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SHORT COMMUNICATION
Size Effects in the Thermal Variations of the Hall Coefficient

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A number of experimental investigations of the temperature T dependence of the Hall coefficient R_{HF} of thin metallic films have been reported[1–9] in the past few years. However some data have only been interpreted in terms of the temperature dependence of the electron density η[7,8,9], even if the transport properties of these metallic films agree[3,6,7,9,10] with the well known Fuchs–Sondheimer theory.[11]

In a previous paper[12] we have derived an expression for the correction in the temperature coefficient, β_{RH} of the Hall coefficient R_{HF} of thin film arising from the temperature dependence of the bulk mean free path l_0. Unfortunately in the case of partially specular scattering of electrons on the external surfaces the expression of β_{RH} is somewhat complicated and its numerical evaluation requires the use of a digital computer.

In order to allow an easier calculation this letter proposes an alternative approach to derive an analytical expression for β_{RH}: it is taken into account that for nearly specular scattering on external surfaces the surface scattering can be treated with good approximation by the Cottey method[13] which states that the film resistivity ρ_F and its temperature coefficient β_F are respectively given by:[13,14]

$$\rho_0/\rho_F = F(\mu) = \tfrac{3}{2}\mu\{\mu - \tfrac{1}{2} + (1-\mu^2)\ln(1+\mu^{-1})\} \quad (1)$$

$$\beta_F/\rho_0 = 1 - G(\mu)\cdot[F(\mu)]^{-1} \quad (2)$$

where:[14]

$$G(\mu) = \mu\,\frac{dF(\mu)}{d\mu} = \frac{3}{2}\mu\left\{3\mu - \frac{3}{2} + (1-3\mu^2)\ln(1+\mu^{-1})\right\} \quad (3)$$

Here β_0 is the temperature coefficient (t.c.r.) of the bulk resistivity ρ_0 which can be written in the form:[15]

$$\beta_0 = d\ln\rho_0/dT = -d\ln l_0/dT \quad (4)$$

The physical parameter μ is related to the specularity parameter p and to the reduced thickness k by:[13]

$$\mu = k\cdot\left[\ln\frac{1}{p}\right]^{-1} \approx k\cdot[1-p]^{-1};\ p \geqslant 0.5 \quad (5)$$

with: $k = a/l_0$.

We have previously shown[16] that in the case of transverse magnetic field the Hall coefficient of thin metallic R_{HF} can be expressed in terms of film resistivity ρ_F and its t.c.r. β_F:

$$R_{HF}/R_{HO} \approx (\rho_F\cdot\beta_F)/(\rho_0\cdot\beta_0) \quad (6)$$

where R_{HO} is the Hall coefficient in bulk material.

By combining Eqs. (1), (2) and (6) we get:

$$R_{HF}/R_{HO} = [1 - D(\mu)]\cdot[F(\mu)]^{-1} \quad (7)$$

$$\text{with: } D(\mu) = G(\mu)/F(\mu) \quad (8)$$

Taking into account the value of the bulk Hall coefficient R_{HO}:[7,17]

$$R_{HO} = -\frac{1}{\eta e}$$

we get:

$$\frac{dR_{HO}}{R_{HO}} = -\frac{d\eta}{\eta} \quad (9)$$

Moreover Eq. (5) yields:

$$\frac{d\mu}{\mu} = -\frac{dl_0}{l_0} \quad (10)$$

assuming that thermal variations in thickness a and specularity parameter can be neglected; the validity of these assumptions has been recently established.[18]

Logarithmic differentiation of Eq. (7) leads to:

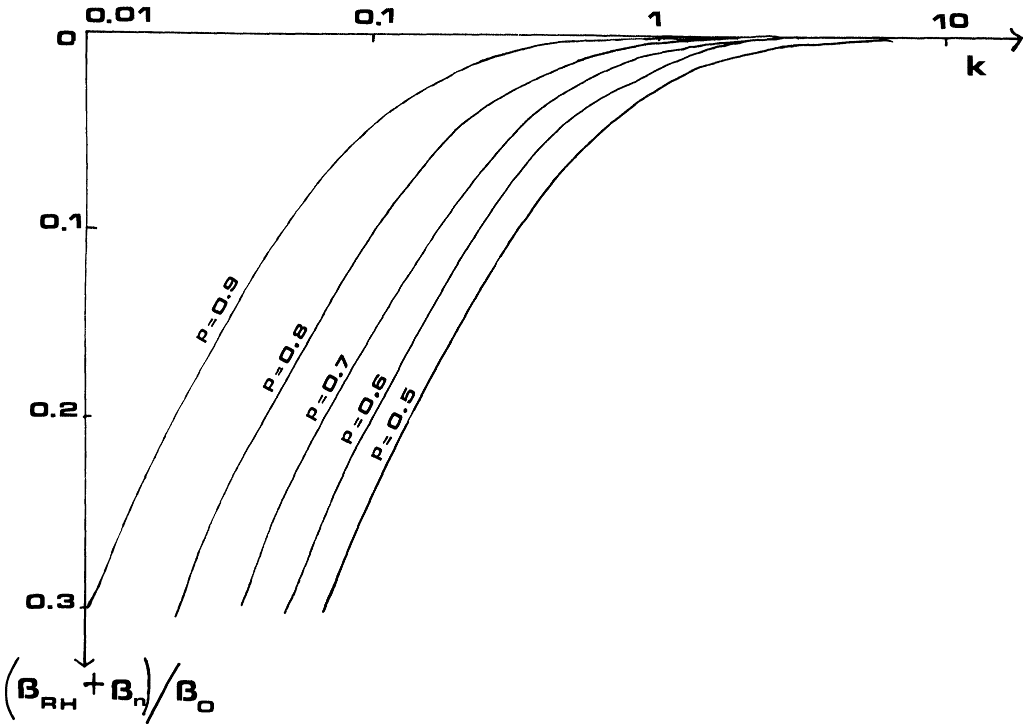

FIGURE 1 Theoretical plots of the ratio $(\beta_{RH} + \beta_\eta)/\beta_0$ versus the reduced thickness k for some values of the specularity parameter p.

$$\frac{\mathrm{d}R_{HF}}{R_{HF}} = -\frac{\mathrm{d}\eta}{\eta} + \frac{\mathrm{d}l_0}{l_0}\,\frac{D(\mu)}{1-D(\mu)}\left\{\frac{\mu}{G(\mu)}\,\frac{\mathrm{d}G(\mu)}{\mathrm{d}\mu} - \frac{\mu}{F(\mu)}\,\frac{\mathrm{d}F(\mu)}{\mathrm{d}\mu}\right\} + \frac{\mathrm{d}l_0}{l_0}\left\{\frac{\mu}{F(\mu)}\cdot\frac{\mathrm{d}F(\mu)}{\mathrm{d}\mu}\right\} \quad (11)$$

For thin films which exhibit linear R_{HF} versus T plots as previously reported by some authors[7] we may define the temperature coefficient β_{RH} of Hall coefficients as:

$$\beta_{RH} = \mathrm{d}\ln R_{HF}/\mathrm{d}T \quad (12)$$

by combining Eqs. (3), (4) and (11) and rearranging we finally obtain:

$$\beta_{RH} = -\beta_\eta - \beta_0 \cdot D(\mu)\left\{\frac{C(\mu)-D(\mu)}{1-D(\mu)} + 1\right\} \quad (13)$$

where:

$$C(\mu) = \frac{\mu}{G(\mu)} \cdot \frac{\mathrm{d}G(\mu)}{\mathrm{d}\mu} \quad (14)$$

and:

$$\mu\frac{\mathrm{d}G(\mu)}{\mathrm{d}\mu} = \frac{3}{2}\,\mu\left\{G(\mu) - \frac{3}{2} + (1-9\mu^2)\ln(1+\mu^{-1}) - \frac{1-3\mu^2}{1+\mu}\right\} \quad (15)$$

β_η is the temperature coefficient of the electronic density η previously defined as:[12]

$$\beta_\eta = \mathrm{d}\ln\eta/\mathrm{d}T \quad (16)$$

Eq. (13) predicts that β_{RH} depends on the reduced thickness k; this behaviour can be easily explained since it has been found that R_{HF} markedly depends on film thickness[15,17] for relatively thin films ($k < 0.5$).

Figure 1 shows theoretical plots of the ratio $(\beta_{RH} + \beta_\eta)/\beta_0$ for different values of the specularity parameter p. It is clear from Eq. (13) that $\beta_{RH} + \beta_\eta$ gives the expression of the change in temperature of the Hall coefficient due to the temperature dependence of the reduced thickness; hence, as expected, the ratio $|(\beta_{RH} + \beta_\eta)/\beta_0|$ decreases (for a given thickness) with increasing values of the specularity parameter p; moreover it appears that the magnitude of $(\beta_{RH} + \beta_\eta)/\beta_0$ rapidly decreases with increasing values of k and finally vanishes for thick films (for example when $p = 0.75$ for $k > 0.1$).

More generally marked size effects could be observed only if the magnitude of β_η is considerably smaller than the generally positive β_0 value. For example for $p = 0.5$ and $k = 0.5$ we obtain a deviation less than 20% until the ratio $|\beta_\eta/\beta_0|$ keeps values greater than 0.2.

As a consequence of this study we estimate, as suggested in a previous paper,[12] that the temperature dependence of the Hall coefficient of thin metallic films may be (in the relatively high temperature range ($T > 150$ K)) interpreted in terms of thermal variations in both the electronic density $\eta(T)$ and the mean free path $\rho_0(T)$; nevertheless, this last effect is significant only if the magnitude of β_η is considerably less than β_0.

Further insight is thus given for the interpretation of thermal variations in Hall coefficient of thin metallic films.

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