

Research Article

A Possible Solution of the Cosmological Constant Problem Based on GW170817 and Planck Observations with Minimal Length Uncertainty

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We propose generalized uncertainty principle (GUP) with an additional term of quadratic momentum motivated by string theory and black hole physics and providing a quantum mechanical framework for the minimal length uncertainty, at the Planck scale. We demonstrate that the GUP parameter, β_0 , could be best constrained by the gravitational wave observations, GW170817 event. To determine the difference between the group velocity of graviton and that of the light, we suggest another proposal based on the modified dispersion relations (MDRs). We conclude that the upper bound of β_0 reads $\approx 10^{60}$. Utilizing features of the UV/IR correspondence and the apparent similarities between GUP (including nongravitating and gravitating impacts on Heisenberg uncertainty principle) and the discrepancy between the theoretical and the observed cosmological constant Λ (obviously manifesting gravitational influences on the vacuum energy density), known as *catastrophe of nongravitating vacuum*, we suggest a possible solution for this long-standing physical problem, $\Lambda \approx 10^{-47} \text{ GeV}^4/\hbar^3 c^3$.

1. Introduction

The cosmological constant, Λ , was introduced by Albert Einstein in 1917 to impose steady-state universe [1] that is neither expanding nor contracting. To counterbalance the effects of gravity, the cosmological constant was introduced to assure static evolution of the universe [2, 3]. This model was subsequently refuted, and accordingly, the Λ -term was abandoned from the Einstein field equation (EFE), especially after the confirmation of the celebrated Hubble observations in 1929 [4], which also have verified the consequences of Friedmann solutions for EFE, at vanishing Λ [5]. Nearly immediate after publishing GR, a matter-free solution for EFE with finite Λ -term was obtained by de Sitter [6]. Later on when it has been realized that the Einstein *static* universe was found unstable for small perturbations [7–9], it was argued that the inclusion of the Λ -term remarkably contributes to the stability of the universe and simultaneously sup-

ports its expansion, especially that the initial singularity of Friedmann-Lemaître-Robertson-Walker (FLRW) models could be improved, as well [10, 11]. Furthermore, the observations of type-Ia high redshift supernovae in the late nineteenth of the last century [12, 13] indicated that the expanding universe is also accelerating, especially at a small Λ -value, which obviously contributes to the cosmic negative pressure [14, 15]. With this regard, we recall that the cosmological constant can be related to the vacuum energy density, ρ , as $\Lambda = 8\pi G\rho/c^2$, where c is the speed of light in vacuum and G is the gravitational constant. In 2018, the Planck observations have provided us with a precise estimation of Λ , namely, $\Lambda_{\text{Planck}} \approx 10^{-47} \text{ GeV}^4/\hbar^3 c^3$ [16]. When comparing this tiny value with the theoretical estimation based on quantum field theory in weakly or nongravitating vacuum, $\Lambda_{\text{QFT}} \approx 10^{74} \text{ GeV}^4/\hbar^3 c^3$, there is, at least, a 121 orders of magnitude difference to be fixed [17–19].

The disagreement between both values is one of the greatest mysteries in physics and known as the cosmological constant problem or *catastrophe of nongravitating vacuum*. This problem represents one of the still puzzling question in physics [18, 20–23]. From different aspects, some possible solutions have been reported [22, 24–26]. In the present work, we utilize the generalized uncertainty principle (GUP), which is an extended version of Heisenberg uncertainty principle (HUP), where a correction term encompassing the gravitational impacts is added, and thus an alternative quantum gravity approach emerges [27, 28]. As a consequence of thought experiments [29], GUP comes up with an extra quadratic term in momentum resulted in various aspects of quantum gravity theories and becomes compatible with string theory and black hole physics [27, 28]; the minimal length uncertainty that was inspired from GUP can be related to the Planck length. Its value can be thought as a length discretization for quantum gravity [30–32]. The proposed minimal length uncertainty exhibits some features of the UV/IR correspondence [33–35], which has been performed in viewpoint of local quantum field theory in the limit of the Planck length restrictions. Thus, it is argued that the UV/IR correspondence is a relevant approach to revealing several aspects of short-distance physics, such as the cosmological constant problem [18, 36–38]. Therefore, a precise estimation of the minimal length uncertainty strongly depends on the proposed upper bound of the GUP parameter, β_0 [39, 40]. Various ratings for β_0 upper bound have been proposed, for example, by comparing quantum gravity corrections to various quantum phenomena with electroweak [41, 42] and astronomical observations [43, 44]. Accordingly, β_0 ranges between 10^{33} and 10^{78} [43–45]. So far, there are various quantum gravity approaches presenting quantum descriptions for different physical phenomena in presence of gravitational fields to be acknowledged here [27, 28].

To summarize, the present attempt is motivated by the similarity of GUP (including nongravitating and gravitating impacts on HUP) and the disagreement between theoretical and observed estimations for Λ (manifesting gravitational influences on the vacuum energy density) and by the remarkable impacts of Λ on early and late evolution of the universe [2, 3, 46]. As a preamble of the present study, we present a novel estimation for β_0 from the binary neutron stars merger, the gravitational wave event GW170817 reported by the Laser Interferometer Gravitational Wave Observatory (LIGO) and the Advanced Virgo collaborations [47]. With this regard, there are different efforts based on the features of the UV/IR correspondence in order to interpret the Λ problem [48–52] with Liouville theorem in the classical limit [48, 53, 54]. Having a novel estimation of β_0 , a solution of the Λ problem, *catastrophe of nongravitating vacuum*, could be best proposed. The upper bound of the dimensionless β_0 could be determined in general relatively and rainbow gravity. The latter is an alternative approach in the special relativity (DSR) [55, 56], in which the dispersion relations is modified. MDRs break the Lorentz symmetries to assure an invariant energy scale [57, 58]. The

geometry of spacetime is also dictated by energy of the cosmic background. As particle passes through space, its energy is interacting with space geometry and forming *rainbow gravity*, like light entering prism. The rainbow gravity is a generalization of DSR to the curved spacetime geometry and supported by various studies [59–62] and observations [63, 64] to integrate the gravitational impacts.

The present paper is organized as follows: Section 2 reviews the basic concepts of the GUP approach with quadratic momentum. The associated modifications of the energy-momentum dispersion relations related to GR and rainbow gravity are also outlined in this section. In Section 3, we show that the dimensionless GUP parameter, β_0 , could be, for instance, constrained to the gravitational wave event GW170817. Section 4 is devoted to calculating the vacuum energy density of states and shows how this contributes to understanding the cosmological constant problem with a quantum gravity approach, the GUP. The final conclusions are outlined in Section 5.

2. Generalized Uncertainty Principle and Modified Dispersion Relations

HUP emerges from commutative phase-space geometry, i.e., $[x_i, x_j] = 0$ and $[p_i, p_j] = 0$. As the energy approaches the Planck scale, HUP likely breaks down to GUP, where the gravitational impacts and noncommutative phase-space geometry are taken into consideration, i.e., $[x_i, x_j] \neq 0$ and $[p_i, p_j] \neq 0$. Various approaches to quantum gravity such as string theory, doubly special relativity, black hole physics help to model the nonzero minimal length uncertainty, which could be related to the Planck scale [27, 28]. The GUP was suggested as [65, 66]

$$\Delta x \Delta p \geq \frac{\hbar}{2} [1 + \alpha(\Delta x)^2 + \beta(\Delta p)^2 + \gamma], \quad (1)$$

where α , β , and γ are independent parameters. This expression leads to nonzero minimal uncertainty in both position and momentum. The corresponding commutation relation

$$[x, p] = i\hbar(1 + \alpha x^2 + \beta p^2), \quad (2)$$

was obtained from quantum group symmetric Heisenberg algebra and Bargmann-Fock representation [66]. It was pointed out that in quantum mechanics and from the sequences of $|\psi_n\rangle$ vector state, both position and momentum eigenstates $|x\rangle$ and $|p\rangle$ can be approximated to an arbitrary precision of increasing localization in position or momentum space [65].

For nonzero minimal uncertainty in position, $((\Delta x)_{|\psi\rangle}^2 = \langle \psi | (x - \langle \psi | x | \psi \rangle)^2 | \psi \rangle \geq \Delta x_0 \forall |\psi\rangle)$, no physical state would exist with such a position eigenstate [65, 66]. γ can be related to the expectation values of position and momentum, $\gamma = \alpha \langle x \rangle^2 + \beta \langle p \rangle^2$. A minimal position uncertainty means that the position operator is no longer self-adjoint and the Heisenberg algebra does not allow for Hilbert space

representation of the position space [65]. In light of this, we restrict ourselves to $\Delta x \neq 0$ and vanishing α [65], i.e., no minimal momentum uncertainty [65, 66]:

$$\Delta x \Delta p \geq \frac{\hbar}{2} [1 + \beta(\Delta p)^2 + \beta(p)^2], \quad (3)$$

where Δx and Δp are the uncertainties in position and momentum, respectively. It should be highlighted that although the KMM work [65] has been succeeded to determine the minimal length uncertainty, it does not consider the limitation on the upper bound of momentum. This would mean that the uncertainty in momentum is going to be divergent and in particular also infinite energy of free particle [65]. This failure has been processed in work of [67] in determination of the maximal test particle's momentum. We have reviewed this point in our reviews [27, 28]. This version of GUP was deduced from black hole physics [68], string theory [30], and supported by different *Gedanken* experiments [69].

The GUP parameter can be expressed as $\beta = \beta_0(\ell_p/\hbar)^2 = \beta_0/(M_p c)^2$, where β_0 is a dimensionless parameter, $\ell_p = 1.616 \times 10^{-35} m$ is the Planck length, and $M_p = 1.22 \times 10^{19} \text{ GeV}/c^2$ is the Planck mass. Equation (3) implies the existence of a minimum length uncertainty, which is related to the Planck scale, $\Delta x_{\min} \approx \hbar\sqrt{\beta} = \ell_p\sqrt{\beta_0}$. It should be noticed that the minimum length uncertainty exhibits features of the UV/IR correspondence [33–35]. Δx is obviously proportional to Δp , where large Δp (UV) becomes proportional to large Δx (IR). Equation (3) is a noncommutative relation, $[\hat{x}_i, \hat{p}_j] = \delta_{ij}\hbar[1 + \beta p^2]$, where both position and momentum operators can be defined as

$$\hat{x}_i = \hat{x}_{0i}, \hat{p}_j = \hat{p}_{0j}(1 + \beta p^2), \quad (4)$$

where \hat{x}_{0i} and \hat{p}_{0j} are the corresponding operators obtained from the canonical commutation relations $[\hat{x}_{0i}, \hat{p}_{0j}] = \delta_{ij}\hbar$ and $p^2 = g_{ij}p^{0i}p^{0j}$.

We can now construct MDR due to quadratic GUP. We start with the background metric in GR gravitational spacetime

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = g_{00}c^2 dt^2 + g_{ij}dx^i dx^j, \quad (5)$$

with $g_{\mu\nu}$ is the Minkowski spacetime metric tensor $(-, +, +, +)$. Accordingly, the modified four-momentum squared is given by

$$\begin{aligned} p_\mu p^\mu &= g_{\mu\mu}p^\mu p^\mu = g_{00}(p^0)^2 + g_{ij}p^{0i}p^{0j}(1 + \beta p^2) \\ &= -(p^0)^2 + p^2 + 2\beta p^2 \cdot p^2. \end{aligned} \quad (6)$$

Comparing this with the conventional dispersion relation, $p_\mu p^\mu = -m^2 c^2$, the time component of the momentum can be defined as $(p^0)^2 = m^2 c^2 + p^2(1 + \beta p^2)$. The energy of the test particle ω can be defined as $\omega/c = -\zeta_\mu p^\mu = -g_{\mu\nu}\zeta^\mu$

p^ν , where the killing vector is given as $\zeta^\mu = (1, 0, 0, 0)$. Therefore, ω could be expressed as $\omega = -g_{00}c(p^0) = c(p^0)$, and the modified dispersion relation in GR gravity with GUP reads

$$\omega^2 = m^2 c^4 + p^2 c^2 (1 + 2\beta p^2). \quad (7)$$

For $\beta \rightarrow 0$, the standard dispersion can be obtained. The rainbow gravity generalizes MDR in doubly special relativity to curved spacetime [70], where the geometry spacetime is explored by a test particle with energy ω [57, 58].

$$\omega^2 f_1\left(\frac{\omega}{\omega_p}\right)^2 - (pc)^2 f_2\left(\frac{\omega}{\omega_p}\right)^2 = (mc^2)^2, \quad (8)$$

where ω_p is the Planck energy and $f_1(\omega/\omega_p)$ and $f_2(\omega/\omega_p)$ are known as the rainbow functions, which are model depending. The rainbow functions can be defined as [63, 71]

$$f_1(\omega/\omega_p) = 1, f_2(\omega/\omega_p) = \sqrt{1 - \eta(\omega/\omega_p)^n}, \quad (9)$$

where η and n are free positive parameters. It was argued that for the logarithmic corrections of black hole entropy [72], the integer n is limited as $n = 1, 2$ [73]. Therefore, it would be eligible to assume that $n = 2$. Thus, MDR for rainbow gravity with GUP can be written as

$$\omega^2 = \frac{(mc^2)^2 + p^2 c^2 (1 + 2\beta p^2)}{1 + \eta [pc/\omega_p]^2 (1 + 2\beta p^2)}. \quad (10)$$

Again, as $\beta \rightarrow 0$, Equation (10) goes back to the standard dispersion relation.

We have constructed two different MDRs for quadratic GUP, namely, Equations (7) and (10) in GR with GUP and in rainbow gravity, respectively. Bounds on the GUP parameter from GW170817 shall be outlined in the section that follows.

3. Bounds on GUP Parameter from GW170817

Instead of violating Lorentz invariance [74], we intend to investigate the speed of the graviton from the GW170817 event. To this end, we use MDRs obtained from the quadratic GUP approaches, discussed in Section 2. Thus, defining an upper bound on the dimensionless GUP parameter β_0 for given bounds on mass and energy of the graviton, where $m_g 4.4 \times 10^{-22} \text{ eV}/c^2$ and $\omega_g = 8.5 \times 10^{-13} \text{ eV}$, respectively, plays an essential role. Assuming that the gravitational waves propagate as free waves, we could, therefore, determine the speed of the mediator, that of the graviton, from the group velocity of the accompanying wavefront, i.e., $v_g = \partial\omega/\partial p$, where ω and p are the energy and momentum of the graviton, respectively [75].

This section intends to define an upper bound on the dimensionless β_0 from the difference between the group velocity of the graviton and the speed of light. Thus, it can be deduced from the MDR approaches, i.e., the difference

between Equation (7) for GR gravity with GUP and Equation (10) for rainbow gravity.

- (i) For GR gravity with GUP: the group velocity, $v_g = \partial \omega / \partial p$, can be given as

$$v_g = \frac{pc^2}{\omega} (1 + 4\beta p^2) = c \left\{ \left[1 - \left(\frac{mc^2}{\omega_g} \right)^2 \right]^{1/2} + 4\beta \frac{\omega_g^2}{c^2} \left[1 - \left(\frac{mc^2}{\omega_g} \right)^2 \right]^{3/2} \right\}. \quad (11)$$

It is obvious that for $\beta \rightarrow 0$, i.e., in absence of GUP impacts, one can estimate the difference between the speed of light and that of the graviton as $|\delta v| = |c - v_g| 1.34 \times 10^{-19} c$. Although small difference is obtained, we are—in the era of gravitational wave (GW) observations—technically able to measure even a such tiny difference, which is apparently comparable with the range of $-3 \times 10^{-15} c < \delta v < +7 \times 10^{-16} c$ reported in ref. [76]. Furthermore, it is found that the GW speed $v_g \sim c$, where $(0.79 c, 1.01 c)$ is the interval due to recent GW observations [77].

In light of this, we could use the results deduced from the GW170817 event, such as the graviton velocity, to set an upper bound on β_0 . For a massless graviton, the difference between the speed of light and that of the graviton in presence of the GUP impacts reads

$$|\delta v_{\text{GUP}}| = \left| 4\beta \frac{\omega^2}{c} \right| = \left| 4\beta_0 \frac{\omega^2}{M_p^2 c^3} \right| \leq 1.941 \times 10^{-80} \beta_0 c. \quad (12)$$

Thus, the upper bound on β_0 of the quadratic GUP can be simply deduced from comparing $|\delta v_{\text{GUP}} / \delta v|$ with

$$\beta_0 \leq 6.89 \times 10^{60}. \quad (13)$$

- (ii) For rainbow gravity: when applying the quadratic GUP approach, Equation (10), the group velocity of the graviton due MDR of the rainbow gravity can be expressed as

$$v_g = \frac{\partial \omega}{\partial p} = \left(\frac{pc^2}{\omega_g} \right) \frac{\left(1 - \eta (\omega_p^2 (mc^2)^2) \right) (1 + 4\beta p^2)}{\left[1 + \eta (cp/\omega_p)^2 (1 + 2\beta p^2) \right]^2}. \quad (14)$$

Similarly, for a massless graviton, one can express the conventional momentum in terms of the GUP parameter.

In order of $\mathcal{O}(\beta)$, we get

$$pc = \omega_g \left[\left(1 - \eta \left(\frac{\omega_g}{\omega_p} \right)^2 \right)^{-1/2} - \beta \frac{\omega_g^2}{c^2} \left(1 - \eta \left(\frac{\omega_g}{\omega_p} \right)^2 \right)^{-3/2} \right]. \quad (15)$$

The investigation of the speed of the graviton from the GW150914 observations [78] specifies the rainbow gravity parameter, $\eta (\omega_g / \omega_p)^2 \leq 3.3 \times 10^{-21}$ [44]. Accordingly, Equation (15) can be reduced to $cp = \omega_g (1 - \beta \omega_g^2 / c^2)$, and the group velocity of the massless graviton is given as

$$v_g = c \left[1 - 5 \frac{\beta \omega^2}{c^2} + \mathcal{O}(\beta^2) \right]. \quad (16)$$

Then, the difference between the speed of light and that of the graviton reads

$$|\delta v_{\text{GUP}}| = \left| 5\beta \frac{\omega^2}{c} \right| \leq 2.43 \times 10^{-80} \beta_0 c. \quad (17)$$

Similarly, one can compare the difference of $|\delta v_{\text{GUP}} / \delta v|$ and get an upper bound of β_0 as

$$\beta_0 \leq 5.5 \times 10^{60}. \quad (18)$$

It is obvious that both results, Equations (13) and (18), are very close to each other, $\beta_0 \leq 10^{60}$. The improved upper bound on β_0 is very similar to the ones reported in refs. [43, 44] for the gravitational wave event GW170814 data and [79] for the gravitational wave event GW170817, which—as well—are depending on astronomical observations. We reestimate the upper bound of β_0 with more recent data of event GW170817 to find more updated constraints. The present results are based on mergers of spinning neutron stars. Thus, it is believed that more accurate observations, the more precise shall be β_0 .

Having set an upper bound on the GUP parameter and counting on the spoken similarities between GUP and the catastrophe of nongravitating vacuum, we can now propose a possible solution of the cosmological constant problem.

Our result on the upper bound on β_0 based on event GW170817 of binary neutron stars merger is very close to the results obtained from the MESSENGER spacecraft which orbited Mercury in 2011-2013 [80] and reported $\beta_0 \leq 10^{69}$ [43]. Table 1 summarizes various upper bounds on β_0 as obtained from different measurements and observations [41, 43]. The present analysis is in excellent agreement, especially with the astronomical observations [43].

4. A Possible Solution of the Cosmological Constant Problem

The cosmological constant can be given as $\Lambda = 3H_0^2 \Omega_\Lambda$, where H_0 and Ω_Λ are the Hubble parameter and the dark energy density, respectively [81]. On the other hand, the

TABLE 1: Possible bounds of the quadratic GUP parameter β_0 .

Phenomena	β_0
Present results based on event GW170817	10^{60}
Quadratic GUP parameter based on event GW170814 [79]	9.63×10^{59}
Light deflection by the sun [43]	10^{78}
Perihelion precession (solar system data) [43]	10^{69}
Perihelion precession (pulsar PRS B 1913+16 data) [43]	10^{71}
Lamb shift [41]	10^{36}
Landau levels [41]	10^{50}
Scanning Tunneling Microscope [41]	10^{21}

origin of the catastrophe of nongravitating vacuum would be understood from the disproportion of the value of Λ in the theoretical calculations, while this is apparently impacting the GW observations [82]. From the most updated Planck observations, the values of $\Omega_\Lambda = 0.6889 \pm 0.0056$ and $H_0 = 67.66 \pm 0.42 \text{ km}\cdot\text{s}^{-1}\cdot\text{Mpc}^{-1}$ [16]. Then, the vacuum energy density is

$$\frac{c^2}{8\pi G} \Lambda = \left(\frac{3H_0^2 c^2}{8\pi G} \right) \Omega_\Lambda = \frac{3\hbar c}{8\pi \ell_p^2 \ell_0} \Omega_\Lambda, \quad (19)$$

where the scale of the visible light is $\ell_0 = c/H_0 \approx 1.368 \times 10^{23} \text{ km}$ [16]. Therefore, one can use Equation (19) to estimate the vacuum energy density in order of $10^{-47} \text{ GeV}^4/(\hbar^3 c^3)$. In quantum field theory, the cosmological constant is to be calculated from sum over the vacuum fluctuation energies corresponding to all particle momentum states [81]. For a massless particle, we obtain

$$\frac{1}{(2\pi\hbar)^3} \int_0^\infty d^3 \vec{p} \left(\frac{\hbar\omega_p}{2} \right) \approx 9.60 \times 10^{74} \text{ GeV}^4/(\hbar^3 c^3). \quad (20)$$

This is clearly infinite integral. But, it is usually cut off, at the Planck scale, $\mu_p = \hbar/\ell_p$. We assume that ω_p is the vacuum energy of quantum harmonic state $\hbar\omega_p = [p^2 c^2 + m_g^2 c^4]^{1/2}$.

To propose a possible solution of the cosmological constant problem, it is initially needed to determine the number of states in the phase-space volume taking into account GUP, Equation (3). An analogy can be found in Liouville theorem in the classical limit. We need to make sure that the size of each quantum mechanical state in phase-space volume is depending on the modified momentum p , especially when taking GUP into consideration, Equation (3). In other words, the number of quantum states in the phase-space volume is assumed not depending on time.

In the classical limit, the relation of the quantum commutation relations and the Poisson brackets is given as $[\hat{A}, \hat{B}] = i\hbar\{A, B\}$. Details on the Poisson bracket in D -dimensions are outlined in the Appendix. Consequently, the modified density of states implies different implications on quantum field theory, such as the cosmological constant problem.

In D -dimensional spherical coordinate systems, the density of states in momentum space is given as [48, 53, 54]

$$\frac{V d^D \vec{p}}{(1 + \beta p^2)^{D+1}}, \quad (21)$$

where V is the volume of phase-space. It should be noticed that in quantum mechanics, the number of quantum states per unit volume is given as $V/(2\pi\hbar)^D$. Therefore, for Liouville theorem, the weight factor in 3-D dimension reads [48, 53, 54] (review the Appendix).

$$\frac{1}{(2\pi\hbar)^3} \frac{d^3 \vec{p}}{(1 + \beta p^2)^4}. \quad (22)$$

In quantum field theory, the modification in the quantum number of state of the phase-space volume should have consequences on different quantum phenomena, such as the cosmological constant problem and the black body radiation. At finite weight factor of GUP, the sum over all momentum states per unit volume of the phase space modifies the vacuum energy density. The cosmological constant, on the other hand, is determined by summing over the vacuum fluctuations, the energies, corresponding to a particular momentum state

$$\begin{aligned} \Lambda_{\text{GUP}}(m) &= \frac{1}{(2\pi\hbar)^3} \int_0^\infty d^3 \vec{p} \rho(p^2) (\hbar\omega_p/2) \\ &= \frac{1}{2(2\pi\hbar)^3} \int \frac{d^3 \vec{p}}{(1 + \beta p^2)^4} \sqrt{p^2 c^2 + m_g^2 c^4}. \end{aligned} \quad (23)$$

With $\cos(\theta) = 1/(1 + \beta p^2)$, this infinite integral for a massless test particle, $m = 0$, could be reduced to a finite integral. With $\theta = [0, \pi/2]$,

$$\Lambda_{\text{GUP}}(m = 0) = \frac{c}{8\pi^2 \beta^2 \hbar^3} \int_0^{\pi/2} \sin(\theta) \cos(\theta) (1 - \cos(\theta)) d\theta. \quad (24)$$

Therefore, the vacuum energy density, which is directly

related to Λ , reads

$$\Lambda_{\text{GUP}}(m=0) = \frac{c \left(M_p^2 c^2 \right)^2}{48\pi^2 \hbar^3 \beta_0^2} = 1.01 \times 10^{-48} \text{GeV}^4 / (\hbar^3 c^3). \quad (25)$$

As expected, the proposed minimal length uncertainty due to GUP exhibits some features of the UV/IR correspondence. We intend that Λ is rendering finite $1/\beta^2$. This result is obtained as a result of the strong suppression on the density of state, at high momenta. We conclude that the connection between the estimated upper bound on β_0 , Equations (13) and (18), deduced from GW170817 event [47] and the most updated observations of the Planck collaboration [16], Equation (20), and our estimation of $\Lambda(m=0)$, Equation (25), gives an plausible interpretation for the cosmological constant problem due to the minimal length uncertainty. The agreement between the observed value, $\Lambda \approx 10^{-47} \text{GeV}^4 / \hbar^3 c^3$, and our calculations based on quantum gravity approach, Equation (25), seems convincing.

5. Conclusions

In the present study, we have proposed GUP with an addition term of quadratic momentum, from which we have driven MDRs for GR with GUP and rainbow gravity, Equation (7) and Equation (10), respectively. Counting on the similarities between GUP (manifesting gravitational impacts on HUP) and the likely origin of the great discrepancy between the theoretical and observed values of the cosmological constant that in the gravitational impacts on the vacuum energy density, the present study suggests a possible solution for the long-standing cosmological constant problem (*catastrophe of nongravitating vacuum*) that $\Lambda \approx 10^{-47} \text{GeV}^4 / \hbar^3 c^3$.

We have assumed that the gravitational waves propagate as a free wave. Therefore, we could drive the group velocity in terms of the GUP parameter for GR and rainbow gravity, Equation (13) and Equation (18), respectively. Moreover, we have used recent results on gravitational waves, the binary neutron stars merger, the GW170817 event, in order to determine the speed of the gravitons. Then, we have calculated the difference between the speed of graviton and that of the light, at finite and vanishing GUP parameter. We have shown that the upper bound on the dimensionless GUP parameter, $\beta_0 \sim 10^{60}$, is merely constrained by such a speed difference. We have concluded that the speed of the graviton is directly related to the GUP approach utilized in.

The cosmological constant problem, which is stemming from the large discrepancy between the QFT-based calculations and the cosmological observations, is tagged as $\Lambda_{\text{QFT}} / \Lambda_{\text{exp}} \sim 10^{121}$. This quite large ratio can be interpreted by features of the UV/IR correspondence and the impacts of gravity. For the earlier, the large Δx (IR) corresponds to a large Δp (UV) in scale of Planck momentum. For the later, the GUP approach, for instance, Equation (3), plays an essential role. We have assumed that in calculating the density of

states where GUP approach is taken into account, a possible solution of the cosmological constant problem, Equation (22), can be proposed. At Planck scale, the resulting density of the states seems to impact the vacuum energy density of each quantum state, Equation (25). A refined value of the cosmological constant we have obtained for a novel upper bound on β_0 , which—in turn—was determined from the GW170817 observations. Finally, the possible matching between the estimation of the upper bound on the GUP parameter deduced from the gravitational waves, the GW170817 event, and the one estimated from the Planck 2018 observations seems to support the conclusion about the great importance of constructing a theory for quantum gravity. This likely helps in explaining various still mysterious phenomena in physics.

Appendix

A. Algebra of Quantum Mechanical Commutators and Poisson Brackets

For a binary set of anticommutative functions on position and momentum, for instance, in D -dimensions, the Poisson bracket expresses their binary operation:

$$\{F(x_1, \dots, x_D; p_1, \dots, p_D), G(x_1, \dots, x_D; p_1, \dots, p_D)\} = \left(\frac{\partial F}{\partial x_i} \frac{\partial G}{\partial p_j} - \frac{\partial F}{\partial p_i} \frac{\partial G}{\partial x_j} \right) \{x_i, p_j\} + \frac{\partial F}{\partial x_i} \frac{\partial G}{\partial x_j} \{x_i, x_j\}. \quad (A.1)$$

During a time duration, δt , the Hamilton's equations of motion for position and momentum can be given as

$$x'_i = x_i + \delta x_i, p'_i = p_i + \delta p_i, \quad (A.2)$$

where

$$\delta x_i = \{x_i, H\} \delta t = \left\{ x_i, p_j \right\} \frac{\partial H}{\partial p_j} + \left\{ x_i, x_j \right\} \frac{\partial H}{\partial x_j}, \quad (A.3)$$

$$\delta p_i = \{p_i, H\} \delta t = - \left\{ x_i, p_j \right\} \frac{\partial H}{\partial x_j}, \quad (A.4)$$

where $H \equiv H(x, p; t)$ is the Hamiltonian, itself.

The estimation of the change in the phase-space volume during the time evolution requires to determine the Jacobian of the transformation from $(x_1, \dots, x_D; p_1, \dots, p_D)$ to $(x'_1, \dots, x'_D; p'_1, \dots, p'_D)$, i.e.,

$$d^D x' d^D p' = \frac{d^D x d^D p}{\mathcal{J}}, \quad (A.5)$$

where \mathcal{J} is the Jacobian of the transformation, which can be

expressed as

$$\mathcal{F} = \left\| \frac{\partial(x'_1, \dots, x'_D; p'_1, \dots, p'_D)}{\partial(x_1, \dots, x_D; p_1, \dots, p_D)} \right\| = 1 + \left(\frac{\partial}{\partial x_i} \frac{\partial(\delta x_i)}{\partial t} + \frac{\partial}{\partial p_i} \frac{\partial(\delta p_i)}{\partial t} \right) \times \delta t. \quad (\text{A.6})$$

The general notations of position and momentum brackets lead to the following algebraic relations:

$$\{x_i, p_j\} = f_{ij}(x, p), \quad \{x_i, x_j\} = g_{ij}(x, p), \quad (\text{A.7})$$

$$\{p_i, p_j\} = h_{ij}(p).$$

Thus, the Jacobian of the transformation is given as [53]

$$\mathcal{F} = \prod_{i=1}^D f_{ii}(x, p) = 1 + \sum_{i=1}^D (f_{ii}(x, p) - 1). \quad (\text{A.8})$$

Therefore, the invariant phase space in D -dimension reads

$$\frac{d^D x d^D p}{(1 + \beta p^2)^{D+1}}. \quad (\text{A.9})$$

Finally, the quantum density of states can be determined from

$$\frac{1}{(2\pi\hbar)^3} \frac{d^3 \vec{p}}{(1 + \beta p^2)^4}. \quad (\text{A.10})$$

Data Availability

All data are included in the attached manuscript.

Ethical Approval

The authors declare that they are in compliance with ethical standards regarding the content of this paper.

Disclosure

The data used to support the findings of this study are included within the article and are cited at relevant places within the text as references. The funding agencies have no role in the design of the study; in the collection, analysis, or interpretation of the data; in the writing of the manuscript; or in the decision to publish the results.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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