Research Article

Bianchi Type-*I* Bulk Viscosity with a DE Cosmological Model

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The finding article presents Bianchi type-*I* universe in the presence of bulk viscous and DE fluid nature of a cosmological model. The solutions of field equations were obtained by assuming hybrid expansion law. The physical significance of the obtained findings illustrates the dominance of bulk viscosity in early and dominance of dark energy fluid emergences in late. This leads to indicate the presence of bulk viscosity nature more effective in early time rather than late times, and also, it shows the dominance of dark energy in late times which grants the current observational result of the universe. Certain physical and geometrical properties of the model are also discussed.

1. Introduction

Experimental results of supernovae type-*I* luminance (Riess et al. [1]; Perlmutter et al. [2]), peaks of cosmic microwave circumstances (Komatsu et al. [3]), oscillations of baryon acoustic (Eisenstein et al. [4]), and structures of large scale (Tegmark et al. [5]) directed a composition of a universe 70% dark energy (Mendola and Tsujikawa [6]; Frieman et al. [7]). The one responsible for the acceleration of the universe by type-*I* supernovae observation is dark energy (DE). In current time, many researchers will approve that the universe was in an accelerating nature. Based on astronomical observations and results, it is estimated that the rate of expansion is increasing with time. A few years ago, the high redshift supernova experiments (Garnavich et al. [8]; Riess et al. [1]; Perlmutter et al. [2, 9, 10]) and cosmic microwave background radiation (Bennett et al. [11]; Riess et al. [12]) have provided the necessary evidence for the accelerated expansion of the universe. This looks inexplicable as for two reasons: the first one for the indication that there must be an unknown and unusual source of energy which acts as antigravity, in order to produce negative pressure to overcome the attractive force of gravity, and the second one for the gravity of general theory which should be modified.

These two scenarios could be identified by the reason of the background of cosmic expansion H and the increase of

large scale structure (Ma et al. [13]). To explain the nature of DE, the amount in terms of density component should considered. Because illustration of type-*I*a supernovae evidences that the existence of almost 2/3 of the total energy density exists in a DE form (Padmanabhan [14]; Copeland et al. [15]; Li et al. [16]). The investigation of DE is possible by the equation of state parameter (EoS) $\omega_{de} = p_{de}/\rho_{de}$. The value of EoS for quintessence($\omega_{de}>-1$),ΛCDM($\omega_{de} = -1$), and phantom scenarios($\omega_{de}<-1$)is shown. Even though cosmological constant (Λ) looks as a suitable candidate for the DE, it suffers from the problem of fine-tuning and the cosmic coincidence. Both of these are connected to the DE density. This is the reason due to different aspects of dynamically changing DE such as quintessence, k-essence, tachyon, phantom, ghost condensate, and quintom.

From the mentioned scalar fields, quintessence with the EoS parameter varying as $-1 < \omega_{de}$ and phantom having $\omega_{de} < -1$ are obtained, and the specific case of $\omega_{de} \ll -1$ is elevated by findings of Riess et al. [12]. However, since current observations show that the DE of equation of state parameter could be less than -1 [17, 18], the quintessence is unmentionable and the phantom field invisible from ultraviolet quantum uncertainty [19] cannot be suitable for DE options explained within a region of $\omega_{de} < -1$. The present status is that none of these models offer a satisfactory solution to the dark energy problem because they either suffer from severe

fine tuning problems, leading to instabilities ruled out, or they are awaiting further analysis.

Bulk viscous cosmology is also an alternative to gravity modifying theories (Nojiri and Odintsov [20]) in that it alters the right hand side of Einstein's field equations instead of the left hand side. In this situation based on the Eckart theorem (Eckart [21]), the consideration of the DE fluid with viscous is important. The evolution of universe involves sequence of dissipative process. In isotropic and homogeneous model, the process of dissipative is modeled as a bulk viscosity (Ren and Meng [22]; Hu and Meng [23]; Meng and Duo [24]). Brevik et al. [25] investigated the overall cases of viscous cosmology in early and late time universe. Norman and Brevik [26] investigated the properties of the characteristic of two different viscous cosmological models for the future universe. Norman and Brevik [27] derived observance of the bulk viscous and approximated the bulk viscosity of the cosmic fluid. The finding of viscosity dominance by late epoch of the universe with accelerated expansion was studied by Padmanabhan and Chitre [28]. Velten et al. [29] have investigated phantom DE as an effect of bulk viscosity. It is illustrated by Brevik and Gorbunova [30] that the fluid which lies in the quintessence region can minimize its pressure and cross the barrier $\omega_{de} = -1$, and behaves like a phantom fluid which leads to the inclusion of large bulk viscosity in a sufficient way.

The characterization by shear and bulk viscosities leads to the effect of dissipation at microscopic interactions. As expansion fluid leaves its equilibrium state, the energy density and the pressure decrease. If there is no bulk viscosity, then, the fluid relaxes instantaneously with pressure and density related by ω. Bulk viscosity slightly met this nature by leading a certain relaxation time scale, but producing a shift between the equation of state pressure and the absolute pressure. Bulk viscosity becomes essential only for such effects where fluid compressibility is essential. Researchers have contributed a necessity role to bulk viscous fluid matter which is different from the traditional case. The effects of both shear and bulk viscosity were explained by Hoogeveen et al. [31] using kinetic theory during early time. The detailed implement for the origin of bulk viscosity in the universe is not correctly understood yet.

Based on the hypothetical view, the bulk viscosity can be derived from local thermodynamic equilibrium, the manifestation as an effective pressure to bring back the system to its thermal equilibrium, which was broken when the cosmological fluid expands. The bulk viscosity pressure thus generated ceases as soon as the fluid reaches equilibrium condition. Very recently, the concept of bulk viscosity is introduced into a DE study. It is important to develop a cosmological model. The concept of viscosity has come from fluid mechanics, and it is related to the velocity gradient of the fluid. Misner [32] revealed that during cosmic evolution when neutrinos decouple from the cosmic fluid, bulk viscosity could arise and lead to an effective mechanism of entropy production. The isotropic homogeneous spatially flat cosmological model with bulk viscous fluid was discussed by Murphy [33]. Bulk viscosity related to the grand unified-theory phase transition (Langacher [34]) may lead to explain the cosmic acceleration.

The presence of bulk viscosity leads to an inflationary-like solution in Friedmann-Robertson-Walker (FRW) spacetime obtained by Padmanabhan and Chitre [28].

There are theoretical views that reveal the recent acceleration without invoking the exotic DE component. The researchers have explained that a bulk viscous dark matter can bring an accelerated expansion of the universe. It is mentioned that in early universe the impact of viscosity is very small whereas in a future universe the influence is significant (Brevik et al. [25, 35]). As viscosity is shown to be important in dissipative phenomena in FRW spacetime, then, it is expected that the cosmological model with bulk viscosity fluid would produce certain results in the two fluid situations. At the inflationary phase, viscosity of model of cosmology shows a great contribution of bulk viscosity (Barrow [36]; Zimdahl [37]; Bafaluy and Pavon [38]). The inflation of bulk viscous leads to a negative pressure, which results in repulsive gravity and ultimately became a cause for the rapid expansion of the universe (Tripathy et al. [39]; Maartens [40]; Lima et al. [41]). The viscosity of bulk gives to the cosmic pressure and plays the role of accelerating the universe. In an expanding system, relaxation processes associated with bulk viscosity effectively reduce the pressure as compared to the value prescribed by the equation of state. For a sufficiently large bulk viscosity, the effective pressure becomes negative and could mimic a DE behavior. The idea of having the bulk viscosity drives the acceleration of the universe (Padmanabhan and Chitre [28]).

At larger scales, being isotropic and homogeneous is the nature of our universe. The well definiteness of the model of ΛCDM in Planck collaboration revealed the property of the universe as isotropic and homogeneity in the geometry of FRW. However, Ade et al. [42, 43] have investigated a poor fit at low multipoles. This implies that the isotropy and homogeneity were not the essential features of the early universe. Hence, the current Planck data results motivate me to construct and analyze the cosmological models with anisotropic geometry to investigate the evolution of the universe by considering Bianch type-I spacetime.

Different scholars investigated models in the backdrop of general relativity within different Bianchi forms. For type-I Bianchi, DE models using constant deceleration parameter have been investigated by Akarsu and Kilinc [44, 45]. Yadav et al. [46] constructed Bianchi type-I DE cosmological models with the deceleration parameter with a variable of EoS parameter. Two fluid DE models either minimally interacting or interacting have been investigated widely (Sheykhi and Setare [47]; Amirhashchi [48]; Amirhashchi et al. [49]; Tripathy et al. [50] and Kumar [51]). Santhi et al. [52] have studied bulk viscous string cosmological models in $f(R)$ gravity. Two fluids of DE cosmological models were constructed in different general scale factors (Mishra et al. [53]). So, all above information gives us motivation to investigate Bianchi type-I spacetime bulk viscous fluid with dark energy cosmological model using hybrid scale factor. Redefining the effective pressure, $\bar{p}$, as $\bar{p} = p - 3\zeta H$, where H is the Hubble parameter and ζ is the bulk viscosity coefficient. In this case, I present an alternative model of DE that includes the effects of bulk viscosity of the universe in general relativity.

The paper is organized as follows: in Section 2, I introduce the metric and field equations, respectively. I present the solutions of field equations by using hybrid scale factor in Section 3. In Section 4, I discussed some physical and geometrical properties of the model. Section 5 comprises the conclusion of the findings.

2. The Metric and Field Equations

We consider the spatially homogeneous and anisotropic Bianchi type-I spacetime in the form

$$ds^2 = dt^2 - A^2 dx^2 - B^2 dy^2 - C^2 d\zeta^2, \tag{1}$$

where A, B, and C are metric functions of cosmic time t alone.

Einstein's field equations are given by

$$R_i^j - \frac{1}{2} R \delta_i^j = -\frac{8\pi G}{c^4} T_i^j = -T_i^j, \tag{2}$$

where R_i^j is the Ricci tensor, $R = R_i^i$ is the Ricci scalar, δ_i^j is the unit four tensor, G is the Newton's gravitational constant, c is the speed of light in the vacuum, and T_i^j is the overall energy momentum tensor that describes the physical ingredients of the Universe. For convenience, we assumed that natural units $c = 8\pi G = 1$.

We assume that the overall energy momentum tensor T_i^j has two different fluid components as bulk viscous $(T_{(v)i}^j)$ and dark energy $(T_{(de)i}^j)$ given by

$$T_i^j = T_{(v)i}^j + T_{(de)i}^j, \tag{3}$$

where the energy momentum tensor of barotropic bulk viscous fluid is

$$T_{(v)i}^j = (\rho + \bar{p}) u_i u^j - \bar{p} \delta_i^j \tag{4}$$

and energy momentum tensor of dark energy fluid is

$$
\begin{aligned}
T_{(de)i}^j &= \mathrm{diag}\left[\rho_{de}, -p_{(de)x}, -p_{(de)y}, -p_{(de)z}\right] \\
&= \mathrm{diag}\left[1, -\omega_{(de)x}, -\omega_{(de)y}, -\omega_{(de)z}\right]\rho_{de} \\
&= \mathrm{diag}\left[1, -(\omega_{de} + \delta), -(\omega_{de} + \gamma), -(\omega_{de} + \eta)\right]\rho_{de}.
\end{aligned}
\tag{5}
$$

Here, ρ is the energy density of matter, $\bar{p}$ is the effective pressure, $u^i = (1, 0, 0, 0)$ is the four-velocity vector satisfying $u_i u^i = 1$ in a comoving coordinate system, $\rho_{(de)}$ is the DE density, and $p_{(de)x}$, $p_{(de)y}$, and $p_{(de)\zeta}$ are the directional pressures and $\omega_{(de)x}$, $\omega_{(de)y}$, and $\omega_{(de)\zeta}$ are the directional EoS parameters of the DE fluid along x, y, and ζ axes, respectively. The parameters of skewness δ on the x-axis, γ on the y-axis, and η on the ζ-axis are deviations from the EoS parameter $(\omega_{de} = p_{de}/\rho_{de})$ on these three directions.

The proper pressure p is connected to the energy density by means of an equation of state as barotropic cosmic fluid, given as $p = \xi\rho$, where ξ is the coefficient of bulk viscosity. Moreover, the bulk viscosity related to energy density with the help of Hubble's parameter (H) as $3\zeta H = \varepsilon_0 \rho$, where ε_0 is the proportionality constant and $3\zeta H$ is usually termed as bulk viscous pressure.

The effective pressure which is the combination of proper pressure and barotropic bulk viscous pressure can be written as

$$\bar{p} = p - 3\zeta H = (\zeta - \varepsilon_0)\rho = \varepsilon\rho, \tag{6}$$

where ε can be considered the effective bulk viscous coefficient.

In Einstein's field equation (2) for metric equation (1) with the help of equations (4) and (5), the system of differential equations is as follows:

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} = -p + 3\zeta H - (\omega_{de} + \delta)\rho_{de}, \tag{7}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} = -p + 3\zeta H - (\omega_{de} + \gamma)\rho_{de}, \tag{8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} = -p + 3\zeta H - (\omega_{de} + \eta)\rho_{de}, \tag{9}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} = \rho + \rho_{de}, \tag{10}$$

where dot metric represents the derivatives of corresponding field variable with respect to cosmic time t.

Substituting $(p - 3\zeta H)$ in field equations as $\bar{p}$,

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} = -\bar{p} - (\omega_{de} + \delta)\rho_{de}, \tag{11}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} = -\bar{p} - (\omega_{de} + \gamma)\rho_{de}, \tag{12}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} = -\bar{p} - (\omega_{de} + \eta)\rho_{de}, \tag{13}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} = \rho + \rho_{de}. \tag{14}$$

The energy conservation equation $(T_{(v);j}^{ij} + T_{(de);j}^{ij} = 0)$ yields

$$
\begin{aligned}
&\dot{\rho} + 3(\bar{p} + \rho)\frac{\dot{a}}{a} + \dot{\rho}_{de} + 3\rho_{de}(\omega_{de} + 1)\frac{\dot{a}}{a} \\
&+ \rho_{de}\left(\delta H_x + \gamma H_y + \eta H_z\right) = 0.
\end{aligned}
\tag{15}
$$

Assume that DE and bulk viscous fluid are minimally interacting. The energy conservation equation (15) for the

two fluids in a separate way, $T^{ij}_{(v);j} = 0$ and DE fluid, $T^{ij}_{(de);j} = 0$, leads to the following equations

$$\dot{\rho} + 3(\bar{p} + \rho)\frac{\dot{a}}{a} = 0, \qquad (16)$$

$$\dot{\rho}_{de} + 3\rho_{de}(\omega_{de} + 1)\frac{\dot{a}}{a} + \rho_{de}(\delta H_x + \gamma H_y + \eta H_z) = 0. \qquad (17)$$

As the inclusion of the barotropic bulk viscous pressure within the contributions is from both the usual cosmic fluid and the effective coefficient of bulk viscosityε, hence, the contribution from bulk viscosity to cosmic pressure is assumed to be proportional to the rest energy density of universe [39].

From the equation of $\bar{p} = \varepsilon\rho$, if contribution from bulk viscosity becomes more than the usual perfect fluid pressure, then, the total effective pressure becomes negative with a negative ε. The accelerated expansion in the present epoch is usually attributed to a fluid with negative pressure, and hence, it can be thought that the contribution coming from the bulk viscosity is greater than the usual pressure. However, the usual pressure from perfect fluid is equal to the contribution from cosmic bulk viscosity; then, the cosmic fluid in the model behaves like a pressure-less dusty universe. But the presence of an exotic DE form leads to a negative pressure of the universe.

3. Solutions of the Field Equations

The system consists of four independent differential equations (11)–(14) having unknowns (A, B, C, $\bar{p}$, ρ, ω_{de}, ρ_{de}, δ, γ, and η). For the purpose of solving unpredictability system, certain additional constraints are needed. First, let us assume that shear scalar (sigma) in the model is proportional to expansion scalar (θ). This leads to

$$B = C^b, \qquad (18)$$

where B and C are the metric potentials and b is a positive constant. The above mathematical analysis was made as suggested by Thorne [54]. Collins et al. [55] have also observed that for spatially homogeneous metric, the normal congruence to the homogeneous expansion maintains the ratio of (σ/θ). Second, considering hybrid expansion law and the relation leads to a time-dependent deceleration parameter which describes the transition of the universe from early decelerating phase to the current accelerating phase. Hence, the average scale factor a is given as the combination of power and exponential law [56]

$$a(t) = t^g e^{ht}, \qquad (19)$$

where $g = ((b + 1)/2)l$ and $h = ((b + 1)/2)\xi$ are positive constants.

The spatial volume of the model is

$$V = ABC = a^3 = t^{3g} e^{3ht}. \qquad (20)$$

The relation which is given by equation (19) shows the combination of exponential and power law, which is commonly called hybrid expansion law. It is revealed that dynamical DE models play an important role in describing the accelerated expansion of the universe. For $h = 0$ and $g = 0$, it can be obtained as a power and exponential expansion law from equation (19), respectively. If g and h are both nonzero, this shows the evolution of the universe with a variable parameter of deceleration. From equation (20), it is concluded that the spatial volume increases with cosmic time which stimulated the current framework of the universe.

It can be assumed that the Hubble parameter and the directional Hubble's parameter along the x-axis are the same, i.e., $H = \dot{a}/a = H_x$ [57]. Hence, we get $A = a$. Using equations (19) and (20), we obtained

$$\begin{aligned} A &= t^g e^{ht}, \\ B &= t^{g(2b/(b+1))} e^{(2b/(b+1))ht}, \\ C &= t^{g(2/(b+1))} e^{(2/(b+1))ht}. \end{aligned} \qquad (21)$$

The metric given by equation (1) with the help of (21) can be written as

$$\begin{aligned} ds^2 &= dt^2 - \left(t^{2g} e^{2ht}\right) dx^2 - t^{g(4b/(b+1))} e^{(4b/(b+1))ht} dy^2 \\ &\quad - t^{g(4/(b+1))} e^{(4/(b+1))ht} dz^2. \end{aligned} \qquad (22)$$

4. Some Physical and Geometrical Properties of the Model

Here, various physical parameters such as Hubble parameter (H), the expansion scalar (θ), the mean anisotropy parameter (A_m), and the shear scalar (σ^2) for the metric equation (1) can be discussed as follows:

For the model (22), H and its the directional can obtained as follows, respectively:

$$\begin{aligned} H &= H_x = h + \frac{g}{t}, \\ H_y &= \frac{2b}{b+1}\left(h + \frac{g}{t}\right), \\ H_z &= \frac{2}{b+1}\left(h + \frac{g}{t}\right). \end{aligned} \qquad (23)$$

The scalar expansion (θ) is

$$\theta = 3H = 3\left(h + \frac{g}{t}\right). \qquad (24)$$

The shear scalar (σ^2) is

$$\sigma^2 = \frac{1}{2}\left(H_x^2 + H_y^2 + H_z^2 - \frac{\theta^2}{3}\right) = \left(\frac{b-1}{b+1}\right)^2\left(h + \frac{g}{t}\right)^2. \qquad (25)$$

The anisotropic parameter (A_m) is

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 = \frac{2}{3}\left(\frac{b-1}{b+1}\right)^2, \quad (26)$$

where $H_i - H = \Delta H_i, i = 1, 2, 3$.

From equation (26), A_m converges to a constant value for large values of cosmic time t. It can be expected from an accelerating model with $b = 1$ that the anisotropic parameter converges to zero and A_m converges to a finite nonzero value for all cosmic time t values. Parameter b is essential to determine the anisotropic behavior of the model, and it became isotropic for $b = 1$. Anisotropic parameter $A_m \neq 0$; it can be inferred that $b \neq 0$. This shows that the universe is anisotropic. Campanelli et al. [58] predicted for $|\sqrt{A_m}| \approx 10^{-5}$ in case of the present universe from the Wilkinson Microwave Anisotropy Probe (WMAP) data for consistency with our model by appropriate choices of parameter b. From equation (25), $b = 1$ implies that the shear scalar vanishes. Hence, the model becomes shear free. The H, θ, and σ^2 parameters start with an infinite large value and then decreasing with during expansion.

By using obtained values of the metric functions A, B, and C from equation (21), equations (11)–(14) become

$$2\dot{H} + \frac{4(b^2 + b + 1)}{(b+1)^2}H^2 = -\bar{p} - (\omega_{de} + \delta)\rho_{de}, \quad (27)$$

$$\left(\frac{b+3}{b+1}\right)\dot{H} + \frac{(b^2 + 4b + 7)}{(b+1)^2}H^2 = -\bar{p} - (\omega_{de} + \gamma)\rho_{de}, \quad (28)$$

$$\left(\frac{3b+1}{b+1}\right)\dot{H} + \frac{(7b^2 + 4b + 1)}{(b+1)^2}H^2 = -\bar{p} - (\omega_{de} + \eta)\rho_{de}, \quad (29)$$

$$\frac{2(b^2 + 4b + 1)}{(b+1)^2}H^2 = \rho + \rho_{de}. \quad (30)$$

From equation (16) using the H and scale factor, ρ for bulk viscous fluid can be obtained

$$\rho = \frac{\rho_0}{\left[e^{\int H dt}\right]^{3(\varepsilon+1)}} = \rho_0\left(t^l e^{\xi t}\right)^{-3/2(b+1)(\varepsilon+1)}, \quad (31)$$

where the rest energy density of present time is equal to ρ_0.

If the time variation is determined, then, it is possible to get the rate of the mean Hubble parameter. This leads to the determination of the rest energy density of the universe from equation (31) for a certain parameter ε.

From equations (30) and (31), the DE density is given by

$$\rho_{de} = \frac{2(b^2 + 4b + 1)}{(b+1)^2}H^2 - \rho_0 a^{-3(\varepsilon+1)}$$
$$= \frac{(b^2 + 4b + 1)}{2t^2}(l + \xi t)^2 - \rho_0\left(t^l e^{\xi t}\right)^{-3/2(b+1)(\varepsilon+1)}. \quad (32)$$

It can be concluded that ρ and ρ_{de} are both decreasing functions as cosmic time slowly increases. The decrease in ρ_{de} is determined by two different factors. These are $1/t^2$ and $1/(t^{(3/2)l(b+1)(\varepsilon+1)})$ with the second one having a bulk viscous coefficient for the density of DE. If $\varepsilon = -1$, benefaction for the DE density from cosmic fluid becomes time dependent. Because ε acts as an EoS parameter $(\bar{p} = \varepsilon\rho)$ for the matter of bulk viscous, it implies ε reduces to vacuum in the absence of the DE model. If $\varepsilon = -1/3$, then there is the dominance of radiation model. Understanding the nature of matter the whole, it is chosen that $\varepsilon = -2/3$ ranged within $[-1, -1/3]$. The parameters of skewness become constant and time independent for $b = 1$, and hence, it implies that the isotropic pressure in all directions does not vary.

By using the deviation free part of equation (17) and equations (27)–(29), the skewness parameters can be obtained as

$$\delta = -\left(\frac{2(b-1)}{3\rho_{de}}\right)\mu(b)F(a), \quad (33)$$

$$\gamma = \left(\frac{b+5}{3\rho_{de}}\right)\mu(b)F(a), \quad (34)$$

$$\eta = -\left(\frac{5b+1}{3\rho_{de}}\right)\mu(b)F(a), \quad (35)$$

where $\mu(b) = (b-1)/(b+1)^2$ and $F(a) = a/a + 2\dot{a}^2/a^2$.

Using equation (31), the proper pressure p is given by

$$p = \xi\rho = \xi\rho_0\left(t^l e^{\xi t}\right)^{-3/2(b+1)(\varepsilon+1)}, \quad (36)$$

where $0 \leq \xi \leq 1$.

Effectiveness pressure $\bar{p}$ is the mixture of both proper pressure and barotropic bulk viscous obtained as

$$\bar{p} = p - 3\xi H = \xi\left(\rho_0\left(t^l e^{\xi t}\right)^{-3/2(b+1)(\varepsilon+1)} - \frac{3}{2t}(b+1)(l + \xi t)\right). \quad (37)$$

4.1. EoS Parameter.
EoS parameter of a DE fluid related by $\omega_{de} = p_{de}/\rho_{de}$. Its values correspond to epochs of different universe in early and late phases of expansions. It includes stiff fluid, radiation, and matter dominated which correspond to $\omega_{de} = 1$, $\omega_{de} = 1/3$, and $\omega_{de} = 0$, respectively. The ranges of quintessence $(-1 < \omega_{de})$, cosmological constant $(\omega_{de} = -1)$, and the phantom region $(\omega_{de} < -1)$ are shown.

Applying equations (27), (32), (33), and (37), we obtain the EoS parameter of dark energy (ω_{de}) as

$$
\begin{aligned}
\omega_{de} &= \frac{-\left[\left(4\dot{H} + 6H^2\right)\left(b^2 + 4b + 1\right) + 3\bar{p}(b+1)^2\right]}{3\rho_{de}(b+1)^2} \\
&= \frac{-\left[\left(\left((3/2)((l+\xi t)/t)^2\right) - \left(2/\left((b+1)t^2\right)\right)\right)\left(b^2 + 4b + 1\right) + 3\xi(\rho - (3/2t)(b+1)(l+\xi t))\right]}{(3/2)\left(b^2 + 4b + 1\right)((l+\xi t)/t)^2 - 3\rho_0\left(t^l e^{\xi t}\right)^{(-3/2)(b+1)(\varepsilon+1)}},
\end{aligned}
\tag{38}
$$

where $\rho = \rho_0(t^l e^{\xi t})^{-(3/2)(b+1)(\varepsilon+1)}$.

4.2. Energy Density Parameters.

The behaviors of energy densities of the fluids are also used to decide whether our model is realistic or not. DE density parameter (Ω_{de}), the matter density parameter (Ω_m) of bulk viscous, and t total density parameter (Ω) are obtained as follows:

$$
\begin{aligned}
\Omega_{de} &= \frac{\rho_{de}}{3H^2} = \frac{2}{3}\frac{\left(b^2 + 4b + 1\right)}{(b+1)^2} \\
&\quad - \frac{4\rho_0 t^2}{3(b+1)^2(\xi t + 1)^2}\left(t^l e^{\xi t}\right)^{(-3/2)(\varepsilon+1)(b+1)},
\end{aligned}
\tag{39}
$$

$$
\Omega_m = \frac{\rho}{3H^2} = \frac{4\rho_0 t^2 \left(t^l e^{\xi t}\right)^{(-3/2)(b+1)(\varepsilon+1)}}{3(b+1)^2(\xi t + l)^2}.
\tag{40}
$$

Summing (39) and (40), we get total energy density parameter

$$
\Omega = \Omega_m + \Omega_{de} = \frac{2}{3}\frac{\left(b^2 + 4b + 1\right)}{(b+1)^2}.
\tag{41}
$$

Here, for large cosmic time t values, the energy density parameter of bulk viscous fluid approaches zero. It can be seen that the total energy density parameter for late cosmic time t approximates to the finite value. Since the total density matter of flat universe approaches 1, it can be obtained by choosing appropriate parameter b.

4.3. Deceleration Parameter.

The deceleration denoted by q is a dimensional measure of cosmic acceleration of the expanding universe and is given by

$$
q = -\frac{a\ddot{a}}{\dot{a}^2} = -1 + \frac{2l}{(b+1)(\xi t + l)^2}.
\tag{42}
$$

The "positive" q implies decelerating universe while its "negative" value of q leads to the accelerated universe. Results of observed high red shift supernova, type-Ia supernova, baryon acoustic oscillations, and the cosmic microwave background reveal that obtained models transit from early decelerating to late time accelerating universe. Based on current data, a value for q is ranged in -0.81 ± 0.14. In this case, the model has $q = -1 + g/((ht + g)^2)$. Hence, at an early phase, $t \to 0\, q \to -1 + (1/g)$, and at late phase, $t \to \infty\, q \approx -1$. To get a transient universe, parameter g is in the range of $0 < g < (1/3)$ [59].

Figure 1 shows the nature of q against cosmic time t for $\xi = 0.5$, 0.6, 0.7, $b = 0.5$, and $l = 0.6$, respectively. From the model, it is clear that the exhibition from early ($q > 0$) to late ($q < 0$) in smooth transition way. Also, the present value of cosmic time is $t_0 = 13.82$ Gyr in this case is $q_0 = -0.73$ [60] which indicates the approaching value to be -1. Results of SNe-Ia reveal that the current accelerating value of q is in the range of $-1 \leq q < 0$.

From Figure 2, the phase of the matter of the energy density of bulk viscous fluid from early phase to late is seen to be decreasing and approaching zero. In Figure 3, it is observed that the DE density (ρ_{de}) remains positive till the late phase of evolution for $\varepsilon = -0.67$. Hence, it is similar which holds true for other choices of $\varepsilon = -0.33$ and -0.9. Therefore, it stipulates energy conditions of weak and null for the obtained model. Here, ρ_{de} has inverse relation with cosmic time and positive value obtained in current situation which is near to zero [61]. This makes us to conclude that fluid of bulk viscous has smaller effect on ρ_{de} but this cannot be rejected. The value of ρ_{de} becomes closer to zero with a small positive value which leads to the fluid affecting DE density. Regardless of the bulk viscous coefficient, the nature of ρ_{de} remains alike.

Figure 4 describes the behavior of EoS parameter (ω_{de}) for choices of bulk viscous coefficients ($\varepsilon = -0.33$, -0.67, and -0.9), and it is an increasing function of cosmic time. As the model of de Sitter and power law by Mishra et al. [62], the EoS parameter is proportional to the values of ε. As it is shown in Figure 4, the green dot line ($\varepsilon = -0.33$) reveals that the greater ε, the more ω_{de} within range of quintessence region. As increase in ε, the model collects certain energy in early phase and act diverse. It is observed that for choices of bulk viscous coefficients, the variation of EoS parameter in phantom region is forceful in the early phase of evolution and lies in quintessence region late times. For larger bulk viscosity ($\varepsilon = -0.33$), the result of the cosmological constant is $\omega_{de} = -1$. Hence, the bulk viscosity with larger viscous coefficient of EoS parameter is more important to solve the cosmic problem of singularity and determine modes of evolution of the future universe.

The DE density dominance appears in the model even if the bulk viscous density is available. Therefore, it implies a
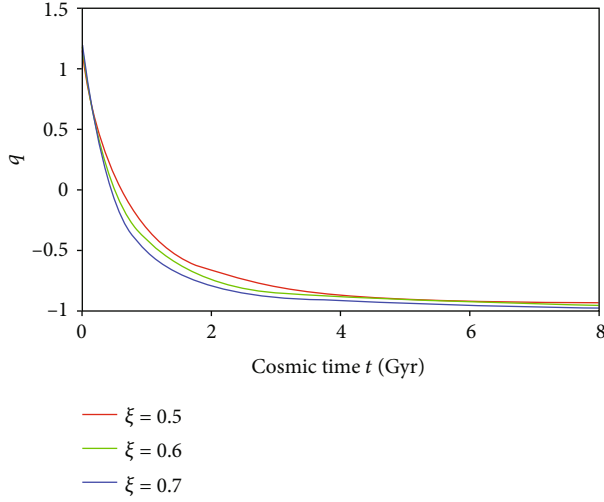
FIGURE 1: The plot of deceleration parameter (q) versus cosmic time t for $\xi = 0.5, 0.6, 0.7$, $b = 0.5$, and $l = 0.6$, respectively.
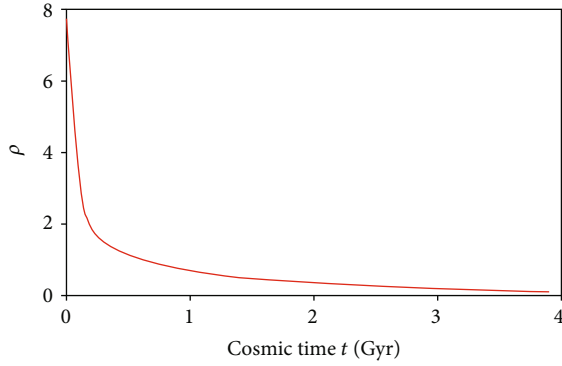

FIGURE 2: The plot of energy density (ρ) of bulk viscous matter versus cosmic time t for $\xi = 0.6$, $l = 0.6$, $\varepsilon = -0.67$, $b = 0.5$, and $\rho_0 = 1$, respectively.
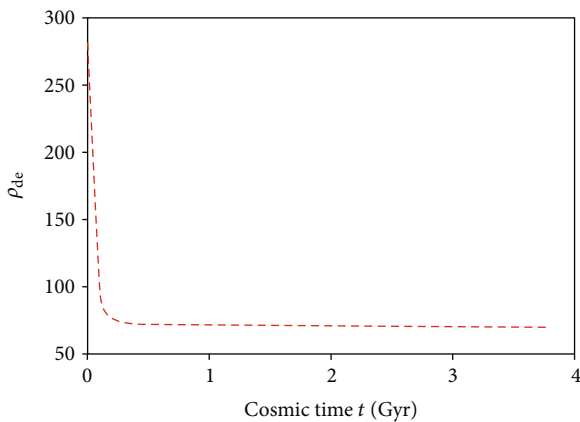

FIGURE 3: The plot of dark energy density (ρ_{de}) versus cosmic time t for $\xi = 0.6$, $l = 0.6$, $\varepsilon = -0.67$, $b = 0.5$, and $\rho_0 = 1$, respectively.

FIGURE 4: Plot of EoS parameter versus cosmic time t for $\varepsilon = -0.33$, $-0.67, -0.9$, $\xi = 0.6$, $l = 0.6$, $b = 0.5$, and $\rho_0 = 1$, respectively.
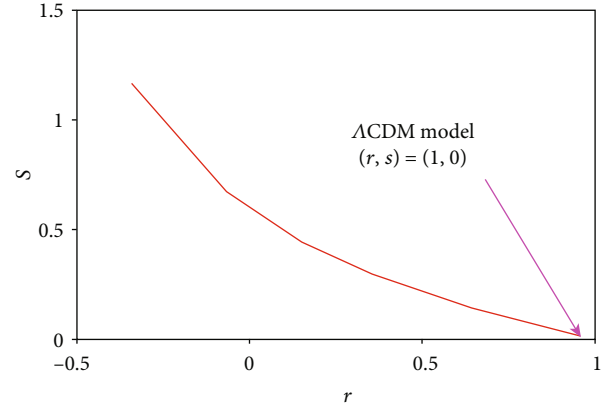
FIGURE 5: The plot of r versus s plane for $\xi = 0.6$, $l = 0.6$, and $b = 0.5$.

other parameters. The obtained ω_{de} is similar for bulk viscous coefficient different values. Moreover, it can be concluded that the parameter of skewness is similarly treated in the case of the isotropic universe. The skewness parameters of δ and η show similar behavior, but γ is different in direction and magnitude from both of them. Also, the effects of anisotropic parameter (A_m) is investigated with respect to cosmic time t, but it does not bring any effect on skewness of parameter.

4.4. Statefinder Diagnostic. Statefinder diagnostic is a reliable approach in distinguishing various DE models. It is based on a pair of new geometrical variables which are related to the third derivative of scale factor with respect to time. To investigate the viability of these models, Sahni et al. [63] forecasted the parameter of state finder (r, s) on cosmological $r - s$ plane. It compares the distance of the ΛCDM limit to DE model. The plane parameters explain different regions in cosmology. For example, $s > 0$ and $r < 1$ lead to the region of phantom and quintessence DE eras, $(r, s) = (1, 0)$ predicts

little impact of bulk viscous on the dynamics of EoS parameter and is at early time. DE EoS parameters can also be useful in testing the model with respect to cosmic time of certain
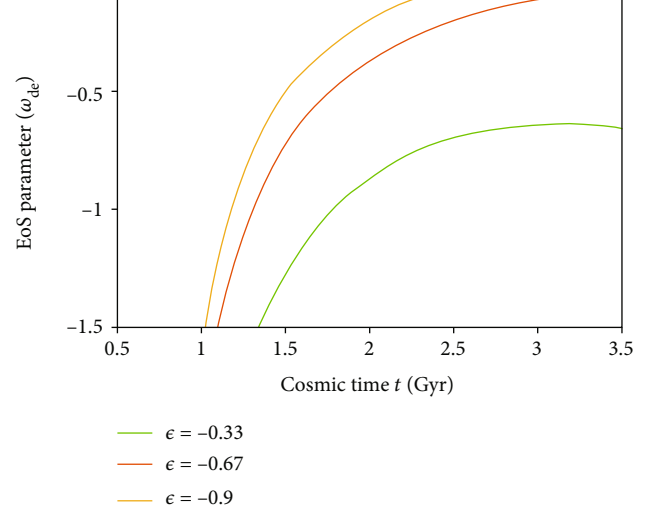
the ΛCDM limit, $(r, s) = (1, 1)$ shows the CDM limit, and $s < 0$ and $r > 1$ assign the Chaplygin gas nature. The parameter of statefinder of the model becomes

$$r = \frac{\ddot{a}}{aH^3} = 1 - \frac{3g}{t^2((g/t) + h)^2} + \frac{2g}{t^3((g/t) + h)^3}$$
$$= 1 - \frac{6l}{(b + 1)(\zeta t + l)^2} + \frac{8l}{(b + 1)^2(\zeta t + l)^3}, \quad (43)$$

$$s = \frac{r - 1}{3(q - (1/2))} = \frac{-12(b + 1)(\zeta t + l)l + 16l}{12(b + 1)(\zeta t + l)l - 9(b + 1)^2(\zeta t + l)^3}. \quad (44)$$

The nature of (r, s) relies on the chosen parameters b, ζ, and l. The parameter of statefinder becomes $1 - ((6l(b + 1) - 8)/((b + 1)^2 l^2))$, $((-12l(b + 1) + 16)/(12(b + 1)l - 9(b + 1) l^2))$. Hence, at late time, the model approaches the ΛCDM model based on suitable choices of b. Figure 5 reveals the nature of the statefinder $r - s$ on the plane. The diagnosis of each DE model was done by comparing ΛCDM limit. The resulting plane obtained by plotting r versus s is shown in Figure 5, and it shows the current model corresponds to ΛCDM limit at late times.

5. Conclusion

The bulk viscous fluid takes part a significant role in the explanation of the DE model. Hence, I have obtained a Bianchi type-I cosmological model filled with bulk viscous and DE fluid. The field equations have been solved for the corresponding model using the hybrid scale factor. It is observed that the volume rose in size with cosmic time, and the constancy of parameters H and θ at late times indicate unchanging in spatial universe expansion. This leads to the nature of viscosity of bulk which plays a role at late times and mimics a DE behavior.

Some behavior of the physical cosmological parameters is investigated through their graphical representation. Here, we have the following interesting observations in the model:

(1) The energy densities of bulk viscous (ρ) and dark energy (ρ_{de}) are always positive and decrease with cosmic time as shown in Figures 2 and 3 which leads to be a realistic model. The energy densities of bulk viscous and dark energy approach zero and positive constant at late times which shows the dominance of dark energy. Furthermore, in the constructed model, the obtained fact is that ρ_{de} dominates at the beginning and dominates later

(2) The behavior of deceleration parameter (q) reveals that the obtained model displays a transitional phase of the universe from early deceleration to present acceleration. From Figure 1, it is observed that the deceleration parameter approaches -1 at late times which is consistent with recent observations

(3) The behavior of EoS parameter indicates that the model starts in the phantom region and lies in quintessence in late times which is shown in Figure 4. For the three values of bulk viscous coefficients ($\varepsilon = -0.33$, -0.67, and -0.9), ω_{de} with $\varepsilon = -0.33$ lies in the quintessence region which reveals that the greater the bulk viscous coefficient value, the higher the probability of occurring ω_{de} in the range of observed regions of quintessence

(4) Based on the behavior of the evolutionary of the statefinder trajectory which is presented in Figure 5, I can put conclusion that the model meets the ΛCDM limit. The above conclusion shows that our model is in good agreement with the recent scenario to describe the dynamics of universe at present epoch

Data Availability

The data used to support the findings of this study are included within the article and are cited at relevant places within the text as references.

Conflicts of Interest

The author declares that there are no conflicts of interest.

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