Research Article

On the Impulsive Synchronization Control for a Class of Chaotic Systems

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The problem on chaos synchronization for a class of chaotic system is addressed. Based on impulsive control theory and by constructing a novel Lyapunov functional, new impulsive synchronization strategies are presented and possess more practical application value. Finally some typical numerical simulation examples are included to demonstrate the effectiveness of the theoretical results.

1. Introduction

Chaos synchronization can be found in many areas, such as laser physics and chemical reactor. Up to now many kinds of methods have been developed to realize chaos synchronization, such as feedback control method [1–3], sliding control method [4–6], backstepping control method [7, 8], fuzzy control method [9–12], and adaptive control method [13–15].

Recently impulsive control theory and its application in chaos synchronization have become a research hotspot. For instance, based on impulsive control strategies, the reduced-order observer for the synchronization of generalized Lorenz chaotic systems is built in [16], the adaptive modified function projective synchronization of multiple time-delayed chaotic Rossler systems is discussed in [17], the hybrid synchronization of Lü hyperchaotic system with disturbances is investigated in [18], and the robust synchronization of perturbed Chen's fractional-order chaotic systems is studied in [19]. However, these results are just about one kind of chaotic systems, which limits their applied scope. Hence how to design the impulsive strategy to realize the synchronization suitable for more chaotic systems activates our research.

In this paper, the following class of chaotic system is introduced:

$$\dot{z} = f(z),$$
$$z(t_0^+) = z_0, \tag{1}$$

where $z(t) \in R^n$ is the system state vector and $f(z)$ is the nonlinear function below second order and can be transformed as follows:

$$\dot{z} = C + Az + B(z)z,$$
$$z(t_0^+) = z_0,$$
$$B_{ij}(z) = \frac{1}{2}\frac{d(f_i(z))}{dz_j}, \tag{2}$$

where A and C are linear constant matrices and $B(z)$ is the linear matrix about z.

System model (2) includes some chaotic systems, such as Lorenz system, Rossler system, Chen system, and Lü system. Hence, the research on system model (2) will possess more practical application value, compared with those focused on one kind of chaotic system.

The remainder of this paper is organized as follows. The model description and preliminaries are proposed in Section 2. Based on impulsive control theory and Lyapunov method, we shall try to propose a new and practical impulsive strategy to realize the synchronization for a class of chaotic systems in Section 3. Finally, some typical examples will be included to show the correctness of the theoretical results in Section 4, and the paper will be concluded in Section 5.

2. Model Description and Preliminaries

Consider the following main chaotic system:

$$\dot{y} = C + Ay + B(y)y,$$
$$y(t_0{}^+) = y_0,$$
$$B_{ij}(y) = \frac{1}{2}\frac{d(f_i(y))}{dy_j},$$

(3)

where $y(t) \in R^n$ is the state variable of the main system.

The slave system is

$$\dot{x} = C + Ax + B(x)x, \quad t \neq t_k,$$
$$\Delta x = B_k(x - y), \quad t = t_k,$$
$$x(t_0{}^+) = x_0,$$
$$B_{ij}(x) = \frac{1}{2}\frac{d(f_i(x))}{dx_j}.$$

(4)

Define the error state variable as

$$e(t) = x(t) - y(t).$$

(5)

We get the following error dynamical system:

$$\dot{e} = Ae + B(x)x - B(y)y, \quad t \neq t_k,$$
$$\Delta e = B_k e, \quad t = t_k,$$
$$e(t_0{}^+) = e_0.$$

(6)

Hence, the problem to be addressed in this paper is to design an impulsive control method such that the tracking error variable satisfies

$$\lim_{t \to \infty} \|e(t)\| = \lim_{t \to \infty} \|x(t) - y(t)\| \longrightarrow 0.$$

(7)

3. Main Results

In this section, based on Lyapunov method and impulsive control theory, the following theoretical results are presented.

Theorem 1. *For the given class of chaotic systems, based on the following impulsive control strategy*

$$a = \lambda_{\max}\left(\frac{(u_k - 1)}{\Delta_k u_k} + \Pi^T + \Pi\right) < 0,$$

(8)

where

$$\Pi = \frac{1}{u_k}(A + B(x) + B(y)),$$
$$u_k = \lambda_{\max}\left[(I + B_k)^T (I + B_k)\right],$$
$$B_k < 0,$$
$$\Delta_k = t_{k+1} - t_k,$$

(9)

the slave chaotic system (4) with any initial conditions will synchronize the master chaotic system (3).

Proof. Choose the following Lyapunov functional candidate:

$$V(t) = g(t)e^T(t)e(t)$$

(10)

with

$$g(t) = \frac{(u_k - 1)}{(t_{k+1} - t_k)u_k}(t_k - t) + \frac{1}{u_k},$$
$$t \in [t_k, t_{k+1}).$$

(11)

Considering $\Delta_k = t_{k+1} - t_k$, we have

$$g(t) = \frac{(u_k - 1)}{\Delta_k u_k}(t_k - t) + \frac{1}{u_k},$$
$$t \in [t_k, t_{k+1}).$$

(12)

When $t \in (t_k, t_{k+1})$, the time derivative of $V(t)$ along the trajectories of the error dynamical system is given by

$$\dot{V}(t) = \dot{g}(t)e^T(t)e(t) + 2g(t)e^T(t)\dot{e}(t)$$
$$= \frac{(u_k - 1)}{\Delta_k u_k}e^T(t)e(t) + 2g(t)e^T(t)\dot{e}(t).$$

(13)

For $B(e)x = B(x)e$, the error dynamical system (6) can be transformed as

$$\dot{e}(t) = (A + B(x) + B(y))e(t), \quad t \neq t_k,$$
$$\Delta e(t) = B_k e(t), \quad t = t_k,$$
$$e(t_0{}^+) = e_0.$$

(14)

Hence

$$\dot{V}(t) \leq e^T(t)\left(\frac{(u_k - 1)}{\Delta_k u_k} + g(t)(A + B(x) + B(y))^T \right.$$
$$\left. + g(t)(A + B(x) + B(y))\right)e(t).$$

(15)

It can be concluded that

$$\dot{V}(t) \leq e^T(t)\left(\frac{(u_k - 1)}{\Delta_k u_k} + \Pi^T + \Pi\right)e(t)$$
$$\leq aV(t),$$

(16)

where

$$\Pi = \frac{1}{u_k}\left(A + B(x) + B(y)\right). \tag{17}$$

Hence

$$\left(V(t)e^{-at}\right)' = \left(\dot{V}(t) - aV(t)\right)e^{-at} \le 0. \tag{18}$$

Integrating the above inequality from t_k to t, we have

$$V(t)e^{-at} \le V(t_k)e^{-at_k}. \tag{19}$$

Then the following inequality can be obtained:

$$V(t) \le V\left(t_k{}^+\right)\exp\left(a(t - t_k)\right). \tag{20}$$

Considering

$$
\begin{aligned}
V\left(t_k{}^+\right) &= g\left(t_k{}^+\right)e^T\left(t_k{}^+\right)e\left(t_k{}^+\right) \\
&= g\left(t_k{}^+\right)\left[(I + B_k)e\left(t_k{}^-\right)\right]^T\left[(I + B_k)e\left(t_k{}^-\right)\right] \\
&= V\left(t_k{}^-\right),
\end{aligned} \tag{21}
$$

we have

$$V(t) \le V\left(t_0{}^+\right)\exp\left(a(t - t_0)\right). \tag{22}$$

One can obtain

$$\|e(t)\| \le e_0\exp\left(\frac{1}{2}a(t - t_0)\right). \tag{23}$$

Hence, it can be concluded that the error dynamical system (6) is stable. That means that the slave system (4) can synchronize with the master system (3) based on the given impulsive control strategy and this completes the proof. □

Next, let $B_k = bI$ and $\Delta_k = \Delta$, and based on Theorem 1, we can derive the following theoretical results, and its proof is omitted.

Theorem 2. *For the given class of chaotic system, based on the following impulsive control strategy*

$$a = \lambda_{\max}\left(\frac{(u - 1)}{\Delta \cdot u} + \Pi^T + \Pi\right) < 0 \tag{24}$$

with

$$
\begin{aligned}
\Pi &= \frac{1}{u}\left(A + B(x) + B(y)\right), \\
u &= \lambda_{\max}\left[(I + bI)^T(I + bI)\right] \\
&\quad b < 0,
\end{aligned} \tag{25}
$$

the slave chaotic system (4) with any initial conditions will synchronize with the master chaotic system (3).
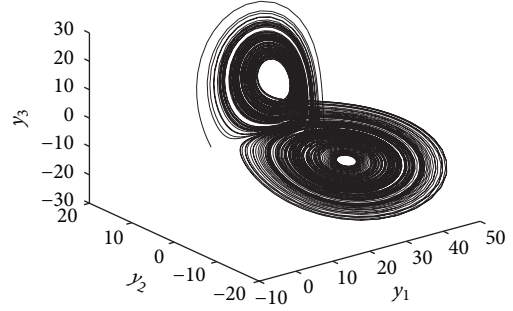

FIGURE 1: Dynamic behavior of the chaotic system in case 1.

4. Example and Simulation

In this section, we will verify the proposed methodology by giving two illustrative examples. First consider the unified chaotic system as follows:

$$
\begin{aligned}
\dot{z}_1 &= (25r + 10)(z_2 - z_1), \\
\dot{z}_2 &= (28 - 35r)z_1 - z_1z_3 + (29r - 1)z_2, \\
\dot{z}_3 &= z_1z_2 - \frac{r + 8}{3}z_3.
\end{aligned} \tag{26}
$$

It can be transformed as

$$\dot{z} = C + Az + B(z)z \tag{27}$$

with

$$
A = \begin{bmatrix} -25r - 10 & 25r + 10 & 0 \\ 28 - 35r & 29r - 1 & 0 \\ 0 & 0 & -\dfrac{r + 8}{3} \end{bmatrix},
$$

$$
B(z) = \frac{1}{2}\begin{bmatrix} 0 & 0 & 0 \\ -z_3 & 0 & -z_1 \\ z_2 & z_1 & 0 \end{bmatrix}, \qquad C = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \tag{28}
$$

Unified chaotic system has different dynamic behavior with different parameter r. For instance, the unified chaotic system represents Lü system when $r = 0.8$, Lorenz system when $0 \le r < 0.8$, and Chen system when $0.8 < r \le 1$.

The numerical simulation is with initial condition $x_0 = [7, 9, 2]^T$, $y_0 = [-1, 3, 5]^T$, and the simulation step 0.001 second. Based on Theorem 2, we choose the impulsive control parameters $\Delta = 0.02$, $b = -0.8$.

Remark 3. Figure 1 depicts the dynamic behavior of the chaotic system in case 1. It can be seen that the state variable moves in a scope and will never converge to a constant with the lapse of time. Figure 2 depicts the time response of the error variable of master-salve system in case 1. It can be seen that the error variable closes to zero quickly based on the given impulse control method, which verifies the correctness of our theoretical results.
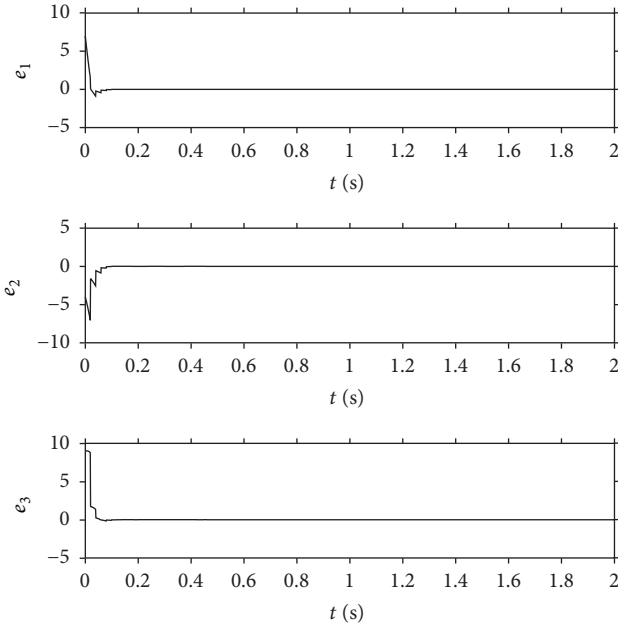
FIGURE 2: Time response of the error variable in case 1.

Next, consider the following Rossler system

$$
\begin{aligned}
\dot{z}_1 &= -z_2 - z_3, \\
\dot{z}_2 &= z_1 + 0.15 z_2, \\
\dot{z}_3 &= 0.2 + z_3 (z_1 - 10).
\end{aligned} \tag{29}
$$

It can be transformed as follows:

$$
\dot{z} = C + Az + B(z) z \tag{30}
$$

with

$$
A = \begin{bmatrix} 0 & -1 & -1 \\ 1 & 0.15 & 0 \\ 0 & 0 & -10 \end{bmatrix}, \qquad B(z) = \frac{1}{2} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ z_3 & 0 & z_1 \end{bmatrix},
$$

$$
C = \begin{bmatrix} 0 \\ 0 \\ 0.2 \end{bmatrix}. \tag{31}
$$

The numerical simulation is with initial condition $x_0 = [-1, 5, -4]^T$, $y_0 = [7, -6, 2]^T$ and the simulation step 0.001 second. Based on Theorem 2, we choose the impulsive control parameters $\Delta = 0.02$ and $b = -0.8$.

Remark 4. Figure 3 depicts the dynamics of the chaotic system in case 2. Figure 4 depicts the time response of the error variable of master-salve system in case 2. It can be seen that the error variable converges to zero quickly based on the proposed impulse control method.

From the above numerical simulations, it can be concluded that our impulsive synchronization strategy has more practical application value compared with those specific to one kind of chaotic system.
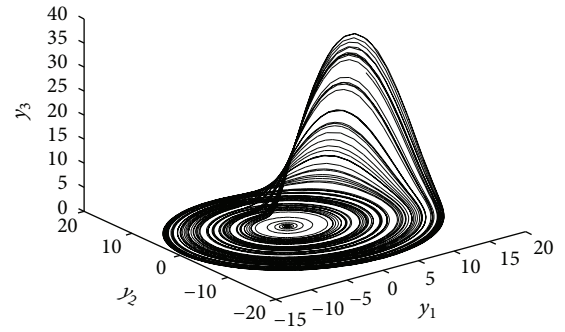

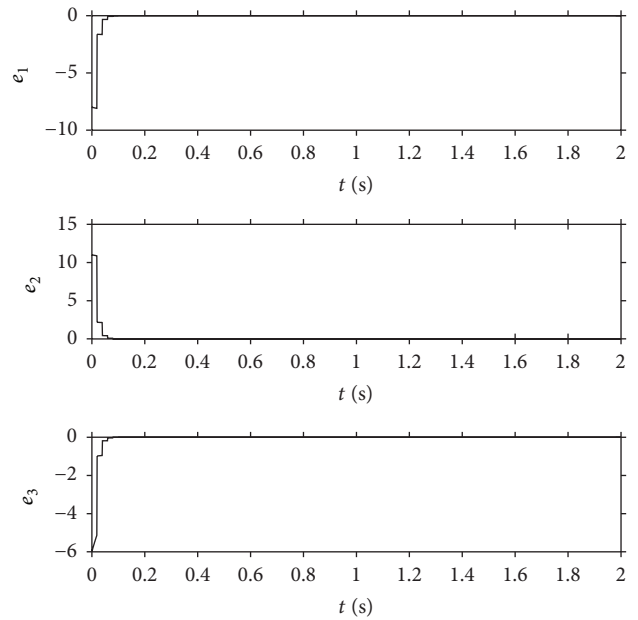FIGURE 3: Dynamical behavior of the chaotic system in case 2.

FIGURE 4: Time response of the error state variable in case 2.

5. Conclusions

This paper focuses on the chaos synchronization problem and tries to figure out the strategies suitable for more chaotic systems. Based on the impulsive control technique and Lyapunov stability theory, we have presented the new impulsive synchronization strategies for a class of chaotic systems. Finally some numerical simulations have been carried out to demonstrate the effectiveness of our theoretical results.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.
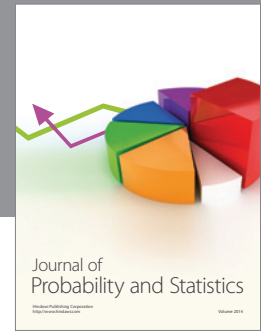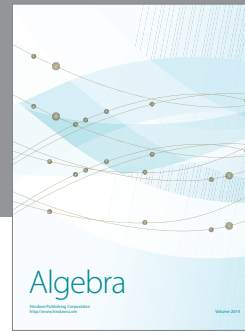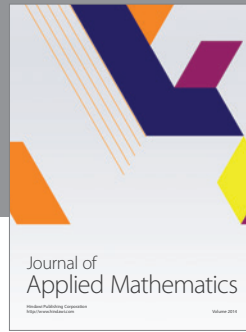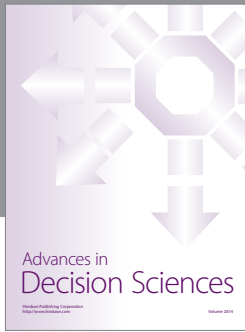
Acknowledgments

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