

Research Article

Quantum Spectrum of Tachyonic Black Holes in a Brane-Anti-Brane System

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Recently, some authors have considered the quantum spectrum of black holes. This consideration is extended to tachyonic black holes in a brane-anti-brane system. In this study, black holes are constructed from two branes which are connected by a tachyonic tube. As the branes come closer to each other, they evolve and make a transition to thermal black branes. It will be shown that the spectrum of these black holes depends on the tachyonic potential and the separation distance between the branes. By decreasing the separation distance, more energy emerges and the spectrum of the black hole increases.

1. Introduction

Of late, some scientists suggested a more exact black hole effective temperature with reference to the quantum spectrum of black holes [1, 2]. This temperature includes both the nonthermal Hawking radiation and the radiation of subsequent Hawking quanta. In [1, 2], it was shown that the quantization depends on the quantum quasi-normal modes of the black hole, but there were certain approximations implicitly made in those calculations. In [3], Corda extended the previous calculations by removing these approximations, and obtained corrected expressions for the quantization and thereby also for the Bekenstein-Hawking entropy. Other researchers [4, 5] using different methods have also obtained the black hole spectrum. Motivated by these works, we consider the quantum spectrum of tachyonic black holes in a brane-anti-brane system. These black holes are constructed from a pair of branes and anti-branes which are connected by a tachyonic tube [6–8]. By decreasing the separation distance between branes, the tachyonic potential between them grows and the tachyonic black holes emit more spectra.

The outline of this paper is as follows: In Section 2, we calculate the quantum spectrum for tachyonic black holes, which are constructed from a brane, an anti-brane, and a tachyonic tube. In Section 3, we generalize this discussion

to thermal black holes. The last section is devoted to a summary and conclusion.

2. The Quantum Spectrum for Tachyonic Black Holes:

In this section, we will firstly consider a system of a brane and an anti-brane which are connected by a tachyonic tube. By increasing the tachyonic potential between the branes, this system evolves to a black hole. We will calculate the spectrum of this tachyonic black hole. In [3], it was shown that the entropy for the black hole is given by

$$S_{\text{BH}} = 4\pi \left(M^2 - \frac{n}{2} \right), \quad (1)$$

where n is the number of quantum states, and M is the energy of the black hole. Now, we wish to calculate the energy of the tachyonic black hole. For a black hole in the brane-anti-brane system, the total potential energy can be obtained by summing over the potentials of the branes and the spaces between them:

$$V_{\text{tot}} = V_{\text{brane}} + V_{\text{extra}}, \quad (2)$$

The extra potential is a function of the fields which can move between the branes. Such fields transmit forces between

the branes, playing a major role in the evolution of the black holes located on the branes. These fields turn out to be tachyons.

To build a tachyonic black hole in this theory and calculate the tachyonic potential, consider a set of $D3-\bar{D}3$ -brane pairs situated at $z_1 = l/2$ and $z_2 = -l/2$, respectively, as shown in Figure 1. z is the transverse coordinate to the branes, and σ is the radius on the world-volume. The induced metric on the brane is

$$\gamma_{ab}d\sigma^a d\sigma^b = -d\tau^2 + \left(1 + z'(\sigma)^2\right)d\sigma^2 + \sigma^2(d\theta^2 + \sin^2\theta d\phi^2). \quad (3)$$

For the case of a single $D3-\bar{D}3$ -brane pair with open string tachyon, the action is [9, 10]:

$$S_{\text{tot-extra}} = -\tau_3 \int d^9\sigma \sum_{i=1}^2 V(TA, l) e^{-\phi} \left(\sqrt{-\det A_i}\right) \quad (4)$$

$$\begin{aligned} (A_i)_{ab} = & \left(g_{MN} - \frac{TA^2 l^2}{Q} g_{Mz} g_{zN} \right) \partial_a x_i^M \partial_b x_i^N + F_{ab}^i \\ & + \frac{1}{2Q} ((D_a TA)(D_b TA)^* + (D_a TA)^*(D_b TA)) \\ & + il(g_{az} + \partial_a z_i g_{zz})(TA(D_b TA)^* - TA^*(D_b TA)) \\ & + il(TA(D_a TA)^* - TA^*(D_a TA))(g_{bz} + \partial_b z_i g_{zz}), \end{aligned} \quad (5)$$

where

$$\begin{aligned} Q &= 1 + TA^2 l^2 g_{zz}, \\ D_a TA &= \partial_a TA - i(A_{2,a} - A_{1,a})TA, \quad V(TA, l) \\ &= g_s V(TA) \sqrt{Q}, \\ e^\phi &= g_s \left(1 + \frac{R^4}{z^4}\right)^{-1/2}, \end{aligned} \quad (6)$$

The quantities ϕ , $A_{2,a}$, and F_{ab}^i are the dilaton field, gauge fields, and field strength, respectively, on the world-volume of the non-BPS brane. TA is the tachyon field, τ_3 the brane tension, and $V(TA)$ the tachyon potential. Indices a, b stand for the tangent directions of the D -branes, whereas the indices M, N run over the background ten-dimensional space-time directions. The indices $i=1$ and 2 represent the Dp -brane and the anti- Dp -brane, respectively. Then, the distance between the D -branes is given by $z_2 - z_1 = l$. In the above action, we use units such that $2\pi\alpha' = 1$.

In writing the action of the $D3$ -brane, we assume that that σ is dependant only on the tachyon field TA , and that the gauge fields are zero. Thus, in the region $r > R$ and $TA' \sim \text{constant}$, the action (5) is

$$S_{D3} \simeq -\frac{\tau_3}{g_s} \int dt \int d\sigma \sigma^2 V(TA) \left(\sqrt{D_{1,TA}} + \sqrt{D_{2,TA}}\right), \quad (7)$$

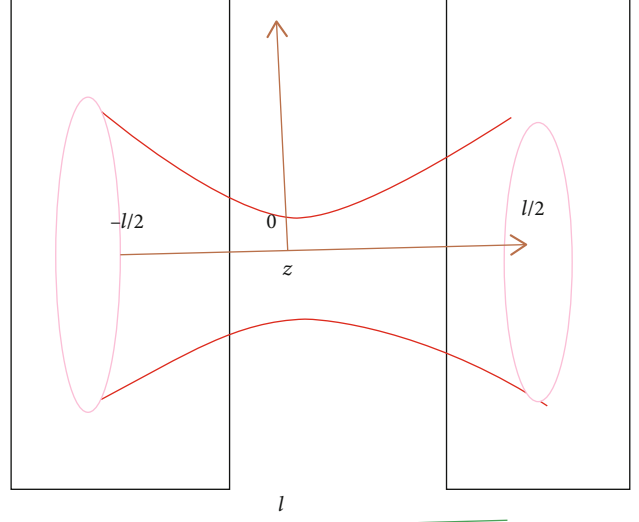


FIGURE 1: Pair of $D3-\bar{D}3$ -brane pairs at $z_1 = l/2$ and $z_2 = -l/2$.

where $D_{1,TA} = D_{2,TA} \equiv D_{TA}$, $V_3 = 4\pi^2/3$ is the volume of a unit sphere S^3 and

$$D_{TA} = 1 + \frac{l'(\sigma)^2}{4} + TA^2 l^2, \quad (8)$$

where a prime denotes a derivative with respect to σ . We make use of the potential [11–13]:

$$V(TA) = \frac{\tau_3}{\cosh \sqrt{\pi} TA}. \quad (9)$$

To calculate the energy-momentum tensor, we have to take the functional derivative of the action with respect to metric g_{MN} , i.e., $T^{MN} = (2/\sqrt{-\det g})(\delta S/\delta g_{MN})$. We get [7, 8],

$$T_{i,\text{brane}}^{00} = V(TA) \sqrt{D_i}, \quad (10)$$

After doing some calculations and using some approximations, we obtain

$$T_{i,\text{brane}}^{00} = \tau_3 + V_{\text{brane}}, \quad (11)$$

where

$$V_{\text{brane}} = \tau_3 \left[\frac{\sqrt{\pi} TA}{2} \right] \left[1 + e^{-2\sqrt{\pi} TA} \right]^{-1} \times \left[\frac{l'(\sigma)^2}{4} + TA^2 l^2 \right]. \quad (12)$$

This potential depends on the distance between the two branes and on the tachyon. The effects of the other branes have to be taken into account to get the change of the parameters with time. It will be shown that as the branes approach each other; the tachyons generate a wormhole connecting the branes which then transmits energy into the black hole from the extra dimensions.

Thus far, we have assumed that the tachyon field changes slowly ($TA \sim t^4/t^3 = t$), whilst neglecting $TA' = \partial TA/\partial \sigma$ and $\dot{TA} = \partial TA/\partial t$. Now, we consider the tachyon field to be changing rapidly as the distance between the brane and anti-brane decreases. So we cannot neglect TA' and \dot{TA} . A new wormhole forms. During this time, the black hole changes from a nonphantom phase to a new phantom phase. Thus, the phantom-dominated era of the black hole accelerates, ending up in a big-rip singularity. In such a case, the action (5) becomes:

$$L \simeq -\frac{\tau_3}{g_s} \int d\sigma \sigma^2 V(TA) \left(\sqrt{D_{1,TA}} + \sqrt{D_{2,TA}} \right), \quad (13)$$

where

$$D_{1,TA} = D_{2,TA} \equiv D_{TA} = 1 + \frac{l'(\sigma)^2}{4} + \dot{TA}^2 - TA'^2 + TA^2 l'^2, \quad (14)$$

and where it is assumed that $TA \ll TA'$. Next, the Hamiltonian related to the above Lagrangian is studied. The canonical momentum density is needed to derive the Hamiltonian, i.e., $\Pi = \partial L/\partial \dot{TA}$ associated with the tachyon, that is

$$\Pi = \frac{V(TA)\dot{TA}}{\sqrt{1 + \left(l'(\sigma)^2/4 \right) + \dot{TA}^2 - TA'^2}}, \quad (15)$$

and the Hamiltonian is

$$H_{\text{DBI}} = 4\pi \int d\sigma \sigma^2 \Pi \dot{TA} - L. \quad (16)$$

Now, we choose $\dot{TA} = 2TA'$, and obtain

$$H_{\text{DBI}} = 4\pi \int d\sigma \sigma^2 \left[\Pi \left(\dot{TA} - \frac{1}{2} TA' \right) \right] + \frac{1}{2} TA \partial_\sigma (\Pi \sigma^2) - L. \quad (17)$$

In the second step of the above equation, we have integrated the term proportional to \dot{TA} by parts. This indicates that the tachyon can be studied as a Lagrange multiplier by imposing the constraint $\partial_\sigma (\Pi \sigma^2 V(TA)) = 0$ on the canonical momentum. By solving the above equation, we get:

$$\Pi = \frac{\beta}{4\pi\sigma^2}, \quad (18)$$

where $\beta = \text{constant}$. By (16) and (18), we get:

$$H_{\text{DBI}} = \int d\sigma V(TA) \sqrt{1 + \frac{l'(\sigma)^2}{4} + \dot{TA}^2 + TA^2 l'^2} F_{\text{DBI}}, \quad (19)$$

$$F_{\text{DBI}} = \sigma^2 \sqrt{1 + \frac{\beta}{\sigma^4}}. \quad (20)$$

We then vary (20), and calculate the equation of motion for $l(\sigma)$:

$$\left(\frac{l' F_{\text{DBI}}}{4\sqrt{1 + (l'(\sigma)^2/4)}} \right) = 0. \quad (21)$$

The solution to this equation is

$$l(\sigma) = 4 \int_\sigma^\infty d\sigma \left(\frac{F_{\text{DBI}}(\sigma)}{F_{\text{DBI}}(\sigma_0)} - 1 \right)^{-1/2} = 4 \int_\sigma^\infty d\sigma' \left(\frac{\sqrt{\sigma_0^4 + \beta^2}}{\sqrt{\sigma'^4 - \sigma_0^4}} \right). \quad (22)$$

This solution represents a wormhole with a finite size throat for non-zero σ_0 , (see Figure 1). Using equations (15), (18), and (22) and assuming that $\dot{TA}^2 = TA'^2$, we obtain

$$TA \sim \int d\sigma \left[\frac{\beta}{4\pi\sigma^2} \right] \left[\sqrt{1 + \left[\frac{\sqrt{\sigma_0^4 + \beta^2}}{\sqrt{\sigma'^4 - \sigma_0^4}} \right]^2} \right]. \quad (23)$$

We see from this that the tachyons depend on the coordinates of the branes and the size of the throat of the wormhole. By decreasing the distance between the branes, the tachyons expand and more energy is transmitted from the extra dimensions into the brane and thus the black hole expands.

The potential between branes can be obtained from equation (20):

$$H_{\text{DBI}} = T + V_{\text{tot}} \quad (24)$$

$$V_{\text{tot}} \simeq \frac{3\tau_3}{\sigma^3} \int d\sigma V_{\text{brane}} F_{\text{DBI}}, \quad (25)$$

$$F_{\text{DBI}} = \sigma^2 \sqrt{1 + \frac{\beta}{\sigma^4}}. \quad (26)$$

The energy density may be calculated from equations (20) and (26):

$$T_{i,\text{brane}}^{00} = \int d\sigma T_{i,\text{brane+extra}}^{00} = \int d\sigma V(TA) \sqrt{D_{TA}} F_{\text{DBI}} \simeq V_{\text{tot}}, \quad (27)$$

where $T_{i,\text{brane}}^{00}$ is the energy of the brane and $T_{i,\text{brane+extra}}^{00}$ is the energy of the brane-anti-brane and tube. Putting the energy density in equation (27) equal to the energy density in equation (11), we obtain:

$$\tau_3 + V_{\text{brane}} = V_{\text{tot}}, \quad (28)$$

$$\tau_3 + V_{\text{brane}} = \frac{3\tau_3}{\sigma^3} \int d\sigma V_{\text{brane}} \sigma^2 \sqrt{1 + \frac{\beta}{\sigma^4}}. \quad (29)$$

By multiplying equation (29) by σ^3 , and by differentiating with respect to the cosmic time, we get

$$3\sigma^2\dot{\sigma}V_{\text{brane}} + \sigma^3\dot{V} = -3\tau_3\sigma^2\dot{\sigma} + 3\tau_3\sigma^2V_{\text{brane}}\dot{\sigma}\sqrt{1 + \frac{\beta}{\sigma^4}}. \quad (30)$$

For $\beta \ll 1$, we obtain

$$\frac{\dot{\sigma}}{\sigma} = \frac{-\dot{V}_{\text{brane}}}{3\tau_3 + [3 - 3\tau_3]V_{\text{brane}}}. \quad (31)$$

Solving equations (32), (22), (23), (29), and (31) simultaneously, we obtain

$$V_{\text{tot}} = \tau_3 + \tau_3 \left[\frac{\sqrt{\pi}[\beta/4\pi[t_0^2 - t^2]] \left[\sqrt{1 + \left[\left(\frac{\sqrt{t_0^4 + \beta^2}/\sqrt{t^4 - t_0^4}} \right)^2} \right]} \right]}{2} \right] \\ \times \left[1 + e^{-2\sqrt{\pi} \left[\frac{\beta}{4\pi[t_0^2 - t^2]} \right]} \left[\sqrt{1 + \left[\left(\frac{\sqrt{t_0^4 + \beta^2}}{\sqrt{t^4 - t_0^4}} \right)^2} \right]} \right] \right]^{-1} \\ \times \left[\frac{1}{4} \left[\left(\frac{\sqrt{t_0^4 + \beta^2}}{\sqrt{t_0^4 - t^4}} \right)^2 \right] + \left[\frac{\beta}{4\pi[t_0^2 - t^2]} \right] \left[\sqrt{1 + \left[\left(\frac{\sqrt{t_0^4 + \beta^2}}{\sqrt{t^4 - t_0^4}} \right)^2} \right]} \right]^2 \right] \\ \times \left[\left(\frac{\sqrt{t_0^4 + \beta^2}}{\sqrt{t_0^4 - t^4}} \right)^2 \right]. \quad (32)$$

Substituting energy (32) in equation (1), we obtain

$$S_{\text{BH}} = 4\pi \left((V_{\text{tot}})^2 - \frac{n}{2} \right), \quad (33) \\ T_{\text{tot}} = \frac{1}{4\pi V_{\text{tot}}}.$$

The above equations show that by decreasing the distance between branes, the tachyonic energy increases. This causes the quantum spectrum of the black hole to grow and the entropy increases.

3. The Quantum Spectrum of Thermal Tachyonic Black Branes

In this section, we will generalize the method in the previous section to thermal black branes. We will show that branes move with high acceleration towards each other. This acceleration produces a curved space-time and creates a horizon around the system. This causes the system to evolve and make a transition to a system of black branes.

To achieve these aims, we begin with the equation of motion for the tachyons as follows:

$$-\frac{\partial^2 \text{TA}}{\partial \tau^2} + \frac{\partial^2 \text{TA}}{\partial \sigma^2} = 0. \quad (34)$$

By using (31), we can write the following reparameterizations

$$\rho = \rho_0 \dot{\sigma}^2 = \rho_0 \frac{\sigma^2}{w^2}, \quad w = \frac{3\tau_3 + [3 - 3\tau_3]V_{\text{brane}}}{\dot{V}_{\text{brane}}}, \quad \bar{\tau} = \gamma \int_0^t d\tau \frac{w}{\sigma} - \gamma \frac{\sigma^2}{2}. \quad (35)$$

Using the above expression and doing the following calculations:

$$\left\{ \left[\left(\frac{\partial \bar{\tau}}{\partial \tau} \right)^2 - \left(\frac{\partial \bar{\tau}}{\partial \sigma} \right)^2 \right] \frac{\partial^2}{\partial \tau^2} + \left[\left(\frac{\partial \rho}{\partial \sigma} \right)^2 - \left(\frac{\partial \rho}{\partial \tau} \right)^2 \right] \frac{\partial^2}{\partial \rho^2} \right\} \text{TA} = 0, \quad (36)$$

we obtain

$$(-g)^{\frac{1}{2}} \frac{\partial}{\partial x_\mu} \left[(-g)^{\frac{1}{2}} g^{\mu\nu} \right] \frac{\partial}{\partial x_\nu} \text{TA} = 0, \quad (37)$$

where $x_1 = \rho$, $x_0 = \bar{\tau}$ and the metric elements become

$$g^{\bar{\tau}\bar{\tau}} \sim -\frac{1}{\beta^2} \left(\frac{w'}{w} \right)^2 \frac{\left(1 - (w/w')^2 (1/\sigma^4) \right)}{\left(1 + (w/w')^2 ((1 + \gamma^{-2})/\sigma^4) \right)^{1/2}} \\ = -\frac{1}{\beta^2} \left(\frac{1 + V_{\text{brane}} \dot{V}_{\text{brane}}^{-2}}{3\tau_3 + [3 - 3\tau_3]V_{\text{brane}}/\dot{V}_{\text{brane}}} \right)^2 \frac{\left(1 - \left((3\tau_3 + [3 - 3\tau_3]V_{\text{brane}}/\dot{V}_{\text{brane}})/1 + V_{\text{brane}} \dot{V}_{\text{brane}}^{-2} \right)^2 (1/\sigma^4) \right)}{\left(1 + \left((3\tau_3 + [3 - 3\tau_3]V_{\text{brane}}/\dot{V}_{\text{brane}})/1 + V_{\text{brane}} \dot{V}_{\text{brane}}^{-2} \right)^2 ((1 + \gamma^{-2})/\sigma^4) \right)^{1/2}} g^{\rho\rho} \\ \sim -(g^{\bar{\tau}\bar{\tau}})^{-1}, \quad (38)$$

where we have assumed $(\partial TA/\partial t = \partial TA/\partial \tau = 2(\partial TA/\partial \sigma))$.

Now, we can compare the elements with the line element of one black $D3$ -brane [14]:

$$ds^2 = D^{-1/2} \bar{H}^{-1/2} (-f dt^2 + dx_1^2) + D^{1/2} \bar{H}^{-1/2} (dx_2^2 + dx_3^2) + D^{-1/2} \bar{H}^{1/2} (f^{-1} dr^2 + r^2 d\Omega_5)^2, \quad (39)$$

where

$$f = 1 - \frac{r_0^4}{r^4}, \quad (40)$$

$$\bar{H} = 1 + \frac{r_0^4}{r^4} \sinh^2 \alpha, \quad (41)$$

$$D^{-1} = \cos^2 \varepsilon + H^{-1} \sin^2 \varepsilon, \quad (42)$$

$$\cos \varepsilon = \frac{1}{\sqrt{1 + (\beta^2/\sigma^4)}}. \quad (43)$$

Equations (38) and (43) lead to

$$\begin{aligned} f &= 1 - \frac{r_0^4}{r^4} \sim 1 - \left(\frac{w}{w'}\right)^2 \frac{1}{\sigma^4} \\ &= 1 - \left(\frac{3\tau_3 + [3 - 3\tau_3]V_{\text{brane}}/\dot{V}_{\text{brane}}}{1 + V_{\text{brane}}\dot{V}_{\text{brane}}^{-2}}\right)^2 \frac{1}{\sigma^4}, \\ \bar{H} &= 1 + \frac{r_0^4}{r^4} \sinh^2 \alpha \sim 1 + \left(\frac{w}{w'}\right)^2 \frac{(1 + \gamma^{-2})}{\sigma^4} \\ &= 1 + \left(\frac{3\tau_3 + [3 - 3\tau_3]V_{\text{brane}}/\dot{V}_{\text{brane}}}{1 + V_{\text{brane}}\dot{V}_{\text{brane}}^{-2}}\right)^2 \frac{(1 + \gamma^{-2})}{\sigma^4}, \\ D^{-1} &= \cos^2 \varepsilon + \bar{H}^{-1} \sin^2 \varepsilon \\ &\simeq 1 \Rightarrow r \sim \sigma, r_0 \sim \left(\frac{w}{w'}\right)^{\frac{1}{2}}, (1 + \gamma^{-2}) \sim \sinh^2 \alpha. \end{aligned} \quad (44)$$

The temperature of the BIon system is $T = 1/\pi r_0 \cosh \alpha$ [7]. As a result, the temperature of the brane-anti-brane system can be calculated as

$$\begin{aligned} T &= \frac{1}{\pi r_0 \cosh \alpha} = \frac{\gamma}{\pi} \left(\frac{w'}{w}\right)^{1/2} \\ &\sim \frac{\gamma}{\pi} \left(\frac{1 + V_{\text{brane}}\dot{V}_{\text{brane}}^{-2}}{3\tau_3 + [3 - 3\tau_3]V_{\text{brane}}/\dot{V}_{\text{brane}}}\right)^{1/2} \end{aligned} \quad (45)$$

The above equation shows that the temperature of thermal tachyonic black branes depends on the tachyonic potential and its changes with respect to time. By increasing the tachyonic potential, the temperature of the system grows and tends to large values.

Like the previous section, we obtain the energy by using the energy-momentum tensor for the black $D3$ -brane [7] and write

$$\begin{aligned} V_{\text{tot}} &= \int d\sigma T^{00} = \int d\sigma \frac{\pi^2}{2} T_{D3}^2 r_0^4 (5 + 4 \sinh^2 \alpha) \\ &\sim \frac{\pi^2}{2} T_{D3}^2 \left(\frac{w}{w'}\right)^{\frac{1}{2}} (9 + \gamma^{-2}) \\ &\sim \frac{\pi^2}{2} T_{D3}^2 \left(\frac{3\tau_3 + [3 - 3\tau_3]V_{\text{brane}}/\dot{V}_{\text{brane}}}{1 + V_{\text{brane}}\dot{V}_{\text{brane}}^{-2}}\right)^{\frac{1}{2}} (9 + \gamma^{-2}), \end{aligned} \quad (46)$$

Substituting the energy (46) in equation (1), we obtain

$$S_{\text{BH}} = 4\pi \left((V_{\text{tot}})^2 - \frac{n}{2} \right). \quad (47)$$

The above equation shows that the spectrum of the thermal tachyonic black branes depends on the tachyonic potential and its changes in terms of time. If the velocity of change in the tachyonic potential increases, branes move towards each other with high acceleration, and greater energy is produced in the system. This extra energy can be seen as the extra spectrum around black branes.

4. Results and Discussion

In this research, we have shown that by decreasing the separation between branes, the tachyonic energy increases. This causes the quantum spectrum of the black hole to grow and the entropy increases. Also, we have found that the spectrum of the thermal tachyonic black branes depends on the tachyonic potential and its change in terms of time. If the velocity of change in the tachyonic potential increases, branes move towards each other with high acceleration, and greater energy is produced in the system. This extra energy can be seen as the extra spectrum around black branes.

A question that arises is whether we could calculate the entropy of a Bohr-like black hole for this system? In the Bohr system, particles like electrons move around the black hole, and the entropy can be calculated by summing over all quantum states. As equation (37) shows, in a brane-anti-brane manifold, particles like scalar, Dirac, and tachyon fields move and experience a curved space-time. Also, the black hole entropy has a thermodynamical relation with the black holes mass:

$$S_{\text{BH}} \sim \frac{M_{\text{BH}}}{T_{\text{BH}}} + \dots, \quad (48)$$

where M_{BH} is the black hole mass, and T_{BH} is the black hole temperature. Usually, the black hole temperature has the relation below with the black hole mass:

$$T_{\text{BH}} \sim \frac{1}{M_{\text{BH}}} + \dots \quad (49)$$

Consequently, the black hole entropy has the relation below with the black hole mass:

$$S_{\text{BH}} \sim (M_{\text{BH}})^2 + \dots \quad (50)$$

The above result is semiclassical. However, by considering quantum states, we again arrive at equation (1). Thus, this model could be applied to tachyonic black holes also.

Another question that may arise is why we choose this special tachyonic potential. This is a very-well known potential in string theory, which is used in most studies for the action of brane-anti-branes. This action produces a potential with one minimum which is a stable result. Although there are more forms for the potentials [15–17], this potential yields better results. In addition, potentials with more than one minimum may affect the stability of the tachyonic black hole. In these conditions, a system may evolve from a state with one minimum tachyonic potential to another one, and many parameters change. These changes can make the system unstable. For these potentials, there are two horizons related to different minima of energy ($r_{\text{horizon1}} = x_1, r_{\text{horizon2}} = x_2$), and three regions ($x < x_1, x > x_2, x_2 < x < x_1$) appear. Some particles are confined between two horizons ($r_{\text{horizon2}} < x < r_{\text{horizon1}}$) and evolve between them. Also, two horizons ($r_{\text{horizon1}}, r_{\text{horizon2}}$) may interact with each other, change and the system becomes unstable.

5. Conclusions

In this research, we have obtained the quantum spectrum of tachyonic black holes in brane-anti-brane systems. These black holes are built from a pair of branes and anti-branes which are connected by a thermal tube. This tube is produced by a tachyonic potential between the branes. By decreasing the distance between the branes, the potential of the tachyons increase, and the energy of the system increases. Consequently, this black hole radiates extra energy, and the quantum spectrum of black hole increases.

Data Availability

No data were used to support this study.

Conflicts of Interest

The author declares that there is no conflict of interest regarding the publication of this paper. This research was performed as part of the employment of the author with Mangosuthu University of Technology, South Africa.

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