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Research Article

Heavy-Light Mesons in the Nonrelativistic Quark Model Using Laplace Transformation Method

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An analytic solution of the N-dimensional radial Schrödinger equation with the combination of vector and scalar potentials via the Laplace transformation method (LTM) is derived. The current potential is extended to encompass the spin hyperfine, spin-orbit, and tensor interactions. The energy eigenvalues and the corresponding eigenfunctions have been obtained in the N-dimensional space. The present results are employed to study the different properties of the heavy-light mesons (HLM). The masses of the scalar, vector, pseudoscalar, and pseudovector for B, B_s, D, and D_s mesons have been calculated in the three-dimensional space. The effect of the dimensional number space is discussed on the masses of the HLM. We observed that the meson mass increases with increasing dimensional space. The decay constants of the pseudoscalar and vector mesons have been computed. In addition, the leptonic decay widths and branching ratio for the B^+, D^+, and B_s^+ mesons have been studied. Therefore, the used method with the current potential gives good results which are in good agreement with experimental data and are improved in comparison with recent theoretical studies.

1. Introduction

One of the most important tasks in nonrelativistic quantum mechanics is to get the solution of the Schrödinger equation. The solution of the Schrödinger equation with spherically symmetric potentials plays a significant role in many fields of physics such as hadronic spectroscopy for understanding the quantum chromodynamics theory. Numerous works have been introduced to get the solution of Schrödinger equation using different methods like the operator algebraic method [1], path integral method [2], the conventional series solution method [3, 4], Fourier transform [5, 6], shifted $(1/N)$ expansion [7, 8], point canonical transformation [9], quasi-linearization method [10], supersymmetric quantum mechanics (SUSQM) [11], Hill determinant method (HDM) [12], and other numerical methods [13–15].

Recently, the study of the different topics has received a great attention from theoretical physicists in the higher dimensional space. In addition, the study is more general and one can obtain the required results in the lower dimensions directly, such as the hydrogen atom [16–18], harmonic oscillator [19, 20], random walks [21], Casimir effects [22], and the quantization of angular momentum [23–27]. The N-dimensional Schrödinger equation has been studied with different forms of spherically symmetric potentials [28–33]. The N-dimensional Schrödinger equation has been investigated with the Cornell potential and extended Cornell potential [34–38] using different methods such as the Nikiforov-Uvarov (NU) method [32, 36, 39, 40], power series technique (PST) [41], the asymptotic iteration method (AIM) [34], Pekeris type approximation (PTA) [41, 42], and the analytical exact iteration method (AEIM) [43, 44].

The LTM is one of the useful methods that contributed to finding the exact solution of Schrödinger equation in one-dimensional space for Morse potential [45, 46], the harmonic oscillator [47], and three-dimensional space with pseudo-harmonic and Mie-type potentials [48] and with noncentral potential [49]. The N-dimensional Schrödinger equation has been solved via the LTM in many studies for Coulomb potential [28], harmonic oscillator [50], Morse potential

[51], pseudoharmonic potential [52], Mie-type potential [53], anharmonic oscillator [54], and generalized Cornell potential [38].

The study of different properties of HLM is very vital for understanding the structure of hadrons and dynamics of heavy quarks. Thus, many theoretical and experimental efforts have been done for understanding distinct characteristics of HLM. In [4, 34, 55], the authors calculated the mass spectra of quarkonium systems as charmonium and bottomonium mesons with the quark-antiquark interaction potential using various methods in many works. Al-Jamel and Widyan [56] studied the spin-averaged mass spectra of heavy quarkonia with Coulomb plus quadratic potential using (NU) method. Abou-Salem [57] has computed the masses and leptonic decay widths of $c\bar{c}$, $b\bar{b}$, $c\bar{s}$, $b\bar{s}$, $b\bar{u}$, and $c\bar{b}$ numerically using Jacobi method. The strong decays, spectroscopy, and radiative transition of heavy-light hadrons have been computed using the quark model predictions [58]. The decay constant of HLM has been calculated using the field correlation method [59]. Moreover, the spectroscopy of HLM has been investigated in the framework of the QCD relativistic quark model [60]. The spectroscopy and Regge trajectories of HLM have been obtained using quasi-potential approach [61]. The decay constants of heavy-light vector mesons [62] and heavy-light pseudoscalar mesons [63] have been calculated with QCD sum rules. A comparative study has been introduced for the mass spectrum and decay properties for the D meson with the quark-antiquark potential using hydrogeometric and Gaussian wave function [64]. In framework of Dirac formalism the mass spectra of D_s [65] and D [66] mesons have been obtained using Martin-light potential in which the hadronic and leptonic decays of D and D_s mesons have been evaluated [67]; besides the rare decays of B^0 and B_s^0 mesons into dimuon ($\mu^+\mu^-$) [68] and the decay constants of B and B_s have been calculated [69]. The mass spectra and decay constants for ground state of pseudoscalar and vector mesons have been obtained using the variational analysis in the light quark model [70]. The spectroscopy of bottomonium and B meson has been studied using the free-form smearing in [71]. The variational method has been employed to compute the masses and decay constants of HLM in [72]. In addition, the decay properties of D and D_s mesons have been investigated using the quark-antiquark potential in [73]. The B and B_s mesons spectra and their decays have been studied with a Coulomb plus exponential type potential in [74]. The leptonic and semileptonic decays of B meson into τ have been studied [75]. The degeneracy of HLM with the same orbital angular momentum has been broken with the spin-orbit interactions [76]. The relativistic quark model has been investigated to study the properties of B and B_s mesons [77] and the excited charm and charm-strange mesons [78]. The perturbation method has been employed to determine the mass spectrum and decay properties of HLM with the mixture of harmonic and Yukawa-type potentials [79]. In [80], the authors have investigated the leptonic decays of seven types of heavy vector and pseudoscalar mesons. The spectra and wave functions of HLM have been calculated within a relativistic quark model by using the Foldy-Wouthuysen transformation [81]. The isospin breaking of heavy meson decay constants had been compared with lattice QCD from QCD sum rules [82]. The decay constants of pseudoscalar and vector B and D mesons have been studied in the light-cone quark model with the variational method [83]. In [84], the authors have calculated the strong decays of newly observed D_J (3000) and D_{sJ} (3040) with two 2P (1^+) quantum number assignments. The leptonic ($D \longrightarrow e^+\nu_e$) and semileptonic ($D \longrightarrow K^{(*)}\ell^+\nu_\ell, D \longrightarrow \pi\ell^+\nu_\ell$) decays have been analyzed using the covariant quark model with infrared confinement within the standard model framework [85]. The weak decays of B, B_s, and B_c into D-wave heavy-light mesons have been studied using Bethe-Salpeter equation [86]. In [87], the decay constant and distribution amplitude for the heavy-light pseudoscalar mesons have been evaluated using the light-front holographic wavefunction. By using the Gaussian wave function with quark-antiquark potential model, the Regge trajectories, spectroscopy, and decay properties have been studied for B and B_s mesons [88], D and D_s mesons [89], and also the radiative transitions and the mixing parameters of the D-meson have been obtained [90]. The dimensional space dependence of the masses of heavy-light mesons has been investigated using the string inspired potential model [91].

The goal of this work is to get the analytic solution of the N-dimensional Schrödinger equation for the mixture of vector and scalar potentials including the spin-spin, spin-orbit, and tensor interactions using LTM in order to obtain the energy eigenvalues in the N-dimensional space and the corresponding eigenfunctions. So far no attempt has been made to solve the N-dimensional Schrödinger equation using the LTM when the spin hyperfine, spin-orbit, and tensor interactions are included. To show the importance of present results, the present results are employed to calculate the mass spectra of the HLM in three-dimensional space and in the higher dimensional space. In addition, the decay constants, leptonic decay widths, and branching fractions of the HLM are calculated.

The paper is systemized as follows: the contributions of previous works are displayed in Section 1. In Section 2, a brief summary of Laplace transformation method is introduced. In Section 3, an analytic solution of the N-dimensional Schrödinger equation is derived. In Section 4, the obtained results are discussed. In Section 5, summary and conclusion are presented.

2. Overview of Laplace Transformation Method

The Laplace transform $\phi(z)$ or $\mathscr{L}$ of a function $f(t)$ is defined by [92]

$$\phi(z) = \mathscr{L}\{f(t)\} = \int_0^\infty e^{-zt} f(t)\,dt. \tag{1}$$

If there is some constant $\sigma \in R$ such that $|e^{-\sigma t} f(t)| \leq M$ for sufficiently large t, the integral in (1) exists for Re $z > \sigma$ for $z > 0$. The Laplace transform may fail to exist because of a sufficiently strong singularity in the function $f(t)$ as $t \longrightarrow 0$. In particular

$$\mathscr{L}\left[\frac{t^{\alpha}}{\Gamma(\alpha+1)}\right]=\frac{1}{z^{\alpha+1}}, \quad \alpha>-1, \tag{2}$$

where Γ is the gamma function. The Laplace transform has the derivative properties

$$\mathscr{L}\left\{f^{(n)}(t)\right\}=z^{n}\mathscr{L}\{f(t)\}-\sum_{k=0}^{n-1}z^{n-1-k}f^{(k)}(0), \tag{3}$$

$$\mathscr{L}\{t^{n}f(t)\}=(-1)^{n}\phi^{(n)}(z), \tag{4}$$

where the superscript (n) stands for the n-th derivative with respect to t for $f^{(n)}(t)$ and with respect to z for $\phi^{(n)}(z)$. If z_0 is the singular point, the Laplace transform behaves near $z \longrightarrow z_0$ as

$$\phi(z)=\frac{1}{(z-z_0)^{\upsilon}}, \tag{5}$$

and then for $t \longrightarrow \infty$

$$f(t)=\frac{1}{\Gamma(\upsilon)}t^{\upsilon-1}e^{z_0 t}. \tag{6}$$

On the other hand, if near origin $f(t)$ behaves like t^{α} with $\alpha>-1$, then $\phi(z)$ behaves near $z \longrightarrow \infty$ as

$$\phi(z)=\frac{\Gamma(\alpha+1)}{z^{\alpha+1}}. \tag{7}$$

3. Analytic Solution of the N-Dimensional Radial Schrödinger Equation

The N-dimensional radial Schrödinger equation that describes the interaction between quark-antiquark systems takes the form [41]

$$\left[\frac{d^2}{dr^2}+\frac{(N-1)}{r}\frac{d}{dr}-\frac{\ell(\ell+N-2)}{r^2}+2\mu\left(E-V_{q\bar{q}}(r)\right)\right]\Psi(r)=0, \tag{8}$$

where ℓ, N represent the angular quantum number and the dimensional number greater than one, respectively, and $\mu = m_q m_{\bar{q}}/(m_q+m_{\bar{q}})$ is the reduced mass of the quark-antiquark system.

In the nonrelativistic quark model, the quark-antiquark potential $V_{q\bar{q}}(r)$ consists of the spin independent potential $V(r)$ and the spin dependent potential $V_{SD}(r)$, respectively:

$$V_{q\bar{q}}(r)=V(r)+V_{SD}(r). \tag{9}$$

The spin independent potential is taken as a combination of vector and scalar parts [93]:

$$V(r)=V_V(r)+V_S(r), \tag{10}$$

$$V_V(r)=\eta\left(ar^2+br\right)-\frac{c}{r}, \tag{11}$$

$$V_S(r)=(1-\eta)\left(ar^2+br\right), \tag{12}$$

where $V_V(r)$ and $V_S(r)$ are the vector and scalar parts, respectively, and η stands for the mixing coefficient. a, b, and c are arbitrary parameters where a, b, and $c>0$ which are fitted with experimental data. The harmonic and linear terms represent the confining part at long distance and the Coulomb term stands for the quark-antiquark interactions through one gluon exchange at short distances which gives better description of quark-antiquark interaction.

The spin dependent potential is extended to three types of interaction terms as [94]

$$V_{SD}(r)=V_{LS}(r)(\mathbf{L}.\mathbf{S})+\mathbf{S}_{12}V_T(r)+V_{SS}(r)(\mathbf{S}_1.\mathbf{S}_2), \tag{13}$$

while the spin-orbit $V_{LS}(r)$ and tensor $V_T(r)$ terms give the fine structure of the states, the spin-spin $V_{SS}(r)$ interaction term describes the hyperfine splitting of the state, and $\mathbf{L}$ is an angular quantum operator, and $\mathbf{S}$ is a spin operator (for detail, see [94]).

$$V_{LS}(r)=\frac{1}{2m_q m_{\bar{q}}r}\left(3\frac{dV_V}{dr}-\frac{dV_S}{dr}\right), \tag{14}$$

$$V_T(r)=\frac{1}{12m_q m_{\bar{q}}}\left(\frac{1}{r}\frac{dV_V}{dr}-\frac{d^2V_V}{dr^2}\right), \tag{15}$$

$$V_{SS}(r)=\frac{2}{3m_q m_{\bar{q}}}\nabla^2 V_V, \tag{16}$$

where ∇^2 is radial Laplace operator.

$$\mathbf{S}_1.\mathbf{S}_2=\frac{1}{2}\left[S(S+1)-\frac{3}{2}\right], \tag{17}$$

$$\langle\mathbf{L}.\mathbf{S}\rangle=\frac{1}{2}[J(J+1)-L(L+1)-S(S+1)], \tag{18}$$

$$\mathbf{S}_{12}=2\left[\mathbf{S}^2-3(\mathbf{S}.\hat{\mathbf{r}})(\mathbf{S}.\hat{\mathbf{r}})\right]. \tag{19}$$

The diagonal elements of the $\mathbf{S}_{12}$ are defined.

$$\langle\mathbf{S}_{12}\rangle=\frac{4}{(2L+3)(2L-1)}\left[\left\langle S^2\right\rangle\left\langle L^2\right\rangle-3\langle\mathbf{L}.\mathbf{S}\rangle^2-\frac{3}{2}\langle\mathbf{L}.\mathbf{S}\rangle\right]. \tag{20}$$

Substituting (11)-(16) into (9) then the nonrelativistic quark-antiquark potential $V_{q\bar{q}}(r)$ takes the form

$$V_{q\bar{q}}(r)=ar^2+br+\delta+\frac{g}{r}+\frac{h}{r^3}, \tag{21}$$

where

$$\delta=\frac{2a}{m_q m_{\bar{q}}}\left[2\eta(\mathbf{S}_1.\mathbf{S}_2)+\left(2\eta-\frac{1}{2}\right)(\mathbf{L}.\mathbf{S})\right], \tag{22}$$

$$g=\frac{b}{m_q m_{\bar{q}}}\left\{\eta\left[\frac{4}{3}(\mathbf{S}_1.\mathbf{S}_2)+\frac{1}{12}\mathbf{S}_{12}\right]+\left(2\eta-\frac{1}{2}\right)(\mathbf{L}.\mathbf{S})\right\}-c, \tag{23}$$

$$h=\frac{3c}{2m_q m_{\bar{q}}}\left[\frac{1}{6}\mathbf{S}_{12}+(\mathbf{L}.\mathbf{S})\right]. \tag{24}$$

Substituting (21) into (8), then

$$\left[\frac{d^2}{dr^2}+\frac{(N-1)}{r}\frac{d}{dr}-\frac{\ell(\ell+N-2)}{r^2}+\varepsilon-Ar^2-Br -2\mu\delta-\frac{G}{r}-\frac{H}{r^3}\right]\Psi(r)=0, \quad (25)$$

where

$$\begin{aligned}\varepsilon&=2\mu E,\\ A&=2\mu a,\\ B&=2\mu b,\\ G&=2\mu g,\\ H&=2\mu h.\end{aligned} \quad (26)$$

The complete solution of (25) takes the form

$$\Psi(r)=r^k e^{-\alpha r^2}f(r),\quad k>0,\ \text{with } \alpha=\sqrt{\frac{\mu a}{2}}, \quad (27)$$

where the term r^k confirms that the solution is bounded at $r=0$. The function $f(r)$ is yet to be determined. From (27) we get

$$\Psi'(r)=r^k e^{-\alpha r^2}\left[f'(r)+\left(\frac{k}{r}-2\alpha r\right)f(r)\right]. \quad (28)$$

$$\Psi''(r)=r^k e^{-\alpha r^2}\left\{f''(r)+\left(\frac{2k}{r}-4\alpha r\right)f'(r)+\left[\frac{k(k-1)}{r^2}+4\alpha^2r^2-4\alpha k-2\alpha\right]f(r)\right\}. \quad (29)$$

Substituting (27), (28), and (29) into (25), then,

$$rf''(r)+\left(\omega-4\alpha r^2\right)f'(r)+\left\{\frac{\lambda}{r}-Br^2+\zeta r-G-\frac{H}{r^2}\right\}f(r)=0, \quad (30)$$

where

$$\omega=2k+N-1, \quad (31)$$

$$\lambda=k(k+N-2)-\ell(\ell+N-2), \quad (32)$$

$$\zeta=\varepsilon-4\alpha k-2\alpha N-2\mu\delta. \quad (33)$$

In order to apply the Laplace transform of the above differential equation, the parametric condition is taken as in [52, 54].

$$k(k+N-2)-\ell(\ell+N-2)=0. \quad (34)$$

Thus, (32) has a solution

$$k_+=\ell,\quad \text{and } k_-=-(\ell+N-2). \quad (35)$$

We take the physical solution of (32) ($k=k_+=\ell$) as in [52, 54].

Substituting (34) into (30) yields

$$rf''(r)+\left(\omega-4\alpha r^2\right)f'(r)+\left\{\zeta r-Br^2-G-\frac{H}{r^2}\right\}f(r)=0. \quad (36)$$

By expanding the term H/r^2 around $y=0$, where $y=r-v$ and v is a parameter as in [36, 56], we get

$$\frac{H}{r^2}=\frac{H}{(y+v)^2}=\frac{H}{v^4}\left(3r^2-8rv+6v^2\right). \quad (37)$$

Substituting (37) into (36) yields

$$rf''(r)+\left(\omega-4\alpha r^2\right)f'(r)+\left\{Qr-Pr^2-C_0\right\}f(r)=0, \quad (38)$$

where

$$Q=\zeta+\frac{8H}{v^3},\quad P=B+\frac{3H}{v^4},\quad \text{and } C_0=G+\frac{6H}{v^2}. \quad (39)$$

The Laplace transform is defined as $\phi(z)=\mathscr{L}\{f(r)\}$ and taking boundary condition $f(0)=0$ yields

$$(z+\tau)\frac{d^2\phi(z)}{dz^2}+\left(\frac{z^2}{4\alpha}+\rho\right)\frac{d\phi(z)}{dz}+\left(\gamma z+\frac{C_0}{4\alpha}\right)\phi(z)=0. \quad (40)$$

Here

$$\tau=\frac{P}{4\alpha},\quad \rho=\frac{Q}{4\alpha}+2,\quad \gamma=\frac{(2-\omega)}{4\alpha}. \quad (41)$$

The singular point of (40) is $z=-\tau$. By using the condition of (5), the solution of (40) takes the form

$$\phi(z)=\frac{C}{(z+\tau)^{n+1}},\quad n=0,1,2,3,\ldots. \quad (42)$$

From (42),

$$\phi'(z)=\frac{-C(n+1)}{(z+\tau)^{n+2}}, \quad (43)$$

$$\phi''(z)=\frac{C(n+1)(n+2)}{(z+\tau)^{n+3}}. \quad (44)$$

Substituting (42)-(44) into (40), we obtain the following relations:

$$\gamma=\frac{n+1}{4\alpha}, \quad (45)$$

$$\gamma\tau+\frac{C_0}{4\alpha}=0, \quad (46)$$

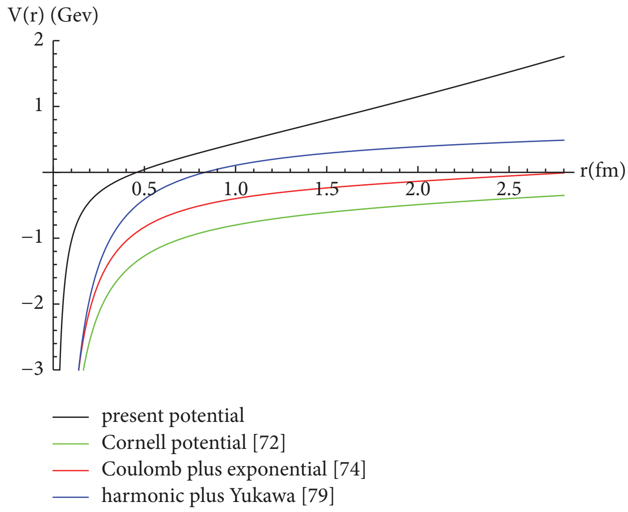

Figure 1: The current potential and other potential models are plotted as functions of distance r.

$$(n+1)(n+2)-\rho(n+1)+\frac{C_0\tau}{4\alpha}=0. \quad (47)$$

Using (26), (39), and (41) and the set of (45)-(47), then, the energy eigenvalue of (8) in the N-dimensional space is given by the relation

$$E_{n\ell N}=\sqrt{\frac{a}{2\mu}}(2n+2\ell+N)-\frac{b^2}{4a}+\delta-\frac{8h}{\upsilon^3} -\frac{h}{a}\left(\frac{9h}{4\upsilon^8}+\frac{3b}{2\upsilon^4}\right). \quad (48)$$

Take the inverse Laplace transform such that $f(r)=\mathscr{L}^{-1}\{\phi(z)\}$. The function $f(r)$ takes the following form:

$$f(r)=\frac{C}{\Gamma(n+1)}r^n e^{-\tau r}. \quad (49)$$

Using (11), (13), and (23), the eigenfunctions of (9) take the following form:

$$\Psi(r)=\frac{C}{\Gamma(n+1)}r^{n+\ell}\exp\left(-\sqrt{\frac{\mu a}{2}}r^2-\sqrt{\frac{\mu}{2a}}br\right). \quad (50)$$

From the condition $\int_0^\infty |\Psi(r)|^2 r^{N-1}dr=1$, the normalization constant C can be computed. In addition, the wave equation $\Psi(r)$ satisfies the boundary condition $\Psi(r=0)=\Psi(r=\infty)=0$.

4. Discussion of Results

In Figure 1, the current potential has been plotted in comparison to other potential models; we see that the present potential is in a qualitative agreement with other potential models [72, 74, 79], in which the confining part is clearly obtained in comparison to Cornell and Coulomb plus exponential potentials. The different states of B and D mesons

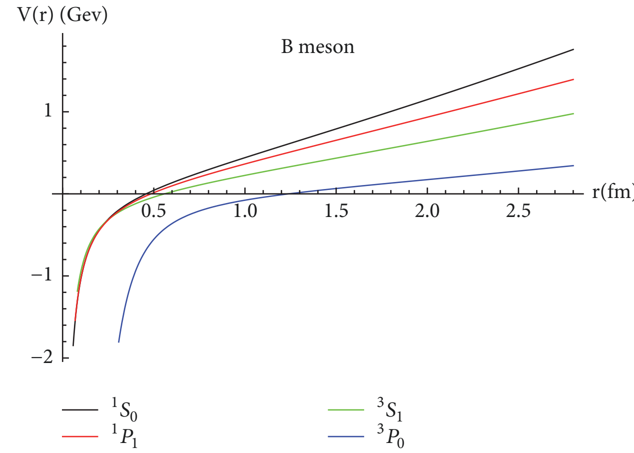

Figure 2: The current potential of B meson for different states.

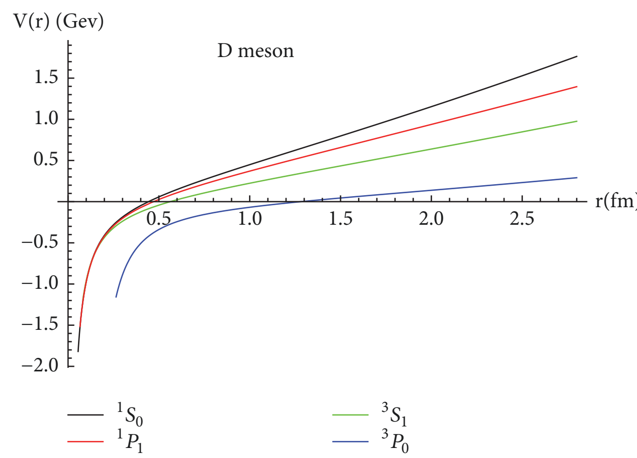

Figure 3: The current potential of D meson for different states.

have been shown in Figures 2 and 3, respectively, in which the principal number of states plays an important role in confining part of potential.

In the following subsections, we employ the obtained results in the previous section to determine the mass spectra of scalar, vector, pseudoscalar, and pseudovector of B, B_s, D, and D_s mesons in the N-dimensional space in comparison with the experimental data (PDG 2016) [95] and with other recent studies. In addition, the decay properties such as decay constants, leptonic decay width, and the branching ratio of HLM are calculated.

4.1. Mass Spectra of Heavy-Light Mesons. The masses of HLM in the N-dimensional space are defined [44]:

$$M_{B,D}=m_q+m_{\bar{q}}+E_{n\ell N}. \quad (51)$$

Substituting (48) into (51), then the mass spectra of HLM in the N- dimensional space can be found from the relation

$$M_{B,D}=m_q+m_{\bar{q}}+\sqrt{\frac{a}{2\mu}}(2n+2\ell+N)-\frac{b^2}{4a}+\delta$$

Table 1: Parameters for HLM.

m_c	m_b	$m_{u,d}$	m_s	η	v
1.45 (GeV)	4.87 (GeV)	0.38 (GeV)	0.48 (GeV)	0.25	1 (GeV^{-1})

Table 2: Masses for pseudoscalar ($^{2S+1}L_J = {}^1S_0$) mesons in GeV. $a = 0.00085\ \text{GeV}^3$, $b = 0.01614\ \text{GeV}^2$, and $c = 0.7$.

Meson	Present Work	Exp. [95]	[72]	[81]	[96]	[88, 89]	[73, 74]	N=4	N=5
D	1.864	1.864	1.895	1.871	1.859	1.884 [89]	1.864 [73]	1.902	1.939
D_s	1.960	1.968	1.962	1.964	1.949	1.965 [89]	1.978 [73]	1.989	2.023
B	5.277	5.280	5.302	5.273	5.262	5.287 [88]	5.272 [74]	5.311	5.346
B_s	5.366	5.366	5.340	5.363	5.337	5.367 [88]	5.385 [74]	5.397	5.428

Table 3: Masses for vector ($^{2S+1}L_J = {}^3S_1$) mesons in GeV. $a = 0.026068\ \text{GeV}^3$, $b = 0.218058\ \text{GeV}^2$, and $c = 8 \times 10^{-3}$.

Meson	Present Work	Exp. [95]	[72]	[81]	[96]	[88, 89]	[73, 74]	N=4	N=5
D	2.010	2.010	2.023	2.008	2.026	2.010 [89]	2.010 [73]	2.218	2.426
D_s	2.100	2.112	2.057	2.107	2.110	2.120 [89]	2.102 [73]	2.244	2.434
B	5.374	5.325	5.356	5.329	5.330	5.323 [88]	5.327 [74]	5.567	5.759
B_s	5.415	5.415	5.384	5.419	5.405	5.413 [88]	5.409 [74]	5.588	5.760

Table 4: Masses for scalar ($^{2S+1}L_J = {}^3P_0$) mesons in GeV. $a = 0043\ \text{GeV}^3$, $b = 0.001\ \text{GeV}^2$, and $c = 10^{-3}$.

Meson	Present Work	Exp. [95]	[72]	[81]	[96]	[88, 89]	[73, 74]	N=4	N=5
D	2.289	2.318±0.029	2.316	2.364	2.357	2.357[89]	2.539[73]	2.374	2.459
D_s	2.350	2.318	2.372	2.437	2.412	2.438[89]	2.311[73]	2.427	2.505
B	5.700	5.710	5.657	5.776	5.740	5.730[88]	5.745[74]	5.736	5.815
B_s	5.720	- - -	5.719	5.811	5.776	5.812[88]	5.843[74]	5.785	5.856

$$- \frac{8h}{v^3} - \frac{h}{a}\left(\frac{9h}{4v^8} + \frac{3b}{2v^4}\right). \tag{52}$$

In Tables 2–6, we have calculated the masses of the HLM in the three-dimensional space in comparison with the experimental data and other recent studies [72–74, 81, 88, 89, 96]. The parameters used in the present calculations are shown in Table 1. In addition, the masses at $N = 4$ and $N = 5$ are calculated. In Tables 2 and 3, we observe that D and B_s meson masses close to experimental data and other meson masses are in good agreement with experimental data and become better in comparison to the results in recent studies [72–74, 81, 88, 89, 96]. In comparison with [72], they used the variational method for the Cornell potential to study the HLM with including the spin-spin and spin-orbit interactions. They ignored the tensor interactions in their calculations. The present results are good in comparison to the results in [72]. In addition, we used the LTM in the present calculations. Yazarloo and Mehiraban used the variational method to study D and D_s mesons for the Cornell potential [73] and used the Nikiforov-Uvarov (NU) method to study B and B_s mesons for the Coulomb plus exponential type potential [74]. The present results are in good agreement with the results of [73, 74]. Kher et al. [89] used a Gaussian wave function to calculate the mass spectra of D and D_s in addition to B and B_s mesons [88] for the Cornell potential. Jing-Bin [81, 96] obtained the spectra of the HLM in the relativistic model from the Bethe-Salpeter equation using the Foldy-Wouthuysen transformation in his works.

We note that the present results for D and B_s meson masses become better in comparison to the results of [81, 88, 89, 96], where the values of pseudoscalar D and B_s mesons are close to the experimental data in Table 2. The values of vector D and B_s mesons close to the experimental data and the values of vector D_s and B mesons are good in comparison to the experimental results in Table 3.

The masses of the scalar mesons are presented in Table 4; the value of D meson is close to the experimental value. The values of D_s and B are in agreement with the experimental values and the value of B_s meson is in good agreement with the theoretical studies [72–74, 81, 88, 89, 96]. In Table 5, we observe that all the values of pseudovector mesons are close to the experimental results except the value of B meson which is in good agreement with the experimental value. The values of vector D_s and B mesons are in good agreement with the experimental results. In Table 6, the results of the p-wave state for the HLM are reported.

The present predictions of D, D_s, B, and B_s mesons are in agreement in comparison to the experimental data and the theoretical studies [73, 74, 81, 88, 89, 96].

In addition, we have investigated the masses of the HLM in the higher dimensions at N=4 and N=5. In Tables 2–6, the effect of the dimensional number is investigated on the masses of the HLM. One can see that the masses increase with increasing dimensional number. The influence of the

Table 5: Masses for pseudovector ($^{2S+1}L_J = {}^1P_1$) mesons in GeV. $a = 0.01359$ GeV^3, $b = 0.08784$ GeV^2, and $c = 0.008$.

Meson	Present Work	Exp. [95]	[72]	[81]	[96]	[88, 89]	[73, 74]	*N*=4	*N*=5
D	2.421	2.421	2.362	2.507	2.434	2.425[89]	2.421[73]	2.571	2.722
D_s	2.460	2.460	2.409	2.558	2.528	2.529[89]	2.429[73]	2.597	2.735
B	5.797	5.726	5.760	5.719	5.736	5.733[88]	5.744[74]	5.936	6.075
B_s	5.828	5.829	5.775	5.819	5.824	5.828[88]	5.841[74]	5.952	6.077

Table 6: Masses for mesons with p-wave state ($^{2S+1}L_J = {}^3P_2$) in GeV. $a = 0.0163$ GeV^3, $b = 0.113$ GeV^2, and $c = 6 \times 10^{-5}$.

Meson	Present work	Exp. [95]	[81]	[96]	[88, 89]	[73, 74]	*N*=4	*N*=5
D	2.463	2.463	2.460	2.482	2.461[89]	2.463[74]	2.628	2.792
D_s	2.500	2.537	2.570	2.575	2.569[89]	2.528[74]	2.641	2.800
B	5.817	5.740	5.739	5.754	5.740[88]	5.743[73]	5.969	6.122
B_s	5.840	5.840	5.838	5.843	5.840[88]	5.840[73]	5.976	6.113

Table 7: The decay constants of pseudoscalar B and D mesons in MeV.

Meson	f_p	$\overline{f}_p$	[72]	[83]	[87]	[97]
D	220	235	228	200 ± 24	$214.2^{+7.6}_{-7.8}$	210 ± 11
D_s	250	243	273	232 ± 17	$253.5^{+6.6}_{-7.1}$	259 ± 10
B	147	201	149	184 ± 32	$191.7^{+7.9}_{-6.5}$	192 ± 13
B_s	174	213	187	215 ± 24	$225.4^{+7.9}_{-5.3}$	230 ± 13

Table 8: The decay constants of vector B and D mesons in MeV.

Meson	f_v	$\overline{f}_v$	[83]	[73, 74]	[79]
D	290	210	247 ± 35	307 [73]	353.8
D_s	310	212	287 ± 29	344 [73]	382.1
B	196	182	210 ± 37	242.4 [74]	234.7
B_s	216	191	239 ± 29	178.8 [74]	244.2

dimensional number is not considered on the masses of the HLM in the works [72–74, 81, 88, 89, 96]. Roy and Choudhury [91] have studied the masses of heavy flavor mesons in the higher dimensional space using string inspired potential. They found that an increase of the dimensional number leads to increase the meson masses. Therefore, the present results of the mass spectra of HLM are in good agreement in comparison with the results of [91].

4.2. Decay Constants. The study of the decay constants is one of the very significant characteristics of the HLM, as it provides a direct source of information on the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. Many theoretical studies have been done for determining the decay constants with different models as relativistic quark model [97–99], lattice QCD [100–102], QCD sum rules [62, 97, 103], and nonrelativistic model [72–74, 79, 97].

The Van Royen-Weisskopf formula [104] can be used to calculate the decay constants of the pseudoscalar and vector mesons f_p and f_v, respectively, in the nonrelativistic limit which is defined as

$$f_{p/v}^2 = \frac{12\,|\Psi(0)|^2}{M_{p/v}}. \tag{53}$$

The Van Royen-Weisskopf formula with the QCD radiative corrections taken into account can be written as [105]

$$\overline{f}_{p/v}^2 = \frac{12\,|\Psi(0)|^2}{M_{p/v}} C^2(\alpha_s), \tag{54}$$

where

$$C(\alpha_s) = 1 - \frac{\alpha_s}{\pi}\left(\Delta_{p/v} - \frac{m_q - m_{\bar{q}}}{m_q + m_{\bar{q}}} \ln \frac{m_q}{m_{\bar{q}}}\right) \tag{55}$$

and $\Delta_p = 2$ and $\Delta_v = 8/3$, for pseudoscalar and vector mesons, respectively.

In Tables 7 and 8, we have determined the decay constants of the pseudoscalar and vector B and D mesons obtained from (53) and (54) in comparison with the results of other recent works. In [87], the authors evaluated the decay constant for the heavy-light pseudoscalar mesons using the helicity-improved light-front holographic wavefunction. In [83], the authors applied the variational method to study the decay constants of the pseudoscalar and vector B and D mesons in the light-cone quark model for the relativistic Hamiltonian with the Gaussian-type function.

In [72], the authors used the variational method to compute the decay constants of HLM from the radial Schrödinger

Table 9: Leptonic decay width of B^+ meson in GeV.

	Present Γ	[74]	[79]	[107]
$B^+ \longrightarrow e^+ \upsilon_e$	2.475×10^{-24}	8.624×10^{-24}	8.094×10^{-24}	5.689×10^{-24}
$B^+ \longrightarrow \mu^+ \upsilon_\mu$	1.086×10^{-19}	3.685×10^{-19}	3.459×10^{-19}	2.439×10^{-19}
$B^+ \longrightarrow \tau^+ \upsilon_\tau$	2.445×10^{-17}	8.196×10^{-17}	7.697×10^{-17}	5.430×10^{-17}

Table 10: Leptonic decay width of D^+ meson in GeV.

	Present Γ	[79]	[108]	[66]
$D^+ \longrightarrow e^+ \upsilon_e$	0.622×10^{-20}	1.488×10^{-20}	1.323×10^{-20}	5.706×10^{-21}
$D^+ \longrightarrow \mu^+ \upsilon_\mu$	2.715×10^{-16}	6.322×10^{-16}	5.641×10^{-16}	2.433×10^{-16}
$D^+ \longrightarrow \tau^+ \upsilon_\tau$	0.668×10^{-15}	1.215×10^{-15}	1.529×10^{-15}	6.157×10^{-16}

Table 11: Leptonic decay width of D_s^+ meson in GeV.

	Present Γ	[79]	[108]	[67]
$D_s^+ \longrightarrow e^+ \upsilon_e$	1.529×10^{-19}	2.962×10^{-19}	3.157×10^{-19}	1.792×10^{-19}
$D_s^+ \longrightarrow \mu^+ \upsilon_\mu$	0.668×10^{-14}	1.259×10^{-14}	1.347×10^{-14}	7.648×10^{-15}
$D_s^+ \longrightarrow \tau^+ \upsilon_\tau$	0.586×10^{-13}	1.296×10^{-13}	1.326×10^{-13}	7.508×10^{-14}

equation with the Cornell potential. Zhi-Gang Wang [97] introduced an analysis of the decay constants of HLM with QCD sum rules. Yazarloo and Mehiraban [79] used the perturbation method to study the decay constants of D, D_s, B, and B_s mesons with the combination of harmonic and Yukawa-type potentials.

In Table 7, the obtained results are in good agreement in comparison to the results of [72, 83, 87, 97]. In Table 8, the present results are compatible with the results of [73, 74, 79, 83]. In addition, the ratio of decay constants for D mesons is ($f_{D_s}/f_D = 1.140$). This value is in good agreement with the experimental value $f_{D_s}/f_D = 1.258 \pm 0.038$ [95]. The present result is in agreement with the obtained values ($f_{D_s}/f_D = 1.195$) in [72] and ($f_{D_s}/f_D = 1.160$) in [83]. Also, we have ($f_{D_s^*}/f_{D^*} = 1.070$) which is in agreement with the calculated values ($f_{D_s^*}/f_{D^*} = 1.183$) in [87] and ($f_{D_s^*}/f_{D^*} = 1.233$) in [97]. The calculated ratio of decay constants for B mesons ($f_{B_s}/f_B = 1.184$) and ($f_{B_s^*}/f_{B^*} = 1.102$) are in good agreement in comparison with ($f_{B_s}/f_B = 1.168$) and ($f_{B_s^*}/f_{B^*} = 1.138$) in [83].

4.3. Leptonic Decay Widths and Branching Ratio. The charged HLM can decay to a charged lepton pair $l^+\nu_l$ via a virtual $W^\pm$ boson. The leptonic decay widths of the HLM can be obtained from the relation [106]

$$\Gamma\left(B^+, D_q \longrightarrow l^+\nu_l\right) = \frac{G_F^2 M_{B,D_q}}{8\pi} m_l^2 \left(1 - \frac{m_l^2}{M_{B,D_q}^2}\right)^2 f_{B,D}^2 \times \begin{cases} \left|V_{ub}\right|^2 & \text{for } B \text{ meson} \\ \left|V_{cq}\right|^2 (q \in d, s), & \text{for } D \text{ meson} \end{cases} \quad (56)$$

where $G_F = 1.664 \times 10^{-5}$ is the Fermi constant and the relevant CKM elements are taken from the PDG [95] as $|V_{ub}| = 0.004$, $|V_{cd}| = 0.227$, and $|V_{cs}| = 0.974$. The leptonic masses m_l are taken as $m_e = 0.501 \times 10^{-3}$ GeV, $m_\mu = 0.105$ GeV, and $m_\tau = 1.776$ GeV. We obtain the decay constants of the HLM from Tables 7 and 8 into (56) to compute leptonic decay widths of the HLM. The obtained results of the leptonic decay width of B^+, D^+, and D_s^+ mesons are shown in Tables 9, 10, and 11, respectively. Vinodkumar et al. [107] calculated the leptonic decay widths of B, B_s mesons besides, D and D_s mesons [66, 67, 108] for the Martin-like potential with Dirac formalism. We have determined the leptonic decay widths of B^+ meson in Table 9 in comparison with the results of the [74, 79, 107], as well as the leptonic decay widths of D^+ meson in Table 10 in comparison with the results of [66, 79, 108] and the leptonic decay widths of D_s^+ meson in Table 11 compared with the results of [66, 79, 108]. We note that the present results are in good agreement with the results of [66, 67, 74, 107, 108].

The branching ratio of the HLM is defined as

$$Br\left(B^+, D_q \longrightarrow l^+\upsilon_l\right) = \Gamma\left(B^+, D_q \longrightarrow l^+\upsilon_l\right) \times \tau_{B^+, D_q} \quad (57)$$

where the lifetime τ of B^+, D^+, and D_s^+ mesons is taken as $\tau_{B^+} = 1.638 ps$, $\tau_{D^+} = 1.040 ps$, and $\tau_{D_s^+} = 0.5 ps$ [95]. We have determined the branching ratio for the B^+, D^+, and D_s^+ mesons compared with the experimental data and with the results of other recent studies [72–74, 88, 89].

In Table 12, we note that the present values of the branching ratio for the B^+ meson are close to experimental data and are in agreement in comparison with the theoretical results [72, 74, 79, 88, 107]. In addition, in Tables 13 and 14, we note that the evaluated results of branching ratio for the D^+ and D_s^+ mesons are close to the experimental data and become better in comparison with works [72, 73, 79, 89, 108].

Table 12: Leptonic branching ratio of B^+ meson.

	Present *Br*	[88]	[79]	[107]	[72]	[74]	**Exp.** [95]
$B^+ \longrightarrow e^+ v_e$	6.162×10^{-12}	8.640×10^{-12}	2.015×10^{-11}	1.419×10^{-11}	6.220×10^{-12}	2.147×10^{-11}	$<9.8 \times 10^{-7}$
$B^+ \longrightarrow \mu^+ v_\mu$	2.705×10^{-7}	0.370×10^{-7}	8.611×10^{-7}	6.085×10^{-7}	2.630×10^{-7}	9.174×10^{-7}	$<1.0 \times 10^{-6}$
$B^+ \longrightarrow \tau^+ v_\tau$	6.088×10^{-5}	0.822×10^{-4}	1.916×10^{-4}	1.354×10^{-4}	1.140×10^{-4}	2.040×10^{-4}	$(1.14 \pm 0.27) \times 10^{-4}$

Table 13: Leptonic branching ratio of D^+ meson.

	Present *Br*	[89]	[79]	[73]	[72]	[108]	**Exp.** [95]
$D^+ \longrightarrow e^+ v_e$	0.984×10^{-8}	0.580×10^{-8}	2.351×10^{-8}	1.77×10^{-8}	1.130×10^{-8}	2.105×10^{-8}	$<8.8 \times 10^{-6}$
$D^+ \longrightarrow \mu^+ v_\mu$	4.293×10^{-4}	2.470×10^{-4}	9.991×10^{-4}	7.54×10^{-4}	4.770×10^{-4}	8.977×10^{-4}	$(3.74 \pm 0.17) \times 10^{-4}$
$D^+ \longrightarrow \tau^+ v_\tau$	1.055×10^{-3}	0.860×10^{-3}	1.920×10^{-3}	1.79×10^{-3}	2.030×10^{-3}	2.933×10^{-3}	$<1.2 \times 10^{-3}$

Table 14: Leptonic branching ratio of D_s^+ meson.

	Present *Br*	[89]	[79]	[73]	[72]	[108]	**Exp.** [95]
$D_s^+ \longrightarrow e^+ v_e$	1.163×10^{-7}	0.940×10^{-7}	2.251×10^{-7}	1.82×10^{-7}	1.630×10^{-7}	1.391×10^{-7}	$<8.3 \times 10^{-5}$
$D_s^+ \longrightarrow \mu^+ v_\mu$	5.078×10^{-3}	4.000×10^{-3}	9.572×10^{-3}	7.74×10^{-3}	6.900×10^{-3}	5.937×10^{-3}	$(5.56 \pm 0.25) \times 10^{-3}$
$D_s^+ \longrightarrow \tau^+ v_\tau$	4.451×10^{-3}	3.780×10^{-3}	9.864×10^{-2}	8.2×10^{-2}	6.490×10^{-2}	5.844×10^{-3}	$(5.55 \pm 0.24)\%$

5. Summary and Conclusion

In this work, we have presented an approximate-analytic solution of the N-dimensional radial Schrödinger equation for the mixture of vector and scalar potentials via the LTM. The spin-spin, spin-orbit, and tensor interactions have been included in the extended Cornell potential model. The energy eigenvalues and the corresponding eigenfunctions have been determined in the N-dimensional space. In three-dimensional space, we have employed the obtained results to study the different properties of the HLM that are not considered in many recent studies. The masses of the scalar, vector, pseudoscalar, and pseudovector for B, B_s, D, and D_s mesons have been calculated in the three-dimensional space and in the higher dimensional space in Tables 2–6. Most of the present calculations are close to the experimental data and are improved in comparison with the recent calculations [72–74, 81, 88, 89, 96]. As well, we have computed the masses of the HLM in the higher dimensional space at N=4 and N=5. The dependence of the masses of HLM on the dimensional number is discussed. We found that the masses increase with increasing dimensional number. This result is obtained in [91]. In Tables 7 and 8, the decay constants of the pseudoscalar and vector mesons have been determined in comparison with the results of [72–74, 79, 83, 87, 97]. The calculated ratios of the decay constants of D mesons (f_{D_s}/f_D = 1.140) and ($f_{D_s^*}/f_{D^*}$ = 1.070) are close to the experimental ratio ($f_{D_s}/f_D = 1.258 \pm 0.038$).

The present results of the decay ratio of B mesons are in good agreement with the results of [72, 83]. The leptonic decay widths of B^+ meson have been studied in comparison with the results of [74, 79, 107] and the leptonic decay widths of D^+ meson in comparison with the results of [66, 79, 108]. In addition, the leptonic decay widths of D_s^+ meson have been studied in comparison with the results of [66, 79, 108].

The obtained results of the leptonic decay widths are compared with the results of [66, 67, 74, 107, 108]. We have determined the branching ratio for the B^+, D^+, and D_s^+ mesons that are in good agreement with the experimental data and with the recent studies [72–74, 88, 89]. Therefore, the current potential with used method gives very good predictions for the heavy-light meson properties. We hope to extend this work to include external force as a future work.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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