model independent upper bound on the WIMP DM mass has been also derived from perturbative unitarity [136] with $M_{\mathrm{WIMP}} \leq 100$ TeV. Recently extensive investigations have been made to explore possible special symmetries underlying the dynamics of DM [137–139].

5.1.1. Direct Detection of Dark Matter. Since DM particles are electrically neutral and cosmologically stable, they are referred to as missing energy at colliders where searches for DM mainly focus on the detection of visible signals like jets and charged leptons. At colliders we can study DM either through investigating its direct detection signals or indirect detection signals. The scalar singlet DM in our model may be discovered through direct and indirect signals. In particular, XENON1T experiment may discover or rule out the scalar singlet DM for reasonable values of DM mass and Higgs portal coupling, rejecting its nonperturbative values higher than 1.5 TeV [140, 141] .

Several terrestrial experiments like CDMS [142, 143], DAMA/NAI [144, 145], XEXNON100 [16], and LUX [18] are still going on around the globe for direct detection of dark matter. These underground detectors are constructed using various targets made up of Xe, Ge, NaI, etc. in an attempt to explore either electronic or nuclear scatterings at low energies. In this case, the recoil energy is usually observed from the scattering between DM particles and nucleons [146] or from scattering between electrons and dark matter. The direct search experiments, XENON100 [16] and LUX, predict an upper bound in the $M_{DM} - \sigma_{DM}$ plane where σ_{DM} represents DM elastic scattering cross section and M_{DM} stands for DM mass. These experiments furnish very stringent bounds on dark matter-nucleon scattering cross section for different DM masses. For example, LUX and XENON100 experiments predict similar DM-nucleon cross section bound at around $10^{-44}\mathrm{cm}^2$ for a DM mass of 1000 GeV whereas XENON1T search predicts a smaller cross section bound $2 \times 10^{-46}\mathrm{cm}^2$ for the same DM mass keeping the DM relic density in the right ballpark [147, 148]. A concise review of current status of scalar singlet dark matter is available in [149] where references to most of the recent experimental and phenomenological investigations are available. In general, for elastic scattering of a DM particle off nucleons, either a standard Higgs or a Z-boson exchange is needed in the t-channel of the dominant tree diagrams. Even though the singlet scalar DM $\xi(1,0,1)$ has no gauge interaction, still it can elastically scatter off nucleons in direct search experiments through Higgs exchange via quartic Higgs portal interaction

$$V_{\mathrm{Port}} = \frac{\lambda_{\phi\xi}}{2}\phi^\dagger\phi\xi^2 + h.c, \tag{39}$$

where the standard Higgs VEV and the portal quartic coupling $\lambda_{\phi\xi}$ contribute directly to the cross section in the lowest order.

Although till today no signals in direct detection experiments have been observed except for the controversial DAMA modulation signal, direct detection searches still have the potential to unravel the mystery of DM because of the fact that if a signal is observed, we can correlate the scattering cross section and mass of the DM particle with its local density.

5.1.2. Indirect Detection of Dark Matter. In indirect dark matter detection (IDMD) experiments, the DM particles may annihilate or decay to standard model particles or other exotic final states in a region of high DM density and finally manifest as a visible signal in form of gamma rays, cosmic rays, neutrinos, and positrons or antiparticles. Such events are expected to exhibit excesses over the desired abundance of the particles in the cosmos. The IDMD searches like Fermi-LAT [19], AMS [150], HESS [151], MAGIC [152], ATIC [153], DAMPE [154], PLANCK [29], ICECUBE [20, 21], etc. basically look for these excesses in the universe to confirm the detection of DM annihilation. For example, DM could be detected through the observation of neutrino fluxes by ICECUBE telescope arising from annihilation dark matter. The IceCube neutrino events have been recently interpreted to be consistent with decaying dark matter mass in the PeV range or larger.

Recently IDMD searches gave several hints for DM detection like lines at 3.5 KeV [155, 156], 130 GeV [157, 158], and the gamma ray excess from the galactic centre [159]. However, no conclusive and consistent information has emerged so far. These signals have been attributed to either astrophysical sources or instrumental effects [160, 161].

Recent data from LUX-2016 and Fermi-LAT [18, 19] have constrained the DM mass as well as its unknown Higgs portal coupling. It can be shown that $\lambda_{\phi\xi} \sim \mathcal{O}(0.01)$ can generate the right relic density with low mass ξ of order 50 GeV. On the other hand direct DM searches from the LUX-16 data have ruled out the existence of scalar DM ξ over a wider mass range $M_\xi \simeq 70-500$ GeV. In summary, the scalar dark matter mass can be on the lower side

$$M_\xi < 60\text{GeV}, \tag{40}$$

contributing prominently to relic density, or on the higher side

$$100\text{TeV} > M_\xi > 500\ \text{GeV}. \tag{41}$$

In (41) the LHS is due to the perturbative unitarity bound [135] and the RHS is due to [18].

5.2. Embedding in SU(5). Besides the SU(5) Higgs representations $5_H, 24_H, 15_H$, and 75_H, we further extend its scalar sector by the scalar singlet DM $\xi(1,0,1)$ which we assume to be also a SU(5) singlet. Obviously it has no direct gauge boson interaction of any kind. But it has interaction with SM Higgs through Higgs portal of the type shown in (39). Then it can have gauge interaction in higher orders. In any theoretical model, the stability of DM must be ensured such that its lifetime is longer than the lifetime of the universe. Usually a discrete symmetry Z_2 is imposed to safeguard the stability.

We assign all the fermions in $\overline{5}_F$, 10_F, and consequently the SM fermions, to possess $Z_2 = -1$. The Higgs representations $5_H, 24_H, 15_H$, and 75_H are assigned $Z_2 = +1$. Needless to mention that the SM Higgs doublet ϕ, $\kappa(3,0,8)$, and $\Delta_L(3,0,1)$ have the same value of $Z_2 = +1$. Out of all the scalars only the DM singlet scalar is assigned odd value of $Z_2 = -1$. This assignment prevents direct Yukawa interaction of ξ and ensures its desired stability.

6. Vacuum Stability in SU(5) through Scalar DM

Despite the above predictions on neutrino masses and mixings, coupling unification, and proton lifetime, the SU(5) model with Higgs representations still has the vacuum instability problem. This problem in the SM arises as the standard Higgs potential solely controlled by the standard Higgs field becomes unstable for large values of the field at scales $\mu \geq M_{\text{Inst.}} = 5\times 10^9$ GeV. As there is no other field so far in the extended SU(5) model for $\mu < M_\Delta (= 10^{12} - 10^{15}\text{GeV})$ to couple through its Higgs portal, the instability problem turns out to be similar to SM. As we have embedded the scalar singlet DM candidate in SU(5) we now investigate the possibility of resolving the vacuum instability through Higgs partial interaction [88, 89, 149].

6.1. RG Equations and Parameters for Higgs Potential. As noted above the standard model Higgs potential

$$V_{SM} = -{\mu_\phi}^2 \phi^\dagger\phi + \lambda_\phi \left(\phi^\dagger\phi\right)^2 \tag{42}$$

develops instability as the Higgs quartic coupling λ_ϕ runs negative at an energy scale $10^9 - 10^{10}$GeV by the renormalization group running. Apart from other interesting suggestions [88, 89] an alternative popular solution to the vacuum instability problem is to extend the SM by a gauge singlet real scalar(ξ) which gives positive contribution to the Higgs quartic coupling and prevents it from becoming negative [88, 162–164]. It is worth mentioning that this scalar singlet can act as potential dark matter candidate termed as weakly interacting massive particle (WIMP) with an extra discrete symmetry $Z_2 : \xi \longrightarrow -\xi$ imposed on it. The scalar singlet is odd under Z_2 symmetry while all other scalars being even and SM fermions being odd under this symmetry. Hence it can not couple to SM particle and become stable. This also matches the discrete symmetry properties of SU(5) representations discussed above. Thus it can serve as a suitable WIMP dark matter particle which is also identified as the SU(5) singlet scalar. The unbroken discrete symmetry of the singlet scalar upto the Planck scale has two important consequences: (i) the ξ VEV is forbidden and (ii) the modified SM potential develops VEV and minima only due to the SM Higgs. The scalar $\kappa(3,0,8)$ has no coupling with ϕ. Even if Δ_L and some of its associates have coupling with ϕ, because of their heavy mass, $M_\Delta \gg M_W$, they are treated to have decoupled from the Lagrangian at energy scales below $\mu \sim M_\Delta$.

$\underline{\mu < M_\Delta}$

The potential of the model is modified in presence of the scalar singlet and a new term arises due to interaction of SM doublet (ϕ) with scalar singlet (ξ) and self-interaction of ξ

$$V(\xi,\phi) = V_{\text{SM}} + \frac{\lambda_{\phi\xi}}{2}\phi^\dagger\phi\xi^2 + \frac{\mu_\xi^2}{2}\xi^2 + \frac{\lambda_\xi}{24}\xi^4 \tag{43}$$

where λ_ξ is dark matter self-coupling, $\lambda_{\phi\xi}$ is standard Higgs and extra Higgs scalar interaction coupling or Higgs portal coupling and μ_ξ is quadratic coupling of extra Higgs scalar. From electroweak scale, up to $\mu = 10^{12}$ GeV, the effective potential is $V'(\xi,\phi) = V_{SM} + V(\xi,\phi)$.

$\mu > M_\Delta$

The introduction of the scalar triplet Δ_L of mass $M_\Delta \sim 10^{12}$ GeV changes the Higgs potential further by additional terms $V(\phi,\Delta_L)$ (arising out of interaction of SM doublet with scalar triplet and self-interaction of scalar triplet) and $V(\xi,\Delta_L)$ (arising out of interaction of scalar singlet DM and scalar triplet)

$$V(\xi,\phi,\Delta_L) = V(\xi,\phi) + V(\phi,\Delta_L) + V(\xi,\Delta_L) \quad (44)$$

where

$$\begin{aligned} V(\phi,\Delta_L) &= M_\Delta^2 \mathrm{Tr}\left(\Delta_L^\dagger \Delta_L\right) + \frac{\lambda_1}{2}\left[\mathrm{Tr}\left(\Delta_L^\dagger\Delta_L\right)\right]^2 \\ &\quad + \frac{\lambda_2}{2}\left(\left[\mathrm{Tr}\left(\Delta_L^\dagger\Delta_L\right)\right]^2 - \mathrm{Tr}\left[\left(\Delta_L^\dagger\Delta_L\right)^2\right]\right) \\ &\quad + \lambda_4\left(\phi^\dagger\phi\right)\mathrm{Tr}\left(\Delta_L^\dagger\Delta_L\right) \\ &\quad + \lambda_5\phi^\dagger\left[\Delta_L^\dagger,\Delta_L\right]\phi \\ &\quad + \left(\mu_\Delta\tilde{\phi}^\dagger\left(\frac{\overrightarrow{\tau}.\overrightarrow{\Delta_L}}{\sqrt{2}}\right)^\dagger\phi + \mathrm{h.c.}\right) \\ V(\xi,\Delta_L) &= \lambda_{\xi\Delta_L}\left(\xi^\dagger\xi\right)\left(\Delta_L^\dagger\Delta_L\right). \end{aligned} \quad (45)$$

Sufficiently below the mass scale $\mu = M_\Delta = 10^{12}$ GeV, our model has two scalars: the first one is the SM Higgs(ϕ) given by $\phi = (1/\sqrt{2})(\phi^+, v + h + i\phi^0)^T$ and the second one is extra scalar singlet(ξ) added to the SM. The mass of the extra singlet is given by

$$M_{DM}^2 = \mu_\xi^2 + \frac{\lambda_{\phi\xi}}{2}v^2. \quad (46)$$

We use the standard Higgs mass $m_h = 125$GeV.

Direct detection experiments [18, 19] impose constraints on the Higgs portal coupling ($\lambda_{\phi\xi}$) and dark matter mass [164, 165] derived from observed DM relic density

$$M_{DM} \sim 3300 \times \lambda_{\phi\xi} \quad (47)$$

or

$$\lambda_{\phi\xi} \sim 0.0003 \times M_{DM}, \quad (48)$$

for $M_{DM} \gg m_{\mathrm{top}}$. To be consistent with (41) we use $M_{DM} = m_\xi \sim 1$TeV throughout this work. Similar analysis can be carried out for all values of DM mass > 500 GeV.

These constraint on $\lambda_{\phi\xi}$ given in (48) can be also considerably relaxed if there is more than one WIMP DM candidate of the same or different species including fermions [110, 166].

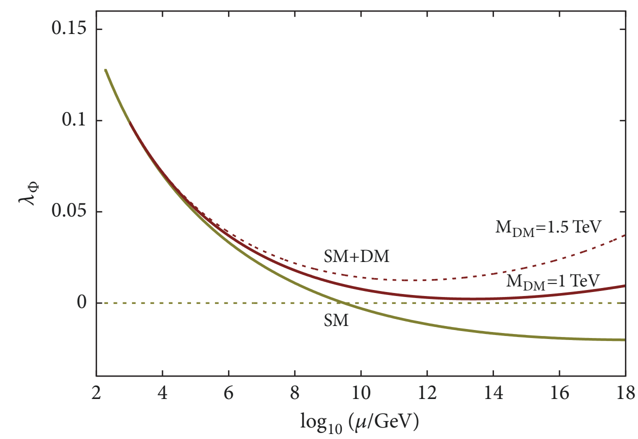


Figure 8: Running of Higgs quartic coupling.

6.2. RG Evolution of Quartic Coupling. Like other couplings of every non-Abelian gauge theory, it is well known that the SM Higgs potential is modified by quantum corrections determined by perturbative renormalization group equations (RGEs) for its running couplings $\eta(\mu)$

$$\frac{d\eta}{dt} = \sum_j \frac{\eta^{(j)}}{(16\pi^2)^j} \quad (49)$$

where $t = \log\mu$, μ is renormalization scale, $\eta(\mu)$ =different couplings (quartic or gauge, or others) at scale μ, and $j = j^{th}$ loop order. The one-loop RG-coefficients of different couplings are presented in the Appendix. For the stability of the Higgs potential (see (43)), the value of self-coupling including corrections must remain positive throughout the course of its evolution up to the Planck scale.

The running of Higgs quartic coupling $\lambda_\phi(\mu)$ with energy scale μ is shown in Figure 8.

In the figure, at first, we have neglected possible threshold effects due to Higgs triplet at $\mu = M_\Delta$ being determined as one of the solutions to neutrino oscillation data. Negligible Δ_L-threshold effect can also result for $\mu_\Delta \ll M_\Delta$. We have used the initial values of different coupling constants at top-quark mass scale ($\mu = m_{\mathrm{top}}$) as given in Table 6 and subsequently evolved them from m_t to Planck scale with the help of RGEs.

From Figure 8, it is clear that the desired quartic coupling remains stable up to the Planck scale for $\lambda_{\phi\xi} = 0.36$ and $M_{DM} = 1$ TeV.

6.3. Higgs Triplet Threshold Effect. Threshold effect due to heavier Higgs masses which couple to ϕ through their portals has been discussed in general [88, 89] and in specific cases [167, 168]. In our case the Higgs triplet mass used to fit the neutrino oscillation data is $M_\Delta \sim 10^{12}$ GeV and its induced VEV is $\mathcal{O}(1-10)$ eV. In such a case the threshold effect caused by the triplet VEV correction term is [88]

$$\Delta\lambda_\phi = \lambda_{\phi\Delta}\frac{v_L^2}{M_\Delta^2} \sim 10^{-36} \quad (50)$$

Table 6: Initial values of coupling constants at top quark mass.

Coupling constants	$\lambda_\phi(m_t)$	$\lambda_\xi(m_t)$	$\lambda_{\phi\xi}(m_t)$	$g_{1Y}(m_t)$	$g_{2L}(m_t)$	$g_{3C}(m_t)$	$y_t(m_t)$
Initial values	0.1296	0.1	0.36	0.35	0.64	1.16	0.94

The remaining threshold effect could be due the self-energy correction or the trilinear term $\mu_\Delta \Delta_L \phi\phi + h.c$ in the Higgs potential giving rise to threshold correction to quartic coupling

$$\Delta\lambda_\phi \equiv \lambda_{\mathrm{TH}} = \frac{\mu_\Delta^2}{M_\Delta^2} \tag{51}$$

Denoting the effective Higgs quartic coupling by $\lambda'(\mu)$ for $\mu \geq M_\Delta$ this is related to the quartic coupling $\lambda_\phi(\mu)$ at $\mu = M_\Delta$ [88]

$$\lambda_\phi(M_\Delta) = \lambda'(M_\Delta) - \lambda_{\mathrm{TH}}. \tag{52}$$

This correction comes into play when the running mass scale is $\mu \sim M_\Delta$ and larger.

We point out that the same values of Majorana Yukawa coupling elements of Y derived in Section 2 are valid up to a scale factor for a wide range of values of trilinear coupling mass parameter $\mu_\Delta < M_\Delta^0$ for which this threshold effect is well within the perturbative regime. We note from (8) that the mass formula gives the scaling relation

$$Y = Y^0 \frac{v_L^0}{v_L} = Y^0 \frac{\mu_\Delta^0}{\mu_\Delta}, \tag{53}$$

where we have used the zero superscript for values at $\mu_\Delta^0 = M_\Delta = 10^{12}$ GeV. Thus, for the values of neutrino mass and mixing given by the oscillation data, a new set of elements of Y are derived for any $\mu_\Delta < \mu_\Delta^0$ by multiplying all the vales given in Tables 2 and 3 by the same scale factor μ_Δ^0/μ_Δ.

In Figure 9 we have presented evolution of Higgs quartic couplings below and above $\mu = M_\Delta$ for $\lambda_{\mathrm{TH}} = 0.1$. Using the notations of the Appendix, we have used the initial values of different coupling constants at scalar triplet mass scale (M_Δ) as $\lambda_1 = \lambda_2 = \lambda_4 = 0.1$ and $\lambda_5 = 0.1$.

For all the three curves given in Figure 9 the scalar DM mass has been fixed at $M_{\mathrm{DM}} = 1$ TeV consistent with LUX:2016 data. The curve labeled as SM+DM+Δ includes threshold effect $\lambda_{\mathrm{TH}} = 0.1$ at $\mu = M_\Delta = 10^{12}$ GeV corresponding to $\mu_\Delta \sim (1/3)M_\Delta$. We have checked that even after including the heavy scalar threshold effect the quartic coupling remains perturbatively positive upto the Planck scale for $\mu_\Delta/M_\Delta \simeq 0.5$. (Denoting $\Phi_H = 24_H$, above the mass scale $\mu > M_{\mathrm{GUT}}$ we impose the well known discrete symmetry $\Phi_H \longrightarrow -\Phi_H$ which is usually assumed in the minimal SU(5) model. Without loss of generality we further assume the Higgs portal coupling $\lambda_{5_H,75_H}$ to be negligible.)

Thus, the issue of vacuum stability of SM Higgs potential is resolved through the embedding of ξ as a WIMP dark matter candidate in SU(5) even after including the heavy Higgs triplet threshold effect which could be verified by direct search experiments and LHC.

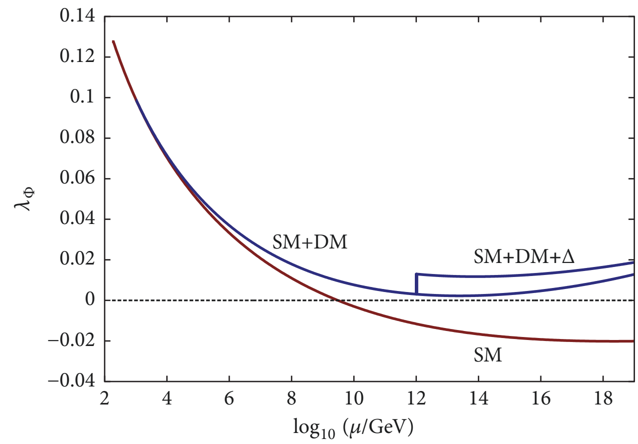


Figure 9: Running of standard Higgs quartic coupling including heavy triplet scalar threshold effect at $\mu = M_\Delta = 10^{12}$ GeV derived from fits to neutrino oscillation data. The curves labeled as SM, SM+DM, and SM+DM+Δ denote contributions due to SM alone, SM plus DM, and SM plus DM plus Higgs triplet threshold effect, respectively, as described in the text. The scalar DM mass has been fixed at $M_{\mathrm{DM}} = 1.0$ TeV consistent with LUX:2016 experimental data.

7. Summary and Conclusion

In this work we have attempted to resolve four limitations of the minimal SU(5) model by extending its scalar sector beyond 5_H and 24_H. Added presence of 15_H and 75_H is noted to account for precision coupling unification with experimentally verifiable proton lifetime for $p \longrightarrow e^+\pi^0$, and type-II seesaw ansatz for neutrino masses. The left-handed triplet Higgs mass in this model is bounded from below ($M_\Delta = M_{15_H}) \geq (M_\kappa = 10^{9.23}$) GeV. Proton lifetime is predicted by taking into account sources of theoretical uncertainties due to GUT threshold effects and those due to electroweak precision parameters. Type-II seesaw scale effect on proton lifetime prediction is also discussed. The limitation due to vacuum stability of the Higgs potential in SU(5) is resolved by the inclusion of a scalar singlet near the TeV scale that acts as a WIMP dark matter candidate. All the fermions and this scalar are assigned to be odd under a dark matter stabilising Z_2 discrete symmetry whereas the SM Higgs is even. The scalar dark matter mass is consistent with current experimental LUX-2016 bound on direct search experiments. Renormalization group evolution of SM Higgs quartic coupling modified by Higgs portal coupling of this scalar DM completely alleviates the vacuum instability problem. We emphasize that no nonstandard Higgs field, except the scalar DM singlet, is present in this model below the κ mass $M_\kappa = 10^{9.23}$ GeV.

We thus conclude that such SM limitations as neutrino mass, coupling unification, proton lifetime, WIMP dark matter, and vacuum stability can be effectively resolved by extending the scalar sector of SU(5) to include $5_H, 24_H, 75_H, 15_H$, and 1_H. At present we need no extension on the established fermion structure of the SM and SU(5) or their minimal gauge structure. The remaining limitations on baryon asymmetry generation and/or the possibility of decaying dark matter projected to manifest as PeV energy IceCube neutrinos will be addressed elsewhere [169, 170].

Appendix

Renormalization Group Equations for Higgs Scalar Couplings

The RGEs for scalar quartic couplings [60–62, 168] in our model at one loop level are given by

$$16\pi^2 \frac{dC}{dt} = \beta_C \quad \left(C = \lambda_\phi, \lambda_{\phi\xi}, \lambda_\xi, \lambda_1, \lambda_2, \lambda_4, \lambda_5\right) \tag{A.1}$$

where

$$\begin{aligned}
&\beta_{\lambda_\phi} \\
&\quad = 24\lambda_\phi^2 + 12\lambda_\phi y_t^2 - 6y_t^4 - 3\lambda_\phi\left(g_{1Y}^2 + 3g_{2L}^2\right) \\
&\qquad + \frac{3}{8}\left[2g_{2L}^4 + \left(g_{1Y}^2 + g_{2L}^2\right)^2\right] + \frac{\lambda_{\phi\xi}^2}{2} \\
&\beta_{\lambda_{\phi\xi}} \\
&\quad = \left\{4\lambda_{\phi\xi} + 12\lambda_\phi + 6y_t^2 - \frac{3}{2}\left(g_{1Y}^2 + 3g_{2L}^2\right) + \lambda_\xi\right\}\lambda_{\phi\xi} \\
&\beta_{\lambda_\xi} = 3\lambda_\xi^2 + 12\lambda_{\phi\xi}^2
\end{aligned} \tag{A.2}$$

For Standard model RG running in the energy scale $\mu < M_{DM}$, the term $\lambda_{\phi\xi}/2$ in β_{λ_ϕ} in (A.3) is to be ignored. The RGEs for SM gauge couplings and top-quark Yukawa coupling at two-loop level are given by

$$\begin{aligned}
\frac{dy_t}{dt} &= \frac{1}{16\pi^2}\left(\frac{9}{2}y_t^2 - \frac{17}{12}g_{1Y}^2 - \frac{9}{4}g_{2L}^2 - 8g_{3C}^2\right)y_t \\
&\quad + \frac{1}{\left(16\pi^2\right)^2}\left[-\frac{23}{4}g_{2L}^4 - \frac{3}{4}g_{2L}^2 g_{1Y}^2 + \frac{1187}{216}g_{1Y}^4\right. \\
&\quad + 9g_{2L}^2 g_{3C}^2 + \frac{19}{9}g_{3C}^2 g_{1Y}^2 - 108g_{3C}^4 \\
&\quad + \left(\frac{225}{16}g_{2L}^2 + \frac{131}{16}g_{1Y}^2 + 36g_{3C}^2\right)y_t^2 \\
&\quad \left. + 6\left(-2y_t^4 - 2y_t^2\lambda_\phi + \lambda_\phi^2\right)\right] \\
\frac{dg_{1Y}}{dt} &= \frac{1}{16\pi^2}\left(\frac{41}{6}g_{1Y}^3\right) + \frac{1}{\left(16\pi^2\right)^2}\left(\frac{199}{18}g_{1Y}^2\right. \\
&\quad \left. + \frac{9}{2}g_{2L}^2 + \frac{44}{3}g_{3C}^2 - \frac{17}{6}y_t^2\right)g_{1Y}^3 \\
\frac{dg_{2L}}{dt} &= \frac{1}{16\pi^2}\left(-\frac{19}{6}g_{2L}^3\right) + \frac{1}{\left(16\pi^2\right)^2}\left(\frac{3}{2}g_{1Y}^2\right. \\
&\quad \left. + \frac{35}{6}g_{2L}^2 + 12g_{3C}^2 - \frac{3}{2}y_t^2\right)g_{2L}^3 \\
\frac{dg_{3C}}{dt} &= \frac{1}{16\pi^2}\left(-7g_{3C}^3\right) + \frac{1}{\left(16\pi^2\right)^2}\left(\frac{11}{6}g_{1Y}^2 + \frac{9}{2}g_{2L}^2\right. \\
&\quad \left. - 26g_{3C}^2 - 2y_t^2\right)g_{3C}^3
\end{aligned} \tag{A.3}$$

After $\mu = 10^{12}$ GeV the scalar triplet Δ_L is introduced and we use the modified RG equations of λ_ϕ and other couplings relevant for this scalar triplet.

$$\begin{aligned}
\beta_{\lambda_\phi} &= \lambda_\phi\left[12\lambda_\phi - \left(\frac{9}{5}g_{1Y}^2 + 9g_{2L}^2\right) + 12y_t^2\right] \\
&\quad + \frac{9}{4}\left(\frac{3}{25}g_{1Y}^4 + \frac{2}{5}g_{1Y}^2 g_{2L}^2 + g_{2L}^4\right) + 6\lambda_4^2 + 4\lambda_5^2 \\
&\quad - 12y_t^4,
\end{aligned} \tag{A.4}$$

$$\begin{aligned}
\beta_{\lambda_1} &= \lambda_1\left[14\lambda_1 + 4\lambda_2 - \left(\frac{36}{5}g_{1Y}^2 + 24g_{2L}^2\right)\right. \\
&\quad \left. + 4\operatorname{tr}[T]\right] + \frac{108}{25}g_{1Y}^4 + \frac{72}{5}g_{1Y}^2 g_{2L}^2 + 18g_{2L}^4 + 2\lambda_2^2 \\
&\quad + 4\lambda_4^2 + 4\lambda_5^2 - 8\operatorname{tr}\left[T^2\right],
\end{aligned} \tag{A.5}$$

$$\begin{aligned}
\beta_{\lambda_2} &= \lambda_2\left[12\lambda_1 + 3\lambda_2 - \left(\frac{36}{5}g_{1Y}^2 + 24g_{2L}^2\right)\right. \\
&\quad \left. + 4\operatorname{tr}[T]\right] - \frac{144}{5}g_{1Y}^2 g_{2L}^2 + 12g_{2L}^4 - 8\lambda_5^2 \\
&\quad + 8\operatorname{tr}\left[T^2\right],
\end{aligned} \tag{A.6}$$

$$\begin{aligned}
\beta_{\lambda_4} &= \lambda_4\left[6\lambda_\phi + 8\lambda_1 + 2\lambda_2 + 4\lambda_4\right. \\
&\quad \left. - \left(\frac{9}{2}g_{1Y}^2 + \frac{33}{2}g_{2L}^2\right) + 6y_t^2 + 2\operatorname{tr}[T]\right] + \frac{27}{25}g_{1Y}^4 \\
&\quad + 6g_{2L}^4 + 8\lambda_5^2 - 4\operatorname{tr}\left[T^2\right],
\end{aligned} \tag{A.7}$$

$$\begin{aligned}
\beta_{\lambda_5} &= \lambda_5\left[2\lambda + 2\lambda_1 - 2\lambda_2 + 8\lambda_4 - \left(\frac{9}{2}g_{1Y}^2 + \frac{33}{2}g_{2L}^2\right)\right. \\
&\quad \left. + 6y_t^2 + 2\operatorname{tr}[T]\right] - \frac{18}{5}g_{1Y}^2 g_{2L}^2 + 4\operatorname{tr}\left[T^2\right],
\end{aligned} \tag{A.8}$$

where T is defined as $T = Y^{\dagger}Y$ and its beta function is expressed through the relation

$$\beta_T = T\left[6T - 3\left(\frac{3}{5}g_{1Y}^2 + 3g_{2L}^2\right) + 2tr\left[T\right]\right]. \tag{A.9}$$

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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