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Research Article

Note on the Persistence of a Nonautonomous Lotka-Volterra Competitive System with Infinite Delay and Feedback Controls

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We study a nonautonomous Lotka-Volterra competitive system with infinite delay and feedback controls. We establish a series of criteria under which a part of n-species of the systems is driven to extinction while the remaining part of the species is persistent. Particularly, as a special case, a series of new sufficient conditions on the persistence for all species of system are obtained. Several examples together with their numerical simulations show the feasibility of our main results.

1. Introduction

In this paper, we consider the following nonautonomous n-species Lotka-Volterra competitive system with infinite delay and feedback controls:

$$\begin{aligned} \dot{x}_i(t) &= x_i(t)\left[r_i(t) - \sum_{j=1}^{n} a_{ij}(t)\int_0^{+\infty} K_{ij}(s)\,x_j(t-s)\,ds \right. \\ &\qquad \left. - b_i(t)\int_0^{+\infty} H_i(s)\,u_i(t-s)\,ds\right], \\ \dot{u}_i(t) &= -c_i(t)\,u_i(t) + d_i(t)\int_0^{+\infty} R_i(s)\,x_i(t-s)\,ds, \\ &\qquad i = 1, 2, \ldots, n, \end{aligned} \tag{1}$$

where $x_i(t)$ $(i = 1, 2, \ldots, n)$ is the density of the ith species at time t and $u_i(t)$ $(i = 1, 2, \ldots, n)$ is the indirect control variable.

In particular, when the coefficients $b_i(t) \equiv 0$, $c_i(t) \equiv 0$, and $d_i(t) \equiv 0$ for all $t \in R$ and $i = 1, 2, \ldots, n$, the system (1) will degenerate into the following *pure delay type* system:

$$\dot{x}_i(t) = x_i(t)\left[r_i(t) - \sum_{j=1}^{n} a_{ij}(t)\int_0^{+\infty} K_{ij}(s)\,x_j(t-s)\,ds\right], \quad i = 1, 2, \ldots, n. \tag{2}$$

As is well known, systems such as (2) without feedback controls are very important mathematical models of multispecies populations dynamics. This is a generalization from Ahmad [1] about two-species system without delays to n-species system of infinite delay. Systems without delays such as [1] have attracted the interest of many researchers (see, e.g., [2–5]), and systems with delays have been studied extensively in the past twenty years, and some good results on the permanence, extinction and persistence or uniform persistence, global stability, and almost periodic solution have been developed (see [6–18]). In [19], Montes de Oca and Pérez provided for us a very interesting work for system (2), who showed that if the coefficients are bounded and continuous and satisfy certain inequalities, then any solution with initial function of system (2) in an appropriate space will have $n-1$ of its components tenting to zero, while the remaining one will stabilize at a certain solution of a logistic differential equation. And for more works about single species dynamic behaviors of infinite delay, one could refer to [20, 21].

On the other hand, as was pointed out by Fan and Wang [22], feedback control is the basic mechanism by which systems, whether mechanical, electrical, or biological, maintain their equilibrium or homeostasis. Many scholars have done works on the ecosystem with feedback controls (see, e.g., [23–29] and the references cited therein). In [23], Shi et al. proposed the feedback control system (1). By using the method of multiple Lyapunov functionals and by developing a new analysis technique, Shi et al. established the sufficient conditions which guarantee part species $x_{r+1}, x_{r+2}, \ldots, x_{r+n}$ of the n-species driven to extinction. But in the paper [23], they did not discuss the survival problems for the remaining species. The main aim of this paper is to study the persistence of the remaining species $x_1, x_2, \ldots, x_r$ of system (1). By the new method motivated by work [11, 27, 28], we will establish new sufficient conditions for which surplus species $x_1, x_2, \ldots, x_r$ of system (1) remain persistent.

The organization of the paper is as follows. In the next section, some assumptions and lemmas are introduced. In Section 3, we state and prove our main results. Finally, several examples with their numerical simulations are presented to show the feasibility of the main results.

2. Preliminaries

Throughout this paper, for system (1), we introduce the following hypotheses.

(H$_1$) $r_i(t)$, $a_{ij}(t)$, $b_i(t)$, $c_i(t)$, and $d_i(t)$ $(i, j = 1, 2, \ldots, n)$ are bounded and continuous, defined on $[0, \infty)$. Furthermore, $a_{ij}(t)$ $(i \neq j)$, $b_i(t)$, $c_i(t)$, and $d_i(t)$ are nonnegative on $[0, \infty)$, and $0 < a_{ii}^l \leq a_{ii}(t) \leq a_{ii}^u < \infty$. Here, we denote $f^l = \inf_{t\geq 0} f(t)$ and $f^u = \sup_{t\geq 0} f(t)$.

(H$_2$) $K_{ij} : [0, \infty) \to [0, \infty)$, $H_i : [0, \infty) \to [0, \infty)$, and $R_i : [0, \infty) \to [0, \infty)$, $i, j = 1, 2, \ldots, n$, are piecewise continuous and satisfy

$$\begin{gathered} \int_0^{+\infty} K_{ij}(s)\,ds = 1, \qquad \mathscr{K}_{ij} = \int_0^{+\infty} sK_{ij}(s)\,ds < \infty, \\ \int_0^{+\infty} H_i(s)\,ds = 1, \qquad \mathscr{H}_i = \int_0^{+\infty} sH_i(s)\,ds < \infty, \\ \int_0^{+\infty} R_i(s)\,ds = 1, \qquad \mathscr{R}_i = \int_0^{+\infty} sR_i(s)\,ds < \infty. \end{gathered} \tag{3}$$

(H$_3$) There exists a positive constant ω such that for each $i = 1, 2, \ldots, n$

$$\liminf_{t\to\infty} \int_t^{t+\omega} r_i(s)\,ds > 0. \tag{4}$$

(H$_4$) There exist positive constants λ and γ such that for each $i = 1, 2, \ldots, n$

$$\begin{gathered} \liminf_{t\to\infty} \int_t^{t+\lambda} c_i(s)\,ds > 0, \\ \liminf_{t\to\infty} \int_t^{t+\gamma} d_i(s)\,ds > 0. \end{gathered} \tag{5}$$

We will consider system (1) together with the initial conditions

$$x_i(\theta) = \phi_i(\theta), \quad u_i(\theta) = \psi_i(\theta), \quad \theta \leq 0,\ i = 1, 2, \ldots, n, \tag{6}$$

where $\phi_i, \psi_i \in BC^+$, $i = 1, 2, \ldots, n$, and

$$\begin{aligned} BC^+ = \{\varphi \in C[(-\infty, 0], [0, +\infty)] : \\ \varphi(0) > 0,\ \varphi \text{ is bounded}\}. \end{aligned} \tag{7}$$

It is easy to verify that solutions of (1) satisfying the initial condition (6) are well defined for all $t \geq 0$ and satisfy

$$x_i(t) > 0, \quad u_i(t) > 0, \quad \forall t \geq 0. \tag{8}$$

We now introduce several lemmas which will be useful in the proofs of the main results.

We consider the following nonautonomous linear equation:

$$\dot{x}(t) = a(t) - b(t)x(t), \tag{9}$$

where nonnegative functions $a(t)$ and $b(t)$ are bounded and continuous, defined on $[0, +\infty)$. We have the following results.

Lemma 1 (see [30]). *Suppose that there exist positive constants η_1 and η_2 such that*

$$\begin{gathered} \liminf_{t\to\infty} \int_t^{t+\eta_1} a(s)\,ds > 0, \\ \liminf_{t\to\infty} \int_t^{t+\eta_2} b(s)\,ds > 0. \end{gathered} \tag{10}$$

Then, there exist positive constants $M \geq m$ such that

$$m \leq \liminf_{t\to\infty} x(t) \leq \limsup_{t\to\infty} x(t) \leq M, \tag{11}$$

for any positive solution $x(t)$ of (9).

Lemma 2 (see [23]). *Suppose that assumptions (H$_1$)–(H$_4$) hold; then there exist constants $\overline{x}_i > 0$ and $\overline{u}_i > 0$ such that*

$$\limsup_{t\to\infty} x_i(t) < \overline{x}_i, \quad \limsup_{t\to\infty} u_i(t) < \overline{u}_i, \quad i = 1, 2, \ldots, n, \tag{12}$$

for any positive solution $(x_1(t), x_2(t), \ldots, x_n(t), u_1(t), u_2(t), \ldots, u_n(t))$ of system (1).

Remark 3. If all parameters $r_i(t)$, $a_{ij}(t)$, $b_i(t)$, $c_i(t)$, and $d_i(t)$ $(i, j = 1, 2, \ldots, n)$ of system (1) have the positive lower bound on $[0, +\infty)$, then, from Lemma 2.2 in [23], we can choose

$$\overline{x}_i = \frac{r_i^u}{a_{ii}^l \int_0^{+\infty} k_{ii}(s) \exp(-r_i^u s)\, ds}, \qquad \overline{u}_i = \frac{d_i^u}{c_i^l} \overline{x}_i. \tag{13}$$

Lemma 4 (see [6]). *Let $x(t) : R \to R$ be a nonnegative and bounded continuous function, and let $k(s) : [0, +\infty) \to [0, +\infty)$ be an integral function satisfying $\int_0^{+\infty} k(s) ds = 1$. Then*

$$\begin{aligned} \liminf_{t\to\infty} x(t) &\le \liminf_{t\to\infty} \int_0^{+\infty} k(s)\, x(t-s)\, ds \\ &\le \limsup_{t\to\infty} \int_0^{+\infty} k(s)\, x(t-s)\, ds \\ &\le \limsup_{t\to\infty} x(t). \end{aligned} \tag{14}$$

3. Main Results

In this section, we discuss the persistence of part species x_i $(1 \le i \le r)$ of system (1), where integer $r \in \{1, 2, \ldots, n\}$. Let functions

$$\begin{aligned} A_{ij}(t) &= \int_0^{+\infty} a_{ij}(t+s) K_{ij}(s)\, ds, \\ B_i(t) &= \int_0^{+\infty} b_i(t+s) H_i(s)\, ds, \\ D_i(t) &= \int_0^{+\infty} d_i(t+s) R_i(s)\, ds, \\ & \qquad i, j = 1, 2, \ldots, n. \end{aligned} \tag{15}$$

Lemma 5. *Suppose that assumptions (H_1)–(H_4) hold and there exists an integer $1 \le r < n$ such that for any $k > r$ there exists an integer $i_k < k$ such that*

$$\limsup_{t\to\infty} \frac{\int_t^{t+w} r_k(s)\, ds}{\int_t^{t+w} r_{i_k}(s)\, ds} < \liminf_{t\to\infty} \frac{A_{kj}(t)}{A_{i_k j}(t)} \quad \forall j \le k,$$

$$\liminf_{t\to\infty} \frac{B_k(t)}{c_k(t)} > \limsup_{t\to\infty} \left(\frac{A_{i_k k}(t)}{D_k(t)} \limsup_{t\to\infty} \frac{\int_t^{t+w} r_k(s)\, ds}{\int_t^{t+w} r_{i_k}(s)\, ds} - \frac{A_{kk}(t)}{D_k(t)} \right),$$

$$\limsup_{t\to\infty} \frac{B_{i_k}(t)}{c_{i_k}(t)} < \liminf_{t\to\infty} \left(\frac{A_{k i_k}(t)}{D_{i_k}(t)} \liminf_{t\to\infty} \frac{\int_t^{t+w} r_{i_k}(s)\, ds}{\int_t^{t+w} r_k(s)\, ds} - \frac{A_{i_k i_k}(t)}{D_{i_k}(t)} \right). \tag{16}$$

Then for each $i = r+1, \ldots, n$ we have

$$\lim_{t\to\infty} x_i(t) = 0, \qquad \lim_{t\to\infty} u_i(t) = 0, \qquad \int_0^{\infty} x_i(t)\, dt < \infty, \tag{17}$$

for any positive solution $(x_1(t), x_2(t), \ldots, x_n(t), u_1(t), u_2(t), \ldots, u_n(t))$ of system (1).

The proof of the extinction of part species $x_{r+1}, x_{r+2}, \ldots, x_{r+n}$ of system (1) could be found in [23] and we hence omit it here.

On the persistence of part species x_i $(1 \le i \le r)$ of system (1), we state and prove the following results.

Theorem 6. *Suppose that all assumptions of Lemma 5 hold and there exists a positive constant $\eta > 0$ such that*

$$\liminf_{t\to\infty} \int_t^{t+\eta} \left[r_i(s) - \sum_{j\ne i}^{r} A_{ij}(s)\, \overline{x}_j - B_i(s)\, \overline{u}_i \right] ds > 0, \qquad \forall i \le r. \tag{18}$$

Then, for each $i = 1, 2, \ldots, r$, there exist positive constants m and M, with $m < M$, such that

$$\begin{aligned} m \le \liminf_{t\to\infty} x_i(t) \le \limsup_{t\to\infty} x_i(t) \le M, \\ m \le \liminf_{t\to\infty} u_i(t) \le \limsup_{t\to\infty} u_i(t) \le M, \end{aligned} \tag{19}$$

for any positive solution $(x(t), u(t)) = (x_1(t), x_2(t), \ldots, x_n(t), u_1(t), u_2(t), \ldots, u_n(t))$ of system (1).

Proof. Let $(x(t), u(t)) = (x_1(t), x_2(t), \ldots, x_n(t), u_1(t), u_2(t), \ldots, u_n(t))$ be any positive solution of system (1). By Lemma 2, let $M = \max_{1\le i\le r}\{\overline{x}_i, \overline{u}_i\}$; for each $i = 1, 2, \ldots, r$, we have $\limsup_{t\to\infty} x_i(t) \le M$ and $\limsup_{t\to\infty} u_i(t) \le M$. So, we only need to prove that there exists a positive constant m such that $\liminf_{t\to\infty} x_i(t) \ge m$ and $\liminf_{t\to\infty} u_i(t) \ge m$ for all $i \le r$.

First of all, assumption (18) implies that there are positive constants α_0 and T_0 such that

$$\int_t^{t+\eta} \left[r_i(s) - \sum_{j\ne i}^{r} A_{ij}(s)\, \overline{x}_j - B_i(s)\, \overline{u}_i \right] ds \ge \alpha_0, \tag{20}$$

for all $t \ge T_0$ and $i \le r$.

By Lemmas 2 and 5, we obtain that, for any constant $\varepsilon > 0$, there is a $T(\varepsilon) > T_0$ such that, for all $t \ge T(\varepsilon)$,

$$\begin{aligned} x_i(t) \le \overline{x}_i + \varepsilon, \quad u_i(t) \le \overline{u}_i + \varepsilon, \quad \forall i \le r, \\ x_i(t) \le \varepsilon, \quad u_i(t) \le \varepsilon, \quad \forall i > r. \end{aligned} \tag{21}$$

Now, for any $i \le r$, we define the Lyapunov function as follows:

$$\begin{aligned} W_i(t) &= x_i(t)\exp\left[-\sum_{j=1}^{n}\int_0^{+\infty}K_{ij}(s)\int_{t-s}^{t}a_{ij}(\theta+s)x_j(\theta)\,d\theta\,ds \right.\\ &\quad \left. -\int_0^{+\infty}H_i(s)\int_{t-s}^{t}b_i(\theta+s)u_i(\theta)\,d\theta\,ds\right]. \end{aligned} \tag{22}$$

By assumptions (H_1) and (H_2) and Lemma 2, we have

$$\begin{aligned} &\sum_{j=1}^{n}\int_0^{+\infty}K_{ij}(s)\int_{t-s}^{t}a_{ij}(\theta+s)x_j(\theta)\,d\theta\,ds \\ &\quad \le \sum_{j=1}^{n}a_{ij}^{u}\int_0^{+\infty}sK_{ij}(s)\,ds\sup_{t\in R}x_j(t) < \infty, \\ &\int_0^{+\infty}H_i(s)\int_{t-s}^{t}b_i(\theta+s)u_i(\theta)\,d\theta\,ds \\ &\quad \le b_i^{u}\int_0^{+\infty}sH_i(s)\,ds\sup_{t\in R}u_i(t) < \infty. \end{aligned} \tag{23}$$

So we see that $W_i(t)$ has definition for all $t \ge 0$. From (23), we can obtain that for any $i \le r$ there is a positive constant $d_i < 1$, and d_i may be dependent on the positive solution of system (1) such that

$$d_i x_i(t) \le W_i(t) \le x_i(t), \quad \forall t \ge 0. \tag{24}$$

Calculating the derivative of $W_i(t)$ with respect to t, we have

$$\begin{aligned} \dot{W}_i(t) &= W_i(t)\left[r_i(t) - \sum_{j=1}^{n}a_{ij}(t)\int_0^{+\infty}K_{ij}(s)x_j(t-s)\,ds \right.\\ &\quad - b_i(t)\int_0^{+\infty}H_i(s)u_i(t-s)\,ds \\ &\quad - \sum_{j=1}^{n}\int_0^{+\infty}K_{ij}(s)a_{ij}(t+s)\,ds x_j(t) \\ &\quad + \sum_{j=1}^{n}\int_0^{+\infty}K_{ij}(s)a_{ij}(t)x_j(t-s)\,ds \\ &\quad - \int_0^{+\infty}H_i(s)b_i(t+s)\,ds u_i(t) \\ &\quad \left. + \int_0^{+\infty}H_i(s)b_i(t)u_i(t-s)\,ds\right] \\ &= W_i(t)\left[r_i(t) - \sum_{j=1}^{n}\int_0^{+\infty}K_{ij}(s)a_{ij}(t+s)\,ds x_j(t) \right.\\ &\quad \left. - \int_0^{+\infty}H_i(s)b_i(t+s)\,ds u_i(t)\right] \\ &= W_i(t)\left[r_i(t) - \sum_{j=1}^{n}A_{ij}(t)x_j(t) - B_i(t)u_i(t)\right], \\ &\qquad \forall i \le r,\ t \ge 0. \end{aligned} \tag{25}$$

Let $\beta_i(t,\varepsilon) = r_i(t) - \sum_{j\ne i}^{n}A_{ij}(t)\varepsilon - \sum_{j\ne i}^{r}A_{ij}(t)\overline{x}_j - B_i(t)(\overline{u}_i + \varepsilon)$. From (21), for all $t \ge T(\varepsilon) > 0$, we have

$$\begin{aligned} \dot{W}_i(t) &\ge W_i(t)\left[\beta_i(t,\varepsilon) - A_{ii}(t)x_i(t)\right] \\ &\ge W_i(t)\left[\beta_i(t,\varepsilon) - A_{ii}(t)d_i^{-1}W_i(t)\right]. \end{aligned} \tag{26}$$

Obviously, from inequality (20), we can find enough small positive constants δ_i and ε_0 such that

$$\int_t^{t+\eta}\left[\beta_i(s,\varepsilon_0) - A_{ii}(s)d_i^{-1}\delta_i\right]ds > \frac{1}{2}\alpha_0, \tag{27}$$

for all $t \ge T_1 = T(\varepsilon_0)$. So for the above ε_0, when $t \ge T_1 = T(\varepsilon_0)$,

$$\dot{W}_i(t) \ge W_i(t)\left[\beta_i(t,\varepsilon_0) - A_{ii}(t)d_i^{-1}W_i(t)\right]. \tag{28}$$

Consider the auxiliary equation

$$\dot{W}_i(t) = W_i(t)\left[\beta_i(t,\varepsilon_0) - A_{ii}(t)d_i^{-1}W_i(t)\right]; \tag{29}$$

then by (28), we obtain that

$$W_i(t) \ge W_i^{*}(t), \quad \forall t \ge T_1, \tag{30}$$

where $W_i^{*}(t)$ is the solution of (29) with the initial condition $W_i(T_1) = W_i^{*}(T_1)$. If $W_i^{*}(t) < \delta_i$ for all $t \ge T_1$, then $W_i^{*}(t)$ is defined on $[T_1, +\infty)$. Integrating inequality (29) from T_1 to t, we obtain

$$\begin{aligned} W_i^{*}(t) &= W_i^{*}(T_1)\exp\int_{T_1}^{t}\left[\beta_i(s,\varepsilon_0) - A_{ii}(s)d_i^{-1}W_i^{*}(s)\right]ds \\ &\ge W_i^{*}(T_1)\exp\int_{T_1}^{t}\left[\beta_i(s,\varepsilon_0) - A_{ii}(s)d_i^{-1}\delta_i\right]ds, \end{aligned} \tag{31}$$

for all $t \geq T_1$. Putting $t = T_1 + m\eta$, $m = 1, 2, \ldots$, then, from (27) and (31), we have

$$W_i^* (T_1 + m\eta) \geq W_i^* (T_1) \exp\left(\frac{1}{2} m\alpha_0\right), \quad m = 1, 2, \ldots. \tag{32}$$

Letting $m \to +\infty$, we have $W_i^*(T_1 + m\eta) \to +\infty$, a contradiction. Hence, there is a $t_i \geq T_1$ such that $W_i^*(t_i) > \delta_i$. Now, we prove that

$$W_i^* (t) \geq \delta_i \exp(-\beta_i(\delta_i, \varepsilon_0)\eta), \quad \forall t \geq t_i, \tag{33}$$

where $\beta_i(\delta_i, \varepsilon_0) = \sup_{t\geq 0}\{|\beta_i(t, \varepsilon_0)| + A_{ii}(t) d_i^{-1}\delta_i\}$, and the definition of $\beta_i(t, \varepsilon_0)$ implies $0 < \beta_i(\delta_i, \varepsilon_0) < \infty$. In fact, if (33) is not true, then there are t_1 and t_2, $t_1 < t_2$, such that

$$\begin{gathered} W_i^* (t_2) < \delta_i \exp(-\beta_i(\delta_i, \varepsilon_0)\eta), \\ W_i^* (t_1) = \delta_i, \qquad W_i^*(t) < \delta_i, \\ \forall t \in (t_1, t_2]. \end{gathered} \tag{34}$$

Choosing the integer $m \geq 0$ such that $t_2 \in (t_1 + m\eta, t_1 + (m+1)\eta]$, then, by (27) and (29), it follows that

$$\begin{aligned} &\delta_i \exp(-\beta_i(\delta_i, \varepsilon_0)\eta) \\ &\quad > W_i^*(t_2) \\ &\quad = W_i^*(t_1) \exp \int_{t_1}^{t_2} \left[\beta_i(t, \varepsilon_0) - A_{ii}(t) d_i^{-1} W_i^*(t)\right] dt \\ &\quad \geq \delta_i \exp\left\{\left\{\int_{t_1}^{t_1+m\eta} + \int_{t_1+m\eta}^{t_2}\right\} \left[\beta_i(t, \varepsilon_0) - A_{ii}(t) d_i^{-1}\delta_i\right] dt\right\} \\ &\quad \geq \delta_i \exp\left\{\int_{t_1+m\eta}^{t_2} \left[\beta_i(t, \varepsilon_0) - A_{ii}(t) d_i^{-1}\delta_i\right] dt\right\} \\ &\quad \geq \delta_i \exp(-\beta_i(\delta_i, \varepsilon_0)\eta), \end{aligned} \tag{35}$$

which is a contradiction.

From (24), (30), and (33), we can obtain that

$$x_i(t) \geq \delta_i \exp(-\beta_i(\delta_i, \varepsilon_0)\eta) \quad \forall t \geq t_i. \tag{36}$$

Finally, we define the constants $m_i = \delta_i \exp(-\beta_i(\delta_i, \varepsilon_0)\eta)$ and $T = \max_{i\leq r}\{t_i\}$; then we have

$$x_i(t) \geq m_i \quad \forall t \geq T,\ i \leq r. \tag{37}$$

Letting $m_i^* = \{\inf_{t\in[0,T]} x_i(t) > 0\}$ and $m^* = \min_{1\leq i\leq r}\{m_i, m_i^*\}$, we have

$$\liminf_{t\to\infty} x_i(t) \geq m^*, \tag{38}$$

for all $i \leq r$.

Further, by Lemma 4 and (38), we can choose constants $\epsilon > 0$ and $T^* > 0$ such that for all $i \leq r$ and $t \geq T^*$

$$\int_0^{+\infty} R_i(s)\, x_i(t-s)\, ds \geq m^* - \epsilon > 0. \tag{39}$$

Considering the second equation of system (1), from (39), for any $t \geq T^*$, we obtain

$$\begin{aligned} \dot{u}_i(t) &= -c_i(t) u_i(t) + d_i(t) \int_0^{+\infty} R_i(s)\, x_i(t-s)\, ds \\ &\geq -c_i(t) u_i(t) + d_i(t)(m^* - \epsilon). \end{aligned} \tag{40}$$

We consider the following auxiliary equation:

$$\dot{v}_i(t) = -c_i(t) v_i(t) + d_i(t)(m^* - \epsilon). \tag{41}$$

Then by assumption (H_4) and applying Lemma 1 there exists a constant $\underline{u}_i > 0$ such that

$$\liminf_{t\to\infty} v_i(t) > \underline{u}_i, \tag{42}$$

for any positive solution $v_i(t)$ of (41). Let $v_i^*(t)$ be the solution of (41) with the initial condition $v_i^*(T^*) = u_i(T^*)$; then by the comparison theorem we have

$$u_i(t) \geq v_i^*(t) \quad \forall t \geq T^*. \tag{43}$$

Thus, we finally obtain

$$\liminf_{t\to\infty} u_i(t) \geq \underline{u}_i. \tag{44}$$

Let $m = \min_{1\leq i\leq r}\{m^*, \underline{u}_i\}$; from (38) and (44), we obtain that $\liminf_{t\to\infty} x_i(t) \geq m$ and $\liminf_{t\to\infty} u_i(t) \geq m$. This completes the proof of Theorem 6. □

As consequences of Theorem 6 we have the following corollaries.

Corollary 7. *If, in system* (1), $b_i(t) = c_i(t) = d_i(t) = 0$ $(i = 1, 2, \ldots, n)$ *for all* $t \geq 0$, *then system* (1) *will be reduced to the following n-species competitive system with infinite delay:*

$$\begin{aligned} \dot{x}_i(t) = x_i(t) \Bigg[r_i(t) - \sum_{j=1}^{n} a_{ij}(t) \\ \times \int_0^{+\infty} K_{ij}(s)\, x_j(t-s)\, ds \Bigg], \quad i = 1, 2, \ldots, n. \end{aligned} \tag{45}$$

Suppose that assumptions (H_1)–(H_3) *hold and there exists an integer* $1 \leq r < n$ *such that for any* $k > r$ *there exists an integer* $i_k < k$ *such that*

$$\limsup_{t\to\infty} \frac{\int_t^{t+w} r_k(s)\, ds}{\int_t^{t+w} r_{i_k}(s)\, ds} < \liminf_{t\to\infty} \frac{A_{kj}(t)}{A_{i_k j}(t)}, \quad \forall j \leq k. \tag{46}$$

Furthermore, there exists a positive constant $\eta > 0$ *such that*

$$\liminf_{t\to\infty} \int_t^{t+\eta} \left[r_i(s) - \sum_{j\neq i}^{r} A_{ij}(s)\, \overline{x}_j \right] ds > 0, \quad \forall i \leq r. \tag{47}$$

Then, for each $i = 1, 2, \ldots, r$, there exist positive constants $m \leq M$ such that

$$m \leq \liminf_{t\to\infty} x_i(t) \leq \limsup_{t\to\infty} x_i(t) \leq M, \quad (48)$$

and for each $i = r+1, \ldots, n$ we have

$$\lim_{t\to\infty} x_i(t) = 0, \qquad \int_0^{\infty} x_i(t)\,dt < \infty, \quad (49)$$

for any positive solution $(x_1(t), x_2(t), \ldots, x_n(t))$ of system (45).

Proof. From the condition,

$$\limsup_{t\to\infty} \frac{\int_t^{t+w} r_k(s)\,ds}{\int_t^{t+w} r_{i_k}(s)\,ds} < \liminf_{t\to\infty} \frac{A_{kj}(t)}{A_{i_k j}(t)}, \quad \forall j \leq k. \quad (50)$$

And the assumptions (H_1)–(H_3) hold; from Corollary 7 in [23], for each $i = r+1, \ldots, n$ we have

$$\lim_{t\to\infty} x_i(t) = 0, \qquad \int_0^{\infty} x_i(t)\,dt < \infty. \quad (51)$$

Further condition

$$\liminf_{t\to\infty} \int_t^{t+\eta} \left[r_i(s) - \sum_{j\neq i}^{r} A_{ij}(s)\,\overline{x}_j \right] ds > 0, \quad \forall i \leq r \quad (52)$$

holds, so we see, from Theorem 6, for each $i = 1, 2, \ldots, r$, that there exist positive constants $m \leq M$ such that

$$m \leq \liminf_{t\to\infty} x_i(t) \leq \limsup_{t\to\infty} x_i(t) \leq M. \quad (53)$$

□

Remark 8. When $r = 1$, the conditions of Corollary 7 will reduce to the assumptions that (H_1)–(H_3) hold and for any $k > 1$ such that

$$\limsup_{t\to\infty} \frac{\int_t^{t+w} r_k(s)\,ds}{\int_t^{t+w} r_1(s)\,ds} < \liminf_{t\to\infty} \frac{A_{kj}(t)}{A_{1j}(t)}, \quad \forall j \leq k. \quad (54)$$

We have that there exist positive constants $m \leq M$ such that

$$m \leq \liminf_{t\to\infty} x_1(t) \leq \limsup_{t\to\infty} x_1(t) \leq M, \quad (55)$$

and for each $i = 2, \ldots, n$ we have

$$\lim_{t\to\infty} x_i(t) = 0, \qquad \int_0^{\infty} x_i(t)\,dt < \infty. \quad (56)$$

In comparison with the assumptions (1.5) together with Proposition 2.2 given by Montes de Oca and Pérez [19], we can see that our assumptions in Corollary 7 are weaker.

Remark 9. When $r = n$, from Corollary 7 we can easily obtain a criterion on the persistence of all species $(x_1(t), x_2(t), \ldots, x_n(t))$ of system (45).

Remark 10. The conclusion of Corollary 7 improves that of Proposition 2.2 given by Montes de Oca and Pérez [19].

Corollary 11. *Suppose that (H_1)–(H_4) hold and there exists a positive constant $\eta > 0$ such that*

$$\liminf_{t\to\infty} \int_t^{t+\eta} \left[r_i(s) - \sum_{j\neq i}^{n} A_{ij}(s)\,\overline{x}_j - B_i(s)\,\overline{u}_i \right] ds > 0, \quad \forall i = 1, 2, \ldots, n. \quad (57)$$

Then, for each $i = 1, 2, \ldots, n$, there exist positive constants m and M, with $m < M$, such that

$$\begin{aligned} m &\leq \liminf_{t\to\infty} x_i(t) \leq \limsup_{t\to\infty} x_i(t) < M, \\ m &\leq \liminf_{t\to\infty} u_i(t) \leq \limsup_{t\to\infty} u_i(t) < M, \end{aligned} \quad (58)$$

for any positive solution $(x_1(t), x_2(t), \ldots, x_n(t), u_1(t), u_2(t), \ldots, u_n(t))$ of system (1).

Remark 12. From Corollary 11 we can easily obtain a criterion on the persistence of all species $(x_1(t), x_2(t), \ldots, x_n(t), u_1(t), u_2(t), \ldots, u_n(t))$ of system (1).

4. Examples

In this section, we will give several examples to illustrate the conclusions of Corollary 7, Theorem 6, and Corollary 11. In the first part we will illustrate the conclusions of Corollary 7, in the second we will illustrate the conclusions of Theorem 6, and in the last we will illustrate the conclusions of Corollary 11.

Example 1. Consider the system

$$\dot{x}_i(t) = x_i(t) \left[r_i(t) - \sum_{j=1}^{3} a_{ij}(t) \int_0^{+\infty} K_{ij}(s)\, x_j(t-s)\,ds \right], \quad i = 1, 2, 3, \quad (59)$$

where

$$\begin{gathered} r_1(t) = 5 + 3\sin t, \qquad r_2(t) = 3 + 3\cos t, \\ r_3(t) = 2 + 2\sin t, \\ a_{11}(t) = 4, \quad a_{12}(t) = 5, \quad a_{13}(t) = 6, \\ a_{21}(t) = 3, \quad a_{22}(t) = 4, \quad a_{23}(t) = 5, \\ a_{31}(t) = 2, \quad a_{32}(t) = 3, \quad a_{33}(t) = 4, \\ K_{11}(t) = e^{-t}, \quad K_{12}(t) = 2e^{-2t}, \quad K_{13}(t) = 3e^{-3t}, \\ K_{21}(t) = e^{-t}, \quad K_{22}(t) = 2e^{-2t}, \quad K_{23}(t) = 3e^{-3t}, \\ K_{31}(t) = e^{-t}, \quad K_{32}(t) = 2e^{-2t}, \quad K_{33}(t) = 3e^{-3t}. \end{gathered} \quad (60)$$

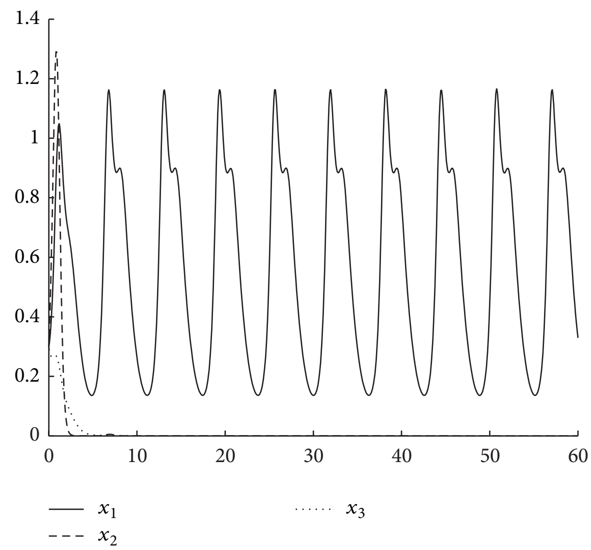

FIGURE 1: Dynamic behaviors of system (59). Here, we take the initial conditions $x_1(\theta) \equiv x_2(\theta) \equiv x_3(\theta) = 0.3$ for all $\theta \in (-\infty, 0]$.

Obviously, we have that the period of system (59) is $\omega = 2\pi$. By calculating, we obtain

$$\begin{gathered}
\limsup_{t\to\infty} \frac{\int_t^{t+w} r_2(s)\,ds}{\int_t^{t+w} r_1(s)\,ds} = \frac{3}{5} < \liminf_{t\to\infty} \frac{A_{21}(t)}{A_{11}(t)} = \frac{3}{4}, \\
\limsup_{t\to\infty} \frac{\int_t^{t+w} r_2(s)\,ds}{\int_t^{t+w} r_1(s)\,ds} = \frac{3}{5} < \liminf_{t\to\infty} \frac{A_{22}(t)}{A_{12}(t)} = \frac{4}{5}, \\
\limsup_{t\to\infty} \frac{\int_t^{t+w} r_3(s)\,ds}{\int_t^{t+w} r_2(s)\,ds} = \frac{2}{5} < \liminf_{t\to\infty} \frac{A_{31}(t)}{A_{21}(t)} = \frac{2}{3}, \\
\limsup_{t\to\infty} \frac{\int_t^{t+w} r_3(s)\,ds}{\int_t^{t+w} r_2(s)\,ds} = \frac{2}{5} < \liminf_{t\to\infty} \frac{A_{32}(t)}{A_{22}(t)} = \frac{3}{4}, \\
\limsup_{t\to\infty} \frac{\int_t^{t+w} r_3(s)\,ds}{\int_t^{t+w} r_2(s)\,ds} = \frac{2}{5} < \liminf_{t\to\infty} \frac{A_{33}(t)}{A_{23}(t)} = \frac{4}{5}.
\end{gathered} \tag{61}$$

We can choose $r = 1$, since all conditions of Corollary 7 hold; therefore, species x_2 and x_3 in system (59) are extinct, and only species x_1 is persistent (see Figure 1). However, conditions (1.5) of Proposition 2.2 given by Montes de Oca and Pérez [19] do not apply in this example.

Example 2. Consider the system

$$\begin{aligned}
\dot{x}_i(t) &= x_i(t)\Bigg[r_i(t) - \sum_{j=1}^{3} a_{ij}(t)\int_0^{+\infty} K_{ij}(s)\,x_j(t-s)\,ds \\
&\qquad - b_i(t)\int_0^{+\infty} H_i(s)\,u_i(t-s)\,ds\Bigg], \\
\dot{u}_i(t) &= -c_i(t)\,u_i(t) + d_i(t)\int_0^{+\infty} R_i(s)\,x_i(t-s)\,ds, \\
&\qquad i = 1, 2, 3,
\end{aligned} \tag{62}$$

where

$$\begin{gathered}
r_1(t) = 2 + \sin t, \qquad a_{11}(t) = 4 + \cos t, \\
a_{12}(t) = \frac{1}{5}, \qquad a_{13}(t) = \frac{1}{6}, \\
b_1(t) = \frac{1}{10}, \qquad r_2(t) = 3 + \cos t, \\
a_{21}(t) = \frac{1}{4}, \qquad a_{22}(t) = 6 + \sin t, \\
a_{23}(t) = \frac{2}{7}, \qquad b_2(t) = \frac{1}{3}, \\
r_3(t) = 1 + \frac{1}{4}\cos t, \qquad a_{31}(t) = \frac{5}{2}, \\
a_{32}(t) = \frac{5}{2}, \qquad a_{33}(t) = 2 + \sin t, \\
b_3(t) = \frac{2}{5}, \qquad c_1(t) = 4 + 2\sin t, \\
c_2(t) = 6 + 2\sin t, \qquad c_3(t) = 5 + 2\sin t, \\
d_1(t) = 1 + \frac{1}{2}\cos t, \qquad d_2(t) = 1 + \frac{1}{3}\cos t, \\
d_3(t) = 1 + \frac{1}{4}\cos t, \\
K_{ij}(t) = H_i(t) = R_i(t) = e^{-t}, \quad i, j = 1, 2, 3.
\end{gathered} \tag{63}$$

Obviously, we have that the period of system (62) is $\omega = 2\pi$. By calculating, we obtain

$$\begin{gathered}
\limsup_{t\to\infty} \frac{\int_t^{t+w} r_3(s)\,ds}{\int_t^{t+w} r_1(s)\,ds} = \frac{1}{2} < \liminf_{t\to\infty} \frac{A_{31}(t)}{A_{11}(t)} = \frac{5}{8+\sqrt{2}}, \\
\limsup_{t\to\infty} \frac{\int_t^{t+w} r_3(s)\,ds}{\int_t^{t+w} r_1(s)\,ds} = \frac{1}{2} < \liminf_{t\to\infty} \frac{A_{32}(t)}{A_{12}(t)} = \frac{25}{2}, \\
\limsup_{t\to\infty} \frac{\int_t^{t+w} r_3(s)\,ds}{\int_t^{t+w} r_1(s)\,ds} = \frac{1}{2} < \liminf_{t\to\infty} \frac{A_{33}(t)}{A_{13}(t)} = 12 - 3\sqrt{2}, \\
\limsup_{t\to\infty} \frac{\int_t^{t+w} r_3(s)\,ds}{\int_t^{t+w} r_2(s)\,ds} = \frac{1}{3} < \liminf_{t\to\infty} \frac{A_{31}(t)}{A_{21}(t)} = 10, \\
\limsup_{t\to\infty} \frac{\int_t^{t+w} r_3(s)\,ds}{\int_t^{t+w} r_2(s)\,ds} = \frac{1}{3} < \liminf_{t\to\infty} \frac{A_{32}(t)}{A_{22}(t)} = \frac{5}{12+\sqrt{2}}, \\
\limsup_{t\to\infty} \frac{\int_t^{t+w} r_3(s)\,ds}{\int_t^{t+w} r_2(s)\,ds} = \frac{1}{3} < \liminf_{t\to\infty} \frac{A_{33}(t)}{A_{23}(t)} = 7 - \frac{7}{4}\sqrt{2}.
\end{gathered} \tag{64}$$

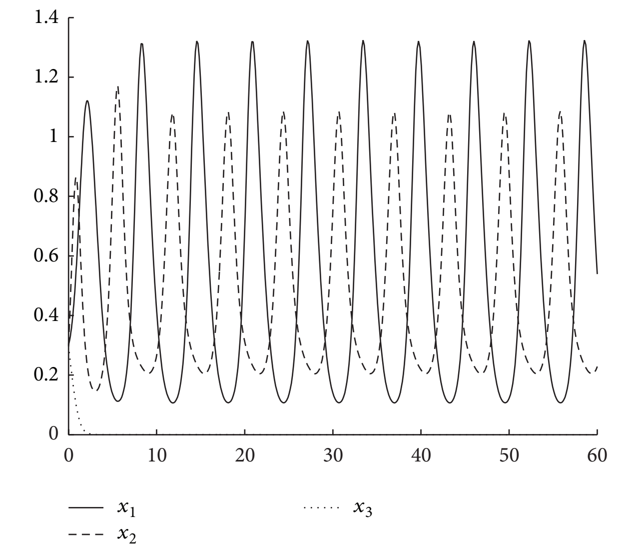

FIGURE 2: Dynamic behaviors of system (62). Here, we take the initial conditions $x_1(\theta) \equiv x_2(\theta) \equiv x_3(\theta) \equiv u_1(\theta) \equiv u_2(\theta) \equiv u_2(\theta) = 0.3$ for all $\theta \in (-\infty, 0]$.

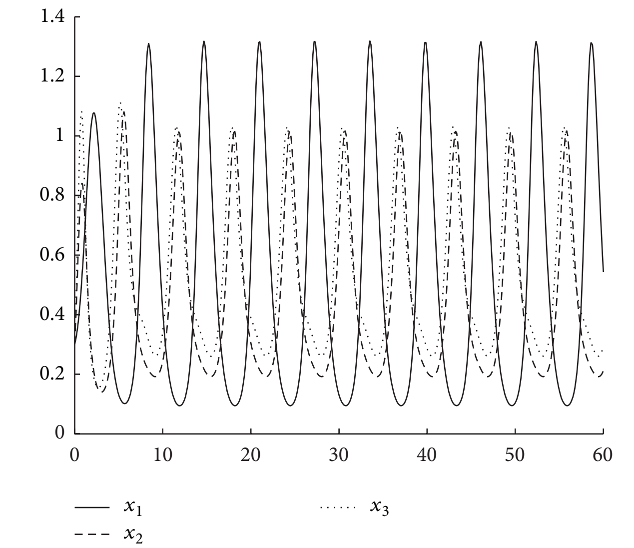

FIGURE 3: Dynamic behaviors of system (66). Here, we take the initial conditions $x_1(\theta) \equiv x_2(\theta) \equiv x_3(\theta) \equiv u_1(\theta) \equiv u_2(\theta) \equiv u_3(\theta) = 0.3$ for all $\theta \in (-\infty, 0]$.

From Remark 3, we can choose $r = 2$, $\overline{x}_1 = 4$, $\overline{x}_2 = 4$, $\overline{x}_3 = 45/16$, $\overline{u}_1 = 3$, $\overline{u}_2 = 4/3$, $\overline{u}_3 = 75/64$, and $\eta = 2\pi$ such that

$$\begin{aligned}
&\liminf_{t\to\infty} \int_t^{t+2\pi} \left[r_1(s) - A_{12}(s)\overline{x}_2 - A_{13}(s)\overline{x}_3 - B_1(s)\overline{u}_1\right] ds \\
&\quad = 0.8625\pi > 0, \\
&\liminf_{t\to\infty} \int_t^{t+2\pi} \left[r_2(s) - A_{21}(s)\overline{x}_1 - A_{23}(s)\overline{x}_3 - B_2(s)\overline{u}_2\right] ds \\
&\quad \approx 1.504\pi > 0.
\end{aligned} \tag{65}$$

All the conditions of Theorem 6 hold; therefore, species x_1 and x_2 coexist, and species x_3 in system (62) is extinct (see Figure 2).

Example 3. Consider the system

$$\begin{aligned}
\dot{x}_i(t) &= x_i(t)\left[r_i(t) - \sum_{j=1}^{3} a_{ij}(t)\int_0^{+\infty} K_{ij}(s)\,x_j(t-s)\,ds \right.\\
&\qquad \left. - b_i(t)\int_0^{+\infty} H_i(s)\,u_i(t-s)\,ds\right], \\
\dot{u}_i(t) &= -c_i(t)\,u_i(t) + d_i(t)\int_0^{+\infty} R_i(s)\,x_i(t-s)\,ds, \\
&\qquad i = 1, 2, 3,
\end{aligned} \tag{66}$$

where

$$\begin{aligned}
r_3(t) &= 4 + \cos t, \qquad a_{31}(t) = \frac{1}{2}, \\
a_{32}(t) &= \frac{2}{7}, \qquad a_{33}(t) = 7 + \sin t,
\end{aligned} \tag{67}$$

and the coefficients and the other kernels are as Example 2. In this case, we can choose $\overline{x}_1 = 4$, $\overline{x}_2 = 4$, $\overline{x}_3 = 5$, $\overline{u}_1 = 3$, $\overline{u}_2 = 4/3$, $\overline{u}_3 = 25/12$, and $\eta = 2\pi$ such that

$$\begin{aligned}
&\liminf_{t\to\infty} \int_t^{t+2\pi} \left[r_1(s) - A_{12}(s)\overline{x}_2 - A_{13}(s)\overline{x}_3 - B_1(s)\overline{u}_1\right] ds \\
&\quad = \frac{2}{15}\pi >, \\
&\liminf_{t\to\infty} \int_t^{t+2\pi} \left[r_2(s) - A_{21}(s)\overline{x}_1 - A_{23}(s)\overline{x}_3 - B_2(s)\overline{u}_2\right] ds \\
&\quad = \frac{16}{63}\pi > 0, \\
&\liminf_{t\to\infty} \int_t^{t+2\pi} \left[r_3(s) - A_{31}(s)\overline{x}_1 - A_{32}(s)\overline{x}_2 - B_3(s)\overline{u}_3\right] ds \\
&\quad = \frac{1}{21}\pi > 0.
\end{aligned} \tag{68}$$

All conditions of Corollary 11 hold, so all the species x_1, x_2, and x_3 are persistent (see Figure 3).

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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