Research Article

Analytical Study of the Holographic Superconductor from Higher Derivative Theory

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In this paper, we analytically study the holographic superconductor models with the high derivative (HD) coupling terms. Using the Sturm-Liouville (S-L) eigenvalue method, we perturbatively calculate the critical temperature. The analytical results are in good agreement with the numerical results. It confirms that the perturbative method in terms of the HD coupling parameters is available. Along the same line, we analytically calculate the value of the condensation near the critical temperature. We find that the phase transition is second order with mean field behavior, which is independent of the HD coupling parameters. Then, in the low-temperature limit, we also calculate the conductivity, which is qualitatively consistent with the numerical one. We find that the superconducting energy gap is proportional to the value of the condensation. But we note that since the condensation changes with the HD coupling parameters, as the function of the HD coupling parameters, the superconducting energy gap follows the same change trend as that of the condensation.

1. Introduction

The mechanism of the high-temperature superconductor is one of the long-standing important and fundamental issues in strongly correlated condensed matter physics. AdS/CFT (Anti-de Sitter/Conformal Field theory) correspondence [1–4] provides a powerful tool and novel mechanism to attack this problem. Great progresses have been made, and the first holographic superconductor model has been constructed in [5]. This model exhibits appealing characteristics, one of which is the superconducting energy gap $\omega_g/T_c \approx 8$. This value roughly approximates that measured in high-temperature superconductor materials [6]. It is in contrast to the one of $\omega_g/T_c \approx 3.5$ from weakly coupled BCS theory.

Lots of works on the holographic superconductor have been fully explored (see [7–9] and references therein). An interesting holographic superconductor model is first constructed in one class of 4-derivative theory [10], which

involves the coupling between the Maxwell field and a Weyl tensor, and after that, lots of extended studies on the holographic superconductor in 4-derivative theory framework are explored in [11–21]. Compared with the usual holographic superconductor, the superconducting energy gap ω_g/T_c runs from 6, approaching the one of weakly coupled BCS theory, to 10, which is beyond that of the usual holographic superconductor and shed a light on the study of the high-temperature superconducting energy gap [10, 11, 20] (The running of superconducting energy gap is also observed in the Gauss-Bonnet (GB) gravity [22, 23] and the quasitopological gravity [24, 25], in which ω_g/T_c is always greater than 8.). Furthermore, the holographic superconductor and its properties, including the superconducting energy gap and Homes' law, have also been studied in 6-derivative theory, where the Maxwell fields couple more Weyl tensor [26]. There are wider superconducting energy gap proximately ranging from 5.5 to 16. And in certain range of parameters of the holographic superconductor models in

both 4- and 6-derivative theory frameworks, the experimental results of Homes' law can be satisfied.

Most of the studies are implemented numerically. To back up the numerical results and especially explore the mechanism behind these phenomena, we can resort to the analytical methods. The analytical matching method is developed in [22, 23, 27–29] to calculate the critical temperature and critical magnetic field. But the validity depends on the choice of the matching point. A particular case is that the analytical method is invalid for the GB holographic superconductor with zero scalar mass since the curvature correction term does not contribute to the analytic approximation [23]. A new analytic procedure by using the second-order Sturm-Liouville (S-L) method is developed to work out the critical temperature in [30], in which the analytic result is in good agreement with the numerical one. And then, this procedure is widely applied to other holographic models and proved to be powerful (see [31–49] and references therein). In addition, the optical conductivity can be also worked out analytically [30]. By this way, the superconducting energy gap can be estimated analytically, which is in good agreement with the numerical result.

In this paper, we will analytically study the holographic superconductor model from higher derivative (HD) theory [10, 26] by using the S-L variational method. The plan of this work is organized as follows. In Section 2, we present a brief introduction of the holographic superconductor models from HD theory. By using the S-L method, we analytically work out the critical temperature of the superconducting phase transition in Section 3. In Section 4, the condensation near the critical temperature is also analytically obtained. Then, we analytically derive the low frequency conductivity in the low-temperature limit in Section 5. In particular, the superconducting energy gap is approximately worked out. Finally, the conclusion and discussion are presented in Section 6.

2. The Holographic Superconductor from HD Theory

In this section, we present a brief review on the holographic superconductor from HD theory. For more details, please refer to [26]. We shall work in the probe limit. So, we shall start with the background geometry of the 4-dimensional SS-AdS black brane:

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + \frac{r^2}{L^2}\left(dx^2 + dy^2\right),$$
$$f(r) = \frac{r^2}{L^2}\left(1 - \frac{r_+^3}{r^3}\right), \tag{1}$$

where r_+ is the horizon of the black brane while the conformal AdS boundary locates at $r \longrightarrow \infty$. The Hawking temperature of this black brane is

$$T = \frac{3r_+}{4\pi L^2}. \tag{2}$$

On top of the above fixed background, we consider the actions including the charged complex scalar field Ψ and the Maxwell field strength $F_{\mu\nu}$ coupling with the Weyl tensor as follows:

$$S_A = \int d^4x \sqrt{-g}\left(-\frac{L^2}{8g_F^2}F_{\mu\nu}X^{\mu\nu\rho\sigma}F_{\rho\sigma}\right), \tag{3}$$

$$S_\Psi = \int d^4x \sqrt{-g}\left(-\left|D_\mu\psi\right|^2 - m^2|\psi|^2\right). \tag{4}$$

In the action S_A, $F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$ with A_μ being the $U(1)$ gauge field. The tensor X comprises an infinite family of HD terms [50] (The HD coupling terms are introduced by the Weyl tensor, which vanishes at the conformal AdS boundary. Such HD coupling terms can give lots of interesting results. For instance, they break the electromagnetic (EM) self-duality in 4 dimensional spacetimes, which corresponds to the particle-vortex duality in the boundary theory [51–56]. In addition, the similar coupling terms can be also introduced to construct the holographic quantum critical phase (QCP) [57–59]. When the backreaction is included, we can construct the metal insulator phase transition [55, 60] and explore the effect on the chaos, the holographic entanglement, the thermodynamics, and the holographic thermalization from the Weyl tensor [60–64]. We can also construct the holographic superconductors with the HD couplings of an $U(1)$ field to a scalar field as in [65–67]. This type of holographic superconductor model exhibits a rich phase structure, and in particular, the transition from the normal to the superconducting phase can be tuned to be of first order or of second order depending of the coupling parameters.):

$$\begin{aligned}X_{\mu\nu}{}^{\rho\sigma} = {}&I_{\mu\nu}{}^{\rho\sigma} - 8\gamma_{1,1}L^2C_{\mu\nu}{}^{\rho\sigma} - 4L^4\gamma_{2,1}C^2I_{\mu\nu}{}^{\rho\sigma}\\&- 8L^4\gamma_{2,2}C_{\mu\nu}{}^{\alpha\beta}C_{\alpha\beta}{}^{\rho\sigma} - 4L^6\gamma_{3,1}C^3I_{\mu\nu}{}^{\rho\sigma}\\&- 8L^6\gamma_{3,2}C^2C_{\mu\nu}{}^{\rho\sigma} - 8L^6\gamma_{3,3}C_{\mu\nu}{}^{\alpha_1\beta_1}C_{\alpha_1\beta_1}{}^{\alpha_2\beta_2}C_{\alpha_2\beta_2}{}^{\rho\sigma}+\cdots,\end{aligned}$$
$$\tag{5}$$

where $I_{\mu\nu}{}^{\rho\sigma} = 2\delta_\mu{}^{[\rho}\delta_\nu{}^{\sigma]}$ is an identity matrix and $C^n = C_{\mu\nu}{}^{\alpha_1\beta_1}C_{\alpha_1\beta_1}{}^{\alpha_2\beta_2}\cdots C_{\alpha_{n-1}\beta_{n-1}}^{\mu\nu}$. The factor of L in Equations (3) and (5) is introduced so that the coupling parameters g_F and $\gamma_{i,j}$ are dimensionless. But for later convenience, we shall set $L = 1$ and $g_F = 1$ below. When $X_{\mu\nu}{}^{\rho\sigma} = I_{\mu\nu}{}^{\rho\sigma}$, the action S_A is just the standard Maxwell theory. For convenience, we write $\gamma_{1,1} = \gamma$ and $\gamma_{2,i} = \gamma_i$ ($i = 1, 2$). In this paper, we shall truncate the X tensor up to the square of the Weyl tensor, which constructs 4- or 6-derivative theory. Since the effect of both 6-derivative terms, i.e., γ_1 and γ_2 terms, is similar [26, 50], it is enough to study γ and γ_1 terms in this paper. Note that in the SS-AdS black brane background, the parameters γ and γ_1 are constrained in the regions $-1/12 \leq \gamma \leq 1/12$ [51, 68] and $\gamma_1 \leq 1/48$ [50], respectively, when other parameters are turned off. Here, we also study the superconductivity in these parameter spaces. Ψ in the action S_Ψ is the charged complex scalar field, which has mass m and charge

q of the gauge field A. It is convenient to write Ψ as $\Psi = \psi e^{i\theta}$, where ψ and θ are the real scalar field and the Stückelberg field, respectively. The covariant derivative D_μ is defined as $D_\mu = \partial_\mu - iqA_\mu$. Choosing the gauge $\theta = 0$, we obtain the EOMs of the gauge field and scalar field as

$$\nabla_\nu \left(X^{\mu\nu\rho\sigma} F_{\rho\sigma} \right) - 4q^2 A^\mu \psi^2 = 0, \tag{6}$$

$$\left[\nabla^2 - \left(m^2 + q^2 A^2 \right) \right] \psi = 0. \tag{7}$$

Further, we assume the following form for both the gauge field and the scalar field as

$$A = (\phi(r), 0, 0, 0),$$
$$\psi = \psi(r), \tag{8}$$

which are only the function of the radial coordinate r. Under this ansatz, the EOMs (6) can be explicitly written as

$$\psi''(r) + \frac{4r^3 - r_+^3}{r^4 - rr_+^3} \psi'(r) - \frac{m^2 \left(r^4 - rr_+^3 \right) - r^2 \phi^2(r)}{\left(r^3 - r_+^3 \right)^2} \psi(r) = 0,$$

$$\phi''(r) + \frac{2 \left(r^6 + 4r^3 r_+^3 \gamma + 96 r_+^6 \gamma_1 \right)}{r^7 - 8r^4 r_+^3 \gamma - 48 rr_+^6 \gamma_1} \phi'(r)$$

$$- \frac{2q^2 r^7 \psi^2(r)}{\left(r^3 - r_+^3 \right) \left(r^6 - 8r^3 r_+^3 \gamma - 48 r_+^6 \gamma_1 \right)} \phi(r) = 0, \tag{9}$$

where the prime denotes the derivative with respect to r. It is more convenient to work in the coordinate $u = r_+/r$, in which $u = 1$ and $u = 0$ are the horizon and the AdS boundary, respectively. Then, the above EOMs become

$$\psi''(u) - \frac{2 + u^3}{u - u^4} \psi'(u) + \frac{m^2 r_+^2 \left(u^3 - 1 \right) + q^2 u^2 \phi^2(u)}{r_+^2 u^2 \left(u^3 - 1 \right)^2} \psi(u) = 0, \tag{10}$$

$$\phi''(u) - \frac{24u^2 \left(\gamma + 12u^3 \gamma_1 \right)}{1 - 8u^3 \gamma - 48u^6 \gamma_1} \phi'(u)$$

$$- \frac{2q^2 \psi^2(u)}{u^2 \left(1 - u^3 \right) \left(1 - 8u^3 \gamma - 48u^6 \gamma_1 \right)} \phi(u) = 0, \tag{11}$$

where the prime now represents the derivative with respect to u. We shall fix the mass of the scalar field to $m^2 = -2$ in this paper. So, the asymptotical behaviors for both ψ and φ at the boundary $u \longrightarrow 0$ are

$$\phi = \mu - \frac{\rho}{r_+} u, \tag{12}$$

$$\psi = u\psi_1 + u^2 \psi_2, \tag{13}$$

where μ and ρ are interpreted as the chemical potential and charge density of the dual boundary field theory, respectively. According to the AdS/CFT correspondence,

either ψ_1 or ψ_2 will act as a condensation operator while the other will be identified as a source. Here, we treat ψ_1 as the source and ψ_2 as the expectation value, which is denoted as $\psi_2 = \langle \mathcal{O}_+ \rangle$.

3. The Critical Temperature

In this section, following the SL method proposed in [30], we analytically work out the critical temperature T_c for the superconducting phase transition. Note that through this paper, we set $q = 1$.

To this end, we consider the system approaching the phase transition point, for which $T \longrightarrow T_c$ such that we can approximately set $\psi = 0$. Then, the Maxwell EOM (11) reduces to

$$\phi''(u) - \frac{24u^2 \left(\gamma + 12u^3 \gamma_1 \right)}{1 - 8u^3 \gamma - 48u^6 \gamma_1} = 0. \tag{14}$$

Since the introduction of the HD coupling term, it is hard to directly obtain the analytic solution of $\phi(u)$ as that in [30]. Alternatively, by treating the coupling parameters γ and γ_1 as the small quantities, we can solve perturbatively the above equation order by order. (Since for the allowed region of the coupling parameters, the denominator $1 - 8u^3 \gamma - 48u^6 \gamma_1$ in Equation (14) is larger than zero, it is safe to solve perturbatively this equation in terms of the order of the coupling parameters.) Up to the first order of γ and γ_1, we obtain $\phi(u)$ as

$$\phi(u) = -(1 - u)\mathcal{C} - 2\gamma \left(1 - u^4 \right) \mathcal{C} - \frac{48}{7} \gamma_1 \mathcal{C} \left(1 - u^7 \right), \tag{15}$$

where C is a constant of integration. When $\gamma = \gamma_1 = 0$, $\phi(u) = C(u - 1)$, which reduces to that without the Weyl coupling term as [30]. The integration constant C can be determined by the UV asymptotic behavior of $\phi(u)$ (Equation (12)), which gives

$$\mathcal{C} = -\frac{\rho}{r_+}. \tag{16}$$

It is convenient to introduce $\lambda = \rho/r_{+c}^2$, for which r_{+c} is the radius of the horizon at $T = T_c$. Then, the solution for $\phi(u)$ (15) can be rewritten as

$$\phi(u) = \lambda r_{+c} \mathcal{C} (1 - u) \left[1 + 2\gamma \xi(u) + \frac{48\gamma_1}{7} \xi_1(u) \right], \tag{17}$$

where $\xi(u) = 1 + u + u^2 + u^3$ and $\xi_1(u) = 1 + u + u^2 + u^3 + u^4 + u^5 + u^6$.

After the solution of $\phi(u)$ is at hand, we turn to solve Equation (10) for the scalar field $\psi(u)$. Near the critical temperature ($T \longrightarrow T_c$), Equation (10) becomes

$$\psi''(u) + \frac{u^3 + 2}{u(u^3 - 1)}\psi'(u) + \frac{1}{u^3 - 1}$$
$$\cdot \left[-\frac{2}{u^2} + \frac{\lambda^2(u-1)(7 + 14\gamma\zeta(u) + 48\gamma_1\zeta_1(u))^2}{49(u^2 + u + 1)}\right]\psi(u) = 0,$$

$$(18)$$

where we have used the solution of $\phi(u)$, i.e., Equation (17). In order to match the behavior at the conformal AdS boundary, we introduce the form for $\psi(u)$ as

$$\psi(u) = \frac{\langle \mathcal{O}_+ \rangle}{\sqrt{2}r_+^2} u^2 F(u). \tag{19}$$

$F(u)$ is introduced in the above equation as a trial function, for which we shall take the formula as

$$F(u) = 1 - \alpha u^2, \tag{20}$$

where α is the parameter to be determined. It satisfies the boundary conditions $F(0) = 1$ and $F'(0) = 0$. Inserting Equation (19) into Equation (18), one obtains a second-order S-L self-adjoint differential equation for $F(u)$ as

$$(u^2 - u^5)F''(u) + (2u - 5u^4)F'(u) - 4u^3F(u)$$
$$- \frac{\lambda^2 u^2 (7 + 14\gamma + 48\gamma_1 - 7u - 14\gamma u^4 - 48\gamma_1 u^7)^2}{49(u^3 - 1)}F(u) = 0.$$

$$(21)$$

The S-L theory indicates that the eigenvalue λ^2 minimizes the expression [37]:

$$\lambda^2 = \frac{\int_0^1 \left\{T(u)\left[F'(u)\right]^2 + P(u)[F(u)]^2\right\}du}{\int_0^1 Q(u)[F(u)]^2 du}, \tag{22}$$

where

$$T(u) = u^2 - u^5,$$
$$P(u) = 4u^3,$$
$$Q(u) = -\frac{u^2 (7 + 14\gamma + 48\gamma_1 - 7u - 14\gamma u^4 - 48\gamma_1 u^7)^2}{49(u^3 - 1)}.$$

$$(23)$$

Then, the critical temperature $\widehat{T}_c \equiv T_c/\sqrt{\rho}$ can be given as

$$\widehat{T}_c = \frac{3}{4\pi\sqrt{\lambda}}. \tag{24}$$

The strategy for calculating the critical temperature is to minimize the expression (22) of the eigenvalue λ^2 with respect to the coefficient α, and then, substituting the

Table 1: The analytical and numerical results for the critical temperature of the holographic superconductors from the 4-derivative theory.

γ	−1/12	−1/24	0	1/24	1/12
$\widehat{T}_c\big|_{\text{Analytical}}$	0.103	0.109	0.117	0.129	0.149
$\widehat{T}_c\big|_{\text{Numerical}}$	0.105	0.111	0.118	0.130	0.150

Table 2: The analytical and numerical results for the critical temperature of the holographic superconductors from the 6-derivative theory.

γ_1	1/48	0	−1/48
$\widehat{T}_c\big|_{\text{Analytical}}$	0.170	0.117	0.106
$\widehat{T}_c\big|_{\text{Numerical}}$	0.183	0.118	0.107

value of λ into Equation (24), we can obtain the value of $\widehat{T}_c$.

When the HD coupling terms vanish, i.e., $\gamma = \gamma_1 = 0$, the eigenvalue λ^2 can be analytically worked out as

$$\lambda^2\big|_{\gamma=0,\,\gamma_1=0}$$
$$= \frac{48\alpha^2 - 80\alpha + 60}{10\sqrt{3}\pi(\alpha^2 - 1) + 30(3 - \ln(3)) - 3\alpha^2(7 + 10\ln(3) + \alpha(130 - 60\ln(9)))}. \tag{25}$$

Its minimum can be achieved at $\alpha \approx 0.6016$. The corresponding minimum value of λ^2 is $\lambda^2 \approx 17.309$. Therefore, the critical temperature $\widehat{T}_c$ reads as $\widehat{T}_c = 0.11704$. This analytical result is in very good agreement with the numerical result: $\widehat{T}_c = 0.118$ [5].

But when $\gamma \neq 0$ or $\gamma_1 \neq 0$, it is hard to work out the analytical expression of λ^2. Therefore, we resort to the perturbative calculation. For the 4-derivative term, we obtain the value of λ^2 up to the order of γ as $\mathcal{O}(\gamma^6)$, while for the 6-derivative term, we obtain its value up to the order of γ_1 as $\mathcal{O}(\gamma_1^8)$. Finally, we present the results for 4- and 6-derivative theories in Tables 1 and 2, respectively. For comparison, we also give the corresponding numerical results in Tables 1 and 2. From the two tables, we see that the analytical results we worked out here are in very good agreement with the numerical ones. It confirms that the perturbative method to calculate the value of λ^2 is available.

4. Condensation

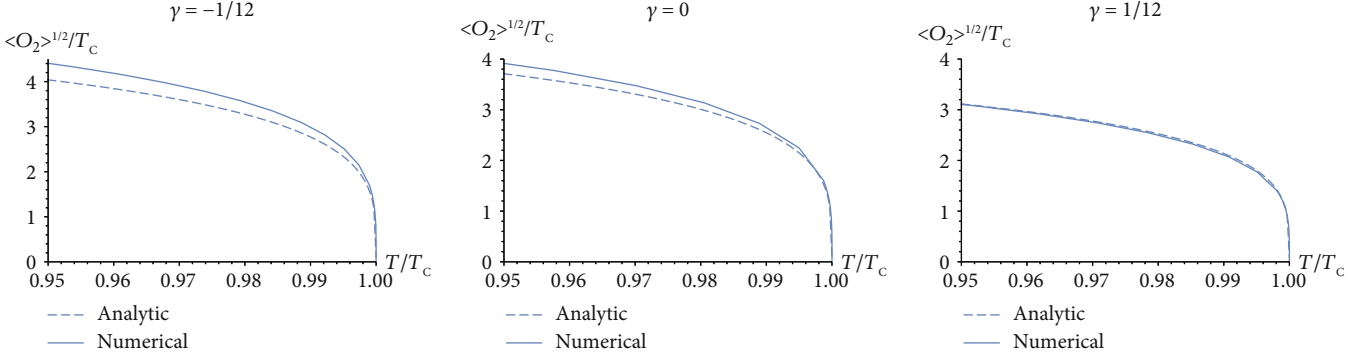In this section, we aim to investigate the condensation operator near the phase transition region. Away from

FIGURE 1: The condensation from 4-derivative theory for different γ obtained analytically and numerically, respectively.

(but close to) the critical temperature T_c, field Equation (11) for ϕ can be rewritten as

$$\phi''(u) + \frac{24u^2\gamma}{(8u^3\gamma - 1)}\phi'(u) - \frac{\langle\mathcal{O}_+\rangle^2 u^2 F^2(u)}{r_+^4(u^3 - 1)(8u^3\gamma - 1)}\phi(u) = 0. \tag{26}$$

Near the phase transition region, the condensation $\langle\mathcal{O}_+\rangle^2 / r_+^4$ is very small. Thus, we can expand $\phi(u)$ perturbatively in terms of the condensation $\langle\mathcal{O}_+\rangle^2 / r_+^4$ as

$$\frac{\phi(u)}{r_+} = \lambda(1 - u)\left[1 + 2\gamma\xi(u) + \frac{48}{7}\gamma_1\xi_1(u)\right] + \frac{\langle\mathcal{O}_+\rangle^2}{r_+^4}\chi(u) + \cdots, \tag{27}$$

where $\chi(u)$ satisfies the boundary condition $\chi(1) = \chi'(1) = 0$. Substituting Equation (27) into Equation (26), we obtain the differential equation for $\chi(u)$ up to the first order of $\langle\mathcal{O}_+\rangle^2 / r_+^4$:

$$\left[\left(1 - 8u^3\gamma - 48u^6\gamma_1\right)\chi'(u)\right]' = \lambda\frac{u^2(7 + 14\gamma\xi(u) + 48\gamma_1\xi_1(u))}{7(u^2 + u + 1)}F(u)^2. \tag{28}$$

Integrating both sides of the above equation in the interval [0,1] with the boundary condition, we obtain

$$\chi'(0) = -\lambda\mathcal{A}, \tag{29}$$

where

$$\mathcal{A} = \int_0^1 \frac{u^2(7 + 14\gamma\xi(u) + 48\gamma_1\xi_1(u))}{7(u^2 + u + 1)}F(u)^2 du. \tag{30}$$

Near the boundary $u = 0$, combining Equations (12) and (27), one has

$$\frac{\mu}{r_+} - \frac{\rho}{r_+^2}u = \lambda(1 - u)\left[1 + 2\gamma\xi(u) + \frac{48}{7}\gamma_1\xi_1(u)\right] + \frac{\langle\mathcal{O}_+\rangle^2}{r_+^4}\left[\chi(0) + u\chi'(0) + \cdots\right]. \tag{31}$$

Comparing the coefficient of u from both sides of the equation, we obtain

$$\frac{\rho}{r_+^2} = \lambda - \frac{\langle\mathcal{O}_+\rangle^2}{r_+^4}\chi'(0) = \lambda\left(1 + \frac{\langle\mathcal{O}_+\rangle^2}{r_+^4}\mathcal{A}\right). \tag{32}$$

Therefore, the condensate near the critical temperature is

$$\langle\mathcal{O}_+\rangle = \kappa T_c^2 \sqrt{1 - \frac{T}{T_c}}, \tag{33}$$

where

$$\kappa = \left(\frac{4\pi}{3}\right)^2 \frac{2}{\sqrt{\mathcal{A}}}. \tag{34}$$

It indicates that the phase transition is second order with mean field behavior, which is independent of the HD coupling parameters. It analytically confirms the results numerically obtained in our previous works [10, 26].

Finally, we also give the analytical and numerical condensation for samples γ and γ_1 in Figures 1 and 2, respectively. Near the critical temperature, the analytical results match well with the numerical ones.

5. The Conductivity at Low-Temperature Region

In this section, we turn to study the conductivity at the low-temperature region. When temperature $T \longrightarrow 0$, the dominant contribution comes from the neighborhood of the boundary ($u = 0$). So, we make a rescaling $\psi(bu)$ and $\phi(bu)$,
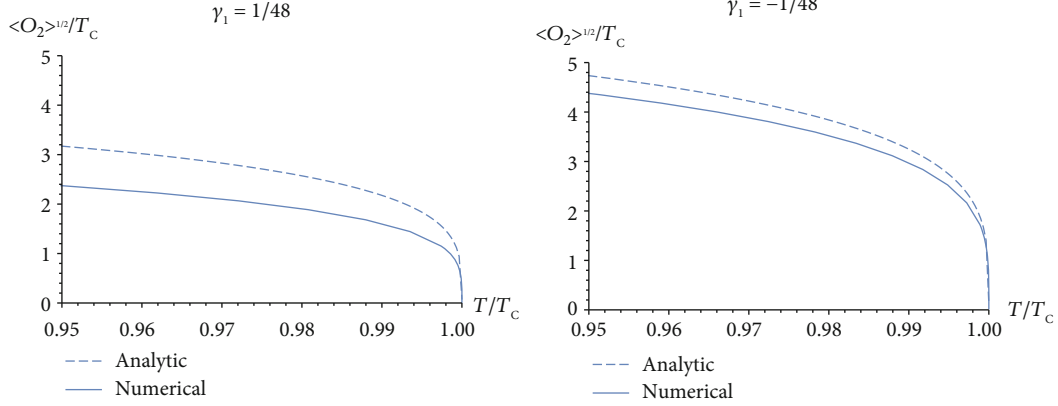
FIGURE 2: The condensation from 6-derivative theory for different γ_1 obtained analytically and numerically, respectively.

and then, $u \longrightarrow u/b$ with letting $b \longrightarrow \infty$. Thus, the field Equations (10) and (11) can be simplified as

$$F''(u) + \frac{2}{u} F'(u) + \frac{\phi^2(u)}{r_+^2} F(u) = 0, \qquad (35)$$

$$\phi''(u) - \frac{\langle O_+ \rangle^2 u^2}{r_+^4} F^2(u)\phi(u) = 0, \qquad (36)$$

where we have used Equation (19) and restored the original coordinate u. We shall solve the above equations subject to the following boundary conditions:

$$3F'(1) + 4F(1) = 0, \quad F(0) = 1, \ F'(0) = 0. \qquad (37)$$

Equation (37) is the natural boundary condition at the horizon, which can be directly deduced from Equation (21).

To proceed, we assume that in the asymptotic regime closing to the conformal boundary, $F(u)$ takes the following power law formula:

$$F(u) \approx \frac{\beta}{bu}, \qquad (38)$$

where β is the parameter to be determined. Then, the solution of field Equation (36) for $\phi(u)$ is

$$\phi(u) = \mathcal{B} r_+ \sqrt{bu} K_{1/2}(bu), \qquad (39)$$

$$b^2 = \frac{\langle \mathcal{O}_+ \rangle \beta}{r_+^2}, \qquad (40)$$

where $\mathcal{B}$ is the integral constant and $K_n(z)$ is the modified Bessel function of the second kind with n denoting the order of the corresponding Bessel function. Near the boundary $u = 0$, using Equation (12), the ratio ρ/r_+^2 can be estimated as

$$\frac{\rho}{r_+^2} = -\frac{\Gamma(-(1/2))}{2^{3/2}} \mathcal{B} b = \sqrt{\frac{\pi}{2}} \mathcal{B} b. \qquad (41)$$

Substituting Equation (39) into Equation (35) and rescaling $u \longrightarrow u/b$, we have

$$F'' + \frac{2}{u} F' + \tilde{\mathcal{B}}^2 u (K_{1/2}(u))^2 F = 0, \quad \tilde{\mathcal{B}} = \frac{\mathcal{B}}{b}. \qquad (42)$$

The above equation is the second-order S-L self-adjoint differential equation. It should be solved in the interval $(0, \infty)$ subject to the boundary condition $F(0) = 1$, $F'(0) = 0$, and $F \longrightarrow 0$ as $u \longrightarrow \infty$. The expression for estimating the minimum eigenvalue of $\tilde{\mathcal{B}}$ is provided by

$$\tilde{\mathcal{B}}^2 = \frac{\int_0^\infty \left[u F'(u) \right]^2 du}{\int_0^\infty u^3 K_{1/2}^2(u) F^2(u) du}. \qquad (43)$$

In order to connect smoothly the asymptotic regime depicted by the power law formula (38) with the boundary condition $F(0) = 1$, we introduce the following trial function:

$$F_\beta(u) = \left(\frac{\beta}{u}\right) \tanh\left(\frac{u}{\beta}\right). \qquad (44)$$

Then, we can minimize expression (43) to fix the value of β. It is easy to find that we have the minimum $\tilde{\mathcal{B}} \approx 1.92$ at $\beta \approx 0.8$.

Once the solution for the scalar field ψ is at hand, we can turn to study the optical conductivity. To this end, we switch on the perturbations of the gauge field along x direction, $A_x (r, t) = A(r) e^{-i\omega t}$. And then, the perturbative gauge field equation can be read as

$$A''(u) + \frac{3u^2 \left(1 - \gamma(4 - 8u^3) - 48\gamma_1(3u^3 - 2)u^3\right)}{(u^3 - 1)(1) + 4\gamma u^3 - 48\gamma_1 u^6} A'(u)$$
$$+ \left[\frac{\omega^2}{r_+^2 (u^3 - 1)^2} + \frac{2\psi(u)^2}{u^2(u^3 - 1)(1 + 4\gamma u^3 - 48\gamma_1 u^6)} \right] A(u) = 0.$$
$$(45)$$

To proceed, we reexpress the above equation as the following form:

$$\left[r_+ \left(1 - u^3 \right) \left(1 + 4\gamma u^3 - 48\gamma_1 u^6 \right) A'(u) \right]'$$
$$+ \frac{\omega^2 \left(1 + 4\gamma u^3 - 48\gamma_1 u^6 \right)}{r_+ \left(1 - u^3 \right)} A(u) = \frac{2r + \psi^2(u)}{u^2} A(u).$$

(46)

We want to recast the above equation as a Schrödinger form. So, we move to the tortoise coordinate, which is defined by

$$r_* = \int \frac{dr}{f(r)} = \frac{1}{6r_+} \left[\ln \frac{(1-u)^3}{1-u^3} - 2\sqrt{3} \tan^{-1} \frac{\sqrt{3}u}{u+2} \right]. \quad (47)$$

The integration constant has been calculated from the boundary condition that $r_* = 0$ at $u = 0$. Then, Equation (46) becomes

$$\frac{d^2 \Phi}{dr_*^2} + \left[\omega^2 - V_{\text{eff}} \right] \Phi = 0, \quad (48)$$

where $\Phi(u) = \sqrt{1 + 4\gamma u^3 - 48\gamma_1 u^6} A(u)$, and the effective potential V_{eff} is

$$V_{\text{eff}} = \frac{2f\psi^2}{-48\gamma_1 u^6 + 4\gamma u^3 + 1} + \frac{6u^3 \left(2\gamma^2 \left(7u^3 - 1 \right) u^3 - \gamma \left(5u^3 - 2 \right) \left(144\gamma_1 u^6 - 1 \right) + 24\gamma_1 u^3 \left(240\gamma_1 u^9 - 96\gamma_1 u^6 - 8u^3 + 5 \right) \right)}{\left(1 + 4\gamma u^3 - 48\gamma_1 u^6 \right)^2} f.$$

(49)

The wave Equation (48) is to be solved subject to ingoing boundary condition at the horizon, i.e., the ω-dependent part of the equation for $V_{\text{eff}} = 0$. The solution reads

$$\Phi(u) \sim e^{-i\omega r_*} \sim (1 - u)^{-i\omega/3r_+}. \quad (50)$$

So, at low frequency, to account for the boundary condition at the horizon, we define

$$A = \frac{(1-u)^{-i\omega/3r_+}}{\sqrt{1 + 4\gamma u^3 - 48\gamma_1 u^6}} \mathscr{G}(u), \quad (51)$$

where $\mathscr{G}(u)$ is regular at the horizon ($u = 1$). Then, Equation (45) becomes

$$\left[\frac{18u \left(24\gamma_1 u^3 \left(30\gamma u^6 + (8 - 12\gamma)u^3 + 48\gamma_1 \left(2 - 5u^3 \right) u^6 - 5 \right) + \gamma \left(-14\gamma u^6 + (2\gamma - 5)u^3 + 2 \right) \right)}{\left(-48\gamma_1 u^6 + 4\gamma u^3 + 1 \right)^2} \right] \mathscr{G}$$
$$+ \left[\frac{6\psi(u)^2}{u^2 \left(1 + 4\gamma u^3 - 48\gamma_1 u^6 \right)} - \frac{i(2u+1)\omega}{r_+} - \frac{(u+2) \left(u^2 + u + 4 \right) \omega^2}{3r_+^2 \left(u^2 + u + 1 \right)} \right] \mathscr{G} - 3 \left(1 - u^3 \right) \mathscr{G}'' + \left[9u^2 - 2 \left(1 + u + u^2 \right) \frac{i\omega}{r_+} \right] \mathscr{G}' = 0.$$

(52)

We can expand the wave function $\mathscr{G}(u)$ in a Taylor series at the horizon, which gives the boundary condition as

$$(3r_+ - 2i\omega)\mathscr{G}'(1) + \left[\frac{2r_+ \left(-9\gamma + 216\gamma_1 + \psi(1)^2 \right)}{4\gamma - 48\gamma_1 + 1} - \frac{2\omega^2}{3r_+} - i\omega \right] \mathscr{G}(1) = 0.$$

(53)

At low temperature, it is convenient to solve Equation (52) by letting $u \longrightarrow u/b$ with $b \longrightarrow \infty$. Then, Equation (52) becomes

$$3\mathscr{G}''(u) + \frac{2i\omega}{r+} \mathscr{G}'(u) - \left[3b^2 \tanh \left(\frac{bu}{\beta} \right)^2 - \frac{8\omega^2}{3r_+^2} \right] \mathscr{G}(u) = 0.$$

(54)

The general solution of Equation (54) can be given in terms of the Legendre function:

$$\mathcal{G}(u) \approx \left(\frac{1 - \tanh (bu/\beta)}{1 + \tanh (bu/\beta)} \right)^{1\omega\beta/6br_+}$$
$$\cdot \left[c + P^{+\beta}_{-(1/2)+(1/2)\sqrt{1+4\beta^2}} \left(\tanh \frac{bu}{\beta} \right) \right.$$
$$\left. + c_- P^{-\beta}_{-(1/2)+(1/2)\sqrt{1+4\beta^2}} \left(\tanh \frac{bu}{\beta} \right) \right], \tag{55}$$

where c_+ and c_- are the integration constants, for which we shall derive them in terms of the boundary condition at the horizon below.

At $u = 1$, we have $\tanh (bu/\beta) \approx 1$, the Legendre functions become

$$P^{\pm\beta}_{-(1/2)+(1/2)\sqrt{1+4\beta^2}} \left(\tanh \frac{bu}{\beta} \right) \approx \frac{2^{\pm\beta/2}}{\Gamma(1\mp\beta)} \left(1 - \tanh \frac{bu}{\beta} \right)^{\mp(\beta/2)}. \tag{56}$$

Therefore, we obtain

$$\mathcal{G}(1) \approx \left[\frac{c_+}{\Gamma(1-\beta)} e^{+b} + \frac{c_-}{\Gamma(1+\beta)} e^{-b} \right] e^{-(i\omega/3r_+)},$$
$$\mathcal{G}'(1) \approx \left[\frac{c_+(b-(i\omega/3r_+))}{\Gamma(1-\beta)} e^{+b} - \frac{c_-(b+(i\omega/3r_+))}{\Gamma(1+\beta)} e^{-b} \right] e^{-(i\omega/3r_+)}. \tag{57}$$

Substituting $\mathcal{G}(1)$ and $\mathcal{G}'(1)$ in the boundary condition (53), we obtain the ratio (c_+/c_-) as

$$\frac{c_+}{c_-} = -e^{-2b} \frac{\Gamma(1-\beta)}{\Gamma(1+\beta)} \mathcal{M}, \tag{58}$$

where

$$\mathcal{M} = \left[\frac{(b^4 - 3b) - 6(3 + 2b)\gamma + 144(3 + b)\gamma_1}{(b^4 + 3b) - 6(3 - 2b)\gamma + 144(3 - b)\gamma_1} \right.$$
$$\left. + \frac{4i\omega b(1 + 4\gamma - 48\gamma_1)(b^4 - 30\gamma - 3 + 576\gamma_1)}{r_+(b^4 + 3b(1 + 4\gamma - 48\gamma_1) - 18(\gamma - 24\gamma_1))^2} \right]. \tag{59}$$

According the definition of conductivity and AdS/CFT correspondence, one has

$$\sigma(\omega) \approx i \frac{\sqrt{\langle \mathcal{O}_+ \rangle}}{\omega} \frac{0.47 - 0.66(c_+/c_-)}{0.85 - 0.30(c_+/c_-)}. \tag{60}$$

So, in the limit of the low frequency ($\omega \longrightarrow 0$), we have

$$\Re\sigma(\omega) \sim e^{-2b} = e^{-Eg/T},$$
$$\Im\sigma(\omega) \approx 0.55 \frac{\sqrt{\langle \mathcal{O}_+ \rangle}}{\omega}. \tag{61}$$

E_g is identified to be the superconducting energy gap, which is determined as

$$E_g \approx \frac{3\sqrt{\beta\langle \mathcal{O}_+ \rangle}}{2\pi} \approx 0.43\sqrt{\langle \mathcal{O}_+ \rangle}. \tag{62}$$

This result reveals that the superconducting energy gap is proportional to the value of the condensation. But we note that since the condensation changes with the HD coupling parameters, as the function of the HD coupling parameters, the superconducting energy gap follows the same change trend as that of the condensation. This analytical result is qualitatively consistent with the numerical one in [10, 26].

Since the introduction of the HD term, which complicates the EOM, we cannot analytically obtain the conductivity as the function of ω at $T \longrightarrow$ as [29]. In the future, we can seek new methods to do this thing. For example, we can study the conductivity as the function of ω at zero temperature by using the semianalytical methods as [69].

6. Conclusions and Discussions

In this paper, we have analytically studied the holographic superconductor models from the HD theory. To achieve this goal, we use the Sturm-Liouville (S-L) eigenvalue method, which has been widely used in holographic models. Different from the usual holographic superconductor models, we cannot derive the analytical expression for the eigenvalue λ^2 due to the introduction of the HD coupling terms. Instead, we develop the perturbative method in terms of the HD coupling parameters to calculate the eigenvalue λ^2 and so the critical temperature. We find that the analytical results are in good agreement with the numerical results, which confirms that the perturbative method is available. Along the same line, we calculate the value of the condensation near the critical temperature. We find that the phase transition is second order with mean field behavior, which is independent of the HD coupling parameters. It analytically confirms the results numerically obtained in our previous works [10, 26]. We also calculate the conductivity in the low-temperature limit, which is qualitatively consistent with the numerical one [10, 26]. We find that the superconducting energy gap is proportional to the value of the condensation. But we note that since the condensation changes with the HD coupling parameters, as the function of the HD coupling parameters, the superconducting energy gap follows the same change trend as that of the condensation.

Data Availability

No data were used to support this study.

Conflicts of Interest

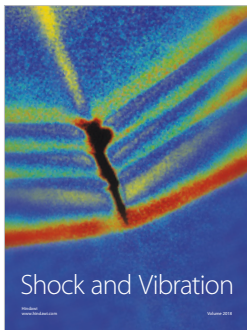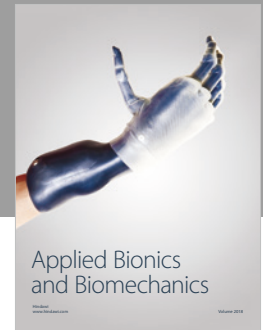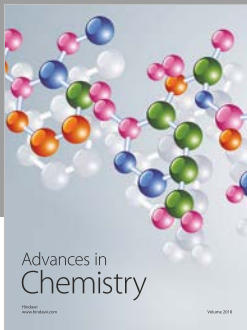The authors declare that they have no conflicts of interest.

Acknowledgments

References

[1] J. M. Maldacena, "The large-N limit of superconformal field theories and supergravity," *International Journal of Theoretical Physics*, vol. 38, no. 4, pp. 1113–1133, 1999.

[2] S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, "Gauge theory correlators from non-critical string theory," *Physics Letters B*, vol. 428, no. 1-2, pp. 105–114, 1998.

[3] E. Witten, "Anti-de sitter space and holography," *Advances in Theoretical and Mathematical Physics*, vol. 2, 1998.

[4] O. Aharony, S. S. Gubser, J. Maldacena, H. Ooguri, and Y. Oz, "Large N field theories, string theory and gravity," *Physics Reports*, vol. 323, no. 3-4, pp. 183–386, 2000.

[5] S. A. Hartnoll, C. P. Herzog, and G. T. Horowitz, "Building a holographic superconductor," *Physical Review Letters*, vol. 101, no. 3, 2008.

[6] K. K. Gomes, A. N. Pasupathy, A. Pushp, S. Ono, Y. Ando, and A. Yazdani, "Visualizing pair formation on the atomic scale in the high-Tc superconductor Bi2Sr2CaCu2O8+δ," *Nature*, vol. 447, no. 7144, pp. 569–572, 2007.

[7] S. A. Hartnoll, C. P. Herzog, and G. T. Horowitz, "Holographic superconductors," *Journal of High Energy Physics*, vol. 2008, no. 12, 2008.

[8] G. T. Horowitz, "Introduction to holographic superconductors," in *From Gravity to Thermal Gauge Theories: The AdS/CFT Correspondence*, pp. 313–347, Springer, Berlin, Heidelberg, 2011.

[9] R. G. Cai, L. Li, L. F. Li, and R. Q. Yang, "Introduction to holographic superconductor models," *Science China - Physics Mechanics & Astronomy*, vol. 58, no. 6, pp. 1–46, 2015.

[10] J. P. Wu, Y. Cao, X. M. Kuang, and W. J. Li, "The 3+1 -holographic superconductor with Weyl corrections," *Physics Letters B*, vol. 697, no. 2, pp. 153–158, 2011.

[11] D. Z. Ma, Y. Cao, and J. P. Wu, "The Stückelberg holographic superconductors with Weyl corrections," *Physics Letters B*, vol. 704, no. 5, pp. 604–611, 2011.

[12] D. Momeni and M. R. Setare, "A note on holographic superconductors with Weyl corrections," *Modern Physics Letters A*, vol. 26, no. 38, pp. 2889–2898, 2011.

[13] D. Momeni, N. Majd, and R. Myrzakulov, "p-wave holographic superconductors with Weyl corrections," *Europhysics Letters*, vol. 97, no. 6, p. 61001, 2012.

[14] D. Momeni, M. R. Setare, and R. Myrzakulov, "Condensation of the scalar field with Stuckelberg and Weyl corrections in the background of a planar AdS–Schwarzschild black Hole," *International Journal of Modern Physics A: Particles and Fields; Gravitation; Cosmology; Nuclear Physics*, vol. 27, no. 22, p. 1250128, 2012.

[15] D. Roychowdhury, "Effect of external magnetic field on holographic superconductors in presence of nonlinear corrections," *Physical Review D*, vol. 86, no. 10, p. 106009, 2012.

[16] Z. Zhao, Q. Pan, and J. Jing, "Holographic insulator/superconductor phase transition with Weyl corrections," *Physics Letters B*, vol. 719, no. 4-5, pp. 440–447, 2013.

[17] D. Momeni, R. Myrzakulov, and M. Raza, "Holographic superconductors with Weyl corrections via gauge/gravity duality," *International Journal of Modern Physics A: Particles and Fields; Gravitation; Cosmology; Nuclear Physics*, vol. 28, no. 19, p. 1350096, 2013.

[18] D. Momeni, M. Raza, and R. Myrzakulov, "Holographic superconductors with Weyl corrections," *International Journal of Geometric Methods in Modern Physics*, vol. 13, no. 01, p. 1550131, 2016.

[19] L. Zhang, Q. Pan, and J. Jing, "Holographic p-wave superconductor models with Weyl corrections," *Physics Letters B*, vol. 743, pp. 104–111, 2015.

[20] S. H. Mansoori, B. Mirza, A. Mokhtari, F. L. Dezaki, and Z. Sherkatghanad, "Weyl holographic superconductor in the Lifshitz black hole background," *Journal of High Energy Physics*, vol. 2016, no. 7, 2016.

[21] Y. Ling and X. Zheng, "Holographic superconductor with momentum relaxation and Weyl correction," *Nuclear Physics B*, vol. 917, pp. 1–18, 2017.

[22] R. Gregory, S. Kanno, and J. Soda, "Holographic superconductors with higher curvature corrections," *Journal of High Energy Physics*, vol. 2009, no. 10, 2009.

[23] Q. Pan, B. Wang, E. Papantonopoulos, J. de Oliveira, and A. B. Pavan, "Holographic superconductors with various condensates in Einstein-Gauss-Bonnet gravity," *Physical Review D*, vol. 81, no. 10, 2010.

[24] X. M. Kuang, W. J. Li, and Y. Ling, "Holographic superconductors in quasi-topological gravity," *Journal of High Energy Physics*, vol. 2010, no. 12, 2010.

[25] X. M. Kuang, W. J. Li, and Y. Ling, "Holographic p-wave superconductors in quasi-topological gravity," *Classical and Quantum Gravity*, vol. 29, no. 8, p. 085015, 2012.

[26] J. P. Wu and P. Liu, "Holographic superconductivity from higher derivative theory," *Physics Letters B*, vol. 774, pp. 527–532, 2017.

[27] S. Kanno, "A note on Gauss–Bonnet holographic superconductors," *Classical and Quantum Gravity*, vol. 28, no. 12, p. 127001, 2011.

[28] X. H. Ge and H. Q. Leng, *Progress in Theoretical Physics*, vol. 128, no. 6, pp. 1211–1228, 2012.

[29] E. J. Chang, C. J. Chou, and Y. Yang, http://arxiv.org/abs/1812.04288.

[30] G. Siopsis and J. Therrien, "Analytic calculation of properties of holographic superconductors," *Journal of High Energy Physics*, vol. 2010, no. 5, 2010.

[31] X. H. Ge, B. Wang, S. F. Wu, and G. H. Yang, "Analytical study on holographic superconductors in external magnetic field," *Journal of High Energy Physics*, vol. 2010, no. 8, 2010.

[32] H. F. Li, R. G. Cai, and H. Q. Zhang, "Analytical studies on holographic superconductors in Gauss-Bonnet gravity," *Journal of High Energy Physics*, vol. 2011, no. 4, 2011.

[33] R. G. Cai, H. F. Li, and H. Q. Zhang, "Analytical studies on holographic insulator/superconductor phase transitions," *Physical Review D*, vol. 83, no. 12, p. 126007, 2011.

[34] Q. Pan, J. Jing, and B. Wang, "Analytical investigation of the phase transition between holographic insulator and superconductor in Gauss-Bonnet gravity," *Journal of High Energy Physics*, vol. 2011, no. 11, 2011.

[35] D. Momeni, E. Nakano, M. R. Setare, and W. Y. Wen, *International Journal of Modern Physics A*, vol. 28, no. 08, p. 1350024, 2013.

[36] J. A. Hutasoit, S. Ganguli, G. Siopsis, and J. Therrien, "Strongly coupled striped superconductor with large modulation," *Journal of High Energy Physics*, vol. 2012, no. 2, 2012.

[37] S. Gangopadhyay and D. Roychowdhury, "Analytic study of properties of holographic p-wave superconductors," *Journal of High Energy Physics*, vol. 2012, no. 8, 2012.

[38] S. Gangopadhyay and D. Roychowdhury, "Analytic study of Gauss-Bonnet holographic superconductors in Born-Infeld electrodynamics," *Journal of High Energy Physics*, vol. 2012, no. 5, 2012.

[39] Q. Pan, J. Jing, B. Wang, and S. Chen, "Analytical study on holographic superconductors with backreactions," *Journal of High Energy Physics*, vol. 2012, no. 6, 2012.

[40] S. Gangopadhyay, "Analytic study of properties of holographic superconductors away from the probe limit," *Physics Letters B*, vol. 724, no. 1-3, pp. 176–181, 2013.

[41] W. Yao and J. Jing, "Analytical study on holographic superconductors for Born-Infeld electrodynamics in Gauss-Bonnet gravity with backreactions," *Journal of High Energy Physics*, vol. 2013, no. 5, 2013.

[42] H. F. Li, "Further studies on holographic insulator/superconductor phase transitions from Sturm-Liouville eigenvalue problems," *Journal of High Energy Physics*, vol. 2013, no. 7, 2013.

[43] L. Nakonieczny, M. Rogatko, and K. I. Wysokinski, "Analytic investigation of holographic phase transitions influenced by dark matter sector," *Physical Review D*, vol. 92, no. 6, 2015.

[44] A. Sheykhi and F. Shaker, "Analytical study of holographic superconductor in Born–Infeld electrodynamics with backreaction," *Physics Letters B*, vol. 754, pp. 281–287, 2016.

[45] A. Sheykhi, H. R. Salahi, and A. Montakhab, "Analytical and numerical study of Gauss-Bonnet holographic superconductors with Power-Maxwell field," *Journal of High Energy Physics*, vol. 2016, no. 4, pp. 1–17, 2016.

[46] C. Lai, Q. Pan, J. Jing, and Y. Wang, *Physics Letters B*, vol. 757, pp. 65–72, 2016.

[47] D. Momeni, M. Raza, and R. Myrzakulov, "Analytical coexistence of s-, p-, (s + p)-phases of a holographic superconductor," *International Journal of Geometric Methods in Modern Physics*, vol. 12, no. 04, p. 1550048, 2015.

[48] D. Momeni, M. Raza, M. R. Setare, and R. Myrzakulov, *International Journal of Theoretical Physics*, vol. 52, no. 8, pp. 2773–2783, 2013.

[49] G. Siopsis, J. Therrien, and S. Musiri, "Holographic superconductors near the Breitenlohner–Freedman bound," *Classical and Quantum Gravity*, vol. 29, no. 8, 2012.

[50] W. Witczak-Krempa, "Quantum critical charge response from higher derivatives in holography," *Physical Review B*, vol. 89, no. 16, p. 161114, 2014.

[51] R. C. Myers, S. Sachdev, and A. Singh, "Holographic quantum critical transport without self-duality," *Physical Review D*, vol. 83, no. 6, 2011.

[52] J. P. Wu, X. M. Kuang, and G. Fu, "Momentum dissipation and holographic transport without self-duality," *The European Physical Journal C*, vol. 78, no. 8, p. 616, 2018.

[53] G. Fu, J. P. Wu, B. Xu, and J. Liu, "Holographic response from higher derivatives with homogeneous disorder," *Physics Letters B*, vol. 769, pp. 569–574, 2017.

[54] J. P. Wu and P. Liu, "Quasi-normal modes of holographic system with Weyl correction and momentum dissipation," *Physics Letters B*, vol. 780, pp. 616–621, 2018.

[55] J. P. Wu, "Transport phenomena and Weyl correction in effective holographic theory of momentum dissipation," *The European Physical Journal C*, vol. 78, no. 4, 2018.

[56] G. Fu and J. P. Wu, http://arxiv.org/abs/1812.11522.

[57] R. C. Myers, T. Sierens, and W. Witczak-Krempa, "A holographic model for quantum critical responses," *Journal of High Energy Physics*, vol. 2016, no. 5, 2016.

[58] A. Lucas, T. Sierens, and W. Witczak-Krempa, "Quantum critical response: from conformal perturbation theory to holography," *Journal of High Energy Physics*, vol. 2017, no. 7, 2017.

[59] J. P. Wu, "Holographic quantum critical conductivity from higher derivative electrodynamics," *Physics Letters B*, vol. 785, pp. 296–300, 2018.

[60] Y. Ling, P. Liu, J. P. Wu, and Z. Zhou, "Holographic metal-insulator transition in higher derivative gravity," *Physics Letters B*, vol. 766, pp. 41–48, 2017.

[61] W. J. Li, P. Liu, and J. P. Wu, "Weyl corrections to diffusion and chaos in holography," *Journal of High Energy Physics*, vol. 2018, no. 4, 2018.

[62] A. Dey, S. Mahapatra, and T. Sarkar, *Physical Review D*, vol. 94, no. 2, 2016.

[63] S. Mahapatra, "Thermodynamics, phase transition and quasinormal modes with Weyl corrections," *Journal of High Energy Physics*, vol. 2016, no. 4, pp. 1–33, 2016.

[64] A. Dey, S. Mahapatra, and T. Sarkar, "Holographic thermalization with Weyl corrections," *Journal of High Energy Physics*, vol. 2016, no. 1, 2016.

[65] X. M. Kuang, E. Papantonopoulos, G. Siopsis, and B. Wang, "Building a holographic superconductor with higher-derivative couplings," *Physical Review D*, vol. 88, no. 8, 2013.

[66] A. Dey, S. Mahapatra, and T. Sarkar, "Generalized holographic superconductors with higher derivative couplings," *Journal of High Energy Physics*, vol. 2014, no. 6, 2014.

[67] A. Dey, S. Mahapatra, and T. Sarkar, "Very general holographic superconductors and entanglement thermodynamics," *Journal of High Energy Physics*, vol. 2014, no. 12, 2014.

[68] A. Ritz and J. Ward, "Weyl corrections to holographic conductivity," *Physical Review D*, vol. 79, no. 6, article 066003, 2009.

[69] R. A. Konoplya and A. Zhidenko, "Holographic conductivity of zero temperature superconductors," *Physics Letters B*, vol. 686, no. 2-3, pp. 199–206, 2010.