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Research Article

Fractal Dimension Analysis of the Julia Sets of Controlled Brusselator Model

Yuqian Deng, Xiuxiong Liu, and Yongping Zhang

School of Mathematics and Statistics, Shandong University at Weihai, Weihai 264209, China

Correspondence should be addressed to Yongping Zhang; ypzhang@sdu.edu.cn

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Fractal theory is a branch of nonlinear scientific research, and its research object is the irregular geometric form in nature. On account of the complexity of the fractal set, the traditional Euclidean dimension is no longer applicable and the measurement method of fractal dimension is required. In the numerous fractal dimension definitions, box-counting dimension is taken to characterize the complexity of Julia set since the calculation of box-counting dimension is relatively achievable. In this paper, the Julia set of Brusselator model which is a class of reaction diffusion equations from the viewpoint of fractal dynamics is discussed, and the control of the Julia set is researched by feedback control method, optimal control method, and gradient control method, respectively. Meanwhile, we calculate the box-counting dimension of the Julia set of controlled Brusselator model in each control method, which is used to describe the complexity of the controlled Julia set and the system. Ultimately we demonstrate the effectiveness of each control method.

1. Introduction

In order to recognize the essence of some extremely sophisticated phenomena, researchers attempt to figure out the regularity and unity which exist behind these phenomena so that they can control and predict them better. In the early 20th century, the fundamental theory of chaos and fractal was proposed. The theory explains the unity of determinacy and randomness and the unity of order and disorder. It is considered to be the third major revolution of science after the theory of relativity and quantum mechanics [1, 2].

Fractal theory, first proposed in the 1970s, comes from the study of nonlinear science. Its primary research object is the geometric form of nature and nonlinear system, which is complex but has some kind of self similarity and regularity. In 1977, Mandelbrot, a professor of mathematics of the Harvard University, published the landmark work *Fractal: Form, Chance and Dimension* [3]. It marked the fractal geometry that had become an independent discipline. Subsequently, he published another work *The Fractal Geometry of Nature* [4], which implied that fractal theory had been basically formed.

Nowadays, with the emergence of some new mathematical tools and methods, especially the combination of the study of fractal theory and computer, the theory has been developed rapidly. In addition, researchers not only constantly establish and improve the theory of fractals, but also apply it in various fields, such as the diffusion processes and chemical kinetics in crowded media, the protein structure and complex vascular branches in biomedicine, dynamical system and hydromechanics in physics, and landforms evolution and earthquake monitoring [5–16]. Even in social and economic activities, the theory of fractals also has numerous applications [17–19].

Considering the complexity of fractal sets, traditional Euclidean geometry dimension cannot accurately depict their geometric forms. Mathematicians propose many definitions of noninteger dimension and use different names to distinguish them. For example, Hausdorff dimension which was proposed by the German mathematician Hausdorff in 1919 has a rigorous mathematical definition. It is established on the basis of Hausdorff measure and can define most fractal sets, so it is easier to deal with in mathematics [20]. Moreover, box-counting dimension is one of the most widely used dimensions. Its popularity is largely due to its relative ease of mathematical calculation and empirical estimation. Besides, other fractal dimensions, such as similar dimension, capacity

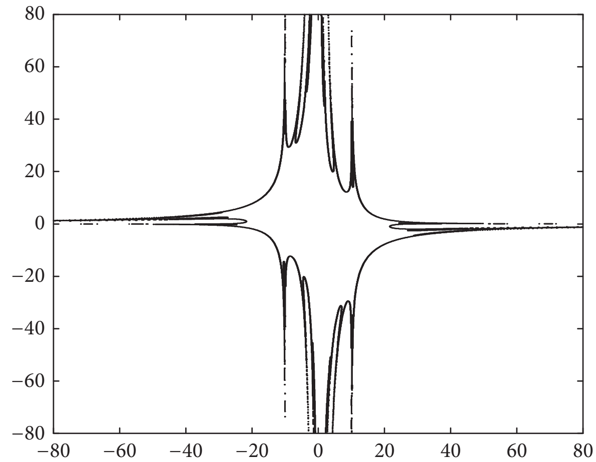

Figure 1: The original Julia set of the system.

dimension, and Lyapunov dimension, also have their own applications in the corresponding fields [21–24].

Brusselator model [25–27] is a kind of reaction diffusion equations which describe the change of chemical elements in the process of chemical reaction [28]. It is significant in the study of the chaos and fractal behavior of nonlinear differential equations. Researchers have studied Brusselator model from different aspects and proven some properties of it [29–33]. These studies of the model have contributed much to the development of nonlinear mathematics. Nowadays, with the development of research, people apply some of the property of the model in all kinds of disciplines and social production activities and have achieved fruitful results.

Notably, in the nonlinear system, the Julia set of the system is an important nonlinear feature. According to the objective requirement, we often need restrict the size of the nonlinear attractive domain. And sometimes it is required that the system possess different or similar behavior and performance in compliance with the actual requirements of technical problems. As a result, how to effectively control the Julia set is particularly critical.

Based on the Julia set of Brusselator model, feedback control method, optimal control method, and gradient control method [34] are taken to control the Julia set of the model. And the box-counting dimension of the Julia set of controlled Brusselator model is calculated in each control method to describe the complexity of the Julia set of the system.

2. Basic Theory

In 1918, Julia Gaston, a famous French mathematician, discovered an important fractal set in fractal theory, when he studied the iteration of complex functions, which was named Julia set. He noticed that functions on the complex plane as simple as $f(z) = z^2 + c$, with a complex constant c, can give rise to fractals of an exotic appearance. The precise definition of Julia set is given below [35].

Take $f : C \to C$ to be a polynomial of degree $n \geq 2$ with complex coefficients, $f(z) = a_0 + a_1 z + \cdots + a_n z^n$. Write f^k for the k-fold composition $f \circ \cdots \circ f$, so that $f^k(w)$ is the kth iterate $f(f(\cdots(f(w))))$ of w. If $f(w) = w$ we call w a fixed point of f, and if $f^p(w) = w$ for some integer $p \geq 1$ we call w a periodic point of f; the least such p is called the period of w. We call $w, f(w), \ldots, f^p(w)$ a period p orbit. Let w be a periodic point of period p, with $(f^p)'(w) = \lambda$, where the prime denotes complex differentiation. The point w is called superattractive, if $\lambda = 0$; attractive, if $0 \leq |\lambda| < 1$; neutral, if $|\lambda| = 1$; and repelling, if $|\lambda| > 1$.

Definition 1. Let $f : C \to C$ be a polynomial of degree $n > 1$; Julia set of f is defined to be the closure of repelling period points of f.

In fractal theory, fractal dimension is one of the most elemental concepts. At present, there are many definitions of fractal dimensions, including Hausdorff dimension, box-counting dimension, similarity dimension. In all kinds of definitions of fractal dimensions, Hausdorff dimension is the basis of the fractal theory. It can even be considered the theoretical basis of the fractal geometry. However, Hausdorff dimension is just suitable for the theoretical analysis of fractal theory, and there is only a small class of fairly solid mathematical regular fractal graphics that can be calculated for their Hausdorff dimension. It is hard to calculate the fractal dimension which is proposed in the practical applications. Therefore, people propose the concept of box-counting dimension. Its popularity is largely due to its relative ease of mathematical calculation and empirical estimation. In fact, in practical applications, the dimension is generally referred to as box-counting dimension. The precise definition of box-counting dimension is as follows.

Definition 2 (see [35]). Let F be any nonempty bounded subset of R^n and let $N_\delta(F)$ be the smallest number of sets of diameter at most δ which can cover F. The *lower* and *upper box-counting dimensions* of F ((1), (2)), respectively, are defined as

$$\underline{\dim}_B F = \underline{\lim}_{\delta \to 0} \frac{\log N_\delta(F)}{-\log \delta}, \tag{1}$$

$$\overline{\dim}_B F = \overline{\lim}_{\delta \to 0} \frac{\log N_\delta(F)}{-\log \delta}. \tag{2}$$

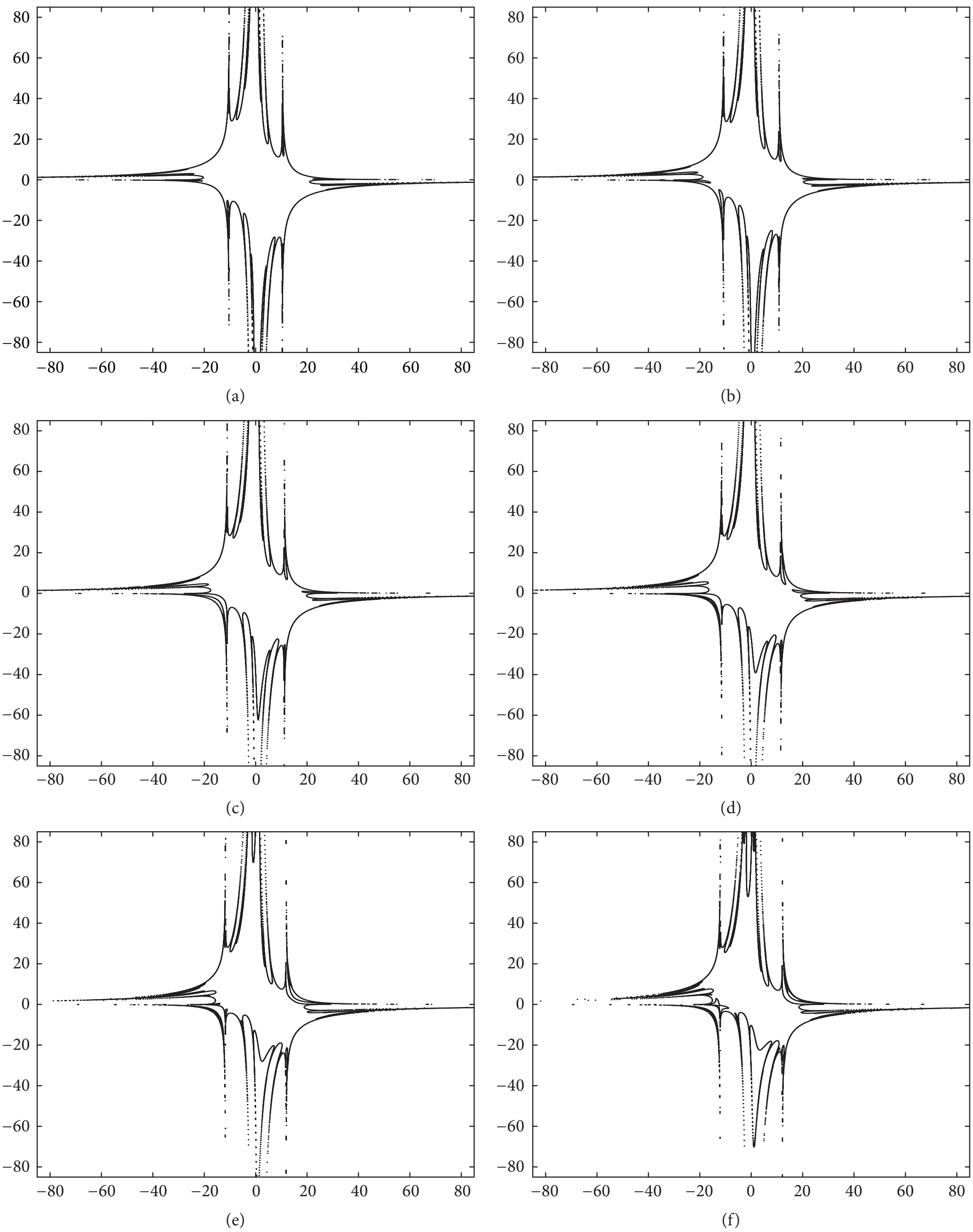

Figure 2: The change of the Julia sets of the controlled system when $a = 0.07$, $b = 0.02$, and $\gamma = 0.01$. (a) $k = 0.06$; (b) $k = 0.14$; (c) $k = 0.22$; (d) $k = 0.30$; (e) $k = 0.38$; (f) $k = 0.45$.

If these are equal we refer to the common value as the *box-counting dimension* of F (3):

$$\dim_B F = \lim_{\delta \to 0} \frac{\log N_\delta (F)}{-\log \delta}. \quad (3)$$

3. The Control of the Julia Set of Brusselator Model

Brusselator model is one of the most fundamental models in nonlinear systems, and the dynamic equations are as follows:

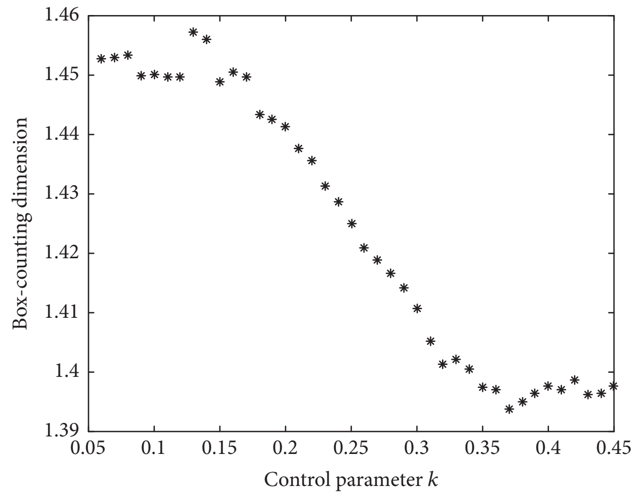

FIGURE 3: The change of box-counting dimensions of the Julia sets of the controlled system when k is from 0.06 to 0.45.

$$\begin{aligned} x_{n+1} &= A - (B+1)\,x_n + x_n^2 y_n, \\ y_{n+1} &= B x_n - x_n^2 y_n, \end{aligned} \tag{4}$$

where x, y denote concentration of reactant in the process of chemical reaction. $A, B > 0$ denote initial concentration of reactant.

Brusselator equations were first discovered by A. Turing in 1952 [36], and then I. Pigogine and Leefver did some systematic studies on it. They pointed out that the Brusselator equations were the most elementary and essential mathematic model which describes the oscillation of biochemistry. They proved that when $B > 1 + A^2$, the equations have stable and unique limit cycle. When $B \leq 1 + A^2$, there is no limit cycle [37]. It can be known that the initial concentration of reactant has an important influence on the system. In fractal theory, Julia set is a set of initial points of the system that satisfy certain conditions. With the same thought, we define the Julia set of Brusselator model. Let $F(x, y) = (A - (B + 1)x_n + x_n^2 y_n, Bx_n - x_n^2 y_n)$.

Definition 3. Set $K = \{(x, y) \in R^2 \mid \{F^n(x, y)\}_{n\geq 1}$ is bounded$\}$ is called the filled Julia set of Brusselator model. The boundary of the filled Julia set is defined to be the Julia set of Brusselator model; that is, $J(F) = \partial K$.

The research results demonstrate that, coupled with a piecewise constant value control function, we can control the size of the limit cycle of (4) [36]. From the definition of Julia set, we found that there is a close relation between the boundedness of iterative orbits and the structure of the Julia set. So if we want to control the Julia set of Brusselator model, how to control the boundedness of the system iterative is critical. We consider especially the stability of the fixed point of Brusselator model, and by designing controllers unstable fixed points are turned into stable fixed points to control the boundedness of iterative orbits effectively. Then the control of the Julia set can be realized.

As was mentioned above, considering the stability of the fixed points of system (5), we try to find controllers to make the fixed point of the system stable. Let the controlled Brusselator model be

$$\begin{aligned} x_{n+1} &= a + (1 - b - \gamma)\,x_n + \gamma x_n^2 y_n + u_n, \\ y_{n+1} &= y_n + b x_n - \gamma x_n^2 y_n + v_n, \end{aligned} \tag{5}$$

where u_n and v_n denote the designed controllers.

3.1. Feedback Control Method. Take $u_n = k(x_n - x^*)$, $v_n = k(y_n - y^*)$, with the control parameter k, and the controlled system is

$$\begin{aligned} x_{n+1} &= a + (1 - b - \gamma)\,x_n + \gamma x_n^2 y_n + k\,(x_n - x^*), \\ y_{n+1} &= y_n + b x_n - \gamma x_n^2 y_n + k\,(y_n - y^*). \end{aligned} \tag{6}$$

Theorem 4. *Let $A_1 = 1 - b - \gamma + 2\gamma x^* y^* + k$, $B_1 = 1 - \gamma x^{*2} + k$, $C_1 = \gamma x^{*2}$, and $D_1 = b - 2\gamma x^* y^*$, where (x^*, y^*) denotes the fixed point of the controlled system (6). If $|(A_1 + B_1) \pm \sqrt{(A_1 - B_1)^2 + 4C_1 D_1}| < 2$, the fixed point of the system is attractive.*

Proof. Write $\widetilde{B}_1(x, y) = (a + (1-b-\gamma)x + \gamma x^2 y + k(x - x^*), y + bx - \gamma x^2 y + k(y - y^*))$. The Jacobi matrix of the controlled system (6) is

$$D\widetilde{B}_1 = \begin{bmatrix} 1 - b - \gamma + 2\gamma x_n y_n + k & \gamma x_n^2 \\ b - 2\gamma x_n y_n & 1 - \gamma x_n^2 + k \end{bmatrix} \tag{7}$$

and its eigenmatrix is

$$D\widetilde{B}_1 - \lambda E = \begin{bmatrix} 1 - b - \gamma + 2\gamma x_n y_n + k - \lambda & \gamma x_n^2 \\ b - 2\gamma x_n y_n & 1 - \gamma x_n^2 + k - \lambda \end{bmatrix}. \tag{8}$$

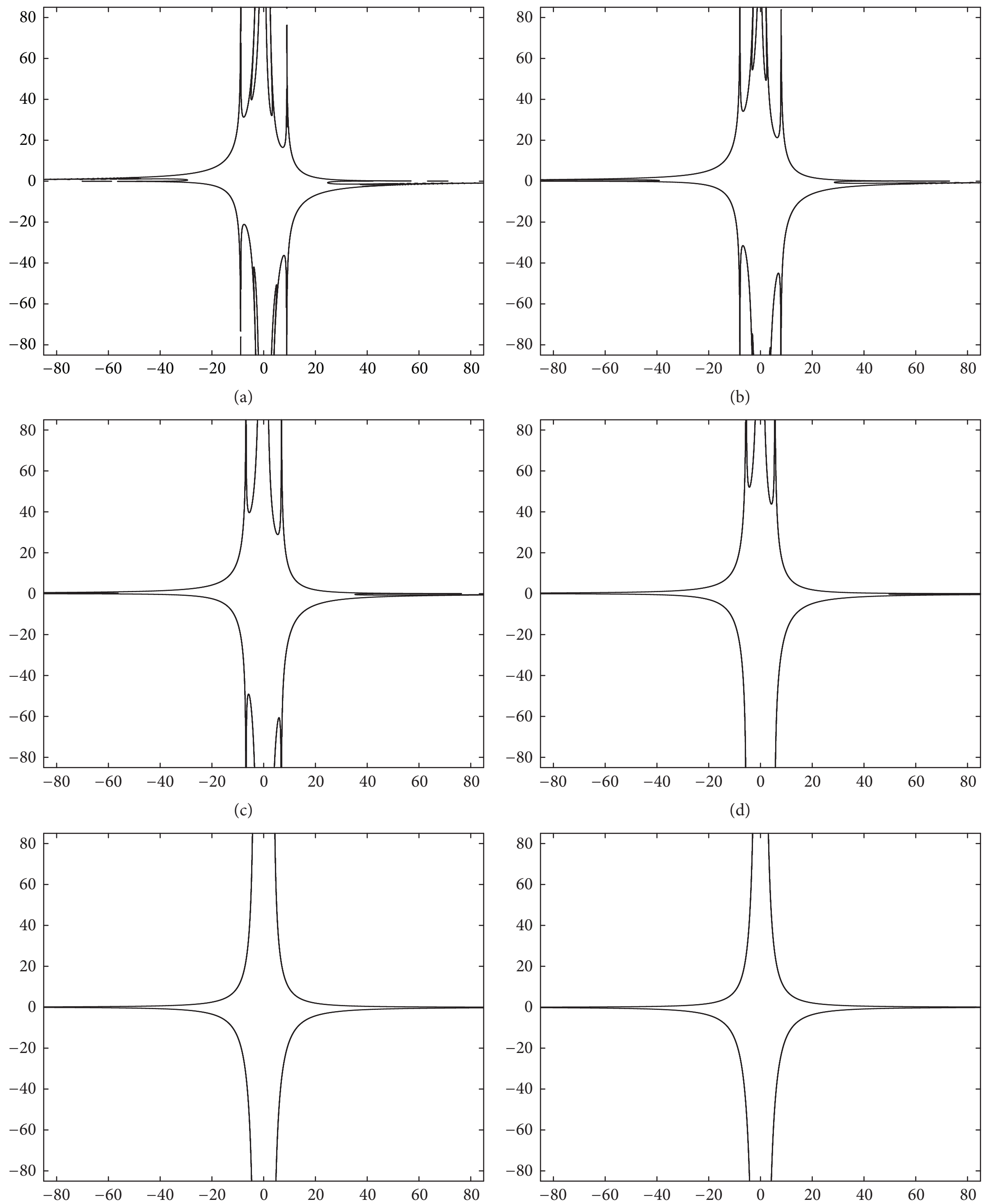

FIGURE 4: The change of the Julia sets of the controlled system when $a = 0.07$, $b = 0.02$, and $\gamma = 0.01$. (a) $k = -0.22$; (b) $k = -0.38$; (c) $k = -0.54$; (d) $k = -0.70$; (e) $k = -0.86$; (f) $k = -1$.

And the characteristic equation is

$$\begin{aligned}\lambda^2 &- \left(2 - b - \gamma + 2\gamma x_n y_n - \gamma x_n^2 + 2k\right)\lambda \\ &+ \left(1 - b - \gamma + 2\gamma x_n y_n + k\right)\left(1 - \gamma x_n^2 + k\right) \\ &- \gamma x_n^2 \left(b - 2\gamma x_n y_n\right) = 0.\end{aligned} \tag{9}$$

When the modulus of the eigenvalues λ_{11}, λ_{12} of the Jacobi matrix at the fixed point is less than 1, the fixed point is attractive:

$$\left|\lambda_{11,12}\right| = \left|\frac{\left(A_1 + B_1\right) \pm \sqrt{\left(A_1 - B_1\right)^2 + 4C_1 D_1}}{2}\right| < 1; \tag{10}$$

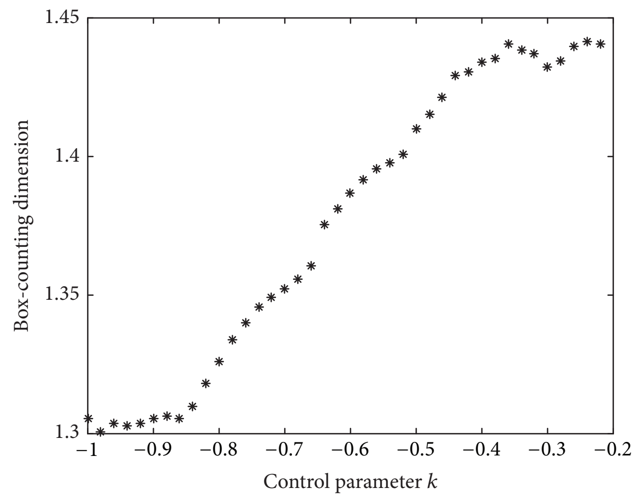

Figure 5: The change of box-counting dimensions of the Julia sets of the controlled system when k is from −1 to −0.22.

that is,

$$\left|(A_1+B_1)\pm\sqrt{(A_1-B_1)^2+4C_1D_1}\right|<2. \tag{11}$$

By Theorem 4, we know that Brusselator model can be controlled by selecting the value of k which satisfies the condition, and then the control of the Julia set can be realized.

For example, take $a = 0.07$, $b = 0.02$, and $\gamma = 0.01$ in system (6) and the initial Julia set is shown in Figure 1; then we get $x^* = 7$, $y^* = 2/7$. By Theorem 4, the range of the value of k is $-1.9895 < k < 0.4695$.

Six simulation diagrams are chosen corresponding to the values of k from 0.06 to 0.45 in Figure 2, and we find that the trend of the change of the Julia sets is obvious. In Figure 3 the box-counting dimensions of the controlled Julia sets are computed in this control method.

In the same control, six simulation diagrams are chosen corresponding to the values of k from −1 to −0.22 in Figure 4 to illustrate the change of the Julia sets in feedback control method. In Figure 5 the box-counting dimensions of the controlled Julia sets are computed in this control method.

In feedback control method, the contraction of the left and the lower parts is faster than the right and the upper parts when the interval of control parameter k is between 0.06 and 0.45. In addition, the complexity of the boundary of the Julia set significantly decreases and the lower part rapidly contracts. When the interval of control parameter k is between −1 and −0.22, the complexity of the boundary of the Julia set significantly decreases, and the Julia set tends to be centrally symmetric. In general, with the absolute value of k increasing, the Julia set contracts to the center gradually.

From the perspective of the change of box-counting dimensions, with the absolute values of k increasing, the change generally shows a monotonic decreasing trend. Particularly when the control parameter k is in the interval from 0.16 to 0.37 and −0.9 to −0.4, the monotonic change of box-counting dimensions is obvious, which indicates the effectiveness of this control method on the Julia set of Brusselator model. □

3.2. Optimal Control Method. Take $u_n = k(f(x_n, y_n) - x_n)$, $v_n = k(g(x_n, y_n) - y_n)$, with $f(x_n, y_n) = a + (1-b-\gamma)x_n + \gamma x_n^2 y_n$, $g(x_n, y_n) = y_n + bx_n - \gamma x_n^2 y_n$, where k is the control parameter. Then the controlled system is

$$\begin{aligned} x_{n+1} &= a + (1-b-\gamma)x_n + \gamma x_n^2 y_n \\ &\quad + k\left(f(x_n, y_n) - x_n\right), \\ y_{n+1} &= y_n + bx_n - \gamma x_n^2 y_n + k\left(g(x_n, y_n) - y_n\right). \end{aligned} \tag{12}$$

Theorem 5. *Let $A_2 = (1-b-\gamma+2\gamma x^* y^*)(k+1)-k$, $B_2 = (1-\gamma x^{*2})(k+1)-k$, $C_2 = \gamma x^{*2}(k+1)$, and $D_2 = (b-2\gamma x^* y^*)(k+1)$, where (x^*, y^*) denotes the fixed point of the controlled system (12). If $|(A_2+B_2)\pm\sqrt{(A_2-B_2)^2+4C_2D_2}| < 2$, the fixed point of the system is attractive.*

Proof. Write $\widetilde{B}_2(x,y) = (a+(1-b-\gamma)x+\gamma x^2 y + k(f(x,y)-x), y+bx-\gamma x^2 y + k(g(x,y)-y))$. The Jacobi matrix of the controlled system (12) is

$$D\widetilde{B}_2 = \begin{bmatrix} (1-b-\gamma+2\gamma x_n y_n)(k+1)-k & \gamma x_n^2(k+1) \\ (b-2\gamma x_n y_n)(k+1) & \left(1-\gamma x_n^2\right)(k+1)-k \end{bmatrix} \tag{13}$$

and its eigenmatrix is

$$D\widetilde{B}_2 - \lambda E = \begin{bmatrix} (1-b-\gamma+2\gamma x_n y_n)(k+1)-k-\lambda & \gamma x_n^2(k+1) \\ (b-2\gamma x_n y_n)(k+1) & \left(1-\gamma x_n^2\right)(k+1)-k-\lambda \end{bmatrix}. \tag{14}$$

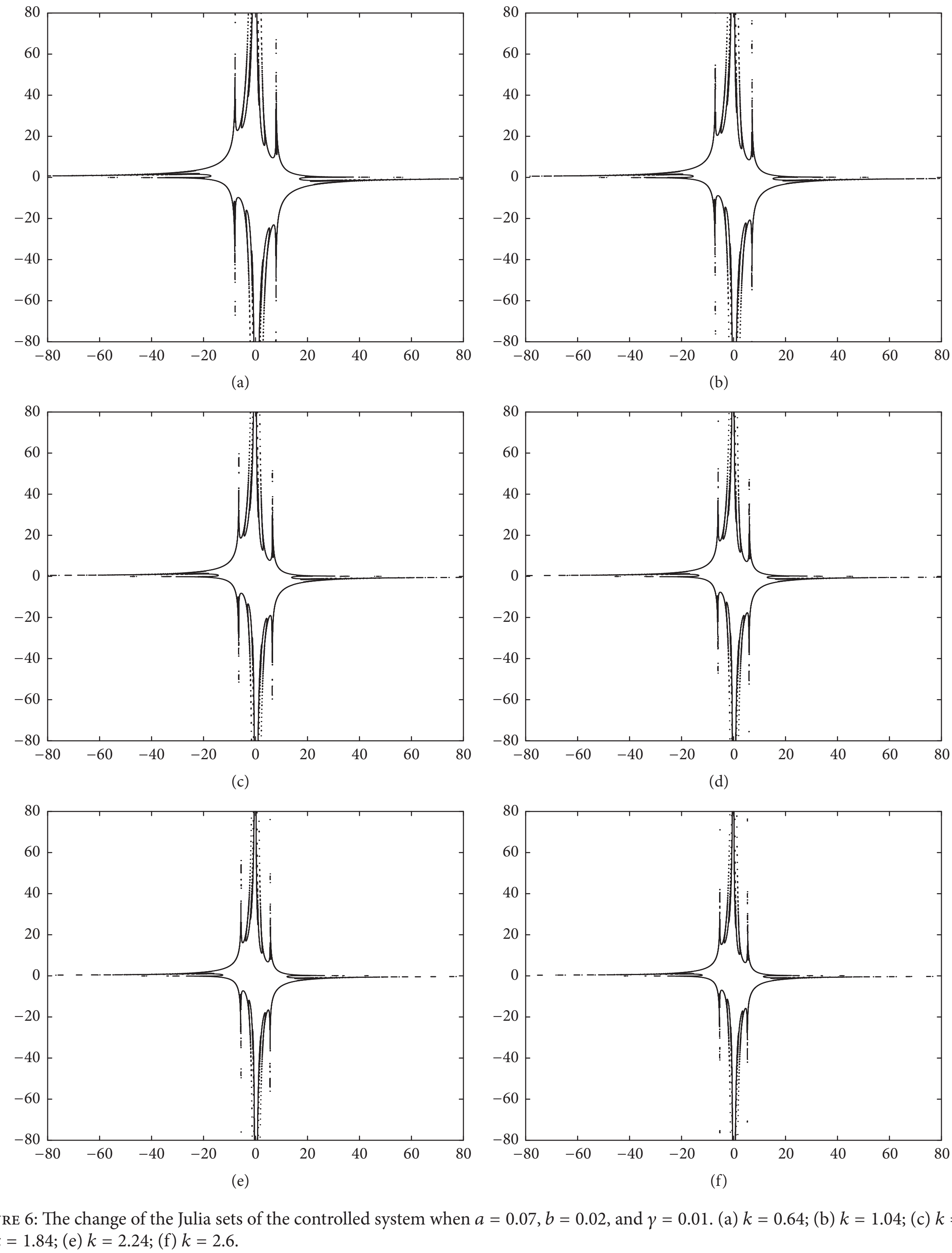

Figure 6: The change of the Julia sets of the controlled system when $a = 0.07$, $b = 0.02$, and $\gamma = 0.01$. (a) $k = 0.64$; (b) $k = 1.04$; (c) $k = 1.44$; (d) $k = 1.84$; (e) $k = 2.24$; (f) $k = 2.6$.

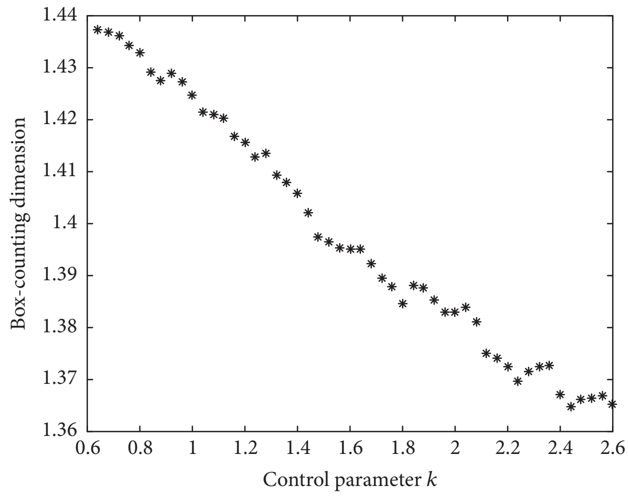

FIGURE 7: The change of box-counting dimensions of the Julia sets of the controlled system when k is from 0.64 to 2.6.

And the characteristic equation is

$$\begin{aligned}\lambda^2 &- \left(\left(2-b-\gamma+2\gamma x_n y_n - \gamma x_n^2\right)(k+1)-2k\right)\lambda \\ &+ \left((1-b-\gamma+2\gamma x_n y_n)(k+1)-k\right) \\ &\cdot \left(\left(1-\gamma x_n^2\right)(k+1)-k\right) - \gamma x_n^2 (b-2\gamma x_n y_n) \\ &\cdot (k+1)^2 = 0.\end{aligned} \tag{15}$$

When the modulus of the eigenvalue $\lambda_{21}, \lambda_{22}$ of the Jacobi matrix at the fixed point is less than 1, the fixed point is attractive:

$$|\lambda_{21,22}| = \left|\frac{(A_2+B_2) \pm \sqrt{(A_2-B_2)^2+4C_2D_2}}{2}\right| < 1; \tag{16}$$

that is,

$$\left|(A_2+B_2) \pm \sqrt{(A_2-B_2)^2+4C_2D_2}\right| < 2. \tag{17}$$

By Theorem 5, we know that Brusselator model can be controlled by selecting the value of k which satisfies the condition, and then the control of the Julia set can be realized.

For example, we take the values of system parameters $a = 0.07$, $b = 0.02$, and $\gamma = 0.01$, which are the same as Section 3.1. Then the range of the value of k is $-1 < k < 190.659$.

Six simulation diagrams are chosen corresponding to the values of k from 0.64 to 2.6 in Figure 6 to illustrate the change of the Julia sets in optimal control method. In Figure 7 the box-counting dimensions of the controlled Julia sets are computed in this control method.

In optimal control method, the effective controlled interval of k is from –1 to 190.6. But when the interval of k is from 0.64 to 2.6, this method has the best controlled effectiveness. In this interval, with the absolute values of k increasing, the Julia sets gradually contract to the center with nearly the same speed, and no significant change in the overall shape of the Julia sets occurs.

From the perspective of the change of box-counting dimensions, box-counting dimensions of the Julia sets generally show a monotonic decreasing trend with the absolute values of k increasing. For the reason that the complexity of the Julia set can be depicted by box-counting dimension, when the complexity of the Julia set decreases with the absolute value of k increasing, it indicates that this control method has great control effectiveness. □

3.3. Gradient Control Method. Take $u_n = k(x_n^2 - x^{*2})$, $v_n = k(y_n^2 - y^{*2})$, with the control parameter k. Therefore the controlled system is

$$\begin{aligned} x_{n+1} &= a + (1-b-\gamma)x_n + \gamma x_n^2 y_n + k\left(x_n^2 - x^{*2}\right), \\ y_{n+1} &= y_n + b x_n - \gamma x_n^2 y_n + k\left(y_n^2 - y^{*2}\right). \end{aligned} \tag{18}$$

Theorem 6. *Let $A_3 = 1 - b - \gamma + 2\gamma x^* y^* + 2kx^*$, $B_3 = 1 - \gamma x^{*2} + 2ky^*$, $C_3 = \gamma x^{*2}$, and $D_3 = b - 2\gamma x^* y^*$, where (x^*, y^*) denotes the fixed point of the controlled system (18). If $|(A_3+B_3) \pm \sqrt{(A_3-B_3)^2 + 4C_3D_3}| < 2$, the fixed point of the system is attractive.*

Proof. Write $\widetilde{B}_3(x,y) = (a + (1-b-\gamma)x + \gamma x^2 y + k(x^2 - x^{*2}), y + bx - \gamma x^2 y + k(y^2 - y^{*2}))$. The Jacobi matrix of the controlled system (18) is

$$D\widetilde{B}_3 = \begin{bmatrix} 1-b-\gamma+2\gamma x_n y_n + 2kx_n & \gamma x_n^2 \\ b - 2\gamma x_n y_n & 1 - \gamma x_n^2 + 2ky_n \end{bmatrix} \tag{19}$$

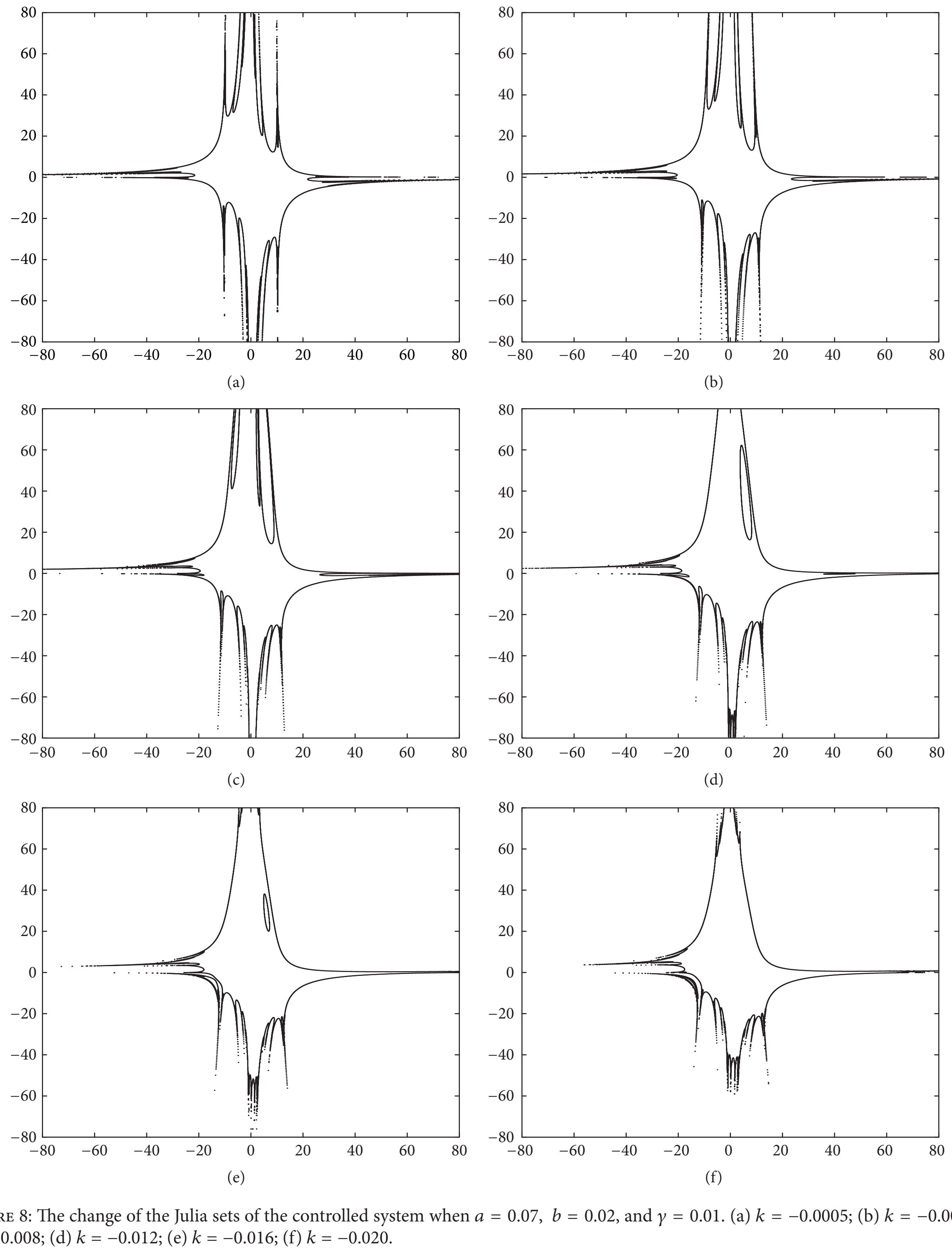

Figure 8: The change of the Julia sets of the controlled system when $a = 0.07$, $b = 0.02$, and $\gamma = 0.01$. (a) $k = -0.0005$; (b) $k = -0.004$; (c) $k = -0.008$; (d) $k = -0.012$; (e) $k = -0.016$; (f) $k = -0.020$.

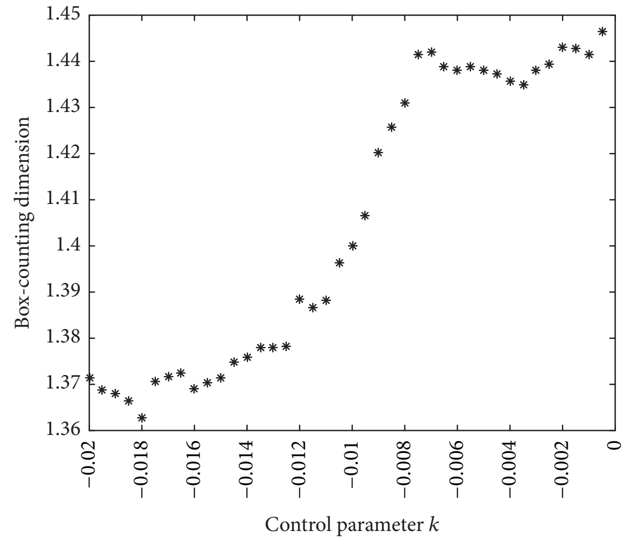

FIGURE 9: The change of box-counting dimensions of the Julia sets of the controlled system when k is from −0.02 to −0.0005.

and its eigenmatrix is

$$D\widetilde{B}_3 - \lambda E = \begin{bmatrix} 1-b-\gamma+2\gamma x_n y_n + 2k x_n - \lambda & \gamma x_n^2 \\ b - 2\gamma x_n y_n & 1-\gamma x_n^2 + 2k y_n - \lambda \end{bmatrix}. \quad (20)$$

And the characteristic equation is

$$\lambda^2 - \left(2-b-\gamma+2\gamma x_n y_n + 2k x_n - \gamma x_n^2 + 2k y_n\right)\lambda + \left(1-b-\gamma+2\gamma x_n y_n + 2k x_n\right)\left(1-\gamma x_n^2 + 2k y_n\right) - \gamma x_n^2 \left(b - 2\gamma x_n y_n\right) = 0. \quad (21)$$

When the modulus of the eigenvalue $\lambda_{31}, \lambda_{32}$ of the Jacobi matrix at the fixed point is less than 1, the fixed point is attractive:

$$\left|\lambda_{31,32}\right| = \left|\frac{(A_3+B_3) \pm \sqrt{(A_3-B_3)^2 + 4C_3D_3}}{2}\right| < 1; \quad (22)$$

that is,

$$\left|(A_3+B_3) \pm \sqrt{(A_3-B_3)^2 + 4C_3D_3}\right| < 2. \quad (23)$$

By Theorem 6, we know that the Brusselator model can be controlled by selecting the value of k which satisfies the condition, and then the control of the Julia set can be realized.

For example, we take the values of system parameters $a = 0.07$, $b = 0.02$, and $\gamma = 0.01$, which are the same as Section 3.1. Then the range of the value of k is $-2.6420 < k < 0.8561$.

Six simulation diagrams are chosen corresponding to the values of k from −0.0005 to −0.02 in Figure 8 to illustrate the change of the Julia sets in gradient control method. In Figure 9 the box-counting dimensions of the controlled Julia sets are computed in this control method.

In the same control, six simulation diagrams are chosen corresponding to the values of k from 0.0005 to 0.02 in Figure 10 to illustrate the change of the Julia sets in gradient control method. In Figure 11 the box-counting dimensions of the controlled Julia sets are computed in this control method.

In gradient control method, we consider two groups of interval of the parameter k. It is worth noticing that the contraction of the left and the lower parts of the Julia set are faster than the right and the upper parts when the interval of control parameter k is from −0.0005 to −0.02, while the right and the upper parts are faster than the left and the lower parts when the interval of control parameter k is from 0.0005 to 0.02. In general, with the absolute values of k increasing, the Julia sets contract to the center gradually, and the complexity of the boundary of the Julia sets decreases.

From the perspective of the change of box-counting dimensions, with the absolute values of k increasing, the box-counting dimensions of the Julia sets generally show a monotonic decreasing trend. Particularly when the control parameter k is in the range from −0.016 to −0.0075 and from 0.0075 to 0.016, the change of box-counting dimensions is obviously monotonic, which indicates the effectiveness of this method on the Julia set of the Brusselator model. □

4. Conclusion

Fractal theory is a hot topic in the research of nonlinear science. Describing the change of chemical elements in the chemical reaction process, Brusselator model, an important class of reaction diffusion equations, is significant in the study of chaotic and fractal behaviors of nonlinear differential equations. In technological applications, it is often required that the behavior and performance of the system can be controlled effectively.

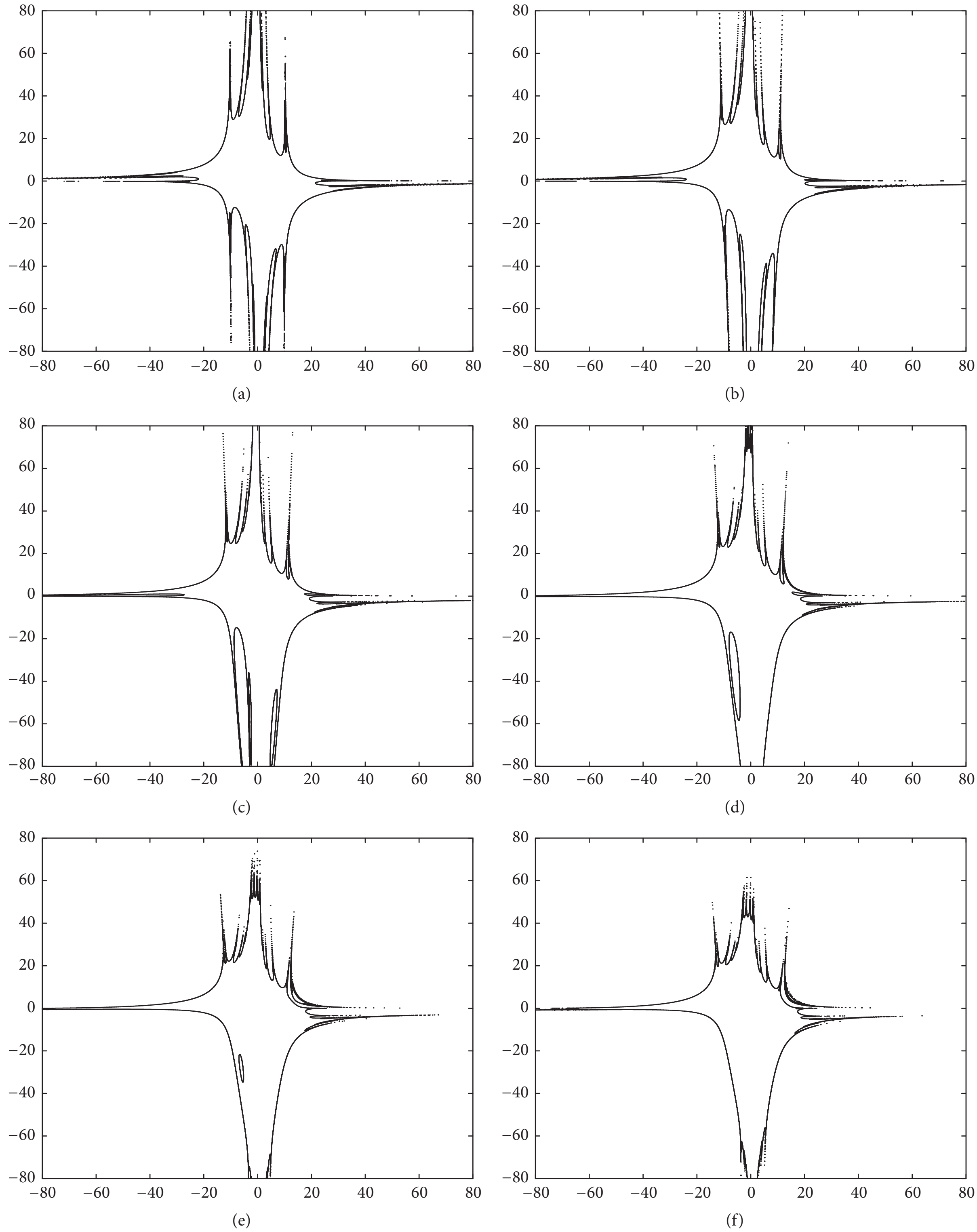

Figure 10: The change of the Julia sets of the controlled system when $a = 0.07$, $b = 0.02$, and $\gamma = 0.01$. (a) $k = 0.0005$; (b) $k = 0.0045$; (c) $k = 0.0085$; (d) $k = 0.0125$; (e) $k = 0.0165$; (f) $k = 0.0200$.

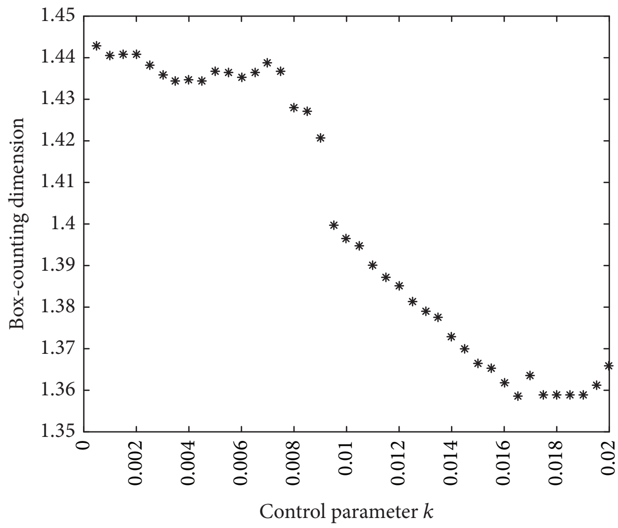

Figure 11: The change of box-counting dimensions of the Julia sets of the controlled system when k is from 0.0005 to 0.02.

In this paper, feedback control method, optimal control method, and gradient control method are taken to control the Julia set of Brusselator model, respectively, and the box-counting dimensions of the Julia set are calculated. In each control method, when the absolute value of control parameter k increases discretely, the box-counting dimension of Julia set decreases and the Julia set contracts to the center gradually. The decrease of box-counting dimension is nearly monotonic, which indicates that the complexity of the Julia set of the system is declined gradually. Thus when the control parameter k is selected, the Julia set of Brusselator model could be controlled. Most importantly, the three control methods have consistency of conclusion and effectiveness.

Competing Interests

The authors declare that they have no competing interests.

Acknowledgments

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References

[1] X. Wang, *Fractal Mechanism of Generalized M-J Sets*, Dalian University of Technology Press, Dalian, China, 2002.

[2] G. Chen and X. Dong, "From chaos to order: methodologies, perspectives and applications," *Methodologies Perspectives & Applications World Scientific*, vol. 31, no. 2, pp. 113–122, 1998.

[3] B. B. Mandelbrot, *Fractal: Form, Chance and Dimension*, W. H. Freeman, San Francisco, Calif, USA, 1977.

[4] B. B. Mandelbrot, *The Fractal Geometry of Nature*, W. H. Freeman and Co., San Francisco, Calif, USA, 1982.

[5] L. Pitulice, D. Craciun, E. Vilaseca et al., "Fractal dimension of the trajectory of a single particle diffusing in crowded media," *Romanian Journal of Physics*, vol. 61, no. 7-8, pp. 1276–1286, 2016.

[6] L. Pitulice, E. Vilaseca, I. Pastor et al., "Monte Carlo simulations of enzymatic reactions in crowded media. Effect of the enzyme-obstacle relative size," *Mathematical Biosciences*, vol. 251, no. 1, pp. 72–82, 2014.

[7] M. Lewis and D. C. Rees, "Fractal surfaces of proteins," *Science*, vol. 230, no. 4730, pp. 1163–1165, 1985.

[8] A. Isvoran, L. Pitulice, C. T. Craescu, and A. Chiriac, "Fractal aspects of calcium binding protein structures," *Chaos, Solitons & Fractals*, vol. 35, no. 5, pp. 960–966, 2008.

[9] P. J. Bentley, "Fractal proteins," *Genetic Programming & Evolvable Machines*, vol. 5, no. 1, pp. 71–101, 2004.

[10] M. Zamir, "Fractal dimensions and multifractility in vascular branching," *Journal of Theoretical Biology*, vol. 212, no. 2, pp. 183–190, 2001.

[11] R. L. Devaney and J. P. Eckmann, "An introduction to chaotic dynamical systems," *Mathematical Gazette*, vol. 74, no. 468, p. 72, 1990.

[12] Z. Huang, "Fractal characteristics of turbulence," *Advances in Mechanics*, vol. 30, no. 4, pp. 581–596, 2000.

[13] M. S. Movahed and E. Hermanis, "Fractal analysis of river flow fluctuations," *Physica A: Statistical Mechanics and Its Applications*, vol. 387, no. 4, pp. 915–932, 2008.

[14] N. Yonaiguchi, Y. Ida, M. Hayakawa, and S. Masuda, "Fractal analysis for VHF electromagnetic noises and the identification of preseismic signature of an earthquake," *Journal of Atmospheric and Solar-Terrestrial Physics*, vol. 69, no. 15, pp. 1825–1832, 2007.

[15] N. Ai, R. Chen, and H. Li, "To the fractal geomorphology," *Geography and Territorial Research*, vol. 15, no. 1, pp. 92–96, 1999.

[16] M. Li, L. Zhu, and H. Long, "Analysis of several problems in the application of fractal in geomorphology," *Earthquake Research*, vol. 25, no. 2, pp. 155–162, 2002.

[17] J. Shu, D. Tan, and J. Wu, "Fractal structure of Chinese stock market," *Journal of Southwest Jiao Tong University*, vol. 38, no. 2, pp. 212–215, 2003.

[18] Y. Lin, "Fractal and its application in security market," *Economic Business*, vol. 8, pp. 44–46, 2001.

[19] Z. Peng, C. Li, and H. Li, "Research on enterprises fractal management model in knowledge economy era ," *Business Research*, vol. 255, no. 10, pp. 33–35, 2002.

[20] W. Zeng, *The Dimension Calculation of Several Special Self-Similar Sets*, Central China Normal University, Wuhan, China, 2007.

[21] K. J. Falconer, *The Geometry of Fractal Sets*, vol. 85 of *Cambridge Tracts in Mathematics*, Cambridge University Press, Cambridge, UK, 1986.

[22] Preiss and David, "Nondifferentiable functions," *Transactions of the American Mathematical Society*, vol. 17, no. 3, pp. 301–325, 1916.

[23] B. B. Mandelbrot, *Les Objets Fractals*, Flammarion, 1984.

[24] X. Gaoan, *Fractal Dimension*, Seismological Press, Beijing, China, 1989.

[25] T. Erneux and E. L. Reiss, "Brussellator isolas," *SIAM Journal on Applied Mathematics*, vol. 43, no. 6, pp. 1240–1246, 1983.

[26] G. Nicolis, "Patterns of spatio-temporal organization in chemical and biochemical kinetics," *American Mathematical Society*, vol. 8, pp. 33–58, 1974.

[27] I. Prigogine and R. Lefever, "Symmetry breaking instabilities in dissipative systems. II," *The Journal of Chemical Physics*, vol. 48, no. 4, pp. 1695–1700, 1968.

[28] F. W. Schneider, "Periodic perturbations of chemical oscillators: experiments," *Annual Review of Physical Chemistry*, vol. 36, no. 36, pp. 347–378, 2003.

[29] R. Lefever, M. Herschkowitz-Kaufman, and J. M. Turner, "The steady-state solutions of the Brusselator model," *Physics Letters A*, vol. 60, no. 3, pp. 389–396, 1977.

[30] P. K. Vani, G. A. Ramanujam, and P. Kaliappan, "Painlevk analysis and particular solutions of a coupled nonlinear reaction diffusion system," *Journal of Physics A: Mathematical and General*, vol. 26, no. 3, pp. L97–L99, 1993.

[31] A. Pickering, "A new truncation in Painleve analysis," *Journal of Physics A: Mathematical and General*, vol. 26, no. 17, pp. 4395–4405, 1998.

[32] I. Ndayirinde and W. Malfliet, "New special solutions of the 'Brusselator' reaction model," *Journal of Physics A: Mathematical and General*, vol. 30, no. 14, pp. 5151–5157, 1997.

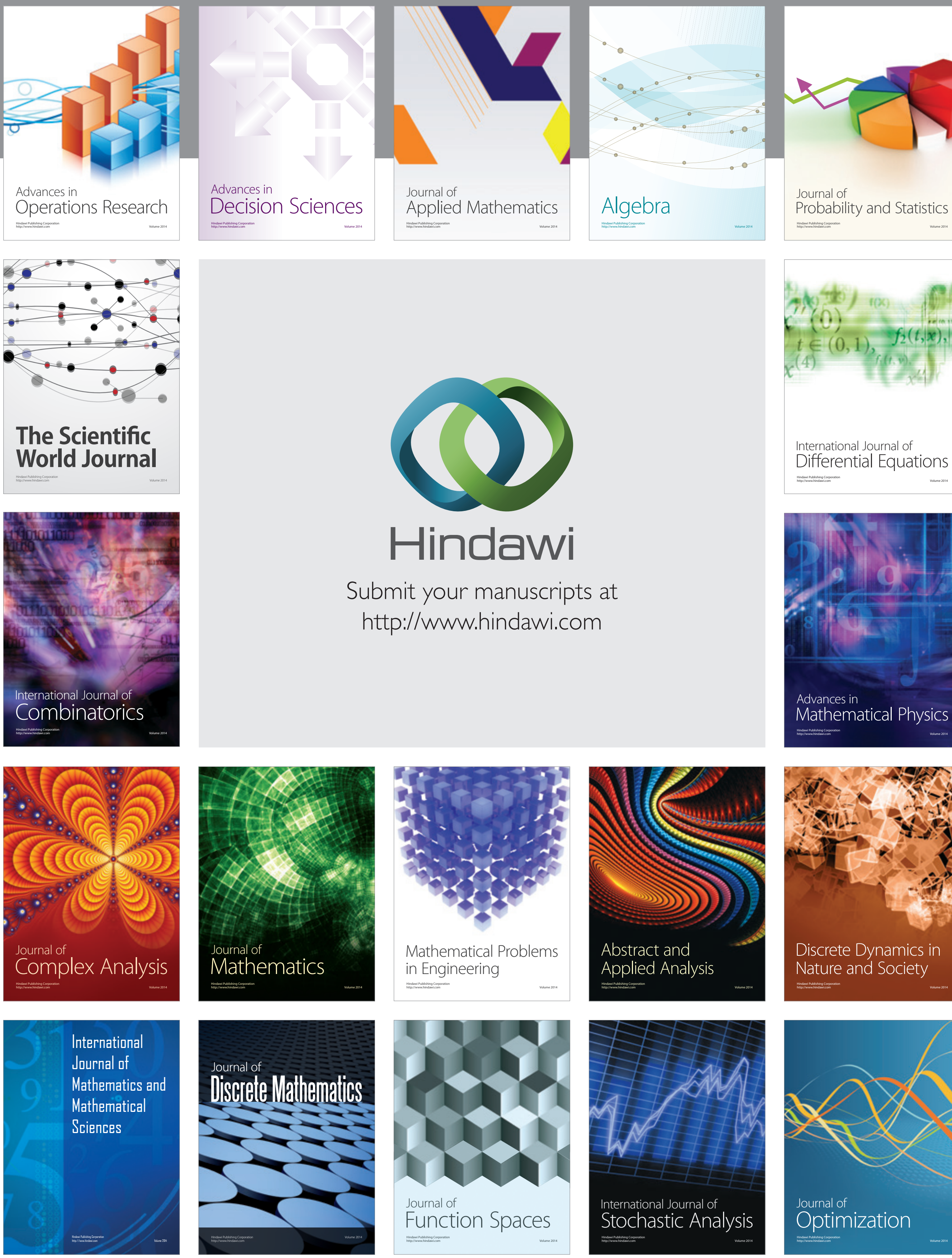
[33] A. L. Larsen, "Weiss approach to a pair of coupled nonlinear reaction-diffusion equations," *Physics Letters. A*, vol. 179, no. 4-5, pp. 284–290, 1993.

[34] Y. Zhang, *Applications and Control of Fractals*, Shandong University, 2008.

[35] K. J. Falconer, "Fractal geometry—mathematical foundations and applications," *Biometrics*, vol. 46, no. 4, p. 499, 1990.

[36] A. Y. Pogromsky, "Introduction to control of oscillations and chaos," *Modern Language Journal*, vol. 23, no. 92, pp. 3–5, 2015.

[37] Y. Qin and X. Zeng, "Qualitative study of the Brussels oscillator equation in Biochemistry," *Chinese Science Bulletin*, vol. 25, no. 8, pp. 337–339, 1980.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