

## Research Article

# Fundamental Units of Measurement and Extra Dimensions

Askar Abdukadyrov 

Kyrgyz State Technical University, Bishkek 720044, Kyrgyzstan

Correspondence should be addressed to Askar Abdukadyrov; [abdukadyrov64@mail.ru](mailto:abdukadyrov64@mail.ru)

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The space available to our perception is three-dimensional with full evidence. The development of physics led to the hypothesis of extra dimensions. It is believed that an important role in the unification of physics should play by the Planck units of mass, length and time, built on the universal constants  $c$  (the speed of light in a vacuum),  $G$  (the gravitational constant), and  $\hbar$  (the reduced Planck constant). In August 2021, published work in which it is shown that the fundamental role in the unification of physics, in fact, was played by the Stoney units, built on the universal constants  $c - G - e$  or  $c - G - \hbar$  and  $\alpha$  (where  $e$  is the elementary electric charge, and  $\alpha$  is the fine-structure constant). Using this result, the presented work offers a possible solution to the riddle of extra dimensions; it is shown that any additional spatial dimension can be expressed in terms of the fundamental length or the product of the fundamental time and the speed of light in a vacuum.

## 1. Introduction

In everyday life, we use three dimensions of space: length, width, and height.

When physicists or mathematicians talk about dimensions, they mean the independent coordinates that are necessary to describe the position of any point in a given space. Usually, coordinates in three-dimensional space are denoted by  $x$ ,  $y$ , and  $z$  (Cartesian coordinates).

Einstein's theory of relativity treats time as a coordinate in the geometry of spacetime. Thus, since time ( $t$ ) is also a coordinate, instead of the three coordinates ( $x_1, x_2, x_3$ ) needed to describe a point in space, there are four coordinates for describing an event in spacetime. Four-dimensional (4D) spacetime is usually denoted by  $x$ ,  $y$ ,  $z$ , and  $ct$  (where  $c$  is the speed of light in a vacuum), or  $x_1, x_2, x_3, x_4$  or  $x^1, x^2, x^3, x^4$ .

After Einstein created the general theory of relativity (theory of gravity), the program of a unified field theory combining gravity and electromagnetism began to develop.

In 1921, the German mathematician Kaluza suggested that an ordinary four-dimensional spacetime has an extra spatial dimension, and in general, spacetime has five dimensions:  $x_1, x_2, x_3, x_4$ , and  $x_5$ . As a result, he was able to unite

the two fundamental forces of gravitation and electromagnetism in the five-dimensional (5D) theory [1].

However, the Kaluza theory remains mysterious in the sense that we do not perceive the fourth dimension of space at all. The space surrounding us remains three-dimensional with complete evidence and immutability. The question naturally arises: if the fourth dimension of space exists, then where is it?

In 1926, Swedish physicist Klein introduced the hypothesis that the additional fifth dimension was curled up and microscopic, namely, that the geometry of the fifth dimension could take the form of a circle, with the radius of  $10^{-30}$  cm [2, 3].

Thus, Klein gave the classical five-dimensional Kaluza theory a quantum interpretation; therefore, it is commonly called the Kaluza-Klein theory [4] and considered an important precursor to string theory in which point particles in particle physics are replaced by one-dimensional objects called strings. In string theory, the characteristic length scale of strings is assumed to be on the order of the Planck length, or  $10^{-35}$  meters.

String theory was considered in the late 1960s as a theory of strong nuclear interaction, before being abandoned in favor of quantum chromodynamics (QCD). Nevertheless,

it turned out that the string theory is a promising candidate for the quantum theory of gravity.

However, for string theory to be mathematically consistent, the strings must be in a universe of ten dimensions [5]. This contradicts the experience that our real universe has four dimensions. To get around the difficulty, string theorists added the explanation that the additional six dimensions exist but cannot be detected directly.

The earliest string theory describes the only bosons, particles like the photon. Studies have shown that string theory can include fermions in its spectrum [6]. Such string theories are now known as superstring theories.

In the mid-1990s, theoretical physicists have shown that different superstring theories represent different limiting cases of the 11 dimensional theory, which became known as M-theory [7]. However, so far, no experimental support of the M-theory exists.

In this paper, we suggest a possible solution to solve the mystery of the extra dimensions.

## 2. Method

Creation of a unified theory of fundamental interactions is a difficult task of modern physics.

It is believed that an important role in this theory should play the system of Planck units of mass ( $m_p$ ), length ( $l_p$ ), and time ( $t_p$ ), built on the universal constants  $c$ ,  $G$  (the gravitational constant), and  $\hbar$  (the reduced Planck constant), and originally proposed in 1899 by German physicist Planck [8]:

$$m_p = \left(\frac{\hbar c}{G}\right)^{1/2} = 2.176 \times 10^{-8} \text{ kg}, \quad (1)$$

$$l_p = \left(\frac{G\hbar}{c^3}\right)^{1/2} = 1.616 \times 10^{-35} \text{ m}, \quad (2)$$

$$t_p = \frac{l_p}{c} = \left(\frac{G\hbar}{c^5}\right)^{1/2} = 5.39 \times 10^{-44} \text{ s}. \quad (3)$$

The Planck length  $l_p$  is the so-called gravitational radius of the Planck mass  $R_{g(p)}$ :

$$l_p = R_{g(p)} = \frac{m_p G}{c^2} \quad (4)$$

(we insist that, in this case, the gravitational radius of the body  $R_g = mG/c^2$  should not be confused with its Schwarzschild radius  $R = 2R_g = 2mG/c^2$ ).

Therefore, it is believed that the Planck length  $l_p$  is uniquely related to the Planck mass  $m_p$ . However, this is an erroneous opinion.

In August 2021, was published the article [9] which shows the following. There is a mass,

$$m_a = m_p \cdot \alpha = \frac{(2.176 \times 10^{-8} \text{ kg})}{137} = 1.588 \times 10^{-10} \text{ kg}, \quad (5)$$

where  $\alpha = ke^2/\hbar c \approx 1/137$  is the fine-structure constant,  $e$  is the elementary electric charge, and  $k$  is the coefficient of proportionality.

A charged elementary particle of mass  $m_a$  has the so-called classical radius  $R_{0(a)}$  is equal to the gravitational radius of the Planck mass (i.e., the Planck length):

$$R_{0(a)} = \frac{ke^2}{m_a c^2} = \frac{\alpha m_p^2 G}{\alpha m_p c^2} = \frac{m_p G}{c^2} = R_{g(p)} \quad (6)$$

or

$$R_{g(p)} = l_p = R_{0(a)}, \quad (7)$$

where  $ke^2 = \alpha m_p^2 G$ .

Thus, the Planck length is associated with two completely different masses,  $m_p \approx 2 \times 10^{-8} \text{ kg}$  and  $m_a \approx 1.58 \times 10^{-10} \text{ kg}$ . Therefore, the Planck units are not necessarily the only unique units of measurement.

In the paper [9] is shown that the fundamental role in the unification of physics, in fact, is played by the system of the unit measurement built on the universal constants  $c - G - e$  (or  $c - G - \hbar$  and  $\alpha$ ):

$$m_s = \left(\frac{\alpha \hbar c}{G}\right)^{1/2} = \left(\frac{ke^2}{G}\right)^{1/2} = 1.859 \times 10^{-9} \text{ kg}, \quad (8)$$

$$l_0 = \left(\frac{\alpha G \hbar}{c^3}\right)^{1/2} = \left(\frac{ke^2 G}{c^4}\right)^{1/2} = 1.38 \times 10^{-36} \text{ m}, \quad (9)$$

$$t_0 = (\alpha G \hbar / c^5)^{1/2} = (ke^2 G / c^6)^{1/2} = 4.6 \times 10^{-45} \text{ s}. \quad (10)$$

Thus, in fact, we are talking about a system of natural units proposed in 1874 Irish scientist Stoney [10] (he is most famous for introducing the term electron as the "fundamental unit quantity of electricity").

In the indicated paper we showed that the mass  $m_s$  (the modern value of the Stoney mass) is boundary of the macrocosm and microcosm; in other words, this mass is the lower limit for the masses of ordinary bodies and the upper limit for the masses of the elementary particles. The length  $l_0$  and time  $t_0$  are the elementary length and elementary time, i.e., they are the smallest values of length and time that exist in nature.

As it has been stated before, Klein had believed that the geometry of the fifth dimension could take the form of a circle.

Developing Klein's idea, we consider that a physical point in three-dimensional (3D) space is actually a tiny ball with a radius  $r$  equal to the elementary length  $l_0$  [11]:  $r = l_0$ .

The distance  $s$  from the center of coordinates to the center of our point is the fourth coordinate  $x_4$ , which is determined by the Pythagorean theorem:

$$s = x_4 = (x_1^2 + x_2^2 + x_3^2)^{1/2} = ct. \quad (11)$$

The minimum distance  $ds$  by which the trial point can be brought closer to the coordinate center is the radius of this point  $r$ :  $ds = r$ .

Therefore, the fifth coordinate  $x_5$  is the *physical point radius*  $r$  equal to the fundamental length  $l_0$  and is the limit of the fourth coordinate  $x_4$ :

$$x_5 = r = \lim x_4 = l_0. \quad (12)$$

In the six-dimensional (6D) theory, which has all the advantages of a five-dimensional theory, the sixth coordinate  $x_6$  is also the limit of the fourth coordinate  $x_4$  but is expressed as the product of the fundamental time  $t_0$  and the speed of light  $c$ :

$$x_6 = r = \lim x_4 = \lim ct = ct_0. \quad (13)$$

Thus, in the six-dimensional theory, the fifth and sixth coordinates are equal to each other:

$$x_5 = l_0 = ct_0 = x_6. \quad (14)$$

It is clear that in the same way can be entered 6 additional dimensions (by 2 dimensions for each spatial axis).

If we add 6 additional dimensions to the four coordinates  $(x^1, x^2, x^3, x^4)$ , we get 10 dimensions of string theory:

$$x^5 = \lim x^1 = l_0, \quad x^6 = \lim x^1 = ct_0, \quad (15)$$

$$x^7 = \lim x^2 = l_0, \quad x^8 = \lim x^2 = ct_0, \quad (16)$$

$$x^9 = \lim x^3 = l_0, \quad x^{10} = \lim x^3 = ct_0. \quad (17)$$

If we add 6 additional dimensions to the five coordinates  $(x_1, x_2, x_3, x_4, x_5)$ , we get 11 dimensions of the M-theory:

$$x_6 = \lim x_1 = l_0, \quad x_7 = \lim x_1 = ct_0, \quad (18)$$

$$x_8 = \lim x_2 = l_0, \quad x_9 = \lim x_2 = ct_0, \quad (19)$$

$$x_{10} = \lim x_3 = l_0, \quad x_{11} = \lim x_3 = ct_0. \quad (20)$$

So, in our opinion, the mystery of additional dimensions is revealed.

### 3. Conclusions

The space available to our perception is three-dimensional (3D). Any additional spatial dimension  $x^D (D > 3)$  is the radius of the physical point  $r$ , which can be expressed in terms of the fundamental length  $l_0$  or the product of the fundamental time  $t_0$  and speed of light in a vacuum  $c$ :

$$x^D (D > 3) = r = l_0 \text{ or } x^D (D > 3) = r = ct_0. \quad (21)$$

### Data Availability

No data were used in this study.

### Conflicts of Interest

The author declares no competing interests.

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