Hindawi
Advances in High Energy Physics
Volume 2018, Article ID 7843730, 13 pages
https://doi.org/10.1155/2018/7843730

Research Article

Quaternionic Approach to Dual Magnetohydrodynamics of Dyonic Cold Plasma

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Received 23 June 2018; Accepted 7 August 2018; Published 14 August 2018

Academic Editor: Antonio J. Accioly

The dual magnetohydrodynamics of dyonic plasma describes the study of electrodynamics equations along with the transport equations in the presence of electrons and magnetic monopoles. In this paper, we formulate the quaternionic dual fields equations, namely, the hydroelectric and hydromagnetic fields equations which are an analogous to the generalized Lamb vector field and vorticity field equations of dyonic cold plasma fluid. Further, we derive the quaternionic Dirac-Maxwell equations for dual magnetohydrodynamics of dyonic cold plasma. We also obtain the quaternionic dual continuity equations that describe the transport of dyonic fluid. Finally, we establish an analogy of Alfven wave equation which may generate from the flow of magnetic monopoles in the dyonic field of cold plasma. The present quaternionic formulation for dyonic cold plasma is well invariant under the duality, Lorentz, and CPT transformations.

1. Introduction

In the past few decades, astronomers predicted that the universe was composed almost entirely of the baryonic matter (ordinary matter). According to Bachynski [1], more than 99% of the matter in the universe is in plasma state. This type of matter may consist of baryonic and nonbaryonic matter. The first experimental evidence of the existence of plasma was given by American Physicists [2]. In plasma, consisting of charged and neutral particles, the interionic force between particles shows electromagnetic in nature. Therefore, due to the long range order of Coulomb force charged particles interact with all other charged particles resulting in a collective behavior of plasma. In 1942, Alfven [3] gave the theory of magnetohydrodynamics (MHD) and suggested that electrically conducting fluid can support the propagation of shear waves called the *Alfven waves*. Basically, MHD describes the behavior of electrically conducting fluid in the presence of magnetic field [4]. It is macroscopic theory that assumes the electrons, ions, and charged particles move together and treated them as a single fluid component known as single fluid theory. The plasma along with MHD is simply described by a single temperature, velocity, and density. However, when the MHD wave propagates faster than plasma thermal speed then the effect of temperature can be neglected [5]. This is called a cold plasma approximation (i.e., in cold plasma approximation, temperature does not take into account). In this approximation, there is no wave related to pressure fluctuation (e.g., sound waves). On the other hand, the hot and warm plasmas are another sates of plasma where the collision between electrons and gas molecules are so frequent that there is a thermal equilibrium between electron and the gas molecules.

Meyer-Vernet [6] discussed the role of magnetic monopole in conducting fluid (plasma). The magnetic monopole proposed by Dirac [7], is a hypothetical elementary particle having only one magnetic pole. Dirac also pointed out that if there exists any monopole in the universe then all the electric charge in the universe will be quantized [8]. Schwinger [9, 10], an exception to the argument against the existence of monopole, formulated relativistically covariant quantum field theory of magnetic monopoles which maintained complete symmetry between electric and magnetic fields. Therefore, the name of particles carrying simultaneously the electric and magnetic charges is dyons. Further, the theoretical approach of Schwinger [9, 10] and

Zwanziger [11] describes the theory of dyonic particles. Peres [12] pointed out the controversial nature [13] of the singular lines of magnetic monopoles and established the charged quantization condition in purely group theoretical manner without using them. In view of mathematical physics, the study of four-dimensional particles (dyons) in distinguish mediums can be explained by division algebras. There are four types of divisions algebras [14], namely, the real, complex, quaternion, and octonion algebras. The complex algebra is an extension of real numbers; the quaternion is an extension of complex numbers while the octonion is an extension of quaternions. Quaternionic algebra [15] can also express by the four-dimensional Euclidean spaces [16, 17], and it has vast applications in the multiple branches of physics.

Further, Rajput [18] pointed out an effective unified theory for quaternionic generalized electromagnetic and gravitational fields of dyons by using the quaternion algebra. The quaternionic form of classical and quantum electrodynamics has been already discussed [19–22]. Many authors [23–29] have studied the role of hypercomplex algebras in various branches of physics. Recently, Chanyal [30, 31] independently proposed a novel approach on the quaternionic covariant theory for relativistic quantum mechanics and established the quantized Dirac-Maxwell equations for dyons. Besides, in literature [32–34], the reformulation of incompressible plasma fluids and MHD equations has been discussed in terms of hypercomplex numbers. Keeping in view the importance of quaternionic algebras, we establish the MHD field equations for dyonic cold plasma. Starting with the definitions of one-fluid and two-fluid theory of plasma, we identify the cold plasma approximation where the thermal effects (or pressure effects) of conducting fluid will be neglected. Further, we introduce the dual MHD equations of dyonic plasma consisted with electrons, magnetic monopoles, and their counter partners, namely, ions and magnetoions. In this study, we clarify that the dominating aspect for the dyonic cold plasma approximation is the dynamics of electrons along with magnetic monopoles. As we know the generalized Dirac-Maxwell like equations are primary equations to explain the dynamics of dyonic cold plasma. Therefore, undertaking the quaternionic dual-velocity and dual-enthalpy of dyonic cold plasma, we have made an attempt to formulate the quaternionic hydroelectric and hydromagnetic fields equations, which are an analogous to the generalized Lamb vector field and vorticity field of conducting dyonic fluid. The Lorenz gauge conditions for dyonic cold plasma fluid are also obtained. Further, we derive the generalized quaternionic Dirac-Maxwell equations to the case of dual magnetohydrodynamics of dyonic cold plasma. We have discussed that these Dirac-Maxwell equations for dyonic cold plasma are well invariant under the duality, Lorentz, and CPT transformations. Finally, the Alfven wave like equation is established which may propagate from the flow of magnetic monopoles in the dyonic cold plasma.

2. The Quaternions

Through the extension of the set of natural numbers to the integers, a complex number $\mathbb{C}$ is defined by the set of all real linear combinations of the unit elements $(1, i)$, such that

$$\mathbb{C} \longmapsto \{\alpha = \alpha_1 + i\alpha_2 \mid (\alpha_1, \alpha_2 \in \mathbb{R})\}, \tag{1}$$

where the real number α_1 is called the real part and α_2 is called the imaginary part of a complex number α. If the real part $\mathrm{Re}(\alpha) = 0$, then we can say that α is purely imaginary. As such, the Euclidean scalar product as $\mathbb{C} \times \mathbb{C} \longmapsto \mathbb{R}$ is then defined by

$$\langle \alpha, \beta \rangle = \mathrm{Re}\left(\alpha \cdot \overline{\beta}\right) = \alpha_1\beta_1 + \alpha_2\beta_2, \tag{2}$$

where $\alpha = \alpha_1 + i\alpha_2$ and $\beta = \beta_1 + i\beta_2$ are two complex numbers. The modulus of any complex number is also defined by $|\alpha| = \sqrt{\alpha \cdot \overline{\alpha}} = \sqrt{\alpha_1^2 + \alpha_2^2}$.

However, a complex field $\mathbb{C}$ is a finite dimensional real vector space, so that we can easily extend the complex number into the quaternionic field $\mathbb{H}$ by losing the commutativity of multiplication. Thus, the quaternion represents the natural extension of complex numbers and forms an algebra under addition and multiplication. Hamilton [15] described a four-dimensional quaternionic algebra and applied it to mechanics in three-dimensional space. A striking feature of quaternions is that the product of two quaternions is noncommutative, meaning that the product of two quaternions depends on which factor is to the left of the multiplication sign and which factor is to the right.

Thus the allowed four-dimensional Hamilton vector space is defined by quaternion algebra $\mathbb{H}$ over the field of real numbers $\mathbb{R}$ as

$$\mathbb{H} \longmapsto \left\{\alpha = \sum_{j=0}^{3} e_j\alpha_j = e_0\alpha_0 + e_1\alpha_1 + e_2\alpha_2 + e_3\alpha_3 \mid \forall \alpha_j \in \mathbb{R}\right\}, \tag{3}$$

where the Hamilton vector space $(\mathbb{H})$ has the quaternionic elements $(e_0, e_1, e_2, \text{and } e_3)$, which are called quaternion basis elements while α_0, α_1, α_2, and α_3 are the real quarterate of a quaternion. As such the addition of two quaternions $\alpha = e_0\alpha_0 + e_1\alpha_1 + e_2\alpha_2 + e_3\alpha_3$ and $\beta = e_0\beta_0 + e_1\beta_1 + e_2\beta_2 + e_3\beta_3$ is given by

$$\begin{aligned} \alpha + \beta &= e_0\left(\alpha_0 + \beta_0\right) + e_1\left(\alpha_1 + \beta_1\right) + e_2\left(\alpha_2 + \beta_2\right) \\ &\quad + e_3\left(\alpha_3 + \beta_3\right), \quad \forall \left(\alpha, \beta\right) \in \mathbb{H}. \end{aligned} \tag{4}$$

Here, the quaternionic addition is clearly associative and commutative. The additive identity element is defined by the zero element; i.e.,

$$0 = e_0 0 + e_1 0 + e_2 0 + e_3 0, \tag{5}$$

and the additive inverse of $\alpha \in \mathbb{H}$ is given by

$$-\alpha = e_0(-\alpha_0) + e_1(-\alpha_1) + e_2(-\alpha_2) + e_3(-\alpha_3). \quad (6)$$

Correspondingly, the product of two quaternions, i.e., $(\alpha \circ \beta) \in \mathbb{H}$, can be expressed by

$$\begin{aligned}\alpha \circ \beta = {} & e_0(\alpha_0\beta_0 - \alpha_1\beta_1 - \alpha_2\beta_2 - \alpha_3\beta_3) \\ & + e_1(\alpha_0\beta_1 + \alpha_1\beta_0 + \alpha_2\beta_3 - \alpha_3\beta_2) \\ & + e_2(\alpha_0\beta_2 - \alpha_1\beta_3 + \alpha_2\beta_0 + \alpha_3\beta_1) \\ & + e_3(\alpha_0\beta_3 + \alpha_1\beta_2 - \alpha_2\beta_1 + \alpha_3\beta_0).\end{aligned} \quad (7)$$

We may notice that this quaternionic product is associative, but not commutative. The quaternionic unit elements (e_0, e_1, e_2, e_3) followed the given relations,

$$\begin{aligned} e_0^2 &= 1, \\ e_A^2 &= -1, \\ e_0 e_A &= e_A e_0 = e_A, \\ e_A e_B &= -\delta_{AB} e_0 + f_{ABC} e_C, \quad (\forall A, B, C = 1, 2, 3)\end{aligned} \quad (8)$$

where δ_{AB} is the delta symbol and f_{ABC} is the Levi Civita three-index symbol having value $f_{ABC} = +1$ for cyclic permutation, $f_{ABC} = -1$ for anticyclic permutation, and $f_{ABC} = 0$ for any two repeated indices. Further, we also may write the following relations to quaternion basis elements

$$\begin{aligned} [e_A,\ e_B] &= 2 f_{ABC} e_C, \\ \{e_A,\ e_B\} &= -2\delta_{AB} e_0, \\ e_A(e_B e_C) &= (e_A e_B) e_C,\end{aligned} \quad (9)$$

where the brackets [,] and { , } are used, respectively, for commutation and the anticommutation relations. Thus the above multiplication rules governed the ordinary dot and cross product; i.e.,

$$\alpha \circ \beta = (\alpha_0\beta_0 - \boldsymbol{\alpha} \cdot \boldsymbol{\beta},\ \alpha_0\boldsymbol{\beta} + \beta_0\boldsymbol{\alpha} + (\boldsymbol{\alpha} \times \boldsymbol{\beta})), \quad (10)$$

where we take $\boldsymbol{\alpha} \times \boldsymbol{\beta} \neq 0$ for noncommutative product of quaternion. The quaternionic product with the scalar quantity ξ is given by

$$\xi \circ \alpha = e_0(\xi\alpha_0) + e_1(\xi\alpha_1) + e_2(\xi\alpha_2) + e_3(\xi\alpha_3). \quad (11)$$

As such, the multiplication identity element can expressed by the unit elements,

$$1 = e_0 1 + e_1 0 + e_2 0 + e_3 0. \quad (12)$$

Moreover, a quaternion can also be decomposed in terms of scalar $(S(\alpha))$ and vector $(\mathbf{V}(\alpha))$ parts as

$$S(\alpha) = \frac{1}{2}(\alpha + \overline{\alpha}), \quad (13)$$

$$\mathbf{V}(\alpha) = \frac{1}{2}(\alpha - \overline{\alpha}), \quad (14)$$

where the quaternionic conjugate $\overline{\alpha}$ is expressed by

$$\overline{\alpha} = e_0\alpha_0 - (e_1\alpha_1 + e_2\alpha_2 + e_3\alpha_3). \quad (15)$$

The real and imaginary parts of α can be written as

$$\mathrm{Re}(\mathbb{H}) = \alpha_0, \quad (16)$$

$$\mathrm{Im}(\mathbb{H}) = \{e_1\alpha_1 + e_2\alpha_2 + e_3\alpha_3 \mid \forall\alpha_{j=1,2,3} \in \mathbb{R}\} \subseteq \mathbb{H}. \quad (17)$$

If $\mathrm{Re}(\mathbb{H}) = 0$ and $\alpha \neq 0$, then α is said to be purely imaginary quaternions. Therefore, all quaternions with zero real are simplified as imaginary space of $\mathbb{H}$, where the imaginary space $\mathrm{Im}(\mathbb{H}) \in \mathbb{R}^3$ is a three-dimensional real vector space,

$$\begin{aligned}\mathrm{Im}(\alpha) &= (\alpha_1, \alpha_2, \alpha_3) \\ \mathrm{Im}(\alpha)^\dagger &= \begin{pmatrix}\alpha_1 \\ \alpha_2 \\ \alpha_3\end{pmatrix}.\end{aligned} \quad (18)$$

Interestingly, we may write the following form of quaternion as

$$\alpha = \mathrm{Re}(\alpha) + \sum_{j=1}^{3} e_j \mathrm{Im}(\alpha_j). \quad (19)$$

The quaternionic Euclidean scalar product $\mathbb{H} \times \mathbb{H} \longmapsto \mathbb{R}$ can also be expressed as

$$\langle \alpha, \beta \rangle = \mathrm{Re}(\alpha \circ \overline{\beta}) = \alpha_0\beta_0 + \alpha_1\beta_1 + \alpha_2\beta_2 + \alpha_3\beta_3. \quad (20)$$

Like complex numbers, the modulus of quaternion α is then defined as

$$|\alpha| = \sqrt{\alpha_0^2 + \alpha_1^2 + \alpha_2^2 + \alpha_3^2}. \quad (21)$$

Since there exists the norm $N(\alpha) = \alpha \circ \overline{\alpha}$ of a quaternion, we have a division; i.e., every α has an inverse of a quaternion and is expressed as

$$\alpha^{-1} = \frac{\overline{\alpha}}{|\alpha|}. \quad (22)$$

The quaternion conjugation satisfies the following property:

$$\overline{\alpha_1 \circ \alpha_2} = \overline{\alpha_1} \circ \overline{\alpha_2}. \quad (23)$$

The norm of the quaternion is positive definite and obeys the composition law

$$N(\alpha_1 \circ \alpha_2) = N(\alpha_1) \circ N(\alpha_2). \quad (24)$$

The quaternion elements are non-Abelian in nature and thus represent a noncommutative division ring. Quaternion is an important fundamental mathematical tool appropriate for four-dimensional world.

3. Magnetohydrodynamics of Cold Plasma

Let us start with the basic parameters of the plasma. As we know that the plasma exists in many more forms in nature which has a wide spread use in the science and technology. The theory of plasma is divided into three categories [35], namely, the microscopic theory, kinetic theory, and the fluid theory. In briefly, the microscopic theory is based on the motion of all the individual particles (e.g., electrons, ions, atoms, molecules, and radicals). According to Klimontovich [36], the time evolution of the particle density ($\rho_s \longmapsto \rho_s(\mathbf{r}, \mathbf{v}, t)$) is expressed by

$$\frac{\partial \rho_s}{\partial t} + \mathbf{v} \cdot \nabla \rho_s + \frac{q_s}{m_s} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla \rho_s = 0, \quad (25)$$

where $\mathbf{v}$ is the velocity of particles, (q_s, m_s) are the effective charge and mass of the s–species particles, and $(\mathbf{E}, \mathbf{B})$ are the electric and magnetic field produced by the microscopic particles. Besides, the collisionless kinetic theory of plasma proposed by Vlasov [37], which has included the Boltzmann distribution function $f_s \simeq \langle \rho_s \rangle$ as [35], is as follows:

$$\frac{\partial f_s}{\partial t} + \mathbf{v} . \nabla f_s + \frac{q_s}{m_s} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) . \nabla f_s = 0. \quad (26)$$

In (25) and (26), we may consider that the two dominating particles (i.e., electrons and ions both) constitute the dynamics of plasma, called the two-fluid theory of plasma [35–38]. For the two-fluid theory of plasma, at a given position (x), the mass and charge densities become

$$\rho_M (x) = m_e n_e (x) + m_i n_i (x), \quad (27)$$

$$\rho_c (x) = q_e n_e (x) + q_i n_i (x), \quad (28)$$

where m_e, n_e, and q_e are defined as the mass, total number, and charge of electrons while m_i, n_i, and q_i are defined as the mass, total number, and charge of ions, respectively. The center of mass fluid velocity can be expressed as

$$\mathbf{v} = \frac{1}{\rho_M (x)} (\mathbf{v}_e m_e n_e (x) + \mathbf{v}_i m_i n_i (x)), \quad (29)$$

and the current density becomes

$$\mathbf{J} = q_e n_e \mathbf{v}_e + q_i n_i \mathbf{v}_i. \quad (30)$$

The continuity equations can be written as

$$\frac{\partial \rho_M}{\partial t} + \nabla \cdot (\rho_M \mathbf{v}) = 0, \quad \text{(mass conservation law)} \quad (31)$$

$$\frac{\partial \rho_c}{\partial t} + \nabla \cdot \mathbf{J} = 0, \quad \text{(charge conservation law)} \quad (32)$$

As such, the momentum equation for plasma fluid is expressed as [35]

$$\rho_M \left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right) \mathbf{v} = (\mathbf{J} \times \mathbf{B}) + \rho_c \mathbf{E} - \nabla p, \quad (33)$$

where ∇p is the pressure force introduced due to the inhomogeneity of the plasma and $(\mathbf{J} \times \mathbf{B})$ is a Lorentz force per unit volume element. Now, we introduce an acceleration to the conducting fluid,

$$\frac{\partial \mathbf{v}}{\partial t} \longmapsto \left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right) \mathbf{v}, \quad (34)$$

where the term $(\mathbf{v} \cdot \nabla)\mathbf{v}$ is used for the convective acceleration of fluid. Furthermore, the generalized Ohm's law becomes [35]

$$\frac{m_e m_i}{\rho_M e^2} \frac{\partial \mathbf{J}}{\partial t} = \frac{m_i}{2 \rho_M e} \nabla p + \mathbf{E} + (\mathbf{v} \times \mathbf{B}) - \frac{m_i}{\rho_M e} (\mathbf{J} \times \mathbf{B}) - \frac{\mathbf{J}}{\sigma}, \quad (35)$$

where σ denotes the conductivity of fluid. One can define Maxwell's equations with natural unit ($\hbar = c = 1$) as

$$\nabla \cdot \mathbf{E} = \rho_c, \quad (36)$$

$$\nabla \cdot \mathbf{B} = 0, \quad (37)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad (38)$$

$$\nabla \times \mathbf{B} = \frac{\partial \mathbf{E}}{\partial t} + \mathbf{J}. \quad (39)$$

Interestingly, if we combine together the conducting fluidic field and electromagnetic field then the relevant theory comes out which is called MHD. The MHD of cold plasma is an approximation theory of fluid dynamics where we neglect temperature effect and combine the electron equation with ionic equation to form a one-fluid model [39]. For the cold plasma model, many researchers [40, 41] suggested that, at a given position, all particle-species (mostly ions and electrons) have comparable temperatures (T), energies ($\mathscr{E}$) (equivalent to masses), and velocities ($\mathbf{v}$). It follows that the fluid velocity is identical for particle velocity. Now, we may summarize the following conditions for the cold plasma approximation, i.e.,

$$\begin{aligned} &T_e \sim T_i \quad \text{(neglected)} \\ &\mathscr{E}_e \sim \mathscr{E}_i \\ &\mathbf{v}_e \sim \mathbf{v}_i \\ &\rho_e \sim \rho_i \\ &\nabla p \sim 0. \end{aligned} \quad (40)$$

We consider that the effected behavior of electrons are comparable to the ions, while their temperatures and pressure-gradients are taken negligible in case of homogeneous cold plasmas. Thus, using approximation (40), the average mass and charge densities to cold plasma are expressed as

$$\varrho \longmapsto \rho_M (x) \simeq m_e n_e (x) \equiv m_i n_i (x), \quad (41)$$

$$\rho \longmapsto \rho_c (x) \simeq q_e n_e (x) \equiv q_i n_i (x). \quad (42)$$

As such, the Navier-Stokes and Ohm's equations become

$$\varrho\left(\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla\right)\mathbf{v}=\rho\mathbf{E}, \tag{43}$$

$$\mathbf{J}=\sigma\left(\mathbf{E}+\mathbf{v}\times\mathbf{B}\right), \tag{44}$$

where $(\mathbf{J}\times\mathbf{B})\sim 0$ to the case if the current is small compared to $(\mathbf{v}\times\mathbf{B})$. The ideal MHD equations $(\rho\sim 0)$ for cold plasma may then be expressed as

$$\frac{\partial\varrho}{\partial t}+\nabla\cdot(\varrho\mathbf{v})=0, \tag{45}$$

$$\varrho\left(\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla\right)\mathbf{v}=0, \tag{46}$$

$$\nabla\times(\mathbf{v}\times\mathbf{B})=\frac{\partial\mathbf{B}}{\partial t}, \tag{47}$$

$$\nabla\times\mathbf{B}=\frac{\partial\mathbf{E}}{\partial t}+\mathbf{J}. \tag{48}$$

To consider wave behavior of cold particles, the cold plasma wave has temperature independent dispersion relation. If $\mathbf{v}_A$ is Alfven velocity, then the dispersion relation for cold plasma waves become [35] $\omega^2=\kappa^2\mathbf{v}_A^2/(1+\mathbf{v}_A^2)$. Interestingly, the cold plasma waves propagate like as Alfven waves which are independent of temperature.

4. Dual MHD Equations for Dyonic Cold Plasma

The dual MHD field not only consists of electrons and ions but also has the magnetic monopole and their ionic partners magnetoions [42]. Generally, the composition of an electron and a magnetic monopole referred a dyon [25]. In this study, we may neglect the magnetoionic contribution like ions to continue the dyonic cold plasma approximations. Dirac [8] proposed the symmetrized field equations by postulating the existence of magnetic monopoles; i.e.,

$$\nabla\cdot\mathbf{E}=\rho^e, \tag{49}$$

$$\nabla\cdot\mathbf{B}=\rho^{\mathrm{m}}, \tag{50}$$

$$\nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}-\mathbf{J}^{\mathrm{m}}, \tag{51}$$

$$\nabla\times\mathbf{B}=\frac{\partial\mathbf{E}}{\partial t}+\mathbf{J}^e. \tag{52}$$

In the above generalized Dirac-Maxwell's equations, ρ^e and ρ^{m} are the electric and magnetic charge densities while $\mathbf{J}^e$ and $\mathbf{J}^{\mathrm{m}}$ are the corresponding current densities. To study the dyonic cold plasma field, there are a couple of masses and charges species in presence of dyons. Thus, the generalized dual densities (mass and charge densities) may be expressed for one-fluid theory of dyonic cold plasma as

$$\varrho^D\left(\varrho^e,\varrho^{\mathrm{m}}\right)\longmapsto\left(m^e n^e+m^{\mathrm{m}}n^{\mathrm{m}}\right), \tag{53}$$

$$\rho^D\left(\rho^e,\rho^{\mathrm{m}}\right)\longmapsto\left(q^e n^e+q^{\mathrm{m}}n^{\mathrm{m}}\right), \tag{54}$$

where m^{m}, n^{m}, and q^{m} are defined as the mass, total number, and charge of magnetic monopoles, respectively. As such, we can express the center of mass velocity of dyonic fluid in cold plasma as

$$\mathbf{v}^D\simeq\frac{1}{\varrho^D}\left(\mathbf{v}^e m^e n^e(x)+\mathbf{v}^{\mathrm{m}}m^{\mathrm{m}}n^{\mathrm{m}}(x)\right), \tag{55}$$

whereupon the dual-current densities (electric and magnetic) are defined by

$$\begin{aligned}\mathbf{J}^e&=q^e n^e\mathbf{v}^e,\\ \text{and }\mathbf{J}^{\mathrm{m}}&=q^{\mathrm{m}}n^{\mathrm{m}}\mathbf{v}^{\mathrm{m}}.\end{aligned} \tag{56}$$

The conservation laws for the dynamics of dyonic cold plasma can be written as

$$\begin{gathered}\frac{\partial\varrho^D}{\partial t}+\nabla\cdot\left(\varrho^D\mathbf{v}^D\right)=0,\\ (\text{dyons mass conservation law})\end{gathered} \tag{57}$$

$$\begin{gathered}\frac{\partial\rho^e}{\partial t}+\nabla\cdot\mathbf{J}^e=0,\\ (\text{electric charge conservation law})\end{gathered} \tag{58}$$

$$\begin{gathered}\frac{\partial\rho^{\mathrm{m}}}{\partial t}+\nabla\cdot\mathbf{J}^{\mathrm{m}}=0,\\ (\text{magnetic charge conservation law}).\end{gathered} \tag{59}$$

The generalized Navier-Stokes force equation can also be exhibited in presence of magnetic monopole; i.e.,

$$\begin{aligned}\varrho^D\left(\frac{\partial}{\partial t}+\mathbf{v}^D\cdot\nabla\right)\mathbf{v}^D&=\left(\mathbf{J}^e\times\mathbf{B}\right)-\left(\mathbf{J}^{\mathrm{m}}\times\mathbf{E}\right)+\rho^e\mathbf{E}\\&\quad+\rho^{\mathrm{m}}\mathbf{B}-(\nabla p)^D,\end{aligned} \tag{60}$$

where the duality invariant Lorentz force equation for dyons is

$$\mathbf{F}^D=\rho^e\mathbf{E}+\left(\mathbf{J}^e\times\mathbf{B}\right)+\rho^{\mathrm{m}}\mathbf{B}-\left(\mathbf{J}^{\mathrm{m}}\times\mathbf{E}\right) \tag{61}$$

and the dyonic pressure gradient term $(\nabla p)^D$ is negligible to the case of cold plasma approximation. Conditionally, if the influence of dyonic current is small then the force equation can be written as

$$\varrho^D\left(\frac{\partial}{\partial t}+\mathbf{v}^D\cdot\nabla\right)\mathbf{v}^D=\rho^e\mathbf{E}+\rho^{\mathrm{m}}\mathbf{B}. \tag{62}$$

In the same way, Ohm's law for the dyonic cold plasma is expressed as

$$\mathbf{J}^e=\sigma^e\left(\mathbf{E}+\mathbf{v}\times\mathbf{B}\right), \tag{63}$$

$$\mathbf{J}^{\mathrm{m}}=\sigma^{\mathrm{m}}\left(\mathbf{B}-\mathbf{v}\times\mathbf{E}\right). \tag{64}$$

where σ^{m} is the magnetic conductivity. Therefore, from (63) to (64), we can conclude that for infinite conductivity of dyons $(\sigma^{e,\mathrm{m}}\longrightarrow\infty)$ the electric and magnetic field vectors constitute from the rotation of each other, i.e., $\mathbf{E}=-(\mathbf{v}\times\mathbf{B})$, and $\mathbf{B}=(\mathbf{v}\times\mathbf{E})$. The above classical field equations given by (49) to (64) of dyons are referred to dual MHD field equations of dyonic cold plasma.

Table 1: Analogies between electrodynamics and hydrodynamics in presence of dyons.

Electrodynamics case	Hydrodynamics case
$\mathscr{A}$ (electric vector potential) $\longmapsto$	$\mathbf{u}$ (electric velocity of the fluid)
$\mathscr{B}$ (magnetic vector potential) $\longmapsto$	$\boldsymbol{v}$ (magnetic velocity of the fluid)
ϕ^e (electric scalar potential) $\longmapsto$	h (electric enthalpy of the fluid)
ϕ^{m} (magnetic scalar potential) $\longmapsto$	k (magnetic enthalpy of the fluid)

5. Quaternionic Formulation to Dual Fields of Dyonic Cold Plasma

In order to write the dual MHD field equations for dyonic cold plasma, we may start with quaternionic two-velocity $(\mathbf{u}, \boldsymbol{v})$ and two-enthalpy (h, k) of dyons for plasma fluid dynamics as

$$\mathbb{U}(e_1, e_2, e_3, e_0) = \left\{u_x, u_y, u_z, -\frac{i}{a_0}h\right\}, \tag{65}$$

$$\mathbb{V}(e_1, e_2, e_3, e_0) = \left\{v_x, v_y, v_z, -ia_0 k\right\}, \tag{66}$$

where $(\mathbb{U}, \mathbb{V})$ are quaternionic variables associated with two four-velocities of electrons and magnetic monopoles of dyons and a_0 denoted the speed of particles (dyons) moving in conducting cold plasma. Here, we have taken the two-enthalpy of dyons, i.e., the internal energy of dyons associated with electrons and magnetic monopoles. Like many physicists [32, 43, 44], there is an analogy between the electromagnetic and hydrodynamic. Thus, we may write the analogy of two four-potentials $(\mathbb{A}, \mathbb{B})$ of dyons as

$$\mathbb{A}\left(\mathscr{A}, -\frac{i}{c}\phi^e\right) \longmapsto \mathbb{U}\left(\mathbf{u}, -\frac{i}{a_0}h\right), \tag{67}$$

$$\mathbb{B}\left(\mathscr{B}, -ic\phi^{\mathrm{m}}\right) \longmapsto \mathbb{V}\left(\boldsymbol{v}, -ia_0 k\right), \tag{68}$$

where the vector components $\mathbf{u} \longrightarrow (u_x, u_y, u_z), \boldsymbol{v} \longrightarrow (v_x, v_y, v_z)$ are analogous to electric and magnetic vector potentials of dyons while the scalar components (h, k) are analogous to their scalar potentials. It should be notice that the role of quaternionic two four-velocities of dyonic fluid in generalized hydrodynamics of cold plasma is the same as the quaternionic two four-potentials of dyons in generalized electrodynamics. Now, we may summarize the dyonic potentials corresponding to its fluid behavior in Table 1.

The unified structure of quaternionic two four-velocities $(\mathbb{W} \in \mathbb{H})$ for the generalized fields of dyonic cold plasma can be written as

$$\begin{aligned}\mathbb{W} &= \left(\mathbb{U} - \frac{i}{a_0}\mathbb{V}\right) \\ &= e_1\left(u_x - \frac{i}{a_0}v_x\right) + e_2\left(u_y - \frac{i}{a_0}v_y\right) \\ &\quad + e_3\left(u_z - \frac{i}{a_0}v_z\right) - \frac{i}{a_0}e_0\left(h - ia_0 k\right),\end{aligned} \tag{69}$$

and it reduces to

$$\begin{aligned}\mathbb{W} &= \sum_{j=1}^{3} e_j w_j - \frac{i}{a_0}e_0\Omega_0 \\ &= \sum_{j=1}^{3} e_j\left(u_j - \frac{i}{a_0}v_j\right) - \frac{i}{a_0}e_0\left(h - ia_0 k\right),\end{aligned} \tag{70}$$

where $\mathbf{w} \longrightarrow (\mathbf{u} - (i/a_0)\boldsymbol{v})$ and $\Omega_0 \longrightarrow (h - ia_0 k)$ are dyonic fluid-velocity and dyonic enthalpy in cold plasma, respectively. Here, the scalar component (Ω_0) represents the amount of dyonic internal energy required to move one kilogram of the fluid element. Now, to formulate the quaternionic dual MHD field equations for dyonic cold plasma, it is necessary to define quaternionic space-time differential operator as

$$\mathbb{D} = \left(\boldsymbol{\nabla}, -\frac{i}{a_0}\frac{\partial}{\partial t}\right) \simeq e_1\frac{\partial}{\partial x} + e_2\frac{\partial}{\partial y} + e_3\frac{\partial}{\partial z} - \frac{i}{a}e_0\frac{\partial}{\partial t}, \tag{71}$$

and its quaternionic conjugate is

$$\begin{aligned}\overline{\mathbb{D}} &= \left(-\boldsymbol{\nabla}, -\frac{i}{a_0}\frac{\partial}{\partial t}\right) \\ &\simeq -e_1\frac{\partial}{\partial x} - e_2\frac{\partial}{\partial y} - e_3\frac{\partial}{\partial z} - \frac{i}{a}e_0\frac{\partial}{\partial t}.\end{aligned} \tag{72}$$

The quaternionic product of $\mathbb{D} \circ \overline{\mathbb{D}}$ will be

$$\begin{aligned}\mathbb{D} \circ \overline{\mathbb{D}} &= \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{a_0^2}\frac{\partial^2}{\partial t^2} = \boldsymbol{\nabla}^2 - \frac{1}{a_0^2}\frac{\partial^2}{\partial t^2} \\ &= \overline{\mathbb{D}} \circ \mathbb{D},\end{aligned} \tag{73}$$

where $\mathbb{D} \circ \overline{\mathbb{D}}$ or $\overline{\mathbb{D}} \circ \mathbb{D}$ is defined by the D' Alembert operator $\square$. In order to emphasize the variation of quaternionic space-time to two four-velocities of dyonic fluid plasma, we may operate the quaternionic differential operator $(\mathbb{D})$ on generalized two four-velocities $(\mathbb{W})$ as

$$\begin{aligned}\mathbb{D} \circ \mathbb{W} = e_1 &\left\{\left(\frac{\partial u_z}{\partial y} - \frac{\partial u_y}{\partial z} - \frac{1}{a_0^2}\frac{\partial v_x}{\partial t} - \frac{\partial k}{\partial x}\right)\right. \\ &\left. + \frac{i}{a_0}\left(-\frac{\partial v_z}{\partial y} + \frac{\partial v_y}{\partial z} - \frac{\partial u_x}{\partial t} - \frac{\partial h}{\partial x}\right)\right\}\end{aligned}$$

$$+ e_2 \left\{ \left(\frac{\partial u_x}{\partial z} - \frac{\partial u_z}{\partial x} - \frac{1}{a_0^2} \frac{\partial v_y}{\partial t} - \frac{\partial k}{\partial y} \right) \right.$$

$$\left. + \frac{i}{a_0} \left(- \frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x} - \frac{\partial u_y}{\partial t} - \frac{\partial h}{\partial y} \right) \right\}$$

$$+ e_3 \left\{ \left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y} - \frac{1}{a_0^2} \frac{\partial v_z}{\partial t} - \frac{\partial k}{\partial z} \right) \right.$$

$$\left. + \frac{i}{a_0} \left(- \frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y} - \frac{\partial u_z}{\partial t} - \frac{\partial h}{\partial z} \right) \right\}$$

$$- e_0 \left\{ \left(\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} + \frac{1}{a_0^2} \frac{\partial h}{\partial t} \right) \right.$$

$$\left. - \frac{i}{a_0} \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} + \frac{\partial k}{\partial t} \right) \right\}. \tag{74}$$

Equation (74) governed the following quaternionic hydrodynamics field equation for dyonic cold plasma, i.e.,

$$\mathbb{D} \circ \mathbb{W} = \boldsymbol{\Psi} \simeq e_1 \psi_1 + e_2 \psi_2 + e_3 \psi_3 + e_0 \chi, \tag{75}$$

where $\boldsymbol{\psi} \longrightarrow (\psi_1, \psi_2, \psi_3)$ and χ are the vector and scalar fields connected to the hydrodynamics of dyonic cold plasma, respectively. Further, the unified structure of quaternionic hydrodynamics field components can be expressed as

$$\psi_1 = \left\{ (\nabla \times \mathbf{u})_x - \frac{1}{a_0^2} \frac{\partial v_x}{\partial t} - \frac{\partial k}{\partial x} \right\} + \frac{i}{a_0} \left\{ - (\nabla \times \boldsymbol{v})_x - \frac{\partial u_x}{\partial t} - \frac{\partial h}{\partial x} \right\}, \tag{76}$$

$$\psi_2 = \left\{ (\nabla \times \mathbf{u})_y - \frac{1}{a_0^2} \frac{\partial v_y}{\partial t} - \frac{\partial k}{\partial y} \right\} + \frac{i}{a_0} \left\{ - (\nabla \times \boldsymbol{v})_y - \frac{\partial u_y}{\partial t} - \frac{\partial h}{\partial y} \right\}, \tag{77}$$

$$\psi_3 = \left\{ (\nabla \times \mathbf{u})_z - \frac{1}{a_0^2} \frac{\partial v_z}{\partial t} - \frac{\partial k}{\partial z} \right\} + \frac{i}{a_0} \left\{ - (\nabla \times \boldsymbol{v})_z - \frac{\partial u_z}{\partial t} - \frac{\partial h}{\partial z} \right\}, \tag{78}$$

$$\chi = - \left\{ \left(\nabla \cdot \mathbf{u} + \frac{1}{a_0^2} \frac{\partial h}{\partial t} \right) - \frac{i}{a_0} \left(\nabla \cdot \boldsymbol{v} + \frac{\partial k}{\partial t} \right) \right\}. \tag{79}$$

We may consider the generalized dual hydrodynamics fields, namely, the hydroelectric and hydromagnetic fields of dyonic fluid associated with the dynamics of electrons and magnetic monopoles in dyonic cold plasma. Thus, the unified fields can be rewrite as

$$\psi_1 \longleftrightarrow \left(B_x + \frac{i}{a_0} E_x \right), \tag{80}$$

$$\psi_2 \longleftrightarrow \left(B_y + \frac{i}{a_0} E_y \right), \tag{81}$$

$$\psi_2 \longleftrightarrow \left(B_y + \frac{i}{a_0} E_y \right), \tag{82}$$

$$\chi \longleftrightarrow - \left(\mathscr{L} - \frac{i}{a_0} \widetilde{\mathscr{L}} \right). \tag{83}$$

The hydroelectric field vector ($\mathbf{E}$) plays as the generalized Lamb vector field and the hydromagnetic field vector ($\mathbf{B}$) plays as the generalized vorticity field [45–47] to the case of dual MHD. The generalized Lamb vector field may be used to accelerate the dyonic fluid flow while the vorticity field is its counterpart. Thus, the generalized dual fields ($\mathbf{E}, \mathbf{B}$) for dyonic fluid become

$$\mathbf{E} = -\nabla \times \boldsymbol{v} - \frac{\partial \mathbf{u}}{\partial t} - \nabla h, \tag{84}$$

$$\mathbf{B} = \nabla \times \mathbf{u} - \frac{1}{a_0^2} \frac{\partial \boldsymbol{v}}{\partial t} - \nabla k, \tag{85}$$

and the dual Lorenz gauge conditions ($\mathscr{L}, \widetilde{\mathscr{L}}$) for the continuous flow of incompressible dyonic fluid plasma are

$$\mathscr{L} \;:\longmapsto\; \nabla \cdot \mathbf{u} + \frac{1}{a_0^2} \frac{\partial h}{\partial t} = 0, \tag{86}$$

$$\widetilde{\mathscr{L}} \;:\longmapsto\; \nabla \cdot \boldsymbol{v} + \frac{\partial k}{\partial t} = 0. \tag{87}$$

The unified quaternionic Lamb-vorticity field vector $\boldsymbol{\Psi}$ (or generalized hydroelectromagnetic field vector) for dyons can be expressed as

$$\boldsymbol{\Psi} = e_1 \left(B_x + \frac{i}{a_0} E_x \right) + e_2 \left(B_y + \frac{i}{a_0} E_y \right) + e_3 \left(B_z + \frac{i}{a_0} E_z \right). \tag{88}$$

Now, applying the quaternionic conjugate of differential operator $\overline{\mathbb{D}}$ to (88), we obtain

$$\overline{\mathbb{D}} \circ \boldsymbol{\Psi} = -e_1 \left[\left\{ (\nabla \times \mathbf{B})_x - \frac{1}{a_0^2} \frac{\partial E_x}{\partial t} \right\} + \frac{i}{a_0} \left\{ (\nabla \times \mathbf{E})_x + \frac{\partial B_x}{\partial t} \right\} \right]$$

$$- e_2 \left[\left\{ (\nabla \times \mathbf{B})_y - \frac{1}{a_0^2} \frac{\partial E_y}{\partial t} \right\} + \frac{i}{a_0} \left\{ (\nabla \times \mathbf{E})_y + \frac{\partial B_y}{\partial t} \right\} \right]$$

$$- e_3 \left[\left\{ (\nabla \times \mathbf{B})_z - \frac{1}{a_0^2} \frac{\partial E_z}{\partial t} \right\} + \frac{i}{a_0} \left\{ (\nabla \times \mathbf{E})_z + \frac{\partial B_z}{\partial t} \right\} \right] + e_0 \left[\nabla \cdot \mathbf{B} + \frac{i}{a_0} \nabla \cdot \mathbf{E} \right]. \quad (89)$$

Equation (89) shows the quaternionic space-time evaluation of generalized Lamb-vorticity fields in the incompressible fluid of dyonic cold plasma. The dynamics of dyonic cold plasma fluid can be expressed by following equation:

$$\overline{\mathbb{D}} \circ \mathbf{\Psi} = -\mathbb{S}\left(\mathbf{S}, \wp\right) \simeq -\left(e_1 S_1 + e_2 S_2 + e_3 S_3 + e_0 \wp\right), \quad (90)$$

where $\mathbb{S}$ is the quaternionic source for the dyonic cold plasma. Moreover, the quaternionic vector and scalar components of dyonic sources, i.e., $(\mathbf{S}, \wp)$, can be written as

$$S_1 \longleftrightarrow \left(\mu J_x^e - \frac{i}{a_0} \frac{J_x^{\mathrm{m}}}{\epsilon} \right), \quad (91)$$

$$S_2 \longleftrightarrow \left(\mu J_y^e - \frac{i}{a_0} \frac{J_y^{\mathrm{m}}}{\epsilon} \right), \quad (92)$$

$$S_3 \longleftrightarrow \left(\mu J_z^e - \frac{i}{a_0} \frac{J_z^{\mathrm{m}}}{\epsilon} \right), \quad (93)$$

$$\wp \longleftrightarrow \left(\mu \rho^{\mathrm{m}} - \frac{i}{a_0} \frac{\rho^e}{\epsilon} \right), \quad (94)$$

where $(\mathbf{J}^e, \rho^e)$ are the quaternionic electric source current and source density associated with the dynamics of hydroelectric field while $(\mathbf{J}^{\mathrm{m}}, \rho^{\mathrm{m}})$ are corresponding magnetic sources associated with the dynamics of hydromagnetic field of dyonic fluid. Therefore, the quaternionic unified hydroelectromagnetic source for dyonic cold plasma can be expressed by

$$\begin{aligned} \mathbb{S} &= \mu \left(e_1 J_x^e + e_2 J_y^e + e_3 J_z^e - e_0 \rho^{\mathrm{m}} \right) \\ &\quad - \frac{i}{a_0} \left(e_1 \frac{J_x^{\mathrm{m}}}{\epsilon} + e_2 \frac{J_y^{\mathrm{m}}}{\epsilon} + e_3 \frac{J_z^{\mathrm{m}}}{\epsilon} + e_0 \frac{\rho^e}{\epsilon} \right) \\ &= \left(\mu e_j \mathbf{J}^e - \frac{i e_0}{a_0} \frac{\rho^e}{\epsilon} \right) - \frac{i}{a_0} \left(e_j \frac{\mathbf{J}^{\mathrm{m}}}{\epsilon} - i e_0 a_0 \mu \rho^{\mathrm{m}} \right) \\ &= \left(\mathbb{J} - \frac{i}{a_0} \mathbb{K} \right). \end{aligned} \quad (95)$$

Here, $\mathbb{J}(e_j, e_0) \longrightarrow (\mu \mathbf{J}^e - (i/a_0)(\rho^e/\epsilon)), \mathbb{K}(e_j, e_0) \longrightarrow ((1/\epsilon)\mathbf{J}^{\mathrm{m}} - i a_0 \mu \rho^{\mathrm{m}})$ are quaternionic two four-fluid sources of dyons and (ϵ, μ) considering the permittivity and permeability satisfy $a_0 = 1/\sqrt{\mu\epsilon}$. Now, equate quaternionic imaginary and real coefficients in (90) and obtain

$$\nabla \cdot \mathbf{E} = \frac{\rho^e}{\epsilon}, \quad \text{(Imaginary part of } e_0\text{)} \quad (96)$$

$$\nabla \cdot \mathbf{B} = \mu \rho^{\mathrm{m}}, \quad \text{(Real part of } e_0\text{)} \quad (97)$$

$$(\nabla \times \mathbf{E})_x = -\frac{\partial B_x}{\partial t} - \frac{J_x^{\mathrm{m}}}{\epsilon}, \quad \text{(Imaginary part of } e_1\text{)} \quad (98)$$

$$(\nabla \times \mathbf{E})_y = -\frac{\partial B_y}{\partial t} - \frac{J_y^{\mathrm{m}}}{\epsilon}, \quad \text{(Imaginary part of } e_2\text{)} \quad (99)$$

$$(\nabla \times \mathbf{E})_z = -\frac{\partial B_z}{\partial t} - \frac{J_z^{\mathrm{m}}}{\epsilon}, \quad \text{(Imaginary part of } e_3\text{)} \quad (100)$$

$$(\nabla \times \mathbf{B})_x = \frac{1}{a_0^2} \frac{\partial E_x}{\partial t} + \mu J_x^e, \quad \text{(Real part of } e_1\text{)} \quad (101)$$

$$(\nabla \times \mathbf{B})_y = \frac{1}{a_0^2} \frac{\partial E_y}{\partial t} + \mu J_y^e, \quad \text{(Real part of } e_2\text{)} \quad (102)$$

$$(\nabla \times \mathbf{B})_z = \frac{1}{a_0^2} \frac{\partial E_z}{\partial t} + \mu J_z^e, \quad \text{(Real part of } e_3\text{)}. \quad (103)$$

The above eight equations represent the quaternionic field equations for hydrodynamics of dyonic cold plasma. These obtained equations are primary equations for dual MHD of dyonic cold plasma, which are exactly the same as the generalized Dirac-Maxwell equations given by (49)-(52). As such, we also may write the unified dual MHD field equations for dyonic cold plasma as

$$\nabla \cdot \mathbf{\Psi} = i\wp, \quad (104)$$

$$\nabla \times \mathbf{\Psi} = -\frac{i}{a_0} \frac{\partial \mathbf{\Psi}}{\partial t} + \mathbf{S}. \quad (105)$$

The present quaternionic formulation describes the macroscopic cold plasma behavior. The solution of differential equations (104)-(105) provides the evolution of generalized lamb vector field and generalized vorticity field to the presence of dyonic cold plasma. Now, we may check the validity of dual MHD field equations for dyonic cold plasma in given subsections.

5.1. Duality Invariant. Let us check the duality invariant symmetry for generalized hydroelectric and hydromagnetic fields of dyonic cold plasma. The duality transformation defines the rotation of hydroelectric and hydromagnetic field components in the quaternionic space such that the physics behind the quantity remains the same after the transformation is performed. Suppose, $F^{\alpha\beta}$ and $\mathscr{F}^{\alpha\beta}$ are the field and dual field tensor, then the duality transformation becomes [48]

$$\begin{aligned} F'^{\alpha\beta} &:\longmapsto F^{\alpha\beta} \cos\theta + \mathscr{F}^{\alpha\beta} \sin\theta, \\ \mathscr{F}'^{\alpha\beta} &:\longmapsto -\mathscr{F}^{\alpha\beta} \sin\theta + F^{\alpha\beta} \cos\theta, \\ &\qquad \left(0 \le \theta \le \frac{\pi}{2} \right). \end{aligned} \quad (106)$$

Correspondingly, the quaternionic hydroelectric and hydromagnetic fields can also be transformed as

$$\begin{pmatrix} \mathbf{E} \\ \mathbf{B} \end{pmatrix} \longmapsto \mathfrak{D}_{2\times 2} \begin{pmatrix} \mathbf{E} \\ \mathbf{B} \end{pmatrix}, \quad (107)$$

where $\mathfrak{D}_{2\times 2} = \begin{pmatrix} \cos\theta & a_0 \sin\theta \\ -(1/a_0)\sin\theta & \cos\theta \end{pmatrix}$ is an unitary matrix called the duality transformation matrix (or simply D-matrix). For general case $\theta = \pi/2$, the generalized dual fields will be transformed as

$$\begin{pmatrix} \mathbf{E} \\ \mathbf{B} \end{pmatrix} = \begin{pmatrix} 0 & a_0 \\ -\dfrac{1}{a_0} & 0 \end{pmatrix} \begin{pmatrix} \mathbf{E} \\ \mathbf{B} \end{pmatrix} :\Longrightarrow \begin{cases} \mathbf{E} \longmapsto a_0 \mathbf{B}, \\ \mathbf{B} \longmapsto -\dfrac{1}{a_0}\mathbf{E}. \end{cases} \tag{108}$$

Here, the D-matrix $\mathfrak{D}_{2\times 2} = \begin{pmatrix} 0 & a_0 \\ -1/a_0 & 0 \end{pmatrix}$. For quaternionic dual-velocity and dual-enthalpy of dyons fluid, the following duality transformation relations governed the streamline flow, i.e.,

$$\begin{pmatrix} \mathbf{u} \\ \boldsymbol{v} \end{pmatrix} = \begin{pmatrix} 0 & a_0 \\ -\dfrac{1}{a_0} & 0 \end{pmatrix} \begin{pmatrix} \mathbf{u} \\ \boldsymbol{v} \end{pmatrix} :\Longrightarrow \begin{cases} \mathbf{u} \longmapsto a_0 \boldsymbol{v}, \\ \boldsymbol{v} \longmapsto -\dfrac{1}{a_0}\mathbf{u}, \end{cases} \tag{109}$$

$$\begin{pmatrix} h \\ k \end{pmatrix} = \begin{pmatrix} 0 & a_0 \\ -\dfrac{1}{a_0} & 0 \end{pmatrix} \begin{pmatrix} h \\ k \end{pmatrix} :\Longrightarrow \begin{cases} h \longmapsto a_0 k, \\ k \longmapsto -\dfrac{1}{a_0}h. \end{cases} \tag{110}$$

Accordingly, the dual-current and dual-density of dyonic plasma will be transformed as

$$\begin{gathered} \begin{pmatrix} \mathbf{J}^e \\ \mathbf{J}^{\mathrm{m}} \end{pmatrix} = \begin{pmatrix} 0 & a_0 \\ -\dfrac{1}{a_0} & 0 \end{pmatrix} \begin{pmatrix} \mathbf{J}^e \\ \mathbf{J}^{\mathrm{m}} \end{pmatrix} : \\ \Longrightarrow \begin{cases} \mathbf{J}^e \longmapsto a_0 \mathbf{J}^{\mathrm{m}}, \\ \mathbf{J}^{\mathrm{m}} \longmapsto -\dfrac{1}{a_0}\mathbf{J}^e, \end{cases} \end{gathered} \tag{111}$$

$$\begin{gathered} \begin{pmatrix} \rho^e \\ \rho^{\mathrm{m}} \end{pmatrix} = \begin{pmatrix} 0 & a_0 \\ -\dfrac{1}{a_0} & 0 \end{pmatrix} \begin{pmatrix} \rho^e \\ \rho^{\mathrm{m}} \end{pmatrix} : \\ \Longrightarrow \begin{cases} \rho^e \longmapsto a_0 \rho^{\mathrm{m}}, \\ \rho^{\mathrm{m}} \longmapsto -\dfrac{1}{a_0}\rho^e. \end{cases} \end{gathered} \tag{112}$$

Interestingly, from relations (108) to (112), we can conclude that the generalized Dirac-Maxwell equations for dyonic fluid of cold plasma are invariant under the duality transformations and showing the highly symmetric nature in presence of dyonic fluid.

5.2. Lorentz Invariant. Let us start with the most usual transformation [49, 50] that preserves the quaternionic intervals $ds^2 = dx^2 + dy^2 + dz^2 - a_0^2 dt^2$; i.e.,

$$X'^{\xi} = \Lambda^{\xi}_{\eta} X^{\eta}, \tag{113}$$

where X is any four-vector and the Lorentz transformation matrix element Λ^{ξ}_{η} is

$$\Lambda^{\xi}_{\eta} \longmapsto \begin{pmatrix} \cosh\phi & 0 & 0 & -i\sinh\phi \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ i\sinh\phi & 0 & 0 & \cosh\phi \end{pmatrix}. \tag{114}$$

Here ϕ is the boost parameter. Using the above Lorentz transformation matrix, we may obtain the following transformation equations for quaternionic four-velocity ($\mathbb{W}$) of dyonic cold plasma which are an analogous to quaternionic potentials of dyons; i.e.,

$$\begin{aligned} w'_x &= \gamma\left(w_x - a_0\Omega_0\right), \\ w'_y &= w_y, \\ w'_z &= w_z, \\ \Omega'_0 &= \gamma\left(\Omega_0 - a_0 w_x\right), \end{aligned} \tag{115}$$

where

$$\begin{gathered} \cosh\phi = \frac{1}{\sqrt{1-\tanh^2\phi}} = \frac{1}{\sqrt{1-a_0^2}} = \gamma, \\ \sinh\phi = a_0\gamma. \end{gathered} \tag{116}$$

If we consider the massive dyonic particles [51], then the transformation relations (115) lead to the energy-momentum transformations for dyonic cold plasma,

$$\begin{aligned} \mathcal{P}'_x &= \gamma\left(\mathcal{P}_x - a_0\mathcal{E}\right), \\ \mathcal{P}'_y &= \mathcal{P}_y, \\ \mathcal{P}'_z &= \mathcal{P}_z, \\ \mathcal{E}' &= \gamma\left(\mathcal{E} - a_0\mathcal{P}_x\right), \end{aligned} \tag{117}$$

where the quaternionic four-momentum is defined by $\mathbb{P}(e_1, e_2, e_3, e_0) = (\mathcal{P}_x, \mathcal{P}_y, \mathcal{P}_z, \mathcal{E})$. It should be notice that the obtained relations (117) are similar to the usual relativistic Lorentz energy-momentum transformation relations [49, 50], where we assume that the speed of dyons (a_0) is comparable to the speed of light ($c \sim 1$). As such, we also may establish the following transformation relations for quaternionic source current and source density, i.e.,

$$\begin{aligned} S'_x &= \gamma\left(S_x - a_0\wp\right), \\ S'_y &= S_y, \\ S'_z &= S_z, \\ \wp' &= \gamma\left(\wp - a_0 S_x\right). \end{aligned} \tag{118}$$

Correspondingly, we obtain the Lorentz transformation relations for unified hydroelectromagnetic field of dyonic cold plasma, so that

$$\begin{aligned}\psi'_x &= \psi_x,\\ \psi'_y &= \gamma\left(\psi_y - i a_0 \psi_z\right),\\ \psi'_z &= \gamma\left(\psi_z + i a_0 \psi_y\right),\end{aligned} \tag{119}$$

along with

$$\begin{aligned}\frac{\partial}{\partial x'} &= \gamma\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial t}\right),\\ \frac{\partial}{\partial t'} &= \gamma\left(\frac{\partial}{\partial t} + a_0^2 \frac{\partial}{\partial x}\right).\end{aligned} \tag{120}$$

The beauty of the transformation relations (118)-(120) is that the generalized Dirac-Maxwell equations for dyonic fluid of cold plasma are well invariant under these Lorentz transformation.

5.3. CPT Invariant. In order to check the CPT invariance [52] for the dual MHD field equations of dyonic cold plasma, we may write the charge conjugation matrix ($\mathscr{C}$) to the case of quaternionic dual-current sources and hydroelectromagnetic fields of dyonic fluid as $\mathscr{C} \longrightarrow \left(\begin{smallmatrix}-1 & 0\\ 0 & -1\end{smallmatrix}\right)$, where the charge conjugation transformation plays as

$$\mathscr{C}: \begin{pmatrix}\mathbf{J}'^{e}\\ \mathbf{J}'^{\mathrm{m}}\end{pmatrix} \longmapsto \begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}\mathbf{J}^{e}\\ \mathbf{J}^{\mathrm{m}}\end{pmatrix}, \tag{121}$$

$$\mathscr{C}: \begin{pmatrix}\mathbf{E}'\\ \mathbf{B}'\end{pmatrix} \longmapsto \begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}\mathbf{E}\\ \mathbf{B}\end{pmatrix}. \tag{122}$$

Correspondingly, the parity matrix $P \longrightarrow \left(\begin{smallmatrix}-1 & 0\\ 0 & 1\end{smallmatrix}\right)$ can govern the following transformations for the dyonic fluid:

$$P: \begin{pmatrix}\mathbf{J}'^{e}\\ \mathbf{J}'^{\mathrm{m}}\end{pmatrix} \longmapsto \begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}\mathbf{J}^{e}\\ \mathbf{J}^{\mathrm{m}}\end{pmatrix}, \tag{123}$$

$$P: \begin{pmatrix}\mathbf{E}'\\ \mathbf{B}'\end{pmatrix} \longmapsto \begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}\mathbf{E}\\ \mathbf{B}\end{pmatrix}. \tag{124}$$

As such, we can write the time reversal matrix, i.e., $T \longrightarrow \left(\begin{smallmatrix}1 & 0\\ 0 & -1\end{smallmatrix}\right)$, and the transformation performs as

$$T: \begin{pmatrix}\mathbf{J}'^{\mathrm{m}}\\ \mathbf{J}'^{e}\end{pmatrix} \longmapsto \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}\mathbf{J}^{\mathrm{m}}\\ \mathbf{J}^{e}\end{pmatrix}, \tag{125}$$

$$T: \begin{pmatrix}\mathbf{E}'\\ \mathbf{B}'\end{pmatrix} \longmapsto \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}\mathbf{E}\\ \mathbf{B}\end{pmatrix}. \tag{126}$$

The forth component of quaternionic sources can also be transformed for charge conjugation, parity, and time reversal as the following ways:

$$\mathscr{C}: \begin{pmatrix}\rho'^{e}\\ \rho'^{\mathrm{m}}\end{pmatrix} \longmapsto \begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}\rho^{e}\\ \rho^{\mathrm{m}}\end{pmatrix}, \tag{127}$$

$$P: \begin{pmatrix}\rho'^{e}\\ \rho'^{\mathrm{m}}\end{pmatrix} \longmapsto \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}\rho^{e}\\ \rho^{\mathrm{m}}\end{pmatrix}, \tag{128}$$

$$T: \begin{pmatrix}\rho'^{e}\\ \rho'^{\mathrm{m}}\end{pmatrix} \longmapsto \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}\rho^{e}\\ \rho^{\mathrm{m}}\end{pmatrix}. \tag{129}$$

We can summarize the quaternionic physical quantities of dual MHD fields and their changes under charge conjugation, parity inversion, and time reversal given by Table 2 [53, 54].

Now, we may apply the CPT transformation relations on generalized Dirac-Maxwell equations for dyonic fluid of cold plasma as [54]

$$\begin{aligned}&\mathscr{C}PT\left(\nabla\cdot\mathbf{E}\right)T^{-1}P^{-1}\mathscr{C}^{-1} = \mathscr{C}PT\left(\frac{\rho^{e}}{\epsilon}\right)T^{-1}P^{-1}\mathscr{C}^{-1},\\ &\mathscr{C}PT\left(\nabla\cdot\mathbf{B}\right)T^{-1}P^{-1}\mathscr{C}^{-1} = \mathscr{C}PT\left(\mu\rho^{\mathrm{m}}\right)T^{-1}P^{-1}\mathscr{C}^{-1},\\ &\mathscr{C}PT\left(\nabla\times\mathbf{B}\right)T^{-1}P^{-1}\mathscr{C}^{-1}\\ &\quad = \mathscr{C}PT\left(\frac{1}{a_0^2}\frac{\partial\mathbf{E}}{\partial t} + \mu\mathbf{J}^{e}\right)T^{-1}P^{-1}\mathscr{C}^{-1}\\ &\qquad + \mathscr{C}PT\left(\mu\mathbf{J}^{e}\right)T^{-1}P^{-1}\mathscr{C}^{-1},\\ &\mathscr{C}PT\left(\nabla\times\mathbf{E}\right)T^{-1}P^{-1}\mathscr{C}^{-1}\\ &\quad = \mathscr{C}PT\left(-\frac{\partial\mathbf{B}}{\partial t}\right)T^{-1}P^{-1}\mathscr{C}^{-1}\\ &\qquad + \mathscr{C}PT\left(-\frac{1}{\epsilon}\mathbf{J}^{\mathrm{m}}\right)T^{-1}P^{-1}\mathscr{C}^{-1}.\end{aligned} \tag{130}$$

Therefore, it may conclude that the generalized Dirac-Maxwell equations for dyonic cold plasma are invariant under CPT transformations.

6. Quaternionic Hydroelectromagnetic Wave Propagation

To establish the dual hydrodynamics wave equations for dyonic cold plasma, we can start with the following quaternionic relation:

$$\mathbb{D}\circ\left(\overline{\mathbb{D}}\circ\mathbf{\Psi}\right) = -\mathbb{D}\circ\mathbb{S}, \tag{131}$$

where the left hand part of (131) can be written as

$$\begin{aligned}\mathbb{D}\circ\left(\overline{\mathbb{D}}\circ\mathbf{\Psi}\right) = e_1&\left\{\left(\frac{\partial^2 B_x}{\partial x^2} - \frac{1}{a_0^2}\frac{\partial^2 B_x}{\partial t^2}\right)\right.\\ &\left.+\frac{i}{a_0}\left(\frac{\partial^2 E_x}{\partial x^2} - \frac{1}{a_0^2}\frac{\partial^2 E_x}{\partial t^2}\right)\right\}\\ &+ e_2\left\{\left(\frac{\partial^2 B_y}{\partial y^2} - \frac{1}{a_0^2}\frac{\partial^2 B_y}{\partial t^2}\right)\right.\\ &\left.+\frac{i}{a_0}\left(\frac{\partial^2 E_y}{\partial y^2} - \frac{1}{a_0^2}\frac{\partial^2 E_y}{\partial t^2}\right)\right\}\end{aligned}$$

Table 2: Quaternionic physical quantities and their CPT transformations.

Physical quantities	**Charge conjugation** ($\mathscr{C}$)	**Parity inversion** (P)	**Time reversal** (T)
∂_t	∂_t	∂_t	$-\partial_t$
∇	∇	$-\nabla$	∇
a_0	a_0	$-a_0$	$-a_0$
$\mathbf{J}^e$	$-\mathbf{J}^e$	$-\mathbf{J}^e$	$-\mathbf{J}^e$
$\mathbf{J}^{\mathrm{m}}$	$-\mathbf{J}^{\mathrm{m}}$	$\mathbf{J}^{\mathrm{m}}$	$\mathbf{J}^{\mathrm{m}}$
$\mathbf{E}$	$-\mathbf{E}$	$-\mathbf{E}$	$\mathbf{E}$
$\mathbf{B}$	$-\mathbf{B}$	$\mathbf{B}$	$-\mathbf{B}$
ρ^e	$-\rho^e$	ρ^e	ρ^e
ρ^{m}	ρ^{m}	$-\rho^{\mathrm{m}}$	ρ^{m}

$$+ e_3 \left\{ \left(\frac{\partial^2 B_z}{\partial z^2} - \frac{1}{a_0^2} \frac{\partial^2 B_z}{\partial t^2} \right) + \frac{i}{a_0} \left(\frac{\partial^2 E_z}{\partial z^2} - \frac{1}{a_0^2} \frac{\partial^2 E_z}{\partial t^2} \right) \right\}. \tag{132}$$

Accordingly, the right hand part of (131) can be expressed as

$$\begin{aligned}
\mathbb{D} \circ \mathbb{S} = {} & e_1 \left\{ \mu \left(\frac{\partial J_z^e}{\partial y} - \frac{\partial J_y^e}{\partial z} - \frac{1}{a_0^2 \mu\epsilon} \frac{\partial J_x^{\mathrm{m}}}{\partial t} - \frac{\partial \rho^{\mathrm{m}}}{\partial x} \right) - \frac{i}{a_0 \epsilon} \left(\frac{\partial J_z^{\mathrm{m}}}{\partial y} - \frac{\partial J_y^{\mathrm{m}}}{\partial z} + \mu\epsilon \frac{\partial J_x^e}{\partial t} + \frac{\partial \rho^e}{\partial x} \right) \right\} \\
& + e_2 \left\{ \mu \left(\frac{\partial J_x^e}{\partial z} - \frac{\partial J_z^e}{\partial x} - \frac{1}{a_0^2 \mu\epsilon} \frac{\partial J_y^{\mathrm{m}}}{\partial t} - \frac{\partial \rho^{\mathrm{m}}}{\partial y} \right) - \frac{i}{a_0 \epsilon} \left(\frac{\partial J_x^{\mathrm{m}}}{\partial z} - \frac{\partial J_z^{\mathrm{m}}}{\partial x} + \mu\epsilon \frac{\partial J_y^e}{\partial t} + \frac{\partial \rho^e}{\partial y} \right) \right\} \\
& + e_3 \left\{ \mu \left(\frac{\partial J_y^e}{\partial x} - \frac{\partial J_x^e}{\partial y} - \frac{1}{a_0^2 \mu\epsilon} \frac{\partial J_z^{\mathrm{m}}}{\partial t} - \frac{\partial \rho^{\mathrm{m}}}{\partial z} \right) - \frac{i}{a_0 \epsilon} \left(\frac{\partial J_y^{\mathrm{m}}}{\partial x} - \frac{\partial J_x^{\mathrm{m}}}{\partial y} + \mu\epsilon \frac{\partial J_z^e}{\partial t} + \frac{\partial \rho^e}{\partial z} \right) \right\} \\
& - e_0 \left\{ \mu \left(\frac{\partial J_x^e}{\partial x} + \frac{\partial J_y^e}{\partial y} + \frac{\partial J_z^e}{\partial z} + \frac{1}{a_0^2 \mu\epsilon} \frac{\partial \rho^e}{\partial t} \right) - \frac{i}{a_0 \epsilon} \left(\frac{\partial J_x^{\mathrm{m}}}{\partial x} + \frac{\partial J_y^{\mathrm{m}}}{\partial y} + \frac{\partial J_z^{\mathrm{m}}}{\partial z} + \mu\epsilon \frac{\partial \rho^{\mathrm{m}}}{\partial t} \right) \right\}.
\end{aligned} \tag{133}$$

Now, equate the real and imaginary parts of quaternionic basis vectors in (131) and obtain the following relations:

$$\nabla \cdot \mathbf{J}^e + \frac{\partial \rho^e}{\partial t} = 0, \tag{134}$$

$$\nabla \cdot \mathbf{J}^{\mathrm{m}} + \frac{1}{a_0^2} \frac{\partial \rho^{\mathrm{m}}}{\partial t} = 0, \tag{135}$$

$$\nabla^2 \mathbf{B} - \frac{1}{a_0^2} \frac{\partial^2 \mathbf{B}}{\partial t^2} - \mu \left(\nabla \rho^{\mathrm{m}} \right) - \frac{1}{a_0^2 \epsilon} \frac{\partial \mathbf{J}^{\mathrm{m}}}{\partial t} + \mu \left(\nabla \times \mathbf{J}^e \right) = 0, \tag{136}$$

$$\nabla^2 \mathbf{E} - \frac{1}{a_0^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} - \frac{1}{\epsilon} \left(\nabla \rho^e \right) - \mu \frac{\partial \mathbf{J}^e}{\partial t} - \frac{1}{\epsilon} \left(\nabla \times \mathbf{J}^{\mathrm{m}} \right) = 0. \tag{137}$$

Equations (134) and (135) defined the well-known dual continuity equations while (136) and (137) represented the generalized hydromagnetic and hydroelectric wave equations for dyonic cold plasma in presence of electrons and magnetic monopoles. The beauty of (136) is that it is an analogous to Alfven wave propagation [55, 56] associated with magnetic monopoles, and the same way (137) describes the counterpart of Alfven wave propagation associated with the electrons. Thus, the unified hydroelectromagnetic wave equations for dyonic fluid of cold plasma can also be expressed as

$$\nabla^2 \boldsymbol{\Psi} - \frac{1}{a_0^2} \frac{\partial^2 \boldsymbol{\Psi}}{\partial t^2} - i \left(\nabla \wp \right) - \frac{i}{a_0} \frac{\partial \mathbf{S}}{\partial t} + \left(\nabla \times \mathbf{S} \right) = 0. \tag{138}$$

Interestingly, the generalized wave equation (138) is invariant under the duality, Lorentz, and CPT transformations.

7. Conclusion

The dyons are high energetic soliton particles existing in the cold plasma. The cold plasma model is the simplest model where we assume negligible plasma temperature, and the corresponding distribution function shows the Dirac delta function centered at the macroscopic flow of linearised velocity. Dyonic cold plasma model can be used in the study of small amplitude electromagnetic waves propagating in the conducting plasma. In this study, we have applied the four-dimensional space-time algebra (quaternionic algebra) to elaborate the dynamics of dyonic fluid in cold plasma field. In Section 2, we have explained in detail the properties of quaternionic algebra. However, the quaternion is an important and appropriate fundamental mathematical tool to understand the four-dimension space-time world. In Sections 3 and 4, the fundamental equations for MHD field and their cold plasma approximation have been defined. The interesting part we have mentioned here that the dual MHD equations for massive dyons consisted of electrons and magnetic monopoles. The generalized equations involving the mass and charge densities are expressed in terms of one-fluid theory of dyonic cold plasma. Accordingly, we have discussed the dual-current densities given by (56). The mass conservation law, dual-charge conservation law, Lorentz force equation, and Ohm's law for dyonic cold plasma have been defined. In Section 5, we have described the quaternionic formulation for moving massive dyonic fluid of incompressible cold plasma. The advantage of the quaternionic formulation is that, it is better to explain two four-velocities, hydroelectric (Lamb vector), and hydromagnetic (vorticity) fields and the dual Lorenz gauge conditions for dyonic cold plasma. It has been emphasized that the dual hydrodynamics field of dyons (i.e., hydroelectric and hydromagnetic fields) deal with both electrohydrodynamic and magnetic-hydrodynamics. In present study, the existence of magnetic monopoles has been visualized to MHD field. It has been shown that the two current sources are also associated with the quaternionic hydroelectric and hydromagnetic fields of dyonic plasma fluid. We have established the eight primary equations of dual MHD field in presence of dyonic fluid. Interestingly, the unified macroscopic Dirac-Maxwell equations (104) and (105) have been obtained in the case of dyonic dual MHD. It has been noticed that like electrodynamics, the Dirac-Maxwell fluid equations are mandatory to describe the dynamics of MHD plasma. The beauty of cold plasma field equations is that these equations are well invariant under the duality, Lorentz, and CPT transformations. In Section 6, we have obtained the quaternionic dual continuity equations for incompressible dyonic fluid. The generalized hydroelectric and hydromagnetic wave equations have been established for dyonic cold plasma in presence of electrons and magnetic monopoles. It has been emphasized that the obtained Alfven wave like equation is associated with magnetic monopoles, while the counterpart of Alfven wave equation plays as electric-plasma waves in presence of electrons.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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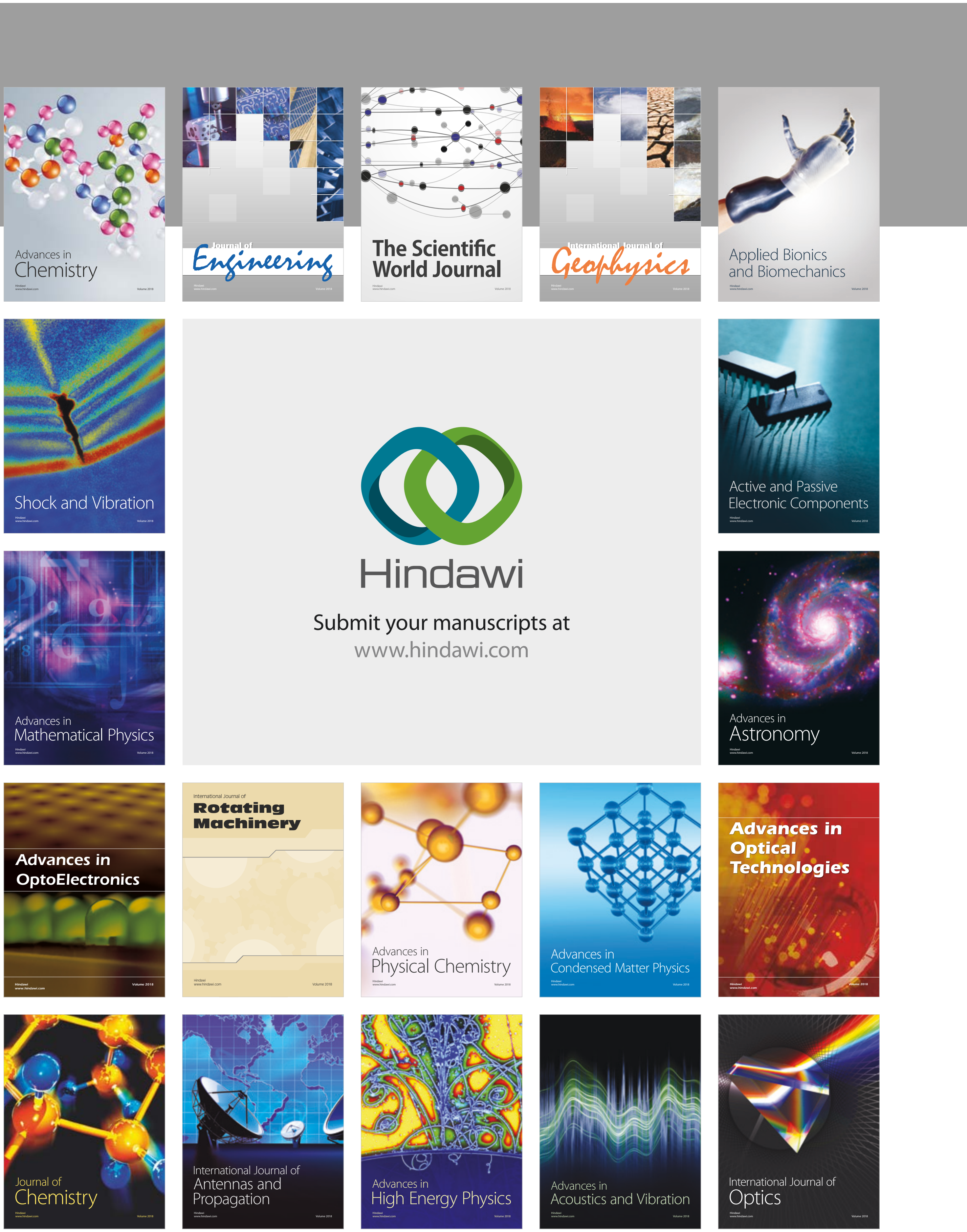
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