NOVEL, USEFUL, AND EFFECTIVE DEFINITIONS FOR FUZZY LINGUISTIC HEDGES

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The main emphasis of this paper is on fuzzy linguistic hedging, used to modify membership functions. This paper investigates the issues of obtaining new definitions for hedges which exceed the traditional definitions given by Zadeh (and others), particularly seeing that the effect of applying these hedges does not cross beyond the reasonable limits of membership values $[0,1]$ and is still meaningful from the point of view of magnitude of membership value and hence be really effective for an application. Some of the most commonly used hedges are presented, these hedges are very, positively, negatively, slightly more, and slightly less. The effects of these hedges on numeric examples are charted.

1. Introduction

In the very common case of knowledge-based systems in which vague concepts and imprecise data must be handled, fuzzy logic provides a useful tool which presents the interest of allowing to manage both imprecision and uncertainty.

In this framework, linguistic modifiers are more important issues in the treatment of data by means of fuzzy logic. These modifiers play the same role in fuzzy modeling as adverbs and adjectives do in language: they both modify qualitative statements.

Zadeh [12], Liu et al. [6] proposed the fuzzy linguistic hedges such as very, more or less slightly to modify the membership functions of the fuzzy sets. The definition of hedges has more to do with common sense knowledge in a domain than with mathematical theory, that is, both the nature of a fuzzy surface and its degree of transformation are associated with people's subjective judgment as to how the region should look, not on mathematical theory of fuzzy surface topology operations. Although a simple linguistic hedge may be used to express different modifications in different applications, the general type of the underlined operation is the same, varying only in terms of detailed parameter

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settings. In Cox's book [3] and Ross's book [9], the linguistic hedges very, more or less, positively, and negatively were introduced and verified in a limited mathematical model.

Since the linguistic hedges were proposed by Zadeh in 1973, only few publications are available dealing with these concepts [1, 2, 4, 5, 8]. Banks [1] used hedge operations to better qualify and emphasized the crisp variables to mix crisp and fuzzy logic applications. Bouchon-Meunier [2] investigated several interesting properties of linguistic hedge, such as

(i) being compatible with symbolic rules,
(ii) avoiding computations and being compatible with the fuzzy logic,
(iii) enhancing the comparison of various available fuzzy implication,
(iv) managing gradual rules in the context of deductive rules.

Novak [8] proposed a horizon shifting model of linguistic hedges, by which the membership function can be shifted as well as its steepness modified. The concept of extended hedge algebras and their application in approximate reasoning was discussed by HO and Wechler [4]. Huang et al. [5] proposed several hedge operators. Their relative hardware realizations in current-mode approach are also found. In addition, [6] Liu et al. proposed a fuzzy logic controller based on linguistic hedges to simplify the membership function construction and the rule developments.

Our investigation exceeds the traditional definitions to provide a loose framework. The remainder of the paper is organized as follows: section two briefly describes Zadeh's fuzzy sets and hedges. Section three proposes a set of novel and useful linguistic hedges with problem illustrations charting the potential of the present research, which differs form those conventionally employed in the literature. The paper is concluded in section four.

2. Fuzzy sets, fuzzy linguistic hedges

Fuzzy sets have been interpreted as membership function μ_F that associates with each element x, of the universe of discourse X a number $\mu_F(x)$ in the interval $[0,1]$. In essence, a fuzzy set F may be represented in the form of

$$F = \int_X \frac{\mu_F(x)}{x}. \tag{2.1}$$

In the case of F having a finite support $x_1, x_2, \ldots, x_n$, the discrete form of (2.1) is [12],

$$F = \sum_{i=1}^{n} \frac{\mu_i}{x_i} = \frac{\mu_1}{x_1} + \frac{\mu_2}{x_2} + \cdots + \frac{\mu_n}{x_n}, \tag{2.2}$$

where μ_i $(i = 1, \ldots, n)$ is the grade of membership of x_i in F. Unlike the crisp set logic that distinguishes the members of given set from no members by binary decision, the fuzzy sets are characterized by their membership functions. In order to manipulate the fuzzy sets as well as ordinary sets with Boolean operations, Zadeh [12] proposed the extension of the ordinary set theory for fuzzy sets.

Let A and B be two fuzzy sets in X with membership functions $\mu_A(x)$ and $\mu_B(x)$, respectively. The fuzzy set operations of union, intersection, and complement are defined as follows.

2.1. Union of fuzzy sets. The union of two fuzzy sets A and B with respective membership functions $\mu_A(x)$ and $\mu_B(x)$ is a fuzzy set C, written as $C = A \cup B$, whose membership function is related to those of A and B by [3]

$$\mu_c(x) = \text{Max}\left[\mu_A(x), \mu_B(x)\right], \quad x \in X. \tag{2.3}$$

2.2. Intersection of fuzzy sets. The intersection of fuzzy sets A and B with respective membership functions $\mu_A(x)$ and $\mu_B(x)$ is a fuzzy set C, written as $C = A \cap B$, whose membership function is related to those of A and B by [3]

$$\mu_c(x) = \text{Min}\left[\mu_A(x), \mu_B(x)\right], \quad x \in X. \tag{2.4}$$

The union of fuzzy sets interpreted as logical "OR" referred to as triangular conorms and the intersection of fuzzy sets modeled as logical "AND" referred to as triangular norms were introduced.

2.3. Complement

$$\overline{\mu}(x) = 1 - \mu_A(x). \tag{2.5}$$

2.4. Fuzzy linguistic hedges. In a fuzzy logic-based system, the information is described linguistically. The linguistic hedge is an operator with an operation like a modifier used to modify the shape of membership functions. Linguistic modifiers have two main behaviors with regard to their effect on the qualifications they modulate. They have either a behavior of reinforcement [such as very, strong] or a behavior of weakening [such as "more or less," "relatively"].

According to the statement in [12], linguistic hedge operations can be classified into three categories: *concentration*, *dilation*, and *contrast intensification*. In this paper, we only focus on the *concentration*-type and the *dilation*-type hedge operations.

2.4.1. Restrictive modifiers/concentrators. In contrast, the effect of dilation is opposite to that of concentration. Applying a concentration operator to a fuzzy set results in the reduction of magnitude to the grade of membership in which it is relatively small for those with a high grade of membership in and relatively large for those with low membership. The reinforcing modifiers provide a characterization, which is stronger than the original one. Zadeh proposed the modifier "very" associated with the transformation $\text{tm}(\mu) = \mu^2$. More generally, we can think of modifiers defined by transformation such that $tm(\mu) \leq \mu$ for any μ [0, 1]. Other examples of these quantifiers include "extremely," "positively." Hedges with mathematical expressions given by Zadeh et al. [3, 7, 9–11] are shown in Table 2.1.

Table 2.1. Concentrators.

Hedge	Operator definition
Very *F*	*F* squared
Plus *F*	*F* to the power 1.25
Extremely *F*	*F* to the power 8 or 3
Very very *F*	*F* to the power 4

Table 2.2. Dilators.

Hedge	Operator definition
More-or-less *F*/fairly	Square root of *F*
Minus *F*	*F* to the power 0.75
Somewhat *F*	*F* to the one-third power

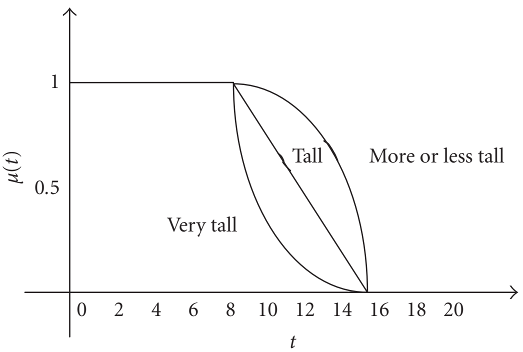

Figure 2.1. Effects of the fuzzy linguistic hedges "very" and "more or less."

2.4.2. Expansive modifiers/dilators. The weakening modifiers provide a new characterization, which is less stronger than the original one. Zadeh introduced the modifier "more or less," associated with the transformation $tm(\mu) = \mu^{1/2}$. Other weakening modifiers can be defined by choosing a transformation such that $tm(\mu) \geq \mu$ for any μ [0, 1]. Other examples of these quantifiers include "more or less," "negatively." Hedges with mathematical expressions given by Zadeh et al. [3, 7, 9–11] are shown in Table 2.2.

The fuzzy sets tall, *very tall*, and *more or less tall* characterized by their membership functions and are shown in Figure 2.1. In this figure, the membership function of the fuzzy set *very tall* is generated by applying the hedge operator *very* to that of the fuzzy set tall while the membership function of the fuzzy set *more or less tall* is generated by applying the hedge operator *more or less* to that of the fuzzy set tall. Obviously, the linguistic hedge *very* tends to narrow the shape of the membership function and decrease the membership degree; the linguistic hedge *more or less* tends to widen the shape of the membership function and increase the membership degree. That is, the members in the fuzzy set *very tall* are closer to the height of tall while the members in the fuzzy set *more or less tall* are farther away from the height of *tall.*

3. Proposed mathematical modeling of linguistic hedges

Discussion will focus on the linguistic hedges very, positively, negatively, slightly more, slightly less, using exponents as a factor for concentration and dilation.

3.1. Very. The very modifier returns the expanded fuzzy value passed as its argument, having raised all the membership values of the fuzzy value by a factor of two, Zadeh [11]. Extremely modifier returns the expanded fuzzy value passed as its argument, having raised all the membership values of the fuzzy value by a factor of eight or three. But the new definitions proposed in this paper are slightly different and yet more meaningful. These new definitions are formulated as follows.

New definitions for concentrators are given as follows:

$$\text{very}^{\,n} = \sqrt{\frac{2^{n+1}}{2}} = F \text{ to the power } \sqrt{2^n},$$
$$\text{very}^{\,n} = \sqrt{\frac{2^{n+1}}{2}} = F \text{ divided by } \sqrt{2^n}, \tag{3.1}$$

when $n = 1$,

$$\text{very} = F \text{ to the power } \sqrt{2^1},$$
$$\text{very} = F \text{ divided by } \sqrt{2^1}, \tag{3.2}$$

when $n = 2$,

$$\text{very very} = F \text{ to the power } \sqrt{2^2},$$
$$\text{very very} = F \text{ divided by } \sqrt{2^2}, \tag{3.3}$$

when $n = 3$,

$$\text{very very very} = F \text{ to the power } \sqrt{2^3},$$
$$\text{very very very} = F \text{ divided by } \sqrt{2^3}. \tag{3.4}$$

Using exponential functions as a factor for concentrators, the following ensure

$$\text{very}^{\,n}F = F \text{ to the power } e^{\sqrt{1/e^{-n}}} = \sqrt{e^n},$$
$$\text{very}^{\,n}F = F \text{ divided by } e^{\sqrt{1/e^{-n}}} = \sqrt{e^n}, \tag{3.5}$$

when $n = 1$,

$$\text{very } F = F \text{ to the power } \sqrt{e^1},$$
$$\text{very } F = \text{ F divided by } \sqrt{e^1}, \tag{3.6}$$

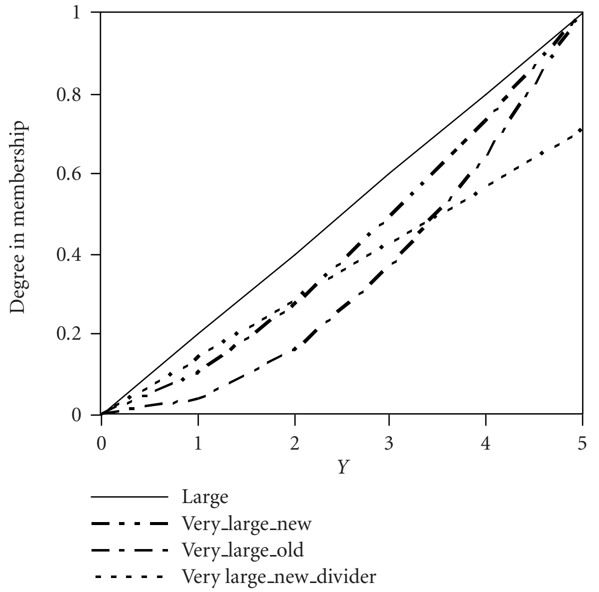

Figure 3.1. Schematic illustration of linguistic hedge "very."

when $n = 2$,

$$\text{very very } F = F \text{ to the power } \sqrt{e^2}, \tag{3.7}$$

$$\text{very very } F = F \text{ divided by } \sqrt{e^2}. \tag{3.8}$$

3.1.1. Problem illustration. Suppose Y is a universe of integers as $Y = \{1,2,3,4,5\}$ taken from [9], then the following linguistic terms are defined as a mapping onto Y:

$$\text{Large} = \left\{\frac{0.2}{1} + \frac{0.4}{2} + \frac{0.6}{3} + \frac{0.8}{4} + \frac{1}{5}\right\}. \tag{3.9}$$

Figure 3.1 shows the effect of applying the hedge "very" to the membership function Large using (3.2) and the results are compared with the older method listed in Table 2.1.

Figure 3.2 shows the effect of applying the hedge "very very" to the membership function Large using (3.3), the results are compared with the older method listed in Table 2.1.

Figure 3.3 shows the effect of applying the hedge "very very very (extremely)" to the membership function Large using (3.4), the results are compared with the older method listed in Table 2.1.

Figure 3.4 shows the effect of applying the hedge "very" to the membership function Large using (3.6), the results are compared with the older method listed in Table 2.1.

Figure 3.5 shows the effect of applying the hedge "very very" to the membership function Large using (3.7) and (3.8), the results are compared with the older method listed in Table 2.1.

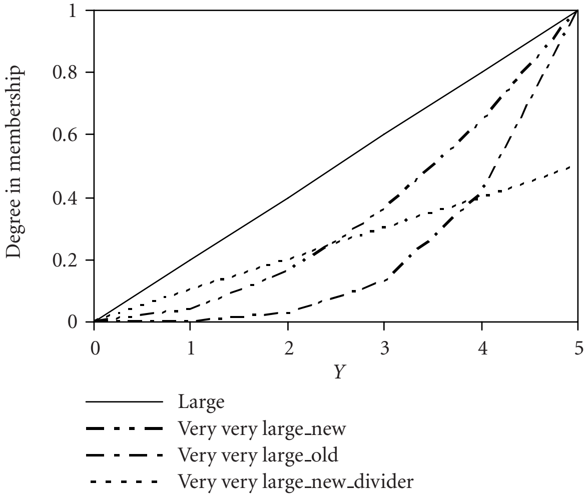

Figure 3.2. Schematic illustration of linguistic hedge “very very.”

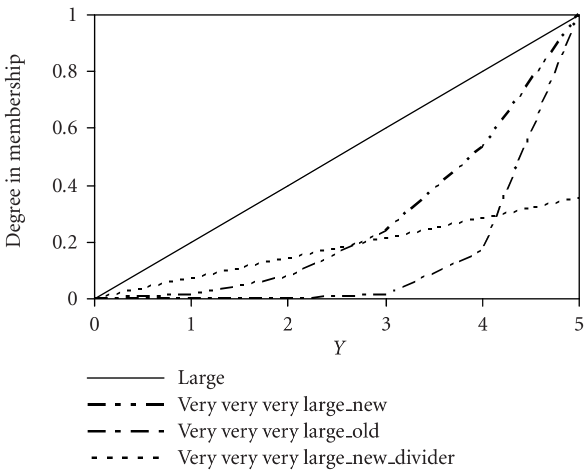

Figure 3.3. Schematic illustration of linguistic hedge “very very very.”

3.1.2. Comparison. Graphs 2, 3, 4, 5, and 6 clearly depict the effect of linguistic hedges “very,” “very very,” and “extremely.” In general, the concentrator “very” reduces the size of original membership values by a factor of $\sqrt{2^n}$, whereas the traditional transformation reduces the set by a factor of 2^n. When more and more concentration is applied, the original set almost disappears under traditional transformations, whereas the new transformation reduces the set by a factor of $\sqrt{2^n}$, hence retaining the set which is meaningful. On the contrary, the divider uniformly reduces the original set by a factor of $\sqrt{2^n}$.

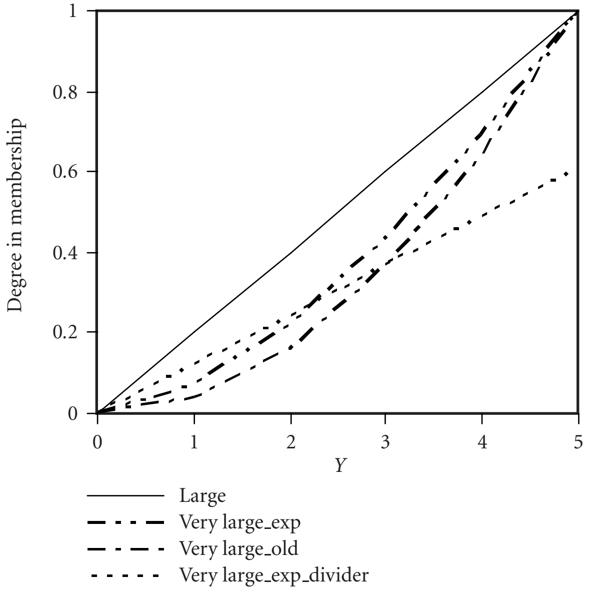

Figure 3.4. Schematic illustration of linguistic hedge “very” using exponentials.

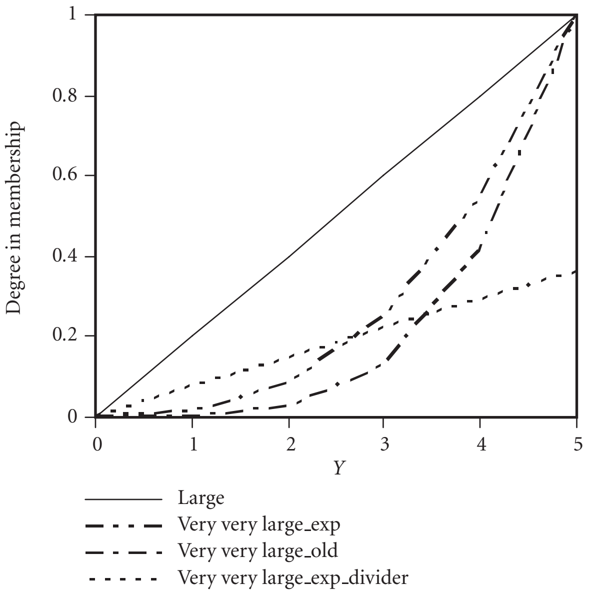

Figure 3.5. Schematic illustration of linguistic hedge “very very” using exponentials.

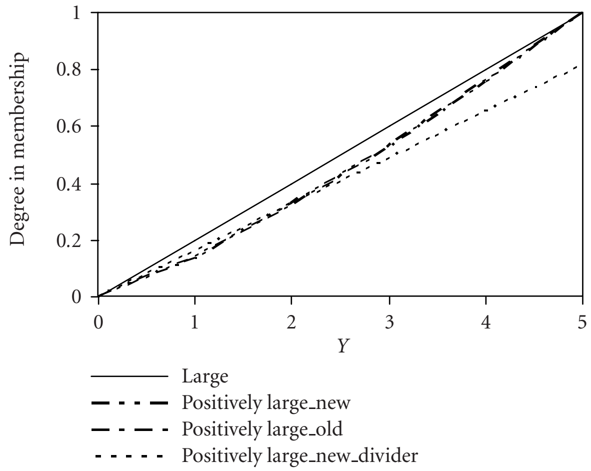

Figure 3.6. Schematic illustration of linguistic hedge "positively."

3.2. Positively (plus). The plus modifier returns the expanded fuzzy value passed as its argument, having raised all the membership values of the fuzzy value by a factor of 1.25. The new definitions are formulated as follows:

$$\begin{aligned}\text{positively } F &= F \text{ to the power } \sqrt{1+0.5} = F \text{ to the power } \sqrt{1.5}\\ &= F \text{ to the power } 1.225,\\ \text{positively } F &= F \text{ divided by } \sqrt{1+0.5} = F \text{ divided by } \sqrt{1.5}\\ &= F \text{ divided by } 1.225.\end{aligned} \tag{3.10}$$

Figure 3.6 shows the effect of applying the hedge "plus" to the membership function Large using (3.10), the results are compared with the older method listed in Table 2.1.

3.3. Negatively (minus). The minus modifier is a dilator that increases the membership values for a given set by a factor of 0.75.

It is possible to use the dilator as given below:

$$\begin{aligned}\text{negatively } F &= F \text{ to the power } \sqrt{1-0.5} = F \text{ to the power } \sqrt{0.5}\\ &= F \text{ to the power } 0.707,\end{aligned}$$

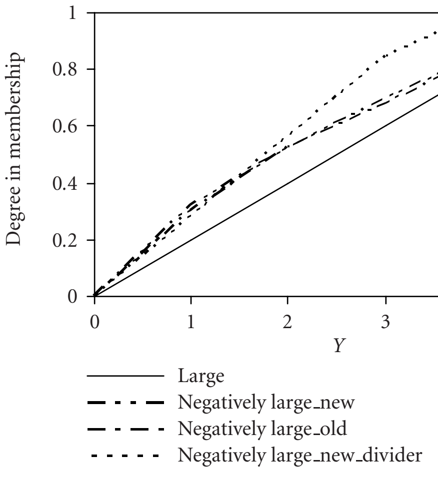

Figure 3.7. Schematic illustration of linguistic hedge "negatively."

$$\text{negatively } F = F \text{ divided by } \sqrt{1-0.5} = F \text{ divided by } \sqrt{0.5}$$
$$= F \text{ divided by } 0.707. \quad (3.11)$$

Figure 3.7 shows the effect of applying the hedge "minus" to the membership function Large using (3.11), the results are compared with the older method given in Table 2.2.

3.3.1. Comparison. These new definitions indirectly justify the arbitrary choice of the power for all original operations. The powers obtained by these new definitions are close enough to the old method and hence there is not much of a difference. Nonetheless, these powers clearly provide an insight into the basic operations they are supposed to perform and hence in a way justify the choice made for the operations and eventually for the exponent.

3.4. Slightly more and slightly less. Fuzzy linguistic hedges "more or less," "somewhat," "rather," and "quite" are dilutors with the same meaning. It is the complement of the linguistic hedge very, originally defined by Zadeh [11],

$$\text{more or less of } F = \text{square root of } F. \quad (3.12)$$

A general version of dilutor is given in Zadeh [11]. This is done by replacing 0.5 in (3.7) with a positive fraction $(1/n)$,

$$\text{more or less of } F = F \text{ to the power } \left(\frac{1}{n}\right). \quad (3.13)$$

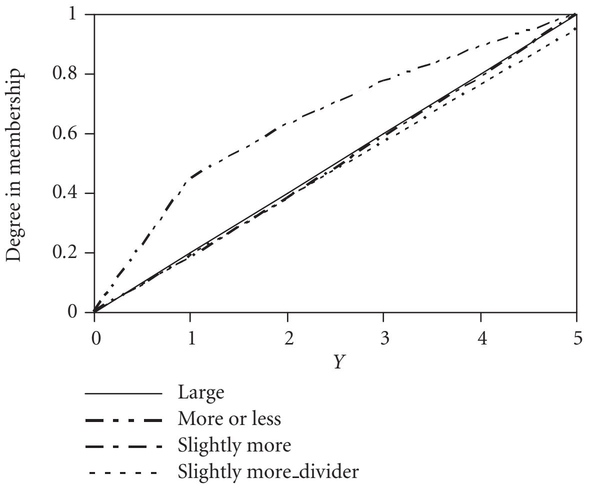

Figure 3.8. Schematic illustration of linguistic hedge "slightly more."

The new definitions are the following:

$$\begin{aligned}\text{slightly more of } F &= F \text{ to the power } \sqrt{1+0.1} = \sqrt{1.1}\\ &= F \text{ to the power } 1.0488,\end{aligned} \tag{3.14}$$

$$\begin{aligned}\text{slightly more of } F &= F \text{ divided by } \sqrt{1+0.1} = \sqrt{1.1}\\ &= F \text{ divided by } 1.0488,\end{aligned} \tag{3.15}$$

$$\begin{aligned}\text{slightly less of } F &= F \text{ to the power } \sqrt{1-0.1} = \sqrt{0.9}\\ &= F \text{ to the power } 0.9487,\end{aligned} \tag{3.16}$$

$$\begin{aligned}\text{slightly less of } F &= F \text{ divided by } \sqrt{1-0.1} = \sqrt{0.9}\\ &= F \text{ divided by } 0.9487.\end{aligned} \tag{3.17}$$

Figure 3.8 shows the effect of applying the hedge "slightly more" to the membership function Large using (3.15) and (3.16), the results are compared with the older method.

Figure 3.9 shows the effect of applying the hedge "slightly less" to the membership function Large using (3.16) and (3.17), the results are compared with the older method.

3.4.1. Comparison. The adjective slightly indicates slight changes from the original values, but the traditional transformation "more or less" tends to dilate more, deviating far form the original set.

In general, all these operators are such that for all membership values less than 0.5, the change is more pronounced and for those which are higher than 0.5, the change is not that pronounced. But one important conclusion that must be placed in proper perspective is that the new definitions do yield meaningful and manageable membership values

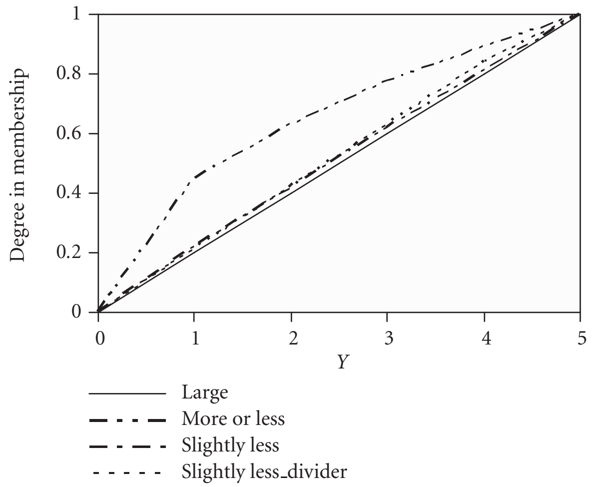

Figure 3.9. Schematic illustration of linguistic hedge "slightly less."

even after multiple operations, whereas the old definitions may not be this effective and transparent.

4. Conclusion

This paper discusses existing definitions of linguistic hedges and exposes their limitations. In this paper, new and more general definitions of hedges are presented and illustrated with graphs. It is hoped that these general formulae will provide more versatility to both fuzzy theorists and application engineers.

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