Hindawi Publishing Corporation
Discrete Dynamics in Nature and Society
Volume 2008, Article ID 143040, 16 pages
doi:10.1155/2008/143040

Research Article

Positive Solutions for Third-Order Nonlinear p-Laplacian m-Point Boundary Value Problems on Time Scales

Fuyi Xu

School of Mathematics and Information Science, Shandong University of Technology, Zibo, Shandong 255049, China

Correspondence should be addressed to Fuyi Xu, zbxufuyi@163.com

Received 7 August 2008; Revised 8 October 2008; Accepted 9 November 2008

Recommended by Leonid Shaikhet

We study the following third-order p-Laplacian m-point boundary value problems on time scales: $(\phi_p(u^{\Delta\nabla}))^{\nabla} + a(t)f(t,u(t)) = 0$, $t \in [0,T]_{\mathbf{T}}$, $\beta u(0) - \gamma u^{\Delta}(0) = 0$, $u(T) = \sum_{i=1}^{m-2} a_i u(\xi_i)$, $\phi_p(u^{\Delta\nabla}(0)) = \sum_{i=1}^{m-2} b_i \phi_p(u^{\Delta\nabla}(\xi_i))$, where $\phi_p(s)$ is p-Laplacian operator, that is, $\phi_p(s) = |s|^{p-2}s$, $p > 1$, $\phi_p^{-1} = \phi_q$, $1/p + 1/q = 1$, $0 < \xi_1 < \cdots < \xi_{m-2} < \rho(T)$. We obtain the existence of positive solutions by using fixed-point theorem in cones. The conclusions in this paper essentially extend and improve the known results.

1. Introduction

The theory of time scales was initiated by Hilger [1] as a means of unifying and extending theories from differential and difference equations. The study of time scales has lead to several important applications in the study of insect population models, neural networks, heat transfer, and epidemic models, see, for example [2–6]. Recently, the boundary value problems with p-Laplacian operator have also been discussed extensively in the literature, for example, see [7–13].

A time scale $\mathbf{T}$ is a nonempty closed subset of $\mathbb{R}$. We make the blanket assumption that $0, T$ are points in $\mathbf{T}$. By an interval $(0, T)$, we always mean the intersection of the real interval $(0, T)$ with the given time scale; that is $(0, T)\cap\mathbf{T}$.

In [14], Anderson considered the the following third-order nonlinear boundary value problem (BVP):

$$\begin{gathered} x'''(t) = f(t, x(t)), \quad t_1 \le t \le t_3, \\ x(t_1) = x'(t_2) = 0, \qquad \gamma x(t_3) + \delta x''(t_3) = 0. \end{gathered} \tag{1.1}$$

Author studied the existence of solutions for the nonlinear boundary value problem by using the Krasnoselskii's fixed point theorem and Leggett and Williams fixed point theorem, respectively.

In [8, 9], He considered the existence of positive solutions of the p-Laplacian dynamic equations on time scales

$$(\phi_p(u^{\Delta}))^{\nabla} + a(t)f(u(t)) = 0, \quad t \in [0, T]_{\mathbf{T}}, \tag{1.2}$$

satisfying the boundary conditions

$$u(0) - B_0(u^{\Delta}(\eta)) = 0, \qquad u^{\Delta}(T) = 0, \tag{1.3}$$

or

$$u^{\Delta}(0) = 0, \qquad u(T) - B_1(u^{\Delta}(\eta)) = 0, \tag{1.4}$$

where $\eta \in (0, \rho(T))$. He obtained the existence of at least double and triple positive solutions of the boundary value problems by using a new double fixed point theorem and triple fixed point theorem, respectively.

In [13], Zhou and Ma firstly studied the existence and iteration of positive solutions for the following third-order generalized right-focal boundary value problem with p-Laplacian operator:

$$\begin{gathered}(\phi_p(u''))'(t) = q(t)f(t, u(t)), \quad 0 \le t \le 1, \\ u(0) = \sum_{i=1}^{m} \alpha_i u(\xi_i), \qquad u'(\eta) = 0, \qquad u''(1) = \sum_{i=1}^{n} \beta_i u''(\theta_i).\end{gathered} \tag{1.5}$$

They established a corresponding iterative scheme for the problem by using the monotone iterative technique.

However, to the best of our knowledge, little work has been done on the existence of positive solutions for third-order p-Laplacian m-point boundary value problems on time scales. This paper attempts to fill this gap in the literature.

In this paper, by using different method, we are concerned with the existence of positive solutions for the following third-order p-Laplacian m-point boundary value problems on time scales:

$$\begin{gathered}(\phi_p(u^{\Delta\nabla}))^{\nabla} + a(t)f(t, u(t)) = 0, \quad t \in [0, T]_{\mathbf{T}}, \\ \beta u(0) - \gamma u^{\Delta}(0) = 0, \qquad u(T) = \sum_{i=1}^{m-2} a_i u(\xi_i), \qquad \phi_p(u^{\Delta\nabla}(0)) = \sum_{i=1}^{m-2} b_i \phi_p(u^{\Delta\nabla}(\xi_i)),\end{gathered} \tag{1.6}$$

where $\phi_p(s)$ is p-Laplacian operator, that is, $\phi_p(s) = |s|^{p-2}s,\ p > 1,\ \phi_p^{-1} = \phi_q,\ (1/p)+(1/q) = 1,$

and a_i, b_i, a, f satisfy

(H_1) $\beta, \gamma \geq 0$, $\beta + \gamma > 0$, $a_i \in [0, +\infty)$, $i = 1, 2, \ldots, m-3$, $a_{m-2} > 0$, $0 < \xi_1 < \cdots < \xi_{m-2} < \rho(T)$, $0 < \sum_{i=1}^{m-2} b_i < 1$, $0 < \sum_{i=1}^{m-2} a_i \xi_i < T$, $d = \beta(T - \sum_{i=1}^{m-2} a_i \xi_i) + \gamma(1 - \sum_{i=1}^{m-2} a_i) > 0$;

(H_2) $f : [0, T]_{\mathbf{T}} \times [0, +\infty) \to R^+$ is continuous, $a \in C_{\text{ld}}((0, T)_{\mathbf{T}}, \mathbb{R}^+)$ and there exists $t_0 \in [\xi_{m-2}, T)_{\mathbf{T}}$ such that $a(t_0) > 0$, where $\mathbb{R}^+ = [0, +\infty)$.

2. Preliminaries and lemmas

For convenience, we list the following definitions which can be found in [1–5].

Definition 2.1. A time scale $\mathbf{T}$ is a nonempty closed subset of real numbers $\mathbb{R}$. For $t < \sup \mathbf{T}$ and $r > \inf \mathbf{T}$, define the forward jump operator σ and backward jump operator ρ, respectively, by

$$\sigma(t) = \inf\{\tau \in \mathbf{T} \mid \tau > t\} \in \mathbf{T},$$
$$\rho(r) = \sup\{\tau \in \mathbf{T} \mid \tau < r\} \in \mathbf{T}, \tag{2.1}$$

for all $t, r \in \mathbf{T}$. If $\sigma(t) > t$, t is said to be right scattered; if $\rho(r) < r$, r is said to be left scattered; if $\sigma(t) = t$, t is said to be right dense; if $\rho(r) = r$, r is said to be left dense. If $\mathbf{T}$ has a right scattered minimum m, define $\mathbf{T}_k = \mathbf{T} - \{m\}$, otherwise set $\mathbf{T}_k = \mathbf{T}$. If $\mathbf{T}$ has a left scattered maximum M, define $\mathbf{T}^k = \mathbf{T} - \{M\}$, otherwise set $\mathbf{T}^k = \mathbf{T}$.

Definition 2.2. For $f : \mathbf{T} \to \mathbb{R}$ and $t \in \mathbf{T}^k$, the delta derivative of f at the point t is defined to be the number $f^{\Delta}(t)$, (provided it exists), with the property that for each $\epsilon > 0$, there is a neighborhood U of t such that

$$\left| f(\sigma(t)) - f(s) - f^{\Delta}(t)(\sigma(t) - s) \right| \leq \epsilon \left| \sigma(t) - s \right|, \tag{2.2}$$

for all $s \in U$.

For $f : \mathbf{T} \to \mathbb{R}$ and $t \in \mathbf{T}_k$, the nabla derivative of f at t, denoted by $f^{\nabla}(t)$ (provided it exists), with the property that for each $\epsilon > 0$, there is a neighborhood U of t such that

$$\left| f(\rho(t)) - f(s) - f^{\nabla}(t)(\rho(t) - s) \right| \leq \epsilon \left| \rho(t) - s \right|, \tag{2.3}$$

for all $s \in U$.

Definition 2.3. A function f is left-dense continuous (i.e., ld-continuous), if f is continuous at each left-dense point in $\mathbf{T}$ and its right-sided limit exists at each right-dense point in $\mathbf{T}$.

Definition 2.4. If $\phi^{\Delta}(t) = f(t)$, then one defines the delta integral by

$$\int_a^b f(t) \Delta t = \phi(b) - \phi(a). \tag{2.4}$$

If $F^{\nabla}(t) = f(t)$, then one defines the nabla integral by

$$\int_a^b f(t)\nabla t = F(b) - F(a). \tag{2.5}$$

Lemma 2.5. *If $d = \beta(T - \sum_{i=1}^{m-2} a_i\xi_i) + \gamma(1 - \sum_{i=1}^{m-2} a_i) > 0$, then for $h \in C_{ld}[0,T]_{\mathbf{T}}$, the boundary value problem (BVP)*

$$\begin{gathered} u^{\Delta\nabla} + h(t) = 0, \quad t \in [0,T]_{\mathbf{T}}, \\ \beta u(0) - \gamma u^{\Delta}(0) = 0, \qquad u(T) = \sum_{i=1}^{m-2} a_i u(\xi_i) \end{gathered} \tag{2.6}$$

has the unique solution

$$\begin{aligned} u(t) = &-\int_0^t (t-s)h(s)\nabla s + \frac{\beta t + \gamma}{d}\int_0^T (T-s)h(s)\nabla s \\ &- \frac{\beta t + \gamma}{d}\sum_{i=1}^{m-2} a_i \int_0^{\xi_i} (\xi_i - s)h(s)\nabla s. \end{aligned} \tag{2.7}$$

Proof. By direct computation, we can easily get (2.7). So, we omit it. □

Lemma 2.6. *If $0 < \sum_{i=1}^{m-2} b_i < 1$, $0 < \sum_{i=1}^{m-2} a_i\xi_i < T$, $d = \beta(T - \sum_{i=1}^{m-2} a_i\xi_i) + \gamma(1 - \sum_{i=1}^{m-2} a_i) > 0$, then for $h \in C_{ld}[0,T]_{\mathbf{T}}$, the boundary value problem (BVP)*

$$\begin{gathered} \left(\phi_p(u^{\Delta\nabla})\right)^{\nabla} + h(t) = 0, \quad t \in [0,T]_{\mathbf{T}}, \\ \beta u(0) - \gamma u^{\Delta}(0) = 0, \qquad u(T) = \sum_{i=1}^{m-2} a_i u(\xi_i), \qquad \phi_p(u^{\Delta\nabla}(0)) = \sum_{i=1}^{m-2} b_i \phi_p(u^{\Delta\nabla}(\xi_i)) \end{gathered} \tag{2.8}$$

has the unique solution

$$\begin{aligned} u(t) = &-\int_0^t (t-s)\phi_q\left(\int_0^s h(r)\nabla r + B\right)\nabla s + \frac{\beta t + \gamma}{d}\int_0^T (T-s)\phi_q\left(\int_0^s h(r)\nabla r + B\right)\nabla s \\ &- \frac{\beta t + \gamma}{d}\sum_{i=1}^{m-2} a_i \int_0^{\xi_i} (\xi_i - s)\phi_q\left(\int_0^s h(r)\nabla r + B\right)\nabla s, \end{aligned} \tag{2.9}$$

where $B = \sum_{i=1}^{m-2} b_i \int_0^{\xi_i} h(r)\nabla r / (1 - \sum_{i=1}^{m-2} b_i)$.

Proof. Integrating both sides of (1.6) on $[0,t]$, we have

$$\phi_p(u^{\Delta\nabla}(t)) = \phi_p(u^{\Delta\nabla}(0)) - \int_0^t h(r)\nabla r. \tag{2.10}$$

So

$$\phi_p(u^{\Delta\nabla}(\xi_i)) = \phi_p(u^{\Delta\nabla}(0)) - \int_0^{\xi_i} h(r)\nabla r. \tag{2.11}$$

By boundary value condition $\phi_p(u^{\Delta\nabla}(0)) = \sum_{i=1}^{m-2} b_i \phi_p(u^{\Delta\nabla}(\xi_i))$, we have

$$\phi_p(u^{\Delta\nabla}(0)) = -\frac{\sum_{i=1}^{m-2} b_i \int_0^{\xi_i} h(r)\nabla r}{1-\sum_{i=1}^{m-2} b_i}. \tag{2.12}$$

By (2.10) and (2.12), we know

$$u^{\Delta\nabla}(t) = -\phi_q\left(\frac{\sum_{i=1}^{m-2} b_i \int_0^{\xi_i} h(r)\nabla r}{1-\sum_{i=1}^{m-2} b_i} + \int_0^t h(r)\nabla r\right). \tag{2.13}$$

This together with Lemma 2.5 implies that

$$\begin{aligned} u(t) = &-\int_0^t (t-s)\phi_q\left(\int_0^s h(r)\nabla r + B\right)\nabla s + \frac{\beta t+\gamma}{d}\int_0^T (T-s)\phi_q\left(\int_0^s h(r)\nabla r + B\right)\nabla s \\ &- \frac{\beta t+\gamma}{d}\sum_{i=1}^{m-2} a_i \int_0^{\xi_i} (\xi_i - s)\phi_q\left(\int_0^s h(r)\nabla r + B\right)\nabla s, \end{aligned} \tag{2.14}$$

where $B = \sum_{i=1}^{m-2} b_i \int_0^{\xi_i} h(r)\nabla r / (1-\sum_{i=1}^{m-2} b_i)$. The proof is complete. □

Lemma 2.7. *Let* $0 < \sum_{i=1}^{m-2} a_i \xi_i < 1$, $d > 0$. *If* $h \in C_{ld}[0,T]_{\mathbf{T}}$ *and* $h(t) \geq 0$, *then the unique solution* u *of* (2.8) *satisfies*

$$u(t) \geq 0. \tag{2.15}$$

Proof. By $u^{\Delta\nabla}(t) = -\phi_q(\sum_{i=1}^{m-2} b_i \int_0^{\xi_i} h(r)\nabla r/(1-\sum_{i=1}^{m-2} b_i)) + \int_0^t h(r)\nabla r) \leq 0$, we can know that the graph of $u(t)$ is concave down on $(0,T)_{\mathbf{T}}$. So we only prove $u(0) \geq 0$, $u(T) \geq 0$.

Firstly, we will prove $u(0) \geq 0$ by the following two perspectives.

(i) If $0 < \sum_{i=1}^{m-2} a_i \leq 1$, we have

$$\begin{aligned} u(0) &= \frac{\gamma}{d}\left[\int_0^T (T-s)\phi_q\left(\int_0^s h(r)\nabla r + B\right)\nabla s - \sum_{i=1}^{m-2} a_i \int_0^{\xi_i} (\xi_i - s)\phi_q\left(\int_0^s h(r)\nabla r + B\right)\nabla s\right] \\ &\geq \frac{\gamma}{d}\left[\int_0^T (T-s)\phi_q\left(\int_0^s h(r)\nabla r + B\right)\nabla s - \sum_{i=1}^{m-2} a_i \int_0^{T} (T - s)\phi_q\left(\int_0^s h(r)\nabla r + B\right)\nabla s\right] \\ &= \frac{\gamma}{d}\left(1-\sum_{i=1}^{m-2} a_i\right)\int_0^T (T-s)\phi_q\left(\int_0^s h(r)\nabla r + B\right)\nabla s \geq 0. \end{aligned} \tag{2.16}$$

(ii) If $\sum_{i=1}^{m-2} a_i > 1$, by (2.8), we have

$$u(0) = \frac{\gamma}{d}\left[\int_0^T (T-s)\phi_q\left(\int_0^s h(r)\nabla r + B\right)\nabla s - \sum_{i=1}^{m-2} a_i \int_0^{\xi_i} (\xi_i - s)\phi_q\left(\int_0^s h(r)\nabla r + B\right)\nabla s\right]$$
$$\geq \frac{\gamma}{d}\left[\int_0^T (T-s)\phi_q\left(\int_0^s h(r)\nabla r + B\right)\nabla s - \sum_{i=1}^{m-2} a_i \int_0^{T} (\xi_i - s)\phi_q\left(\int_0^s h(r)\nabla r + B\right)\nabla s\right]$$
$$= \frac{\gamma}{d}\int_0^T \left[\left(T - \sum_{i=1}^{m-2} a_i\xi_i\right) + \left(\sum_{i=1}^{m-2} a_i - 1\right)s\right]\phi_q\left(\int_0^s h(r)\nabla r + B\right)\nabla s \geq 0. \tag{2.17}$$

On the other hand, we have

$$u(T) = -\int_0^T (T-s)\phi_q\left(\int_0^s h(r)\nabla r + B\right)\nabla s + \frac{\beta+\gamma}{d}\int_0^T (T-s)\phi_q\left(\int_0^s h(r)\nabla r + B\right)\nabla s$$
$$- \frac{\beta+\gamma}{d}\sum_{i=1}^{m-2} a_i \int_0^{\xi_i} (\xi_i - s)\phi_q\left(\int_0^s h(r)\nabla r + B\right)\nabla s$$
$$\geq \frac{\beta}{d}\left[\sum_{i=1}^{m-2} a_i \int_0^{\xi_i} (\xi_i(T-s) - T(\xi_i - s))\phi_q\left(\int_0^s h(r)\nabla r + B\right)\nabla s\right.$$
$$\left. + \sum_{i=1}^{m-2} a_i\xi_i \int_{\xi_i}^T (T-s)\phi_q\left(\int_0^s h(r)\nabla r + B\right)\nabla s\right]$$
$$+ \frac{\gamma}{d}\sum_{i=1}^{m-2} a_i\left[\int_0^T (T-s)\phi_q\left(\int_0^s h(r)\nabla r + B\right)\nabla s - \int_0^T (\xi_i - s)\phi_q\left(\int_0^s h(r)\nabla r + B\right)\nabla s\right]$$
$$= \frac{\beta}{d}\sum_{i=1}^{m-2} a_i\left[\int_0^{\xi_i} (T-\xi_i)s\phi_q\left(\int_0^s h(r)\nabla r + B\right)\nabla s + \xi_i\int_{\xi_i}^T (T-s)\phi_q\left(\int_0^s h(r)\nabla r + B\right)\nabla s\right]$$
$$+ \frac{\gamma}{d}\sum_{i=1}^{m-2} a_i\left[\int_0^T (T-\xi_i)\phi_q\left(\int_0^s h(r)\nabla r + B\right)\nabla s\right] \geq 0. \tag{2.18}$$

The proof is completed. □

Lemma 2.8. *Let $a_i \geq 0$, $i = 1, \ldots, m-2$, $0 < \sum_{i=1}^{m-2} a_i\xi_i < T$, $d > 0$. If $h \in C_{ld}[0,T]_{\mathbf{T}}$ and $h(t) \geq 0$, then the unique positive solution $u(t)$ of (BVP)* (2.8) *satisfies*

$$\inf_{t\in[\xi_{m-2},T]_{\mathbf{T}}} u(t) \geq \sigma\|u\|, \tag{2.19}$$

where $\sigma = \min\{a_{m-2}(T-\xi_{m-2})/(T - a_{m-2}\xi_{m-2}), a_{m-2}\xi_{m-2}/T, \xi_{m-2}/T\}$, $\|u\| = \sup_{t\in[0,T]_{\mathbf{T}}}|u(t)|$.

Proof. Let $u(\bar{t}) = \max_{t\in[0,T]_{\mathbf{T}}} u(t) = \|u\|$, we shall discuss it from the following two perspectives.

Case 1. If $0 < \sum_{i=1}^{m-2} a_i < 1$.

Firstly, assume $\bar{t} < \xi_{m-2} < T$, then $\min_{t\in[\xi_{m-2},T]_{\mathbf{T}}} u(t) = u(T)$. By $u(T) = \sum_{i=1}^{m-2} a_i u(\xi_i) \geq a_{m-2}u(\xi_{m-2})$, we have

$$\begin{aligned} u(\bar{t}) &\leq u(T) + \frac{u(T) - u(\xi_{m-2})}{T - \xi_{m-2}}(0 - T) = u(T) - \frac{T}{T - \xi_{m-2}}u(T) + \frac{T}{T - \xi_{m-2}}u(\xi_{m-2}) \\ &\leq u(T)\left(1 - \frac{T}{T - \xi_{m-2}} + \frac{T}{a_{m-2}(T - \xi_{m-2})}\right) = u(T)\frac{T - a_{m-2}\xi_{m-2}}{a_{m-2}(T - \xi_{m-2})}. \end{aligned} \tag{2.20}$$

So

$$\min_{t\in[\xi_{m-2},T]_{\mathbf{T}}} u(t) \geq \frac{a_{m-2}(T - \xi_{m-2})}{T - a_{m-2}\xi_{m-2}}\|u\|. \tag{2.21}$$

Secondly, assume $\xi_{m-2} < \bar{t} < T$, then $\min_{t\in[\xi_{m-2},T]_{\mathbf{T}}} u(t) = u(T)$. Otherwise, we have $\min_{t\in[\xi_{m-2},T]_{\mathbf{T}}} u(t) = u(\xi_{m-2})$, then $\bar{t} \in [\xi_{m-2}, T]_{\mathbf{T}}$, $u(\xi_{m-2}) \geq u(\xi_{m-1}) \geq \cdots \geq u(\xi_2) \geq u(\xi_1)$. By $0 < \sum_{i=1}^{m-2} a_i < 1$, we have

$$u(T) = \sum_{i=1}^{m-2} a_i u(\xi_i) \leq \sum_{i=1}^{m-2} a_i u(\xi_{m-2}) < u(\xi_{m-2}) \leq u(T), \tag{2.22}$$

a contradiction.

By concave of $u(t)$, we get $u(\xi_{m-2})/\xi_{m-2} \geq u(\bar{t})/\bar{t} \geq u(\bar{t})/T$. In fact, since $u(T) \geq a_{m-2}u(\xi_{m-2})$, then $u(T)/a_{m-2}\xi_{m-2} \geq u(\bar{t})/T$, which implies

$$\min_{t\in[\xi_{m-2},T]_{\mathbf{T}}} u(t) \geq \frac{a_{m-2}\xi_{m-2}}{T}\|u\|. \tag{2.23}$$

Case 2. If $\sum_{i=1}^{m-2} a_i > 1$.

Firstly, assume $u(\xi_{m-2}) \leq u(T)$, then $\min_{t\in[\xi_{m-2},T]_{\mathbf{T}}} u(t) = u(\xi_{m-2})$. By concave of $u(t)$, we have $\bar{t} \in [\xi_{m-2}, t]_{\mathbf{T}}$, which implies $u(\xi_{m-2})/\xi_{m-2} \geq u(\bar{t})/\bar{t} \geq u(\bar{t})/T$, then

$$\min_{t\in[\xi_{m-2},T]_{\mathbf{T}}} u(t) \geq \frac{\xi_{m-2}}{T}\|u\|. \tag{2.24}$$

Secondly, assume $u(\xi_{m-2}) > u(t)$, then $\min_{t\in[\xi_{m-2},T]_{\mathbf{T}}} u(t) = u(T)$, and $\bar{t} \in [\xi_1, T]_{\mathbf{T}}$. If not, $\bar{t} \in [0, \xi_1)$, then $u(\xi_1) \geq \cdots \geq u(\xi_{m-2}) > u(T)$. So, we have

$$u(T) = \sum_{i=1}^{m-2} a_i u(\xi_i) > u(T)\sum_{i=1}^{m-2} a_i \geq u(T), \tag{2.25}$$

a contradiction. By $\sum_{i=1}^{m-2} a_i > 1$, there exists $\bar{\xi} \in \{\xi_1, \xi_2, \ldots, \xi_{m-2}\}$ such that $u(\bar{\xi}) \leq u(T)$, then $u(\xi_1) \leq u(\xi_2) \leq \cdots \leq u(\xi_{m-2}) \leq u(1)$. By concave of $u(t)$, we have $u(1)/\xi_1 \geq u(\xi_1)/\xi_1 \geq u(\bar{t})/\bar{t} \geq u(\bar{t})/T$, then

$$\min_{t \in [\xi_{m-2}, T]_{\mathbf{T}}} u(t) \geq \xi_1 \|u\|. \tag{2.26}$$

Therefore, by (2.21)–(2.26), we have

$$\inf_{t \in [\xi_{m-2}, T]_{\mathbf{T}}} u(t) \geq \sigma \|u\|, \tag{2.27}$$

where $\sigma = \min\{a_{m-2}(T - \xi_{m-2})/(T - a_{m-2}\xi_{m-2}), a_{m-2}\xi_{m-2}/T, \xi_{m-2}/T\}$. The proof is complete. □

Let $E = C_{\mathrm{ld}}[0, T]_{\mathbf{T}}$ be endowed with the ordering $x \leq y$ if $x(t) \leq y(t)$, for all $t \in [0, T]_{\mathbf{T}}$, and $\|u\| = \max_{t \in [0,T]_{\mathbf{T}}} |u(t)|$ is defined as usual by maximum norm. Clearly, it follows that $(E, \|u\|)$ is a Banach space.

We define a cone by

$$K = \left\{ u : u \in E,\ u(t) \text{ is concave, nonnegative on } [0, T]_{\mathbf{T}},\ \inf_{t \in [\xi_{m-2}, T]_{\mathbf{T}}} u(t) \geq \sigma \|u\| \right\}. \tag{2.28}$$

Define an operator $S : K \to E$ by setting

$$\begin{aligned} Su(t) = &- \int_0^t (t - s)\phi_q\left(\int_0^s a(r) f(r, u(r)) \nabla r + A\right) \nabla s \\ &+ \frac{\beta t + \gamma}{d} \int_0^T (T - s)\phi_q\left(\int_0^s a(r) f(r, u(r)) \nabla r + A\right) \nabla s \\ &- \frac{\beta t + \gamma}{d} \sum_{i=1}^{m-2} a_i \int_0^{\xi_i} (\xi_i - s)\phi_q\left(\int_0^s a(r) f(r, u(r)) \nabla r + A\right) \nabla s, \end{aligned} \tag{2.29}$$

where $A = \sum_{i=1}^{m-2} b_i \int_0^{\xi_i} a(r) f(r, u(r)) \nabla r / (1 - \sum_{i=1}^{m-2} b_i)$. Obviously, u is a solution of boundary value problem (1.6) if and only if u is a fixed point of operator S.

Lemma 2.9. *$S : K \to K$ is completely continuous.*

Proof. By (H_2) and Lemmas 2.7-2.8, we easily get $SK \subset K$. By Arzela-Ascoli theorem and Lebesgue dominated convergence theorem, we can easily prove S is completely continuous. □

Lemma 2.10 (see [15]). *Let K be a cone in a Banach space X. Let D be an open bounded subset of X with $D_K = D \cap K \neq \varnothing$ and $\overline{D}_K \neq K$. Assume that $A : \overline{D}_K \to K$ is a compact map such that $x \neq Ax$ for $x \in \partial D_K$. Then the following results hold.*

(1) *If* $\|Ax\| \leq \|x\|$, $x \in \partial D_K$, *then* $i_K(A, D_K) = 1$.

(2) *If there exists* $x_0 \in K \setminus \{0\}$ *such that* $x \neq Ax + \lambda x_0$, *for all* $x \in \partial D_K$ *and all* $\lambda > 0$, *then* $i_K(A, D_K) = 0$.

(3) *Let* U *be open in* X *such that* $\overline{U} \subset D_K$. *If* $i_K(A, D_K) = 1$ *and* $i_K(A, U_K) = 0$, *then* A *has a fixed point in* $D_K \setminus \overline{U}_K$. *The same result holds if* $i_K(A, D_K) = 0$ *and* $i_K(A, U_K) = 1$, *where* $i_K(A, D_K)$ *denotes fixed point index.*

One defines

$$K_\rho = \{u(t) \in K : \|u\| < \rho\}, \qquad \Omega_\rho = \left\{u(t) \in K : \min_{\xi_{m-2} \leq t \leq T} x(t) < \sigma\rho\right\}. \tag{2.30}$$

Lemma 2.11 (see [15]). Ω_ρ *defined above has the following properties:*

(a) $K_{\sigma\rho} \subset \Omega_\rho \subset K_\rho$;

(b) Ω_ρ *is open relative to* K;

(c) $x \in \partial\Omega_\rho$ *if and only if* $\min_{\xi_{m-2} \leq t \leq T} x(t) = \sigma\rho$;

(d) *if* $x \in \partial\Omega_\rho$, *then* $\sigma\rho \leq x(t) \leq \rho$, *for* $t \in [\xi_{m-2}, T]_{\mathbf{T}}$.

For the convenience, we introduce the following notations:

$$\begin{gathered}
f_{\sigma\rho}^{\rho} = \min\left\{\min_{\xi_{m-2} \leq t \leq T} \frac{f(t,u)}{\phi_p(\rho)} : u \in [\sigma\rho, \rho]\right\}, \qquad f_0^{\rho} = \max\left\{\max_{0 \leq t \leq T} \frac{f(t,u)}{\phi_p(\rho)} : u \in [0, \rho]\right\}, \\
f^{\alpha} = \lim_{u \to \alpha} \sup \max_{0 \leq t \leq T} \frac{f(t,u)}{\phi_p(u)}, \qquad f_{\alpha} = \lim_{u \to \alpha} \inf \max_{\xi_{m-2} \leq t \leq T} \frac{f(t,u)}{\phi_p(u)} \qquad (\alpha := \infty \text{ or } 0^+), \\
\frac{1}{m} = \frac{(\beta T + \gamma)}{d} \int_0^T (T - s) \nabla s \phi_q\left(\int_0^T a(r) \nabla r + \frac{\sum_{i=1}^{m-2} b_i \int_0^{\xi_i} a(r) \nabla r}{1 - \sum_{i=1}^{m-2} b_i}\right), \\
\frac{1}{M} = \frac{1}{d} \int_{\xi_{m-2}}^T (T - s) \phi_q\left(\int_{\xi_{m-2}}^s a(r) \nabla r\right) \nabla s \min\left\{\beta\xi_{m-2} + \gamma, \beta \max\left\{\sum_{i=1}^{m-2} a_i \xi_1, a_{m-2}\xi_{m-2}\right\} + \gamma \sum_{i=1}^{m-2} a_i\right\}.
\end{gathered} \tag{2.31}$$

Lemma 2.12. *If* f *satisfies the following condition:*

$$f_0^{\rho} \leq \phi_p(m), \quad u \neq Su, \; u \in \partial K_\rho, \tag{2.32}$$

then

$$i_K(S, K_\rho) = 1. \tag{2.33}$$

Proof. For $u \in \partial K_\rho$, then from (2.32), we have

$$\begin{aligned}
\int_0^s a(r) f(r,u(r))\nabla r + A &= \int_0^s a(r) f(r,u(r))\nabla r + \frac{\sum_{i=1}^{m-2} b_i \int_0^{\xi_i} a(r) f(r,u(r))\nabla r}{1-\sum_{i=1}^{m-2} b_i} \\
&\le \int_0^T a(r) f(r,u(r))\nabla r + \frac{\sum_{i=1}^{m-2} b_i \int_0^{\xi_i} a(r) f(r,u(r))\nabla r}{1-\sum_{i=1}^{m-2} b_i} \\
&\le \phi_p(m\rho)\left(\int_0^T a(r)\nabla r + \frac{\sum_{i=1}^{m-2} b_i \int_0^{\xi_i} a(r)\nabla r}{1-\sum_{i=1}^{m-2} b_i}\right).
\end{aligned} \tag{2.34}$$

So that

$$\phi_q\left(\int_0^s a(r) f(r,u(r))\nabla r + A\right) \le m\rho\phi_q\left(\int_0^T a(r)\nabla r + \frac{\sum_{i=1}^{m-2} b_i \int_0^{\xi_i} a(r)\nabla r}{1-\sum_{i=1}^{m-2} b_i}\right). \tag{2.35}$$

Therefore,

$$\begin{aligned}
Su(t) &\le \frac{\beta t+\gamma}{d}\int_0^T (T-s)\phi_q\left(\int_0^s a(r) f(r,u(r))\nabla r + A\right)\nabla s \\
&\le \frac{(\beta T+\gamma)m\rho}{d}\int_0^T (T-s)\nabla s\phi_q\left(\int_0^T a(r)\nabla r + \frac{\sum_{i=1}^{m-2} b_i \int_0^{\xi_i} a(r)\nabla r}{1-\sum_{i=1}^{m-2} b_i}\right) \\
&= \rho.
\end{aligned} \tag{2.36}$$

This implies that $\|Su\| \le \|u\|$ for $u \in \partial K_\rho$. Hence, by Lemma 2.10(1) it follows that $i_K(S,K_\rho) = 1$. □

Lemma 2.13. *If f satisfies the following condition:*

$$f_{\sigma\rho}^{\rho} \ge \phi_p(M\sigma), \quad u \ne Su,\ u \in \partial\Omega_\rho, \tag{2.37}$$

then

$$i_K(S,\Omega_\rho) = 0. \tag{2.38}$$

Proof. Let $e(t) \equiv 1$ for $t \in [0,T]_{\mathbf{T}}$. Then $e \in \partial K_1$. We claim that

$$u \ne Su + \lambda e, \quad u \in \partial\Omega_\rho,\ \lambda > 0. \tag{2.39}$$

In fact, if not, there exist $u_0 \in \partial\Omega_\rho$ and $\lambda_0 > 0$ such that $u_0 = Su_0 + \lambda_0 e$. By $f_{\sigma\rho}^{\rho} \geq \phi_p(M\sigma)$, we have

$$\begin{aligned}\int_0^s a(r)f(r,u_0(r))\nabla r + A &= \int_0^s a(r)f(r,u_0(r))\nabla r + \frac{\sum_{i=1}^{m-2} b_i \int_0^{\xi_i} a(r)f(r,u_0(r))\nabla r}{1-\sum_{i=1}^{m-2} b_i} \\ &\geq \int_{\xi_{m-2}}^s a(r)f^+(r,u(r))\nabla r \\ &\geq \phi_p(M\sigma\rho)\left(\int_{\xi_{m-2}}^s a(r)\nabla r\right).\end{aligned} \tag{2.40}$$

So that

$$\phi_q\left(\int_0^s a(r)f(r,u(r))\nabla r + A\right) \geq M\sigma\rho\phi_q\left(\int_{\xi_{m-2}}^s a(r)\nabla r\right). \tag{2.41}$$

By [16, Theorem 2.2(iv)], for $t > 0$, we have

$$\left(\frac{\int_0^t (t-s)\phi_q\left(\int_0^s a(r)f(r,u_0(r))\nabla r + A\right)\nabla s}{t}\right)^{\Delta} = \frac{\int_0^t s\phi_q\left(\int_0^s a(r)f(r,u_0(r))\nabla r + A\right)\nabla s}{t\sigma(t)} \geq 0. \tag{2.42}$$

So, for $i = 1, 2, \ldots, m-2$, we have

$$\frac{\int_0^{\xi_{m-2}} (\xi_{m-2}-s)\phi_q\left(\int_0^s a(r)f(r,u_0(r))\nabla r + A\right)\nabla s}{\xi_{m-2}} \geq \frac{\int_0^{\xi_i} (\xi_i-s)\phi_q\left(\int_0^s a(r)f(r,u_0(r))\nabla r + A\right)\nabla s}{\xi_i}. \tag{2.43}$$

Therefore,

$$\begin{aligned}Su_0(\xi_{m-2}) \geq \frac{\beta}{d}\Bigg[&\xi_{m-2}\int_0^T (t-s)\phi_q\left(\int_0^s a(r)f(r,u_0(r))\nabla r + A\right)\nabla s \\ &- T\int_0^{\xi_{m-2}} (\xi_{m-2}-s)\phi_q\left(\int_0^s a(r)f(r,u_0(r))\nabla r + A\right)\nabla s\Bigg] \\ &+ \frac{\gamma}{d}\Bigg[\int_0^T (t-s)\phi_q\left(\int_0^s a(r)f(r,u_0(r))\nabla r + A\right)\nabla s \\ &- \int_0^{\xi_{m-2}} (\xi_{m-2}-s)\phi_q\left(\int_0^s a(r)f(r,u_0(r))\nabla r + A\right)\nabla s\Bigg]\end{aligned}$$

$$\geq \frac{\beta\xi_{m-2}+\gamma}{d}\int_{\xi_{m-2}}^{T}(T-s)\phi_q\left(\int_0^s a(r)f(r,u_0(r))\nabla r + A\right)\nabla s$$
$$\geq \frac{(\beta\xi_{m-2}+\gamma)M\sigma\rho}{d}\int_{\xi_{m-2}}^{T}(T-s)\phi_q\left(\int_{\xi_{m-2}}^s a(r)\nabla r\right)\nabla s, \tag{2.44}$$

$$\begin{aligned}Su_0(T) &\geq \frac{\beta}{d}\sum_{i=1}^{m-2}a_i\left[\int_0^{\xi_i}(t-\xi_i)s\phi_q\left(\int_0^s a(r)f(r,u_0(r))\nabla r + A\right)\nabla s\right.\\ &\qquad\left. + \xi_i\int_{\xi_i}^{T}(T-s)\phi_q\left(\int_0^s a(r)f(r,u_0(r))\nabla r + A\right)\nabla s\right]\\ &\quad + \frac{\gamma}{d}\sum_{i=1}^{m-2}\left[\int_0^{T}(t-s)\phi_q\left(\int_0^s a(r)f(r,u_0(r))\nabla r + A\right)\nabla s\right.\\ &\qquad\left. - \int_0^{\xi_i}(T-s)\phi_q\left(\int_0^s a(r)f(r,u_0(r))\nabla r + A\right)\nabla s\right]\\ &\geq \frac{\beta}{d}\sum_{i=1}^{m-2}a_i\xi_i\int_{\xi_i}^{T}(t-s)\phi_q\left(\int_0^s a(r)f(r,u_0(r))\nabla r + A\right)\nabla s\\ &\quad + \frac{\gamma}{d}\sum_{i=1}^{m-2}\int_{\xi_i}^{T}(t-s)\phi_q\left(\int_0^s a(r)f(r,u_0(r))\nabla r + A\right)\nabla s\\ &\geq \frac{M\sigma\rho}{d}\left(\beta\max\left\{\sum_{i=1}^{m-2}a_i\xi_1, a_{m-2}\xi_{m-2}\right\} + \gamma\sum_{i=1}^{m-2}\right)\int_{\xi_{m-2}}^{T}(T-s)\phi_q\left(\int_{\xi_{m-2}}^s a(r)\nabla r\right)\nabla s.\end{aligned} \tag{2.45}$$

Obviously, we can know

$$\begin{aligned}\min_{t\in[\xi_{m-2},T]_{\mathbf{T}}} Su_0(t) &= \min\left\{Su_0(\xi_{m-2}), Su_0(T)\right\}\\ &\geq \frac{M\sigma\rho}{d}\int_{\xi_{m-2}}^{T}(T-s)\phi_q\left(\int_{\xi_{m-2}}^s a(r)\nabla r\right)\nabla s\\ &\quad\times\min\left\{\beta\xi_{m-2}+\gamma, \beta\max\left\{\sum_{i=1}^{m-2}a_i\xi_1, a_{m-2}\xi_{m-2}\right\} + \gamma\sum_{i=1}^{m-2}\right\}\\ &\geq \sigma\rho.\end{aligned} \tag{2.46}$$

For $t\in[\xi_{m-2},T]_{\mathbf{T}}$, then

$$\begin{aligned}u_0(t) = Su_0(t) + \lambda_0 e(t) &\geq \min_{t\in[\xi_{m-2},T]} Su_0(t) + \lambda_0\\ &= \min\left\{Su_0(\xi_{m-2}), Su_0(T)\right\} + \lambda_0 \geq \sigma\rho + \lambda_0.\end{aligned} \tag{2.47}$$

This together with Lemma 2.11(c) implies that

$$\sigma\rho \geq \sigma\rho + \lambda_0, \tag{2.48}$$

a contradiction. Hence, by Lemma 2.10(2), it follows that $i_K(S, \Omega_\rho) = 0$. □

3. Main results

We now give our results on the existence of positive solutions of BVP (1.6).

Theorem 3.1. *Suppose conditions (H_1) and (H_2) hold, and assume that one of the following conditions holds.*

- (H_3) *There exist $\rho_1, \rho_2 \in (0, +\infty)$ with $\rho_1 < \sigma\rho_2$ such that $f_0^{\rho_1} \leq \phi_p(m)$, $f_{\sigma\rho_2}^{\rho_2} \geq \phi_p(M\sigma)$.*
- (H_4) *There exist $\rho_1, \rho_2 \in (0, +\infty)$ with $\rho_1 < \rho_2$ such that $f_0^{\rho_2} \leq \phi_p(m)$, $f_{\sigma\rho_1}^{\rho_1} \geq \phi_p(M\sigma)$.*

Then, the boundary value problem (1.6) *has at least one positive solution.*

Proof. Assume that (H_3) holds, we show that S has a fixed point u_1 in $\Omega_{\rho_2} \backslash \overline{K}_{\rho_1}$. By $f_0^{\rho_1} \leq \phi_p(m)$ and Lemma 2.12, we have that

$$i_K(S, K_{\rho_1}) = 1. \tag{3.1}$$

By $f_{\sigma\rho_2}^{\rho_2} \geq \phi_p(M\sigma)$ and Lemma 2.13, we have that

$$i_K(S, K_{\rho_2}) = 0. \tag{3.2}$$

By Lemma 2.11(a) and $\rho_1 < \sigma\rho_2$, we have $\overline{K}_{\rho_1} \subset K_{\sigma\rho_2} \subset \Omega_{\rho_2}$. It follows from Lemma 2.10(3) that S has a fixed point u_1 in $\Omega_{\rho_2} \setminus \overline{K}_{\rho_1}$. When condition (H_4) holds, the proof is similar to the above, so we omit it here.

As a special case of Theorem 3.1, we obtain the following result. □

Corollary 3.2. *Suppose conditions (H_1) and (H_2) hold, and assume that one of the following conditions holds.*

- (H_5) $0 \leq f^0 < \phi_p(m)$ *and* $\phi_p(M) < f_\infty \leq \infty$.
- (H_6) $0 \leq f^\infty < \phi_p(m)$ *and* $\phi_p(M) < f_0 \leq \infty$.

Then, the boundary value problem (1.6) *has at least one positive solution.*

Theorem 3.3. *Assume conditions (H_1) and (H_2) hold, and suppose that one of the following conditions holds.*

- (H_7) *There exist ρ_1, ρ_2, and $\rho_3 \in (0, +\infty)$ with $\rho_1 < \sigma\rho_2$ and $\rho_2 < \rho_3$ such that*

$$f_0^{\rho_1} \leq \phi_p(m), \quad f_{\sigma\rho_2}^{\rho_2} \geq \phi_p(M\sigma), \quad u \neq Su, \quad \forall\, u \in \partial\Omega_{\rho_2}, \quad f_0^{\rho_3} \leq \phi_p(m). \tag{3.3}$$

(H_8) *There exist* ρ_1, ρ_2, *and* $\rho_3 \in (0, +\infty)$ *with* $\rho_1 < \rho_2 < \sigma\rho_3$ *such that*

$$f_0^{\rho_2} \le \phi_p(m), \quad f_{\sigma\rho_1}^{\rho_1} \ge \phi_p(M\sigma), \quad u \ne Su, \quad \forall u \in \partial K_{\rho_2}, \quad f_{\sigma\rho_3}^{\rho_3} \ge \phi_p(M\sigma). \tag{3.4}$$

Then, the boundary value problem (1.6) *has at least two positive solutions. Moreover, if in* (H_7) $f_0^{\rho_1} \le \phi_p(m)$ *is replaced by* $f_0^{\rho_1} < \phi_p(m)$, *then the BVP* (1.6) *has a third positive solution* $u_3 \in K_{\rho_1}$.

Proof. Assume that condition (H_7) holds, we show that either S has a fixed point u_1 in ∂K_{ρ_1} or $\Omega_{\rho_2} \setminus \overline{K}_{\rho_1}$. If $u \ne Su$ for $u \in \partial K_{\rho_1} \cup \partial K_{\rho_3}$. By Lemma 2.12 and Lemma 2.13, we have that

$$\begin{aligned} i_K(S, K_{\rho_1}) &= 1, \\ i_K(S, K_{\rho_3}) &= 1, \\ i_K(S, K_{\rho_2}) &= 0. \end{aligned} \tag{3.5}$$

By Lemma 2.11(a) and $\rho_1 < \sigma\rho_2$, we have $\overline{K}_{\rho_1} \subset K_{\sigma\rho_2} \subset \Omega_{\rho_2}$. It follows from Lemma 2.10(3) that S has a fixed point u_1 in $\Omega_{\rho_2} \setminus \overline{K}_{\rho_1}$. Similarly, S has a fixed point u_2 in $K_{\rho_3} \setminus \overline{\Omega}_{\rho_2}$. When condition (H_8) holds, the proof is similar to the above, so we omit it here.

As a special case of Theorem 3.3, we obtain the following result. □

Corollary 3.4. *Assume conditions* (H_1) *and* (H_2) *hold, if there exists* $\rho > 0$ *such that one of the following conditions holds.*

(H_9) $0 \le f^0 < \phi_p(m)$, $f_{\sigma\rho}^{\rho} \ge \phi_p(M\sigma)$, $u \ne Su$, $\forall u \in \partial\Omega_\rho$ *and* $0 \le f^\infty < \phi_p(m)$.

(H_{10}) $\phi_p(M) < f_0 \le \infty$, $f_0^{\rho} \le \phi_p(m)$, $u \ne Su$, $\forall u \in \partial K_\rho$ *and* $\phi_p(M) < f_\infty \le \infty$.

Then, the boundary value problem (1.6) *has at least two positive solutions.*

4. Some examples

In this section, we present some simple examples to explain our results. We only study the case $\mathbf{T} = \mathbb{R}$, $(0, T)_{\mathbf{T}} = (0, 1)$.

Example 4.1. Consider the following three-point boundary value problem with p-Laplacian:

$$\begin{gathered} (\phi_p(u''))' + a(t)f(t, u) = 0, \quad 0 < t < 1, \\ u'(0) = 0, \qquad u(1) = \frac{1}{2}u\left(\frac{1}{3}\right), \qquad (\phi_p(u'')(0)) = \frac{1}{4}\left(\phi_p(u'')\left(\frac{1}{3}\right)\right), \end{gathered} \tag{4.1}$$

where $\beta = 0$, $\gamma = 1$, $a_1 = 1/2$, $b_1 = 1/4$, $\xi_1 = 1/3$, $a(t) = 1$, $p = q = 2$. By computing, we can know $\sigma = 1/6$, $M = 819/16$, $m = 9/10$. Let $\rho_1 = 1$, $\rho_2 = 208$, then $\sigma\rho_1 < \rho_1 < \sigma\rho_2 < \rho_2$. We define a nonlinearity f as follows:

$$f(t,u)=\begin{cases}\dfrac{9t^3}{10}\left(\dfrac{1}{6}-u\right)^3, & 0<t<1,\ u\in\left[0,\dfrac{1}{6}\right],\\ \dfrac{9t^3}{10}\sin\left(\dfrac{6}{5}\dfrac{\pi}{2}u-\dfrac{1}{5}\dfrac{\pi}{2}\right), & 0<t<1,\ u\in\left[\dfrac{1}{6},1\right],\\ \dfrac{9t^3}{10}\left(\dfrac{208}{202}-\dfrac{6}{202}u\right)+\dfrac{819}{96}\left(\dfrac{6}{202}u-\dfrac{6}{202}\right), & 0<t<1,\ u\in\left[1,\dfrac{208}{6}\right],\\ \dfrac{819}{96}+t^3\left(u-\dfrac{208}{6}\right)^2, & 0<t<1,\ u\in\left[\dfrac{208}{6},208\right],\\ \dfrac{819}{96}+t^3\left(208-\dfrac{208}{6}\right)^2[1+(u-208)], & 0<t<1,\ u\in[208,+\infty].\end{cases} \tag{4.2}$$

Then, by the definition of f, we have

(i) $f_0^{\rho_1}\le\phi_p(m)=9/10$;
(ii) $f_{\sigma\rho_2}^{\rho_2}\ge\phi_p(M\sigma)=819/19968$.

So condition (H_3) holds, by Theorem 3.1, boundary value problem (4.1) has at least one positive solution.

Acknowledgment

Project supported by the National Natural Science Foundation of China (10471075).

References

[1] S. Hilger, “Analysis on measure chains—a unified approach to continuous and discrete calculus,” *Results in Mathematics*, vol. 18, no. 1-2, pp. 18–56, 1990.
[2] R. P. Agarwal and D. O’Regan, “Nonlinear boundary value problems on time scales,” *Nonlinear Analysis: Theory, Methods & Applications*, vol. 44, no. 4, pp. 527–535, 2001.
[3] F. M. Atici and G. Sh. Guseinov, “On Green’s functions and positive solutions for boundary value problems on time scales,” *Journal of Computational and Applied Mathematics*, vol. 141, no. 1-2, pp. 75–99, 2002.
[4] H.-R. Sun and W.-T. Li, “Positive solutions for nonlinear three-point boundary value problems on time scales,” *Journal of Mathematical Analysis and Applications*, vol. 299, no. 2, pp. 508–524, 2004.
[5] M. Bohner and A. Peterson, Eds., *Advances in Dynamic Equations on Time Scales*, Birkhäuser, Boston, Mass, USA, 2003.
[6] H. R. Sun and W. T. Li, “Positive solutions for nonlinear m-point boundary value problems on time scales,” *Acta Mathematica Sinica*, vol. 49, no. 2, pp. 369–380, 2006 (Chinese).
[7] H.-R. Sun and W.-T. Li, “Existence theory for positive solutions to one-dimensional p-Laplacian boundary value problems on time scales,” *Journal of Differential Equations*, vol. 240, no. 2, pp. 217–248, 2007.
[8] Z. He, “Double positive solutions of three-point boundary value problems for p-Laplacian dynamic equations on time scales,” *Journal of Computational and Applied Mathematics*, vol. 182, no. 2, pp. 304–315, 2005.
[9] Z. He and X. Jiang, “Triple positive solutions of boundary value problems for p-Laplacian dynamic equations on time scales,” *Journal of Mathematical Analysis and Applications*, vol. 321, no. 2, pp. 911–920, 2006.

[10] D.-X. Ma, Z.-J. Du, and W.-G. Ge, "Existence and iteration of monotone positive solutions for multipoint boundary value problem with p-Laplacian operator," *Computers & Mathematics with Applications*, vol. 50, no. 5-6, pp. 729–739, 2005.

[11] Y. Wang and W. Ge, "Positive solutions for multipoint boundary value problems with a one-dimensional p-Laplacian," *Nonlinear Analysis: Theory, Methods & Applications*, vol. 66, no. 6, pp. 1246–1256, 2007.

[12] Y. Wang and C. Hou, "Existence of multiple positive solutions for one-dimensional p-Laplacian," *Journal of Mathematical Analysis and Applications*, vol. 315, no. 1, pp. 144–153, 2006.

[13] C. Zhou and D. Ma, "Existence and iteration of positive solutions for a generalized right-focal boundary value problem with p-Laplacian operator," *Journal of Mathematical Analysis and Applications*, vol. 324, no. 1, pp. 409–424, 2006.

[14] D. R. Anderson, "Green's function for a third-order generalized right focal problem," *Journal of Mathematical Analysis and Applications*, vol. 288, no. 1, pp. 1–14, 2003.

[15] K. Q. Lan, "Multiple positive solutions of semilinear differential equations with singularities," *Journal of the London Mathematical Society*, vol. 63, no. 3, pp. 690–704, 2001.

[16] M. Bohner and A. Peterson, *Dynamic Equations on Time Scales: An Introduction with Application*, Birkhäuser, Boston, Mass, USA, 2001.


Advances in
Operations Research
Advances in
Decision Sciences
Journal of
Applied Mathematics
Algebra
Journal of
Probability and Statistics
The Scientific
World Journal
International Journal of
Differential Equations
Hindawi
Submit your manuscripts at
http://www.hindawi.com
International Journal of
Combinatorics
Advances in
Mathematical Physics
Journal of
Complex Analysis
Journal of
Mathematics
Mathematical Problems
in Engineering
Abstract and
Applied Analysis
Discrete Dynamics in
Nature and Society
International
Journal of
Mathematics and
Mathematical
Sciences
Journal of
Discrete Mathematics
Journal of
Function Spaces
International Journal of
Stochastic Analysis
Journal of
Optimization