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Research Article

Numerical Simulation of One-Dimensional Fractional Nonsteady Heat Transfer Model Based on the Second Kind Chebyshev Wavelet

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In the current study, a numerical technique for solving one-dimensional fractional nonsteady heat transfer model is presented. We construct the second kind Chebyshev wavelet and then derive the operational matrix of fractional-order integration. The operational matrix of fractional-order integration is utilized to reduce the original problem to a system of linear algebraic equations, and then the numerical solutions obtained by our method are compared with those obtained by CAS wavelet method. Lastly, illustrated examples are included to demonstrate the validity and applicability of the technique.

1. Introduction

Fractional calculus is a branch of mathematics that deals with generalization of the well-known operations of differentiations to arbitrary orders. Many papers on fractional calculus have been published for the real-world applications in science and engineering such as viscoelasticity [1], bioengineering [2], biology [3], and more can be found in [4, 5]. Moreover fractional partial differential equations also are widely used in the areas of signal processing [6], mechanics [7], econometrics [8], fluid dynamics [9], and electromagnetics [10]. As the analytical solutions of fractional partial differential equations are not easy to derive, the scholars are committed to obtain their numerical solutions of these equations.

In recent years, various numerical methods have been proposed for solving fractional diffusion equations, these methods include wavelets methods [11–17], Jacobi, Legendre, and Chebyshev polynomials methods [18–21], spectral methods [22, 23], finite element method [24], wavelet Galerkin method [25], and finite difference methods [26, 27]. In [28], a new matrix method is proposed to solve two-dimensional time-dependent diffusion equations with Dirichlet boundary conditions. In [29], the authors utilize the second kind Chebyshev wavelets to obtain the numerical solutions of the convection diffusion equations. Xie et al. use the Chebyshev operational matrix method to numerically solve one-dimensional fractional convection diffusion equations in [30]. In this paper, we apply the second kind Chebyshev wavelet method to obtain the numerical solutions of one-dimensional fractional nonsteady heat transfer model. The obtained numerical solutions by our method have been compared with those obtained by CAS wavelet method.

The current paper is organized as follows: Section 2 introduces the basic definitions of fractional calculus. In Section 3, the mathematical model of nonsteady heat transfer problem is proposed. Section 4 illustrates the second kind Chebyshev wavelets and their properties. In Section 5, we apply the second kind Chebyshev wavelet for solving fractional nonsteady heat transfer model. Numerical examples are presented to test the proposed method in Section 6. Finally, a conclusion is drawn in Section 7.

2. One-Dimensional Nonsteady Heat Transfer Model

For one infinite plate sample, as shown in Figure 1, the height is δ, the upper surface and the edge are adiabatic,

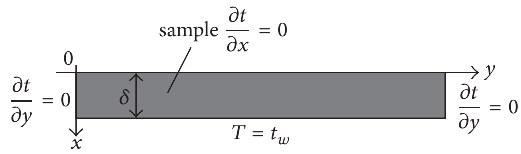

Figure 1: Nonsteady heat transfer model with constant temperature boundary condition.

and the lower surface is contacted with the fluid, which its temperature is t_w. The heat conductivity coefficient of the sample is λ, the density is ρ, and the specific heat capacity is c_p. The initial temperature is t_0, taking the origin of coordinates on the sample adiabatic surfaces, and the nonsteady heat transfer model with the initial-boundary condition can be defined as follows [31]:

$$\begin{aligned} &\frac{\partial t}{\partial \tau} = \frac{\lambda \partial^2 t}{\rho c_p \partial x^2}, \\ &\tau = 0, \\ &t = t_0, \\ &x = 0, \\ &\frac{\partial t}{\partial x} = 0, \\ &x = \sigma, \\ &t = t_w. \end{aligned} \tag{1}$$

Obviously, when the sample density ρ, heat conductivity coefficient λ, specific heat capacity c_p, and thickness δ are known, we can obtain the temperature distribution at any position x and any time τ, which is the nonsteady heat conduction model with constant temperature boundary condition. Based on the above-mentioned model, we give the fractional-order nonsteady heat transfer model of the following form:

$$\frac{\partial T}{\partial t} = \frac{\lambda \partial^\alpha T}{\rho c_p \partial x^\alpha} + g(x,t), \quad 0 \le x \le 1,\ t \ge 0,\ 1 < \alpha \le 2, \tag{2}$$

with the initial condition:

$$T(x,0) = f(x), \quad 0 \le x \le 1, \tag{3}$$

and the boundary conditions:

$$\begin{aligned} T(0,t) &= g_0(t), \\ T(1,t) &= g_1(t), \\ &0 \le t \le 1, \end{aligned} \tag{4}$$

where $g(x,t)$ denotes source term, $f(x)$ is a given function, and $g_0(t)$, $g_1(t)$ are continuous functions with first-order derivative.

3. Preliminaries of the Fractional Calculus

In this section, we give some necessary definitions and mathematical preliminaries on fractional calculus which will be used further in this paper.

Definition 1. The Riemann-Liouville fractional integral operator I^α $(\alpha > 0)$ of a function $f(t)$ is defined as follows [4]:

$$I^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} f(\tau)\, d\tau, \quad \alpha > 0,\ \alpha \in \Re^+. \tag{5}$$

Some properties of the operator I^α are as follows:

$$I^\alpha I^\beta f(t) = I^{\alpha+\beta} f(t), \quad (\alpha > 0,\ \beta > 0), \tag{6}$$

$$I^\alpha t^\gamma = \frac{\Gamma(1+\gamma)}{\Gamma(1+\gamma+\alpha)} t^{\alpha+\gamma}, \quad (\gamma > -1). \tag{7}$$

Definition 2. The Caputo fractional derivative ${}_0D_t^\alpha$ of a function $f(t)$ is defined as follows [4]:

$${}_0D_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t \frac{f^n(\tau)}{(t-\tau)^{n-\alpha+1}}\, d\tau, \quad (n-1 < \alpha \le n,\ n \in N). \tag{8}$$

Some properties of the Caputo fractional derivative are as follows:

$$\begin{aligned} &{}_0D_t^\alpha t^\beta = \frac{\Gamma(1+\beta)}{\Gamma(1+\beta-\alpha)} t^{\beta-\alpha}, \\ &\qquad 0 < \alpha < \beta + 1,\ \beta > -1, \\ &I^\alpha D^\alpha f(t) = f(t) - \sum_{k=0}^{n-1} f^{(k)}(0^+) \frac{t^k}{k!}, \\ &\qquad n-1 < \alpha \le n,\ n \in N. \end{aligned} \tag{9}$$

4. The Second Kind Chebyshev Wavelet and Its Operational Matrix of Fractional Integration

4.1. The Second Kind Chebyshev Wavelet and Its Properties.

The second kind Chebyshev wavelet $\psi_{nm}(t) = \psi(k,n,m,t)$ has four arguments, $n = 1, 2, \dots, 2^{k-1}$, $k \in N^*$. They are defined on the interval $[0,1)$ as follows [19]:

$$\psi_{nm}(t) = \begin{cases} 2^{k/2} \widetilde{U}_m\left(2^k t - 2n + 1\right), & \frac{n-1}{2^{k-1}} \le t < \frac{n}{2^{k-1}}, \\ 0, & \text{o.w.} \end{cases} \tag{10}$$

with

$$\widetilde{U}_m(t) = \sqrt{\frac{2}{\pi}} U_m(t), \quad m = 0, 1, 2, \dots, M-1. \tag{11}$$

Here $U_m(t)$ are the second kind Chebyshev polynomials which are orthogonal with respect to the weight function $w(t) = \sqrt{1-t^2}$ and satisfy the following recursive formula:

$$U_0(t) = 1,$$
$$U_1(t) = 2t, \quad (12)$$
$$U_{m+1}(t) = 2tU_m(t) - U_{m-1}(t), \quad m = 1, 2, \ldots.$$

A function $f(t)$ defined over $[0, 1)$ may be expanded in terms of the second kind Chebyshev wavelet as follows:

$$f(t) \simeq \sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} c_{nm} \psi_{nm}(t) = C^T \Psi(t), \quad (13)$$

where

$$c_{nm} = \left(f(t), \psi_{nm}(t)\right)_{\omega_n} = \int_0^1 \omega_n(t) \psi_{nm}(t) \, \mathrm{d}t, \quad (14)$$

and the weight function $w_n(t) = w(2^k t - 2n + 1)$. Moreover, C and $\Psi(t)$ are $\widehat{m} = (2^{k-1}M)$ column vectors given by

$$C = \left[c_{10}, c_{11}, \ldots, c_{1(M-1)}, c_{20}, c_{21}, \ldots, c_{2(M-1)}, \ldots, c_{2^{k-1}0}, \ldots, c_{2^{k-1}(M-1)}\right]^T,$$
$$\Psi(t) = \left[\psi_{10}, \psi_{11}, \ldots, \psi_{1(M-1)}, \psi_{20}, \psi_{21}, \ldots, \psi_{2(M-1)}, \ldots, \psi_{2^{k-1}0}, \ldots, \psi_{2^{k-1}(M-1)}\right]^T. \quad (15)$$

Take the collocation points as follows:

$$t_i = \frac{2i-1}{2^k M}, \quad i = 1, 2, \ldots, 2^{k-1}M, \; \widehat{m} = 2^{k-1}M. \quad (16)$$

We define the second kind Chebyshev wavelet matrix $\Phi_{\widehat{m}\times\widehat{m}}$ as

$$\Phi_{\widehat{m}\times\widehat{m}} = \left[\Psi\left(\frac{1}{2\widehat{m}}\right), \Psi\left(\frac{3}{2\widehat{m}}\right), \ldots, \Psi\left(\frac{2\widehat{m}-1}{2\widehat{m}}\right)\right]. \quad (17)$$

An arbitrary function of two variables $T(x, t)$ defined over $[0, 1) \times [0, 1)$ may be expanded into Chebyshev wavelets basis as follows:

$$T(x, t) \simeq \sum_{i=1}^{\widehat{m}} \sum_{j=1}^{\widehat{m}} d_{ij} \psi_i(x) \psi_j(t) = \Psi^T(x) D \Psi(t), \quad (18)$$

where $D = [d_{ij}]_{\widehat{m}\times\widehat{m}}$ and $d_{ij} = (\psi_i(x), (T(x, t), \psi_j(t)))$.

The following theorem discusses the convergence and accuracy estimation of the proposed method.

Theorem 3. *Let $f(t)$ be a second-order derivative square-integrable function defined over $[0, 1)$ with bounded second-order derivative, satisfying $|f''(t)| \leq B$ for some constants B; then*

(1) *$f(t)$ can be expanded as an infinite sum of the second kind Chebyshev wavelets and the series converge to $f(t)$ uniformly, that is,*

$$f(t) = \sum_{n=0}^{\infty} \sum_{m\in Z}^{\infty} c_{nm} \psi_{nm}(t), \quad (19)$$

where $c_{nm} = \langle f(t), \psi_{nm}(t) \rangle_{L^2_\omega[0,1)}$.

(2)

$$\sigma_{f,k,M} < \frac{\sqrt{\pi} B}{2^3} \left(\sum_{n=2^{k-1}+1}^{\infty} \frac{1}{n^5} \sum_{m=M}^{\infty} \frac{1}{(m-1)^4} \right)^{1/2}, \quad (20)$$

where $\sigma_{f,k,M} = (\int_0^1 |f(t) - \sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} c_{nm} \psi_{nm}(t)|^2 \omega_n(t) dt)^{1/2}$.

4.2. Operational Matrix of Fractional Integration. On the interval $[0, 1)$, we defined a $\widehat{m}$ – set of block-pulse functions (BPFs) as

$$b_i(t) = \begin{cases} 1, & \dfrac{i}{\widehat{m}} \leq t < \dfrac{i+1}{\widehat{m}}, \\ 0, & \text{o.w.} \end{cases} \quad i = 0, 1, 2, \ldots, \widehat{m} - 1. \quad (21)$$

The functions $\{b_i(t)\}$ are disjoint and orthogonal:

$$b_i(t) b_j(t) = \begin{cases} 0, & i \neq j, \\ b_i(t), & i = j, \end{cases}$$
$$\int_0^1 b_i(s) b_j(s) \, \mathrm{d}s = \begin{cases} 0, & i \neq j, \\ \dfrac{1}{m}, & i = j. \end{cases} \quad (22)$$

Similarly, the second kind Chebyshev wavelet may be expanded into an $\widehat{m}$-term block-pulse functions as

$$\Psi(t) = \Phi_{\widehat{m}\times\widehat{m}} B_{\widehat{m}}(t). \quad (23)$$

Kilicman has given the block-pulse functions operational matrix of fractional integration F^α of following form:

$$\left(I^\alpha B_{\widehat{m}}\right)(t) \approx F^\alpha B_{\widehat{m}}(t), \quad (24)$$

where

$$B_{\widehat{m}}(t) = \left[b_0(t), b_1(t), \ldots, b_{\widehat{m}-1}(t)\right]^T,$$

$$F^\alpha = \frac{1}{\widehat{m}^\alpha} \frac{1}{\Gamma(\alpha+2)} \begin{bmatrix} 1 & \xi_1 & \xi_2 & \xi_3 & \cdots & \xi_{\widehat{m}-1} \\ 0 & 1 & \xi_1 & \xi_2 & \cdots & \xi_{\widehat{m}-2} \\ 0 & 0 & 1 & \xi_1 & \cdots & \xi_{\widehat{m}-3} \\ \vdots & \vdots & \ddots & \ddots & & \vdots \\ 0 & 0 & \cdots & 0 & 1 & \xi_1 \\ 0 & 0 & 0 & \cdots & 0 & 1 \end{bmatrix}. \quad (25)$$

Next, we derive the second kind Chebyshev wavelet operational matrix of fractional integration. Let

$$\left(I^\alpha \Psi\right)(t) = P^\alpha_{\widehat{m}\times\widehat{m}} \Psi(t), \quad (26)$$

where $P^\alpha_{\widehat{m}\times\widehat{m}}$ is called the second kind Chebyshev wavelet operational matrix of fractional integration and it can be given by

$$P^\alpha_{\widehat{m}\times\widehat{m}} = \Phi_{\widehat{m}\times\widehat{m}} F^\alpha \Phi^{-1}_{\widehat{m}\times\widehat{m}}. \quad (27)$$

For More details, see [29].

5. Numerical Implementation

In this section, we use the second kind Chebyshev wavelets method for numerically solving the nonsteady fractional-order heat transfer model with initial-boundary conditions. In order to solve this problem, we assume

$$\frac{\partial^3 T}{\partial t \partial x^2} = \Psi^T(x) D \Psi(t), \tag{28}$$

where $D = (d_{ij})_{\widehat{m}\times\widehat{m}}$ is an unknown matrix which should be determined, and $\Psi(\cdot)$ is the vector defined in (15). By integrating (28) from 0 to t, we obtain

$$\frac{\partial^2 T}{\partial x^2} = \left.\frac{\partial^2 T}{\partial x^2}\right|_{t=0} + \Psi^T(x) D P_{\widehat{m}\times\widehat{m}} \Psi(t). \tag{29}$$

Making use of the initial condition (3) enables one to put (29) in the following form:

$$\frac{\partial^2 T}{\partial x^2} = f''(x) + \Psi^T(x) D P_{\widehat{m}\times\widehat{m}} \Psi(t). \tag{30}$$

Then we have

$$\begin{aligned}\frac{\partial^\alpha T}{\partial x^\alpha} &= I_x^{2-\alpha}\left(\frac{\partial^2 T}{\partial x^2}\right) \\ &= I_x^{2-\alpha}\left(\left.\frac{\partial^2 T}{\partial x^2}\right|_{t=0} + \Psi^T(x) D P_{\widehat{m}\times\widehat{m}} \Psi(t)\right) \\ &= I_x^{2-\alpha} f''(x) + \Psi^T(x)\left(P^{2-\alpha}_{\widehat{m}\times\widehat{m}}\right)^T D P_{\widehat{m}\times\widehat{m}} \Psi(t).\end{aligned} \tag{31}$$

By integrating (30) two times from 0 to x, we obtain

$$\begin{aligned}T(x,t) &= T(0,t) + x \left.\frac{\partial T}{\partial x}\right|_{x=0} + f(x) - f(0) \\ &\quad - x f'(0) + \Psi^T(x)\left(P^{2}_{\widehat{m}\times\widehat{m}}\right)^T D P_{\widehat{m}\times\widehat{m}} \Psi(t),\end{aligned} \tag{32}$$

and, by putting $x = 1$ in (32), we get

$$\begin{aligned}T(x,t) &= T(0,t) + xH(t) + f(x) - f(0) - x f'(0) \\ &\quad + \Psi^T(x)\left(P^{2}_{\widehat{m}\times\widehat{m}}\right)^T D P_{\widehat{m}\times\widehat{m}} \Psi(t),\end{aligned} \tag{33}$$

where

$$\begin{aligned}H(t) &= T(1,t) - T(0,t) + f(0) + f'(0) - f(1) \\ &\quad - \Psi^T(1)\left(P^{2}_{\widehat{m}\times\widehat{m}}\right)^T D P_{\widehat{m}\times\widehat{m}} \Psi(t).\end{aligned} \tag{34}$$

By one time differentiation of (33) with respect to t, we obtain

$$\begin{aligned}\frac{\partial T}{\partial t} &= T'(0,t) + xH'(t) \\ &\quad + \Psi^T(x)\left(P^{2}_{\widehat{m}\times\widehat{m}}\right)^T D P_{\widehat{m}\times\widehat{m}} \Psi(t),\end{aligned} \tag{35}$$

where

$$\begin{aligned}H'(t) &= T'(1,t) - T'(0,t) \\ &\quad - \Psi^T(1)\left(P^{2}_{\widehat{m}\times\widehat{m}}\right)^T D P_{\widehat{m}\times\widehat{m}} \Psi(t).\end{aligned} \tag{36}$$

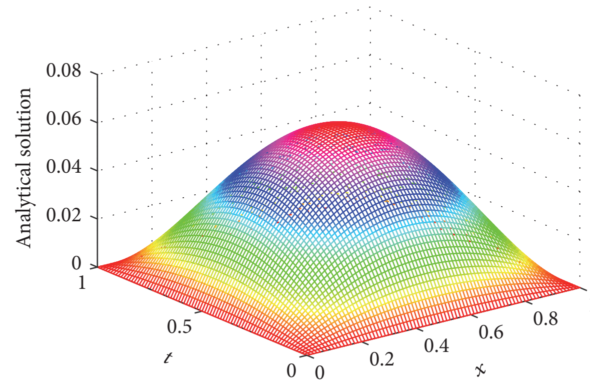

FIGURE 2: Analytical solution.

Now by substituting (31) and (35) into (2) and combining (4) and taking collocation points $x_i = (2i-1)/\widehat{m}, t_j = (2j-1)/\widehat{m}$, $i, j = 1, 2, 3, \ldots, \widehat{m}$, we obtain the following linear system of algebraic equations:

$$\begin{aligned}&T'(0,t_j) + x_i\Big(T'(1,t_j) - T'(0,t_j) \\ &\quad - \Psi^T(1)\left(P^{2}_{\widehat{m}\times\widehat{m}}\right)^T D P_{\widehat{m}\times\widehat{m}} \Psi(t_j)\Big) + \Psi^T(x_i) \\ &\quad \cdot \left(P^{2}_{\widehat{m}\times\widehat{m}}\right)^T D P_{\widehat{m}\times\widehat{m}} \Psi(t_j) = a I_x^{2-\nu} f''(x_i) \\ &\quad + a\Psi^T(x_i)\left(\mathrm{P}^{2-\nu}_{\widehat{m}\times\widehat{m}}\right)^T D P_{\widehat{m}\times\widehat{m}} \Psi(t_j) + g(x_i, t_j), \\ &\qquad\qquad i, j, = 1, 2, 3, \ldots, \widehat{m}.\end{aligned} \tag{37}$$

By solving this system to determine D, we can get the numerical solution of this problem by substituting D into (33).

6. Numerical Simulations

In this section, we use the proposed method to solve the initial-boundary problem of nonsteady heat transfer equations. The following numerical examples are given to show the effectiveness and practicability of the proposed method and the results have been compared with the analytical solution.

Example 4. Consider the following fractional-order nonsteady heat transfer model:

$$\frac{\partial T}{\partial t} = \frac{\lambda \partial^{1.5} T}{\rho \bar{c}_p \partial x^{1.5}} + g(x,t), \quad 0 \le x \le 1, \ t \ge 0, \tag{38}$$

where the parameters $\rho = 7500$, $\bar{c}_p = 0.795$, $\lambda = 800$, and $g(x,t) = x(x-1)(2t-1) - 0.302793571044498 x^{0.5} t(t-1)$ with initial-boundary condition $T(x,0) = T(0,t) = T(1,t) = 0$. The analytical solution of this problem is $T(x,t) = xt(x-1)(t-1)$. The graph of the analytical solution is shown in Figure 2. The graphs of the numerical solutions when $k = M = 3$, $k = M = 4$, $k = M = 5$ are shown in Figures 3–5. From Examples 4, 6, and 7, it can be concluded that the numerical solutions approximate to the analytical solution for

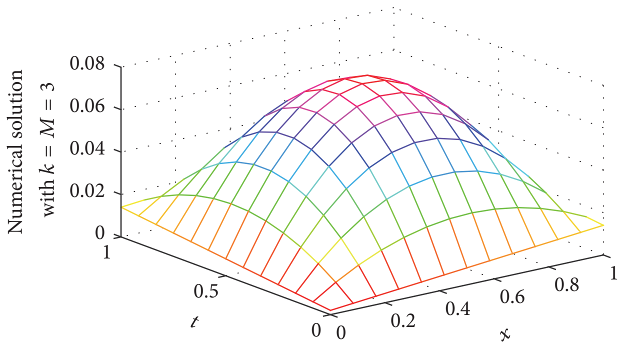

Figure 3: Numerical solution with $k = M = 3$.

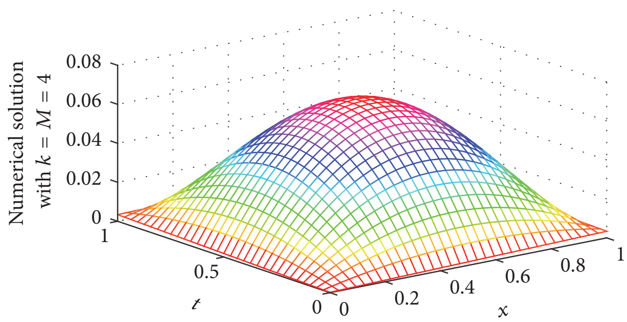

Figure 4: Numerical solution with $k = M = 4$.

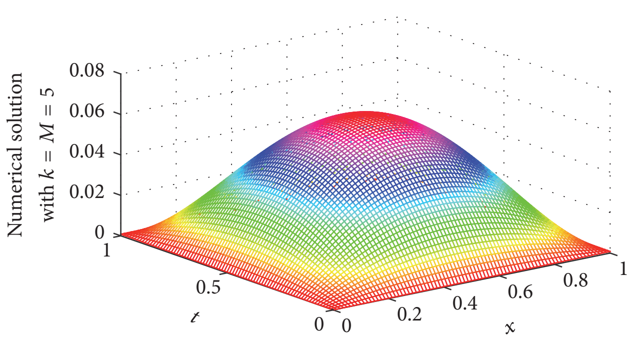

Figure 5: Numerical solution with $k = M = 5$.

a given value k, as M increases, or, for a given value M, as k increases.

Example 5. Consider the following fractional-order nonsteady heat transfer equation:

$$\frac{\partial T}{\partial t} = \frac{\partial^{1.8} T}{\partial x^{1.8}} + 2x^2 t - \frac{2x^{0.2} t^2}{\Gamma(1.2)}, \quad 0 \le x \le 1,\ t \ge 0, \tag{39}$$

with initial-boundary condition $T(x,0) = T(0,t) = 0$, $T(1,t) = t^2$. The analytical solution of this problem is $T(x,t) = x^2 t^2$. When $k = M = 3$, $k = M = 4$, $k = M = 5$, the numerical solutions obtained by our method and those obtained by CAS wavelet method at some values of x, t are listed in Table 1.

Example 6. We consider the following second-order nonsteady heat transfer model:

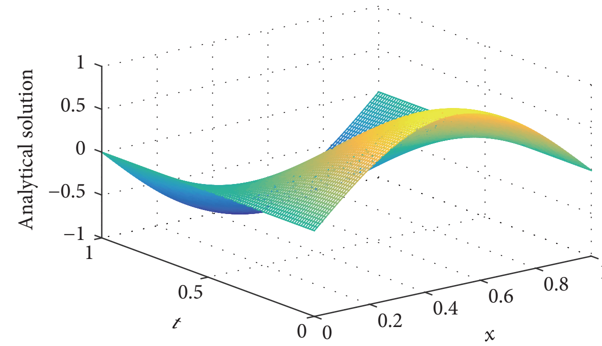

Figure 6: Analytical solution.

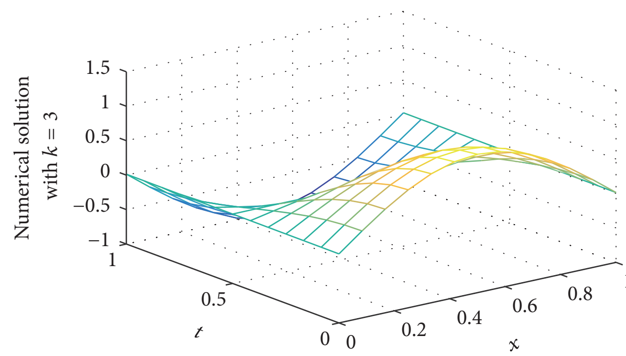

Figure 7: Numerical solution with $k = 3$.

$$\frac{\partial T}{\partial t} = 2\frac{\partial^2 T}{\partial x^2} + 3\sin(x) - \sin(t) - 2\cos(t), \qquad 0 \le x \le 1,\ t > 0, \tag{40}$$

in such a way that $T(x,0) = \sin(x) + 1$, $T(0,t) = \cos(t)$, $T(1,t) = \sin(1) + \cos(t)$. The analytical solution of the system is $T(x,t) = \sin(x) + \cos(t)$. The absolute errors between the numerical and analytical solutions obtained by our method and CAS wavelet method at some values of x, t when $k = 3$, $(M = 3, M = 4, M = 5)$ are shown in Table 2. Table 2 shows that our method has a better approximation than CAS wavelet method.

Example 7. Consider the following second-order nonsteady heat transfer model:

$$\frac{\partial T}{\partial t} = \frac{\lambda \partial^2 T}{\rho c_p \partial x^2} + g(x,t), \quad 0 \le x \le 1,\ t \ge 0, \tag{41}$$

where the parameters $\rho = 7500$, $\bar{c}_p = 0.795$, $\lambda = 1000$, and $g(x,t) = -\pi \sin(\pi x)\sin(\pi t) + 0.167714884696017\pi^2 \sin(\pi x)\cos(\pi t)$, in such a way that $T(x,0) = \sin(\pi x)$, $T(0,t) = T(1,t) = 0$. The analytical solution of this problem is $T(x,t) = \sin(\pi x)\cos(\pi t)$. The graphs of the analytical and numerical solutions, when $M = 3$, $(k = 3, 4, 5)$, are shown in Figures 6–9.

Example 8. Consider (41), with $\alpha = 2, 1.9, 1.8, 1.7$; the numerical solutions when $k = M = 4$ at $t = 0.3, 0.6, 0.95$ are shown in Figure 10. This example is introduced to verify

Table 1: The numerical solutions obtained by our method and those obtained by CAS wavelet method when $k = M = 3$, $k = M = 4$, $k = M = 5$.

t	x	Anal. Sol.	$k = M = 3$		$k = M = 4$		$k = M = 5$	
			Our method	CAS wavelet	Our method	CAS wavelet	Our method	CAS wavelet
	0.3	0.0036000	0.00362673	0.01527126	0.00360257	0.00471281	0.00360019	0.00382719
0.2	0.6	0.0144000	0.01445390	0.03638127	0.01440370	0.01673180	0.01440048	0.01492319
	0.9	0.0324000	0.03248217	0.07371928	0.03240631	0.03631963	0.03240060	0.03273187
	0.3	0.0225000	0.02253176	0.04872121	0.02250487	0.02826189	0.02250046	0.02293819
0.5	0.6	0.0900000	0.09061074	0.12739812	0.09006721	0.09537428	0.09000059	0.09072347
	0.9	0.2025000	0.20257431	0.25873179	0.20250850	0.20736183	0.20250074	0.20301829
	0.3	0.0576000	0.05765362	0.09381981	0.05760489	0.60121872	0.05760062	0.05830218
0.8	0.6	0.2304000	0.23048904	0.28237189	0.23040790	0.23833829	0.23040074	0.23138192
	0.9	0.5184000	0.51851904	0.60381038	0.51841027	0.52478172	0.51840112	0.51953785

Table 2: The absolute errors obtained by our method and CAS wavelet method when $M = 3$, $M = 4$, $M = 5$.

(x, t)	Anal. Sol.	$k = 3, M = 3$		$k = 3, M = 4$		$k = 3, M = 5$	
		Our method	CAS wavelet	Our method	CAS wavelet	Our method	CAS wavelet
(0, 0)	1.00000000	$1.627162e-4$	$2.381923e-2$	$8.719295e-6$	$8.737819e-4$	$2.319280e-6$	$2.648278e-4$
(0.1, 0.1)	1.09483758	$1.738173e-4$	$2.731899e-2$	$5.371912e-6$	$6.271928e-4$	$2.842802e-6$	$3.748217e-4$
(0.2, 0.2)	1.17873590	$2.371827e-4$	$3.759289e-2$	$2.361827e-5$	$3.271929e-3$	$4.830209e-6$	$4.684278e-4$
(0.3, 0.3)	1.25085669	$2.731872e-4$	$4.542767e-2$	$4.731872e-5$	$4.281912e-3$	$5.371982e-6$	$6.472938e-4$
(0.4, 0.4)	1.31047933	$3.261772e-4$	$5.251757e-2$	$5.219289e-5$	$5.381018e-3$	$7.381928e-7$	$7.863982e-4$
(0.5, 0.5)	1.35700810	$8.271985e-5$	$4.378391e-2$	$6.319288e-5$	$6.379843e-3$	$6.238299e-6$	$7.635176e-4$
(0.6, 0.6)	1.38997808	$4.268278e-4$	$8.373456e-3$	$5.738273e-5$	$5.792808e-3$	$8.302930e-6$	$8.368386e-4$
(0.7, 0.7)	1.40905987	$4.791982e-4$	$7.371928e-2$	$7.382093e-5$	$7.728732e-3$	$1.983100e-5$	$9.673817e-4$
(0.8, 0.8)	1.41406280	$5.281928e-4$	$6.367643e-2$	$8.382938e-5$	$8.732763e-3$	$9.381098e-6$	$2.371927e-3$
(0.9, 0.9)	1.40493687	$6.782916e-4$	$7.371892e-2$	$9.381982e-5$	$9.738273e-3$	$2.381983e-5$	$4.281988e-3$
(1.0, 1.0)	1.38177329	$9.381928e-4$	$8.263828e-2$	$9.983787e-5$	$9.425146e-3$	$3.313910e-5$	$8.871999e-4$

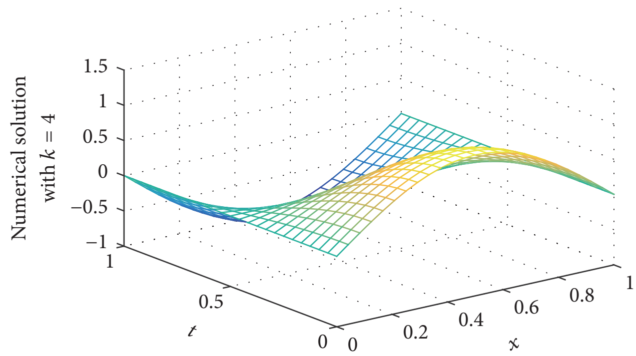

Figure 8: Numerical solution with $k = 4$.

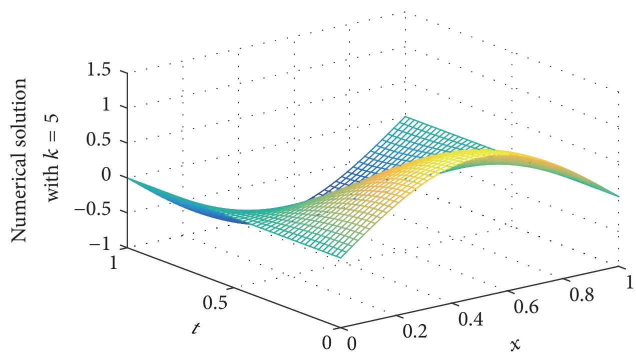

Figure 9: Numerical solution with $k = 5$.

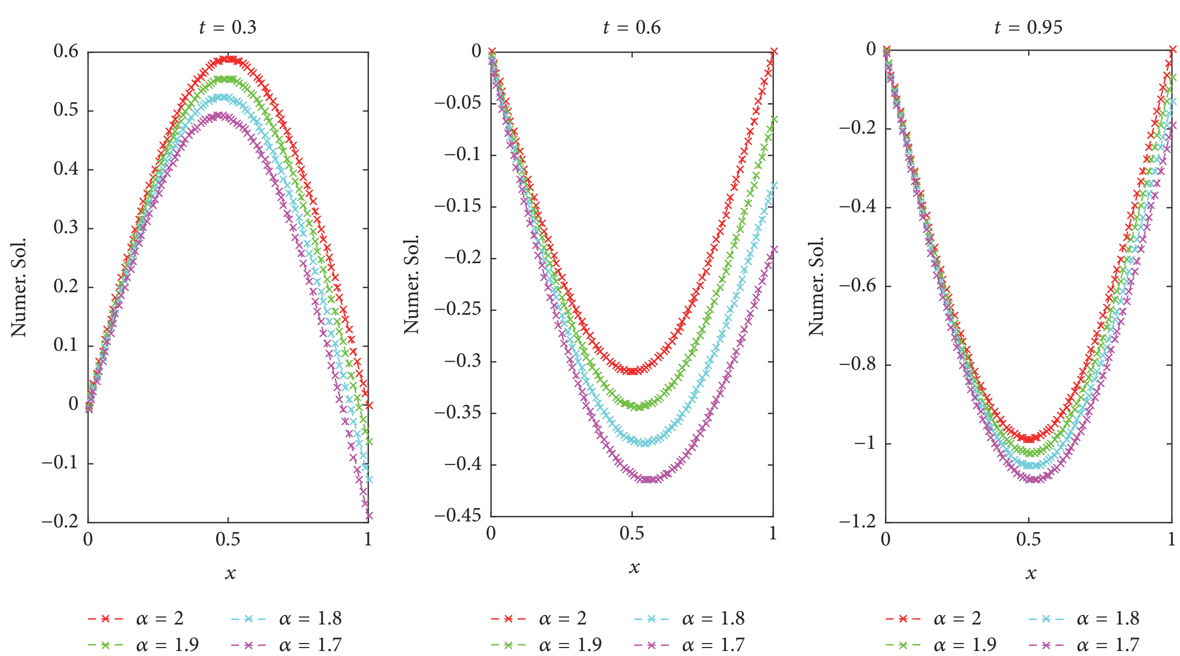

Figure 10: The numerical solutions with $\alpha = 2, 1.9, 1.8, 1.7$ when $k = M = 4$.

the robustness of the proposed method; when the fractional order gradually approaches to 2, the numerical solutions are in agreement with the analytical solution.

7. Conclusions

This paper presents a numerical technique for approximating solutions of one-dimensional fractional nonsteady heat transfer model by combining the second kind Chebyshev wavelet with its operational matrix of fractional-order integration. In the proposed method, a small number of grid points guarantee the necessary accuracy. The main advantage of wavelet method for solving the kinds of equations is that, after dispersing the coefficients, matrix of algebraic equations is sparse. The solution is convenient, even though the size of increment may be large. Several examples are given to demonstrate the powerfulness of the proposed method.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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References

[1] R. L. Bagley and P. J. Torvik, "A theoretical basis for the application of fractional calilus to visoelasticity," *Journal of Rheology*, vol. 27, no. 3, pp. 201–210, 1983.

[2] R. L. Magin, "Fractional calculus in bioengineering," *Critical Reviews in Biomedical Engineering*, vol. 32, no. 1, pp. 1–377, 2004.

[3] D. A. Robinson, "The use of control systems analysis in the neurophysiology of eye movements.," *Annual Review of Neuroscience*, vol. 4, pp. 463–503, 1981.

[4] I. Podlubny, *Fractional Differential Equations*, vol. 198 of *Mathematics in Science and Engineering*, Academic Press, San Diego, Calif, USA, 1999.

[5] K. Diethelm, *The Analysis of Fractional Differential Equations*, vol. 2004 of *Lecture Notes in Mathematics*, Springer-Verlag, Berlin, Germany, 2010.

[6] Y.-M. Chen, Y.-Q. Wei, D.-Y. Liu, D. Boutat, and X.-K. Chen, "Variable-order fractional numerical differentiation for noisy signals by wavelet denoising," *Journal of Computational Physics*, vol. 311, pp. 338–347, 2016.

[7] A. Guerrero and M. A. Moreles, "On the numerical solution of the eigenvalue problem in fractional quantum mechanics," *Communications in Nonlinear Science and Numerical Simulation*, vol. 20, no. 2, pp. 604–613, 2015.

[8] R. T. Baillie, "Long memory processes and fractional integration in econometrics," *Journal of Econometrics*, vol. 73, no. 1, pp. 5–59, 1996.

[9] V. V. Kulish and J. L. Lage, "Application of fractional calculus to fluid mechanics," *Journal of Fluids Engineering*, vol. 124, no. 3, pp. 803–806, 2002.

[10] V. E. Tarasov, "Fractional integro-differential equations for electromagnetic waves in dielectric media," *Theoretical and Mathematical Physics*, vol. 158, no. 3, pp. 355–359, 2009.

[11] T. Liu, "A wavelet multiscale method for the inverse problem of a nonlinear convection–diffusion equation," *Journal of Computational and Applied Mathematics*, vol. 330, pp. 165–176, 2018.

[12] M. H. Heydari, M. R. Hooshmandasl, and F. M. M. Ghaini, "Wavelets method for the time fractional diffusion-wave equation," *Physics Letters A*, vol. 379, no. 3, pp. 71–76, 2015.

[13] Y. Chen, Y. Wu, Y. Cui, Z. Wang, and D. Jin, "Wavelet method for a class of fractional convection-diffusion equation with variable coefficients," *Journal of Computational Science*, vol. 1, no. 3, pp. 146–149, 2010.

[14] G. Hariharan and K. Kannan, "Review of wavelet methods for the solution of reaction-diffusion problems in science and engineering," *Applied Mathematical Modelling*, vol. 38, no. 3, pp. 799–813, 2014.

[15] M. Yi, Y. Ma, and L. Wang, "An efficient method based on the second kind Chebyshev wavelets for solving variable-order fractional convection diffusion equations," *International Journal of Computer Mathematics*, pp. 1–19, 2017.

[16] F. Zhou and X. Xu, "The third kind Chebyshev wavelets collocation method for solving the time-fractional convection diffusion equations with variable coefficients," *Applied Mathematics and Computation*, vol. 280, pp. 11–29, 2016.

[17] M. H. Heydari, M. R. Hooshmandasl, C. Cattani, and G. Hariharan, "An optimization wavelet method for multi variable-order fractional differential equations," *Fundamenta Informaticae*, vol. 151, no. 1-4, pp. 255–273, 2017.

[18] M. Behroozifar and A. Sazmand, "An approximate solution based on Jacobi polynomials for time-fractional convection-diffusion equation," *Applied Mathematics and Computation*, vol. 296, pp. 1–17, 2017.

[19] N. H. Sweilam, A. M. Nagy, and A. A. El-Sayed, "Second kind shifted Chebyshev polynomials for solving space fractional order diffusion equation," *Chaos, Solitons and Fractals*, vol. 73, pp. 141–147, 2015.

[20] N. H. Sweilam, A. M. Nagy, and A. A. El-Sayed, "On the numerical solution of space fractional order diffusion equation via shifted Chebyshev polynomials of the third kind," *Journal of King Saud University - Science*, vol. 28, no. 1, pp. 41–47, 2016.

[21] S. Abbasbandy, S. Kazem, M. S. Alhuthali, and H. H. Alsulami, "Application of the operational matrix of fractional-order Legendre functions for solving the time-fractional convection-diffusion equation," *Applied Mathematics and Computation*, vol. 266, pp. 31–40, 2015.

[22] Y. Yang, Y. Chen, Y. Huang, and H. Wei, "Spectral collocation method for the time-fractional diffusion-wave equation and convergence analysis," *Computers and Mathematics with Applications*, vol. 73, no. 6, pp. 1218–1232, 2017.

[23] E. Pindza and K. M. Owolabi, "Fourier spectral method for higher order space fractional reaction-diffusion equations," *Communications in Nonlinear Science and Numerical Simulation*, vol. 40, pp. 112–128, 2016.

[24] F. Zeng and C. Li, "A new Crank-Nicolson finite element method for the time-fractional subdiffusion equation," *Applied Numerical Mathematics*, vol. 121, pp. 82–95, 2017.

[25] M. H. Heydari, M. R. Hooshmandasl, G. B. Loghmani, and C. Cattani, "Wavelets Galerkin method for solving stochastic heat equation," *International Journal of Computer Mathematics*, vol. 93, no. 9, pp. 1579–1596, 2016.

[26] K. Burrage, A. Cardone, R. D'Ambrosio, and B. Paternoster, "Numerical solution of time fractional diffusion systems," *Applied Numerical Mathematics*, vol. 116, pp. 82–94, 2017.

[27] V. G. Pimenov, A. S. Hendy, and R. H. De Staelen, "On a class of non-linear delay distributed order fractional diffusion equations," *Journal of Computational and Applied Mathematics*, vol. 318, pp. 433–443, 2017.

[28] B. Zogheib and E. Tohidi, "A new matrix method for solving two-dimensional time-dependent diffusion equations with Dirichlet boundary conditions," *Applied Mathematics and Computation*, vol. 291, pp. 1–13, 2016.

[29] F. Zhou and X. Xu, "Numerical solution of the convection diffusion equations by the second kind Chebyshev wavelets," *Applied Mathematics and Computation*, vol. 247, pp. 353–367, 2014.

[30] J. Xie, Q. Huang, and X. Yang, "Numerical solution of the one-dimensional fractional convection diffusion equations based on Chebyshev operational matrix," *SpringerPlus*, vol. 5, no. 1, article no. 1149, 2016.

[31] Q. Chen, Z. Dong, Y. Ma et al., "Test thermo-physical properties of solid material based on one dimensional unsteady heat transfer model in constant temperature boundary condition," *Journal of Central South University*, vol. 46, no. 12, pp. 4686–4692, 2015.