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Research Article

Effects of the Cornell-Type Potential on a Position-Dependent Mass System in Kaluza-Klein Theory

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In this paper, we have investigated a scalar particle with position-dependent mass subject to a uniform magnetic field and a quantum flux, both coming from the background which is governed by the Kaluza-Klein theory. By modifying the mass term of the scalar particle, we insert the Cornell-type potential. In the search for solutions of bound states, we determine the relativistic energy profile of the system in this background of extra dimension. Particular cases of this system are analyzed and a quantum effect can be observed: the dependence of the magnetic field on the quantum numbers of the solutions.

1. Introduction

In quantum mechanics systems, we consider the mass of the particle that is immersed in the system as a constant, for example, the effective mass of the hydrogen atom problem, the mass of the three-dimensional harmonic oscillator, and the mass of an electrically charged particle subject to a perpendicular uniform magnetic field to the plane of motion of the particle (Landau quantization) [1]. These are classic problems and idealized prototypes which provide results that can be approximated with real problems and with large applications. However, recently, several studies have emerged with the proposition that effective mass can be a function of the position [2–16]. The justification for this change in effective mass is based on systems where there are several applications, for example, semiconductor heterostructures [17], electronic properties of the semiconductors [18], quantum wells, wires and dots [19–22], quantum liquids [23], and He clusters [24]. It is noteworthy that quantum systems where mass is a function of the position is known in the literature as a position-dependent mass quantum system [25, 26].

Position-dependent mass quantum systems have been investigated in the relativistic context, for example, the pionic atom [27], in solution of the Dirac equation [28], in implications in atomic physics [29], in the quark-antiquark interaction [30], in effects of external fields on a two-dimensional Klein Gordon particle under pseudo-harmonic oscillator interaction [31], in noncommutative space [32], in the cosmic spacetime [33, 34], in the global monopole spacetime [35], in the rotating cosmic string spacetime [36], in the spacetime with torsion [25, 26, 37], in possible scenarios of Lorentz symmetry violation [38–40], on the Kelin–Gordon oscillator [41–44], on the Majorana fermion [45], in the Som-Raychaudhuri spacetime [46, 47], and in Kaluza-Klein theory [48].

The first attempt to explaining the physics interaction (gravitation and electromagnetism) in a unified theory has been proposed the Kaluza-Klein theory (KKT) [49, 50]. They established that the electromagnetism can be introduced through an extra (compactified) dimension in the spacetime, where the spatial dimension becomes five-dimensional.

Recently, the KKT has been the background for quantum systems research. In the nonrelativistic context, we have studies in global effects due to cosmic defects [51], in Aharonov–Bohm effect for bound states [52] and in Landau levels [53]; in the relativistic context, we have studies in Aharonov–Bohm effect for bound states on the confinement of a scalar particle to a Coulomb-type potential [48], in loop variables for a class of spacetimes, in geometric phases in graphene [55], on the Klein–Gordon oscillator [56] and in the Dirac field [57].

However, one point that has not yet been investigated is the interaction between a position-dependent mass scalar field with a linear potential plus a Coulomb-type potential in an environment described by the KKT theory.

In this paper, we investigate the effects of central potentials on a position-dependent mass scalar field in a spacetime of $(1+4)$ dimensions described by KKT, where we chose a particular case for the gauge field with extra dimension, which characterizes the relativistic Landau gauge [33, 26] and describes the presence of a magnetic quantum flux along the z-axis providing an effect analogous to the Aharanov–Bohm effect for bound states [58, 59]. Given this, we solve the Klein–Gordon equation in this background, where, analytically, we obtain solutions of bound states for a scalar field subject to a linear central potential plus a Coulomb-type potential, that is, a Cornell-type potential. We then detail these solutions of bound states for only the linear potential and only the Coulomb-type potential, where in all cases the influence of the gauge configuration from the extra dimension on the relativistic energy levels of the system is perceptible.

The structure of this paper is as follows: in Section 2, we investigate a position-dependent mass scalar field in a background described by KKT, where, in our first analysis Section 2.1, we describe the relativistic quantum dynamics of this position-dependent mass system subject to Cornell-type interaction, of which, analytically, we determine its energy profile. Then, we analyze particular cases of this system, Section 2.2 where we consider only the Coulomb-type central potential and Section 2.3 only the central linear potential; in Section 3, we present our conclusions.

2. Effects of Central Potentials on a Position-Dependent Mass System in a Kaluza-Klein Theory

The main idea behind the KKT [49, 50] is that the spacetime is five-dimensional with the purpose of unifying electromagnetism and gravitation. In this way, we can work with general relativity in five dimensions. The information about the electromagnetism is given by introducing a gauge field $A_\mu(x)$ in the line element of the spacetime as [48, 53] ($c=\hbar=1$):

$$ds^2 = -dt^2 + d\rho^2 + \rho^2 d\varphi^2 + dz^2 + \left[dw + KA_\mu(x)dx^\mu\right]^2, \quad (1)$$

where $\rho = \left(x^2+y^2\right)^{1/2}$ is radial coordinate, $0 \le \varphi \le 2\pi$, $-\infty < z < \infty$, K is the Kaluza constant [51], and w is the extra dimension. We are interested in investigating a massive scalar field of position-dependent mass subject to a uniform magnetic field and under the Aharonov-Bohm effect for bound states [58, 59]. Then, based on Refs. [48, 56], we can introduce a uniform magnetic field B_0 and a quantum flux Φ through the line element of the Minkowski spacetime (1) in the form

$$ds^2 = -dt^2 + d\rho^2 + \rho^2 d\varphi^2 + dz^2 + \left[dw + \left(\frac{B_0\rho^2}{2} + \frac{\Phi}{2\pi}\right)d\varphi\right]^2, \quad (2)$$

where the gauge field is given by the component

$$A_\varphi = \frac{B_0\rho^2}{2K} + \frac{\Phi}{2\pi K}, \quad (3)$$

which gives rise to a uniform magnetic field $\vec{B} = \nabla \times \vec{A} = K^{-1}B_0\hat{z}$ [53], where $\hat{z}$ is unitary vector in the z-direction. Therefore, for a position-dependent mass scalar field in this five-dimensional spacetime, the Kelin–Gordon equation is written in the form [48]:

$$\partial_\mu\left(g^{\mu\nu}\sqrt{-g}\partial_\nu\right)\phi - \sqrt{-g}\left[m + S\left(\vec{r}\right)\right]^2\phi = 0, \quad (4)$$

where $g^{\mu\nu}$, with $\mu, \nu = 0, 1, 2, 3, 4$, is inverse metric tensor, m is rest mass of the scalar field and $S\left(\vec{r}\right) = S(\rho)$ is scalar central potential. In this way, from Equations (2) and (4) becomes

$$-\frac{\partial^2\phi}{\partial t^2} + \frac{\partial^2\phi}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial\phi}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2\phi}{\partial\varphi^2} - \left(\frac{\Phi}{\pi\rho^2} - B_0\right)\frac{\partial^2\phi}{\partial\varphi\partial w} + \frac{\partial^2\phi}{\partial w^2} + \frac{\partial^2\phi}{\partial z^2} + \frac{\Phi^2}{4\pi^2\rho^2}\frac{\partial^2\phi}{\partial w^2} - \frac{B_0\Phi}{2\pi}\frac{\partial^2\phi}{\partial w^2} + \frac{B_0^2\rho^2}{4}\frac{\partial^2\phi}{\partial w^2} - \left[m + S(\rho)\right]^2 = 0, \quad (5)$$

which represents the interaction of a position-dependent mass scalar field with a uniform magnetic field in the five-dimensional spacetime described by a KKT.

2.1. Cornell-Type Potential. The Cornell potential, which consists of a linear potential plus a Coulomb potential, is a particular case of the quark-antiquark interaction, which has one more harmonic type term [30]. The Coulomb potential is responsible by the interaction at small distances and the linear potential leads to the confinement. Recently, the Cornell potential has been studied in the ground state of three quarks [60]. However, this type of potential is worked on spherical symmetry; in cylindrical symmetry, which is our case, this type of potential is known as Cornell-type potential [46]. This type of interaction has been studied in Refs. [33, 36, 37, 40, 42, 44, 46]. Given this, let us consider the central potential

$$S(\rho) = \frac{a}{\rho} + b\rho, \quad (6)$$

where a and b are constants. In addition, the solution to Equation (5) can be written in the form

$$\phi(\rho, \varphi, z, w, t) = e^{-i\left(\mathscr{E}t - l\varphi - kz - qw\right)}u(\rho), \quad (7)$$

where $l = 0, \pm1, \pm2, \ldots, -\infty < k < \infty$, q is a constant and $u(\rho)$ is a radial wave function. By substituting Equations (6) and (7) into Equation (5), we obtain

$$\frac{d^2u}{d\rho^2} + \frac{1}{\rho}\frac{du}{d\rho} - \frac{\iota^2}{\rho^2}u - \frac{2am}{\rho}u - 2bm\rho u - \Omega^2\rho^2 u + \alpha u = 0, \quad (8)$$

where we define the parameters

$$\alpha = \mathscr{E}^2 - m^2 - k^2 - q^2 - m\omega\left(l - \frac{q\Phi}{2\pi}\right) - 2ab;$$
$$\iota^2 = \left(l - \frac{q\Phi}{2\pi}\right)^2; \ \Omega^2 = b^2 + \frac{m^2\omega^2}{4}, \quad (9)$$

with $\omega = qB_0/m$.

Let us define $\varrho = \sqrt{\Omega}\varrho$, then Equation (8) becomes

$$\frac{d^2u}{d\varrho^2} + \frac{1}{\varrho}\frac{du}{d\varrho} - \frac{\iota^2}{\varrho^2}u - \frac{\beta}{\varrho}u - \delta\varrho u - \varrho^2 u + \frac{\alpha}{\Omega}u = 0, \quad (10)$$

where

$$\beta = \frac{2am}{\sqrt{\Omega}}; \quad \delta = \frac{2bm}{\Omega^{3/2}}. \quad (11)$$

The radial wave function $u(\varrho)$ must be well behaved at $\varrho \to 0$ and at $\varrho \to \infty$. Then, the analysis of the asymptotic behavior of Equation (10) at $\varrho \to 0$ and at $\varrho \to \infty$ gives us the following solution in terms of an unknown function $H(\varrho)$ [33, 35]:

$$u(\varrho) = \varrho^{|\iota|} e^{-(1/2)\varrho(\varrho+\delta)} H(\varrho). \quad (12)$$

Then, by substituting Equation (12) into Equation (10), we have

$$\frac{d^2H}{d\varrho^2} + \left[\frac{2|\iota| + 1}{\varrho} - 2\varrho - \delta\right]\frac{dH}{d\varrho} + \left[\frac{\alpha}{\Omega} + \frac{\delta^2}{4} - 2 - 2|\iota| - \frac{\delta}{2\varrho}(2|\iota| + 1) - \frac{\beta}{\varrho}\right]H = 0, \quad (13)$$

which is known in the literature as biconfluent Heun equation [33, 61] and $H(\varrho)$ is a biconfluent Heun function: $H(\varrho) = H_b(2|\iota|, \delta, \alpha/\Omega + \delta^2/4, 2\beta; \varrho)$.

The biconfluent Heun equation has two singular points, where one is the origin and the other is infinity [33]. The origin is a regular singular point. Given this, Equation (13) has at least one solution around the origin given by the power series [62]:

$$H(\varrho) = \sum_{j=0}^{\infty} c_j \varrho^j. \quad (14)$$

By substituting Equation (14) into Equation (13), we obtain the recurrence relation of the biconfluent Heun series

$$c_{j+2} = \frac{[\tau + \delta(j+1)]c_{j+1} - (\theta - 2j)c_j}{(j+2)(j+2+2|\iota|)}, \quad (15)$$

and the coefficients

$$c_1 = \frac{\tau}{1 + 2|\iota|}; \quad c_2 = \frac{(\tau + \delta)c_1 - \theta c_0}{2(2 + 2|\iota|)} = \frac{c_0}{4(1 + |\iota|)}\left[\frac{(\tau + \delta)\tau}{1 + 2|\iota|} - \theta\right], \quad (16)$$

where we define the new parameters

$$\theta = \frac{\alpha}{\Omega} + \frac{\delta^2}{4} - 2 - 2|\iota|; \quad \tau = \frac{\delta}{2}(2|\iota| + 1) + \beta. \quad (17)$$

As we are interested in solutions of bound states, therefore, to obtain a finite degree polynomial for the biconfluent Heun series, we must truncate the power series, and this is possible through the following conditions [33, 26]:

$$c_{n+1} = 0; \quad \theta = 2n, \quad (18)$$

where $n = 1, 2, 3, \ldots$, represents the radial modes. In order to analyze these conditions, we must assign values to n. In this case, consider $n = 1$, that is, the radial mode corresponding to the lowest energy state of the system. Therefore, the condition $c_{n+1} = 0$ produces $c_2 = 0$, which from Equation (16) we obtain

$$\Omega_{l,1}^3 - \frac{2a^2m^2}{1 + 2|\iota|}\Omega_{l,1}^2 - \frac{4abm^2(1 + |\iota|)}{1 + 2|\iota|}\Omega_{l,1} - \frac{b^2m^2(2|\iota| + 3)}{2} = 0, \quad (19)$$

where we choose the frequency (or the magnetic field) as the parameter of adjustment of the condition $c_{n+1} = 0$, not only for the radial mode $n = 1$, but for any value of n. In addition, as the parameter Ω depends on the magnetic field as established in Equation (20), where we have simplified our notation by labelling:

$$\omega_{l,1} = \frac{2}{m}\sqrt{\Omega_{l,1} - b^2} \leftrightarrow B_0^{l,1} = \frac{2}{q}\sqrt{\Omega_{l,1} - b^2}. \quad (20)$$

It is noteworthy that a third-degree algebraic equation has at least one real solution and it is exactly this solution that gives us the allowed values of the magnetic field for the lowest state of the system, which we do not write because its expression is very long. We can note, from Equation (19) that the possible values of the magnetic field depend on the quantum numbers of the system and the parameters associated with the background governed by a KKT. In addition, for each relativistic energy level, we have a different relation of the magnetic field to the parameters of the gauge field given by KKT, of the parameters associated to the Cornell-type potential and the quantum numbers of the system $\{l, n\}$. For this reason, we have labelled the parameters Ω, ω and B_0 in Equations (19) and (20).

For our analysis to become complete we must take $n = 1$ in the condition $\theta = 2n$, that is, $\theta = 2$, which gives us the expression

$$\mathscr{E}_{k,l,1} = \pm\sqrt{m^2 + k^2 + q^2 + 2ab + m\omega_{l,1}\left(l - \frac{q\Phi}{2\pi}\right) + 2\Omega_{l,1}\left(2 + \left|l - \frac{q\Phi}{2\pi}\right|\right) - \frac{b^2m^2}{\Omega_{l,1}^2}}. \quad (21)$$

Then, by substituting the real solution of Equation (19) into Equation (21) it is possible to obtain the allowed values of the relativistic energy for the radial mode $n = 1$ of a position dependent mass scalar particle in a background governed by the metric given in Equation (2). In contrast to Refs. [51, 56], we can see that the lowest energy state defined by the real solution of algebraic equation given in Equation (19) plus the expression given in Equation (21) is defined by the radial mode $n = 1$, instead of $n = 0$. This effect arises due to the presence of the Cornell-type central potential in the system. Note that it is necessary physically that the lowest energy state is $n = 1$ and not $n = 0$, otherwise the opposite would imply that $c_1 = 0$, which requires that the rest mass of the scalar particle be zero that is contrary to the proposal of this investigation.

We can specify our analysis through the Cornell-type potential, that is, by taking the parameters $a \rightarrow 0$ or $b \rightarrow 0$, imposing that interaction be produced only by the linear potential or the Coulomb-type potential, respectively. We can observe that if we take the limit $a \rightarrow 0$ and $b \rightarrow 0$ in Equation (6), hence, we recover the Kelin–Gordon equation without position-dependent mass in a KKT background defined by the gauge configuration given in Equation (3). Therefore, we would have in Equation (5) the Kelin–Gordon equation for the relativistic Landau quantization and a quantum flux. In this case, the solution to the radial wave equation (8) would be given in terms of the confluent hypergeometric function. This analysis has been made in Refs. [51, 56], where the magnetic field does not have restricted values.

2.2. Coulomb-Type Potential. In this particular case, it means that $b \rightarrow 0$. Thus, the scalar central potential given in Equation (6) is rewritten as:

$$S(\rho) = \frac{a}{\rho}. \tag{22}$$

Equation(22) represents a Coulomb-type potential. This type of potential has been studied in propagation of gravitational waves [63] and quark models [64]. There are also investigations of the Coulomb-type potential in condensed matter systems, an atom with electric quadrupole moment [65] and magnetic quadrupole moment [66], neutral particle with permanent magnetic dipole moment [67], in molecules [68–70] and in pseudo-harmonic interactions [71, 72]. Therefore, in this particular case, Equation (19) is reduced and gives us the allowed values of the magnetic field for the radial mode $n = 1$:

$$\begin{aligned} \omega_{l,1} &= \frac{4a^2 m}{\left[1 + 2\sqrt{(l - q\Phi/2\pi)^2 + a^2}\right]} \leftrightarrow \\ B_0^{l,1} &= \frac{4a^2 m^2}{q\left[1 + 2\sqrt{(l - q\Phi/2\pi)^2 + a^2}\right]}. \end{aligned} \tag{23}$$

We can note that the allowed values of the magnetic field are connected by the quantum flux Φ through the shift in the eigenvalues of the angular momentum, $l_{eff} = \left(l - q^{\Phi}/2\pi\right)$, that is, an effect analogous to the Aharonov-Bohm effect for bound states [58, 59], making them a periodic function with periodicity $\Phi_0 = \pm(2\pi/q)\nu$, with $\nu = 0, 1, 2, \ldots$, that is, $B_0^{l,1}(\Phi + \Phi_0) = B_0^{l\mp\nu,1}(\Phi)$.

Equation (21) is also reduced as follows:

$$\mathscr{E}_{k,l,1}^2 = m^2 + k^2 + q^2 + m\omega_{l,1}\left[2 + \sqrt{\left(l - \frac{q\Phi}{2\pi}\right)^2 + a^2} + l - \frac{q\Phi}{2\pi}\right] \tag{24}$$

Then, by substituting Equation (23) into Equation (24), we obtain

$$\mathscr{E}_{k,l,1} = \pm\sqrt{m^2 + k^2 + q^2 + \frac{4a^2 m^2}{\left[1 + 2\sqrt{(l - q\Phi/2\pi)^2 + a^2}\right]}\left(2 + \sqrt{\left(l - \frac{q\Phi}{2\pi}\right)^2 + a^2} + l - \frac{q\Phi}{2\pi}\right)}. \tag{25}$$

Equation (25) corresponds to the allowed values of relativistic energy for the radial mode $n = 1$ of a position-dependent mass particle subject to the Coulomb-type potential in a possible scenario described by a KKT. By comparing the Equation (25) with the result obtained in Refs. [51, 56], we can note that the presence of the Coulomb-type potential modifies the energy profile of the system. This modification can be explained by the radical breaking of degeneracy of the Landau levels and by the representation of the lowest energy state that is defined by the radial mode $n = 1$, instead of $n = 0$. By comparing Equation (25) with the result obtained in Ref. [48], we can also assume that the presence of a uniform magnetic field modifies the energy profile of the system. In addition, the allowed values of relativistic energy (25) are influenced by the quantum flux Φ through the shift in the eigenvalues of the angular momentum $l_{eff} = (l - q\Phi/2\pi)$, that is, an effect analogous to the Aharonov-Bohm effect for bound states [58, 59], making them a periodic function with periodicity $\Phi_0 = \pm(2\pi/q)\nu$, with $\nu = 0, 1, 2, \ldots$, that is, $\mathscr{E}_{l,1}(\Phi + \Phi_0) = \mathscr{E}_{l\mp\nu,1}(\Phi)$. This latter characteristic may be of interest in persistent current calculations [25, 48, 73]. The result given in Equation (25), less than the term q^2 that stems from the KKT, is analogous to the result obtained in Ref. [37]. However, it is noteworthy that, in the latter case, the Landau gauge is inserted into the Kelin–Gordon equation by the minimum coupling.

2.3. Linear Potential. Now, let us consider the particular case $a \rightarrow 0$. Thus, the scalar central potential given in Equation (6) is rewritten as:

$$S(\rho) = b\rho. \tag{26}$$

Equation (22) represents a linear central potential. There are studies involving the linear potential in atomic and molecular physics [74xref>–78], quantum bouncer [79, 80], motion of a quantum particle in a uniform force field [1, 81] and in relativistic quantum systems [33, 35, 25, 26, 40, 43, 45, 47]. Therefore, in this particular case, Equation (19) is reduced and gives us the allowed values of the magnetic field for the radial mode $n = 1$:

$$B_0^{l,1} = \frac{2}{q}\sqrt{\left[\frac{b^2 m^2}{2}\left(2\left|l - \frac{q\Phi}{2\pi}\right| + 3\right)\right]^{2/3} - b^2}. \tag{27}$$

Again, we can observe that the allowed values of the magnetic field for the radial mode $n = 1$ (27) are influenced by the quantum flux Φ through the shift in the eigenvalues of the angular momentum $l_{eff} = (l - q\Phi/2\pi)$, that is, an effect analogous to the Aharonov-Bohm effect for bound states [58, 59], making them a periodic function with periodicity $\Phi_0 = \pm(2\pi/q)\nu$, with $\nu = 0, 1, 2, \ldots$, that is, $B_0^{l,1}(\Phi + \Phi_0) = B_0^{l\mp\nu,1}(\Phi)$.

Equation (21) is also reduced as follows:

$$\mathscr{E}_{k,l,1}^2 = m^2 + k^2 + q^2 + m\omega_{l,1}\left(l - \frac{q\Phi}{2\pi}\right) + 2\Omega_{l,1}\left(2 + \left|l - \frac{q\Phi}{2\pi}\right|\right) - \frac{b^2m^2}{\Omega_{l,1}^2}. \tag{28}$$

By substituting Equation (27) into Equation (28), we obtain

$$\mathscr{E}_{k,l,1} = \pm\left[m^2 + k^2 + q^2 + 2\left(l - \frac{q\Phi}{2\pi}\right)\sqrt{\left[\frac{b^2m^2}{2}\left(2\left|l - \frac{q\Phi}{2\pi}\right| + 3\right)\right]^{2/3} - b^2} + 2\left(2 + \left|l - \frac{q\Phi}{2\pi}\right|\right) \times \left[\frac{b^2m^2}{2}\left(2\left|l - \frac{q\Phi}{2\pi}\right| + 3\right)\right]^{1/3} - \frac{b^2m^2}{\left[\left(b^2m^2/2\right)\left(2\left|l - q\Phi/2\pi\right| + 3\right)\right]^{2/3}}\right]^{1/2}, \tag{29}$$

which represents the allowed values of relativistic energy for the radial mode $n = 1$ of a position-dependent mass particle subject to a linear central potential in a possible scenario described by a KKT. By comparing Equation (29) with the result obtained in Refs. [51, 56], we can note that the presence of the linear potential modifies the relativistic energy levels of the system. This modification can be explained by the radical breaking of degeneracy of the Landau levels and by the representation of the lowest energy state that is defined by the radial mode $n = 1$, instead of $n = 0$. Again, we can note that the allowed values of relativistic energy (25) are influenced by the quantum flux Φ through the shift in the eigenvalues of the angular momentum $l_{eff} = (l - q\Phi/2\pi)$, that is, an effect analogous to the Aharonov-Bohm effect for bound states [58, 59], making them a periodic function with periodicity $\Phi_0 = \pm(2\pi/q)\nu$, with $\nu = 0, 1, 2, \ldots$, that is, $\mathscr{E}_{l,1}(\Phi + \Phi_0) = \mathscr{E}_{l\mp\nu,1}(\Phi)$. This latter characteristic may be of interest in persistent current calculations [25, 48, 73]. The result given in Equation (5), less than the term q^2 that stems from the KKT, is analogous to the result obtained in Ref. [26]. However, it is noteworthy that, in the latter case, the Landau gauge is inserted into the Klein–Gordon equation by the minimum coupling.

3. Conclusion

We have investigated a position-dependent mass scalar particle subjected to a uniform magnetic field and a quantum flux in a background governed by the KKT. In this scenario, we have analyzed the interaction between a scalar particle and the Cornell-type potential, where, in the search for solutions of bound states, we determine analytically the energy profile of the system, which is influenced by the background. We can note that relativistic energy spectrum is not defined by a closed expression. In fact, it is only possible to determine allowed values of relativistic energy for each radial mode separately. As an example, we analyze the lowest energy state of the system represented by the radial mode $n = 1$, instead of $n = 0$. We can also note that the presence of the Cornell-type potential breaks the degeneracy of the Landau levels. In addition, a quantum effect characterized by the dependence of the magnetic field on the quantum numbers of the system is observed, where we have shown that their possible values are determined by a third-degree algebraic equation.

We have particularized our system through the parameters that characterize the linear term and the Coulomb type term of the Cornell-type potential, where we consider the absence of one or other. First, we consider the absence of the linear central potential, where we determine the allowed values of the magnetic field and the relativistic energy for the radial mode $n = 1$. We can observe that both are influenced by the quantum flux through a shift in the angular momentum eigenvalues, producing an analogous effect to the Aharonov-Bohm effect for bound states, making them as periodic functions of the quantum flux. In addition, we can also note that the presence of the linear central potential breaks the degeneracy of the relativistic Landau levels. Then, we consider the absence of the linear central potential and analyze its energy profile, where, except for the expressions of the allowed values of the magnetic field and the relativistic energy for the lower energy state, which are totally modified, the characteristics of the previous case are analogous.

Data Availability

No data available.

Conflicts of Interest

The authors declare that there is no conflict of interest.

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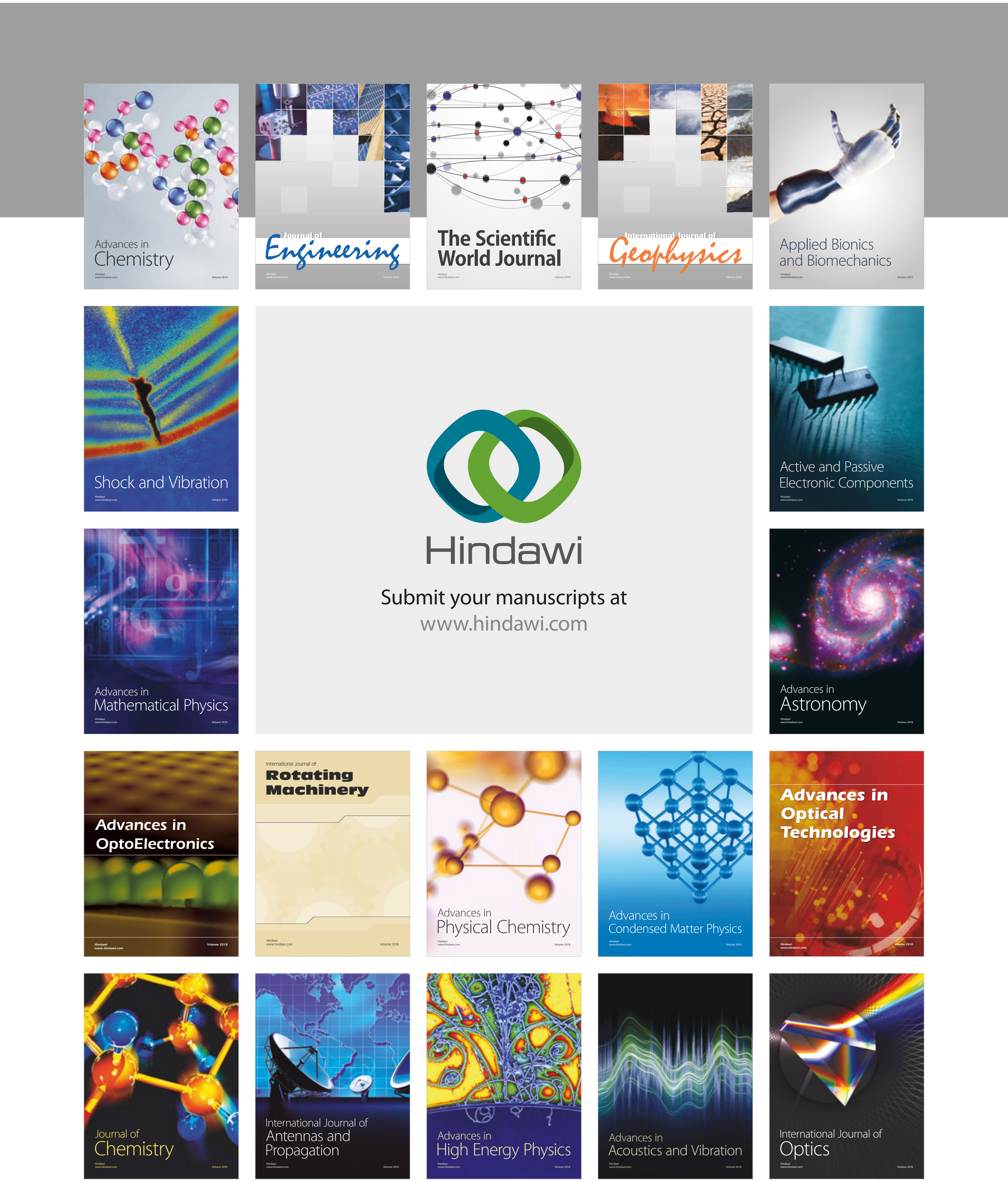
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