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SHORT COMMUNICATION

A Practically Realizable Model of a Uniformly Distributed R-C Network ($\overline{\text{URC}}$)

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An accurate second order approximation to the open circuit voltage transfer function of a uniformly distributed R-C network ($\overline{\text{URC}}$) is presented from which a practically realizable model can be easily developed using standard network synthesis techniques.

The uniformly distributed RC ($\overline{\text{URC}}$) structure occurs in several different forms in integrated circuits, this film as well as monolithic. In monolithic integrated circuits, the $\overline{\text{URC}}$ structure is often unavoidably produced. The one-port and two-port properties of a $\overline{\text{URC}}$ have been studied extensively with special emphasis on its important applications.[5] Neglecting the higher order effects of propagation through the system, the $\overline{\text{URC}}$ can be treated as an electrical transmission line with a constant distributed resistance and capacitance per unit length.[1] The transmission line model is an adequate representation and does agree with experimental observations. However, to simplify the mathematical analysis and to make feasible circuit design with $\overline{\text{URC}}$ elements, it is necessary to replace the infinite RC element system by a finite element lumped network.

Of the large number of attempts made at modelling the $\overline{\text{URC}}$ structure, the best first order lumped two-port model is that of Bhattacharyya and Gupta.[2] A salient drawback of this model lay in the need for choosing and adjusting the coefficients of capacitors each time its application was deemed necessary in a particular situation. Recently, Dutta Roy and Kumar[3] developed a second order two-port lumped model which simulates the exact performance of $\overline{\text{URC}}$ almost precisely over a wide and useful frequency range without the need for mathematical manipulation of the elements involved. However, both these models cannot be realized in actual practice owing to the negative elements involved which are physically impossible to incorporate in the equivalent representation. These models are thus purely mathematical and theoretical and just describe the terminal behaviour. In this communication a simplified second order approximation is derived for the $\overline{\text{URC}}$ open circuit voltage transfer function from which a realistic model can be easily constructed which is physically realizable.

Through continued fraction expansion and optimum truncation of the transcendental functions occurring in the admittance parameters of a $\overline{\text{URC}}$, Dutta Roy and Kumar[3] arrived at the following two-port model parameters:

$$y_{11} = y_{22} = \frac{105 + 45p + p^2}{R(105 + 10p)} \tag{1}$$

$$y_{12} = y_{21} = \frac{-41160 + 2520p - 77p^2}{R(41160 + 4340p)} \tag{2}$$

where $p = ju = j\omega CR, u$ is the normalized frequency variable and R and C are the total resistance and capacitance of a $\overline{\text{URC}}$ line. The corresponding open circuit voltage transfer function can be easily mathematically reduced to the following very simple form

$$T(ju) = \frac{-y_{12}}{y_{22}} = 0.1952 \frac{(p^2 - 33p + 534)}{(p^2 + 45p + 105)} \tag{3}$$

The important result given in Eq. (3) is compared with the exact values for magnitude and phase in Figures 1 and 2 over a large and useful frequency range. The accuracy of the approximation is obvious from the figures. Further this biquadratic form of the transfer function can be readily employed to develop a practically realizable model using op amps, NICs, gyrators or controlled sources with a standard network synthesis approach.[4]

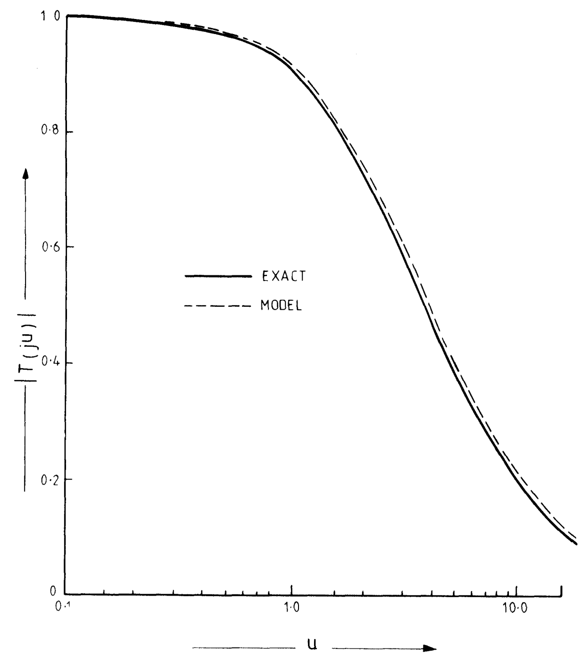

FIGURE 1 Frequency response of | $T(ju)$ | for the $\overline{\mathrm{URC}}$ and the approximate model.

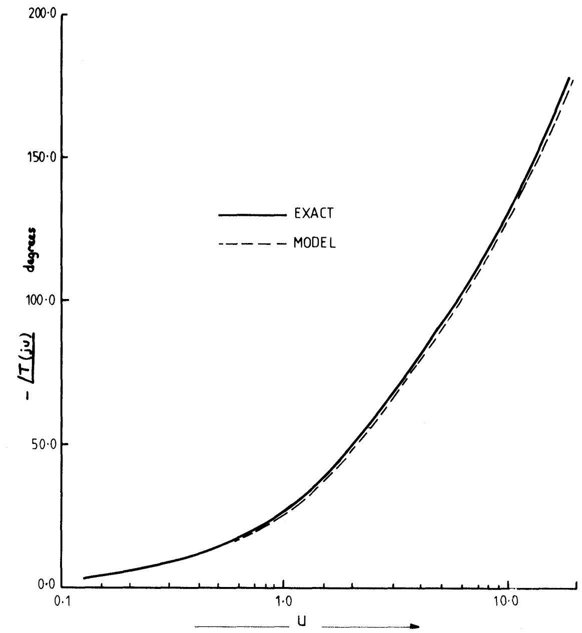

FIGURE 2 Frequency response of $\underline{/\ T(ju)}$ for the $\overline{\mathrm{URC}}$ and approximate model.

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