Hindawi
Discrete Dynamics in Nature and Society
Volume 2017, Article ID 4035068, 8 pages
https://doi.org/10.1155/2017/4035068

Research Article

An Enhanced Dynamic User Optimal Passenger Flow Assignment Model for Metro Networks

Hanchuan Pan, Zhigang Liu, and Hua Hu

School of Urban Rail Transportation, Shanghai University of Engineering Science, 333 Long Teng Road, Shanghai 201620, China

Correspondence should be addressed to Hanchuan Pan; panano@sues.edu.cn

Received 7 March 2017; Accepted 26 April 2017; Published 7 June 2017

Academic Editor: Paolo Renna

By considering the difference between a car driver's route choice behavior on the road and a passenger's route choice behavior in urban rail transit (URT), this paper proposes an enhanced Dynamic User Optimal (DUO) passenger flow assignment model for metro networks. To capture realistic URT phenomena, the model has integrated the train operation disturbance constraint. Real passenger and train data are used to verify the proposed model and algorithm. The results indicate that the DUO-based model is more suitable for describing passenger route choice behavior under uncertain conditions compared to a static model. Moreover, this paper found that passengers under oversaturated conditions are more sensitive to train operation disturbances compared to undersaturated passengers.

1. Introduction

Although the car driver's dynamic route choice behavior has been studied for several decades, little attention has been paid to passenger's route choice behavior in urban rail transit [1]. At present, most previous studies do not consider dynamic features when passengers make decisions in urban rail transit (URT). The limitations of these models include the inability of modeling passengers' departure times and real-time congestion.

The earliest known work related to dynamic features is the bottleneck model proposed by Vickrey [2]. After several decades of developments, many models have been proposed. For example, Yang and Jiang [3] proposed an enhanced route choice model based on cumulative prospect theory to describe dynamic passenger choice behavior. By considering queue spillback, Zhou et al. [4] developed a model to describe dynamic passenger assignment behavior based on AFC Data. However, these models are suitable only for simple networks. Therefore, researchers began to search for new theories to describe passenger route choice behavior. Among them, the optimal dynamic models received considerable attention. Optimal dynamic route choice principles can be classified into two categories: the Dynamic User Optimal (DUO) principle [5–7] and the Dynamic System Optimal (DSO) principle [8–10]. Obviously, managers prefer traffic assignment, which is consistent with the DSO principle. However, DUO can describe the actual passenger distribution in URT more accurately.

DUO is used to represent equilibrium state in a time-varying URT network. This equilibrium can be described as follows: for each instant, travel costs of all used routes between each origin-destination (OD) pair are equal and are less than the unused routes. Dafermos and Sparrow [11] found that such equilibrium ensures that "for each instant, none of the travelers could decrease his/her travel cost by unilaterally changing his/her route."

To develop mathematical models that describe DUO principle, several previous studies had been conducted by researchers. Since the 1990s, considerable attention has been paid to variational inequality (VI) for analyzing DTA problems based on DUO. Friesz et al. [12] first proposed variational inequality to describe DUO conditions. However, this model is very difficult to solve. After several years of research and based on monotonic path cost function, Friesz and Mookherjee [13] proposed a projection algorithm that converges to a DUO solution.

In the aforementioned studies, it is obvious to note that DUO principle is widely used to develop models of road traffic assignment. However, these models cannot be used for describing passenger route choice behavior directly because there are some important differences between them.

(1) Difference of Research Object. In the field of road traffic assignment, we pay attention to a driver's route choice behavior, while, in rail transit, we focus on the choice behavior of passengers. Drivers change their route in an intersection, while passengers change their route in a transfer station.

(2) Difference between Traffic Flow and Passenger Flow. For road traffic, vehicle speed is closely related to the density of traffic flow, while train operation is predetermined by the timetable and experiences little impact from passenger density.

(3) Difference of Path Impedance. In the field of road traffic, Path impedance usually consists of travel time, road condition, and mileage. Meanwhile, for URT, path impedance mainly consists of travel time, comfort, transfer time, train headway, and so forth.

In fact, with the continuous increase in passenger volume in URT, the frequency of train operation disturbance also increases [14–16]. To retain the advantage of DUO models, passengers' reaction to the disturbance should be considered. In this paper, the DUO problem is formulated with a disturbance constraint to capture the realistic URT phenomena. Furthermore, we proposed a feasible algorithm based on Frank-Wolf's one to solve this model.

The remainder of this paper is organized as follows: the formulation of network and impedance function is presented in Section 2. Section 3 proposed a passenger flow assignment model and algorithm. A numerical experiment is given in Section 4. Finally, conclusions are provided in Section 5.

2. Model Formulation

2.1. Network Representation. This study considers an urban metro network that contains several stations. Nodes (set N) and links (set L) are extracted from the practical metro network. The vector (i, j) represents the directed links from station i to station j. Three different lines cross with each other to illustrate a simple network, as shown in Figure 1. Each transfer station consists of several ordinary stations depending on the number of linked lines. For example, transfer station A is composed of node 302 (located on line 3) and node 101 (located on line 1). Then, direct links (302, 101) and (101, 302) represent transfer activity between these two nodes.

2.2. Assumptions. To better model dynamic passenger route choice behavior in URT, three basic assumptions are presented as follows:

(1) Passengers have perfect knowledge of the travel conditions. That is, passengers can change their original path using real-time information about the travel scenario.

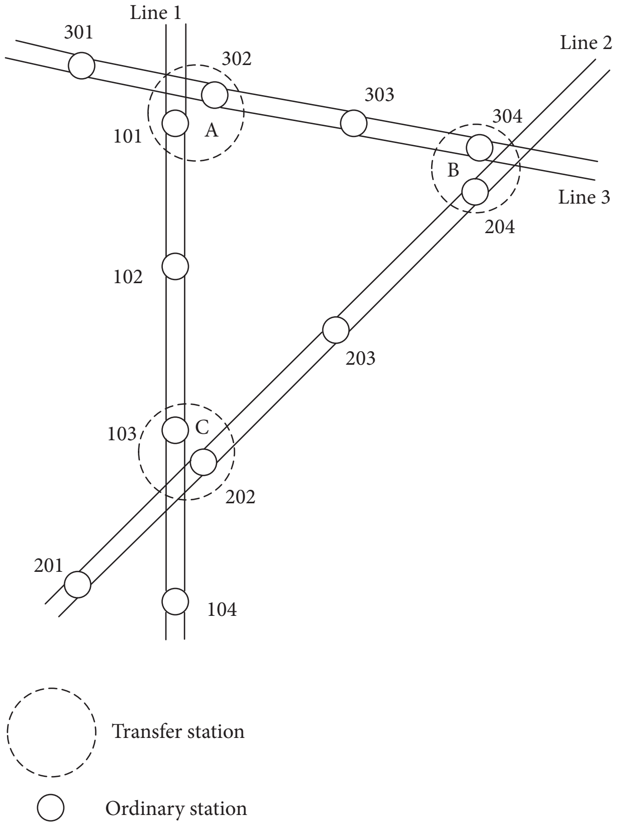


FIGURE 1: The representation of a simple network.

(2) Passengers will not cancel their trip due to the impact of train operation disturbance. That is, no passengers will leave the subway system until they arrive at the destination station.

(3) Transit lines do not have capacity limits; that is, all passengers who are waiting on the platform can board the train.

2.3. Impedance Function. Traditional static passenger assignment models (either deterministic or stochastic) assume that the assigned passenger flow exists in every link of the path between the OD pair. The impedance of the path is generally constant and will not change with passenger flow distribution. Obviously, this assumption is not accurate enough. Therefore, in this paper, we focus on the impedance of links instead of the whole path. Moreover, the impedance is changed with the number of passengers who are traveling through the corresponding link.

The main component of path impedance is travel time. Therefore, in this paper, the impedance of each path contains four parts: (1) travel time on the train; (2) transfer time; (3) waiting time; and (4) overload delays

(1) Travel Time on the Train. Travel time on the train contains train section running time and dwell time, which are predetermined by the timetable. Obviously, the timetable

is not affected by the volume of passenger flows. Therefore, travel time on the train can be calculated as follows:

$$T_{\text{train}} = \sum_{i,j\in R_k} t_{ij} + \sum_{i\in R_k} t_i, \tag{1}$$

where t_{ij} represents train running time from node i to node j and t_i represents train dwell time. R_k represents a path k, which contains a number of nodes.

(2) Transfer Time. Transfer activity occurs when passengers need to change from one line to another. With the development of URT in China, more than fifty percent of passengers need to transfer during their trips. Thus, transfer activity in path finding turns out to be an important component of passenger travel.

In general, transfer time is usually associated with the passenger walking speed and the length of the transfer channel. Therefore, transfer time can be calculated as follows:

$$T_{\text{trans}} = \frac{l_j^i}{v_a}, \tag{2}$$

where l_j^i represents the transfer length from node i to node j and v_a denotes the average walking speed. A penalty needs to be considered for transfer activity to indicate extra transfer cost. Therefore, transfer time can be denoted as follows: $\alpha \cdot T_{\text{trans}}$, where α represents the penalty coefficient.

(3) Waiting Time. Passenger waiting time is related to the train headway and the time that passengers arrive at the platform. According to a previous study [17], the randomness of a passenger's departure time is greatly reduced when the train headway is greater than 12 minutes. They tend to arrange their travel plans according to the train departure time. In contrast, the travel is often random when the interval of the train is less than 12 minutes. That is, as shown in Figure 2, passenger waiting time tends to be a constant when the train interval is long (more than 12 minutes), and passenger waiting time tends to be half of the train headway when the train interval is short (less than 12 minutes). Therefore, the waiting time can be calculated as follows:

$$T_{\text{wait}} = \frac{I_q}{2}, \tag{3}$$

where I_q represents train headway of line q.

(4) Overload Delays. As mentioned above, the average passenger waiting time can be calculated as half of train headway. However, under oversaturated conditions, not all passengers waiting on the platform are able to board the train, and some passengers are left behind. For example, passengers crowded on the platform may cause the platform screen door not to close. Therefore, overload delays are complicated and difficult to calculate. BPR [18] formulation is applied in this study to estimate overload delays as follows:

$$c_a'(t) = \omega \cdot \left(\frac{x_a(t)}{c}\right)^{\rho}, \tag{4}$$

where $c_a'(t)$ is the overload delays for link a at time t and c represents train capacity. ω and ρ are parameters, which can be obtained by actual survey.

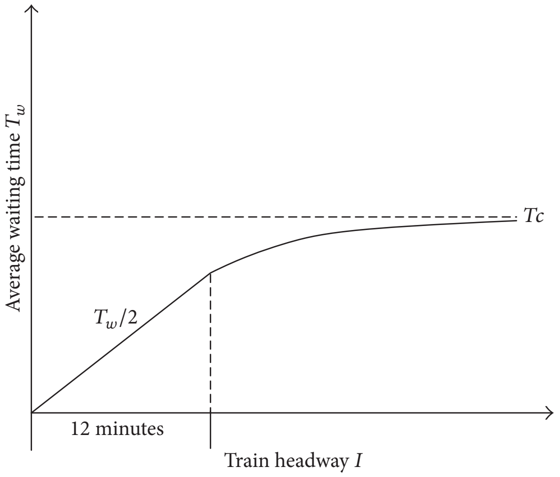


FIGURE 2: Relationships between the average waiting time and the headway.

3. Assignment Model and Algorithm

3.1. Model of Passenger Flow Assignment Based on DUO. The DUO assignment model is well known as a general model that consistently unifies dynamic features and a Wardrop equilibrium. According to the definition of DUO, for each node and instant, impedances of all used paths between each OD pair are equal, which equal the minimum instantaneous impedance. Meanwhile, the instantaneous impedances of all unused paths are more than the used ones.

To capture the disturbance of train operation in an actual network, we developed a DUO-based model. In this paper, we divide the study period into several equal time intervals denoted by t. According to assumption (2), passengers will not cancel the trip when they enter the subway system. Therefore, all inflows into any node must be equal to all outflows from this node, and the flow conservation can be formulated as follows:

$$\sum_{a\in I_l} q_a^n(t) = \sum_{a\in O_l} p_a^s(t), \quad \forall l \neq s, \tag{5}$$

where I_l represents the set of inflows into node l and O_l represents the set of outflows from node l. If the node is destination or origin stations, the above equation should be extended as below:

$$\lambda_l f_l^o(t) + \sum_{a\in I_l} q_a^n(t) = \mu_l f_l^d(t) + \sum_{a\in O_l} p_a^s(t), \quad \forall l \neq s, \tag{6}$$

where λ_l and μ_l are binary parameters. If node l is an origin station, $\lambda_l = 1$; otherwise $\lambda_l = 0$. Similarly, if node l is a destination station, $\mu_l = 1$; otherwise, $\mu_l = 0$.

The next task is to derive a mathematically tractable equation for temporarily calculating the passengers traveling through each directed link at any time. We can obtain the equation by flow conservation, given as follows:

$$x_a(t) = \sum_s x_a^s(t), \quad \forall a, t, \tag{7}$$

where $x_a^s(t)$ denotes the number of passengers who are traveling through direct link a to destination s. $x_a(t)$ denotes the cumulative number of passengers who are traveling through direct link a.

Inflow into link a during interval t can be calculated as

$$p_a(t) = \sum_s p_a^s(t), \quad \forall a, t. \tag{8}$$

Similarly, outflow from link a during interval t can be obtained by

$$q_a(t) = \sum_s q_a^s(t), \quad \forall a, t. \tag{9}$$

Moreover, nonnegative constraints must be satisfied:

$$\begin{aligned} & p_a(t) \geq 0, \\ & q_a(t) \geq 0, \\ & x_a(t) \geq 0, \\ & \qquad \forall a, t \\ & p_a^s(t) \geq 0, \\ & q_a^s(t) \geq 0, \\ & x_a^s(t) \geq 0, \\ & \qquad \forall a, t, s. \end{aligned} \tag{10}$$

In a URT practical network, passengers can board or alight from the train only at the stations (nodes). That is, no passengers could leave the system in direct links. When time is discretized, x_a^s can be calculated as follows:

$$x_a^s(t) = x_a^s(t-1) + p_a^s(t-1) - q_a^s(t-1), \quad \forall a, t, s. \tag{11}$$

In this paper, passengers traveling through the direct links obey the First-In-First-Out (FIFO) principle [19]. In other words, passengers who enter the direct link cannot overtake each other. We denote the impedance of direct link a at time t as $\tau_a(t)$. If passengers enter the direct link at time t, then they will leave the link at time $t + \tau_a(t)$. Similarly, if passengers enter the direct link at time $t + \Delta t$, then they will leave the link at time $t + \Delta t + \tau_a(t + \Delta t)$. Therefore, FIFO principle can be represented as follows:

$$t + \Delta t + \tau_a(t + \Delta t) \geq t + \tau_a(t), \quad \forall a, t. \tag{12}$$

Disturbance of the train operation can affect passenger's predetermined travel plan. For example, it is possible that passenger waiting time is greater than they estimated before departure due to a disturbance. In our model, we add the constraint that limits passengers entering the direct links when they are affected by train disturbance:

$$\begin{aligned} & q_a(t) = 0, \\ & i_a(t) \in S, \\ & \qquad \forall a, t, \end{aligned} \tag{13}$$

where S represents the set of stations influenced by disturbance propagation.

According to the DUO definition by a previous study [20], passengers dynamically change their route in each travel node. It is obvious that a passenger's decision making is dependent on the instantaneous impedance in the network. As we know, the impedance of the direct link is related to the number of passengers who are traveling through the link. Therefore, the objective function of the proposed model is given by

$$\min Z = \int_0^T \sum_a \int_0^{x_a(t)} c_a(w)\, dw\, dt. \tag{14}$$

3.2. Solution. Overall, due to the complexity of the URT network, the proposed passenger flow assignment model based on DUO is difficult to solve by traditional commercial solvers. The Frank-Wolf algorithm, which was first proposed by Frank and Wolfe, is worth considering [21]. This algorithm has been used to address traffic problems since the 1970s, and it is still considered an effective way to solve DUO problems today. The proposed algorithm in this paper is similar to previous ones, but the major differences are their definition of impedance and disturbance of train operation. The Frank-Wolf based algorithm for the DUO problem in URT is outlined as follows.

Step 1 (initialization). Create nodes and direct links from the practical URT network and discrete-time setting, and then let $t = 0$, $x_a^s(0) = 0$, and $q_a^s(0) = 0$ $\forall a, s$.

Step 2. Determine the impedance of each of the links based on the passengers who are traveling through the corresponding link; if there is disturbance of train operation, then modify the corresponding $c_a(t)$.

Step 3. Use an all-or-nothing algorithm to calculate inflow rates for each link in the network, which is a feasible solution.

Step 4. Use (11) to calculate the number of passengers who are traveling through directed link at time t, and use (4) to calculate the impedance of each link. Then, use (5) to calculate outflow of each link at time t. According to the conservation principle, passenger flows are assigned in the network based on an all-or-nothing algorithm.

Step 5. Use equation $q_a^n(t) = 0$ to modify the influence of train disturbance. Then, calculate the instantaneous impedance for each link as follows:

$$c_a^n(t) = c_a\left[x_a^n(t), p_a^n(t), q_a^n(t)\right] + \varepsilon_m^s(t) - \varepsilon_l^s(t), \tag{15}$$

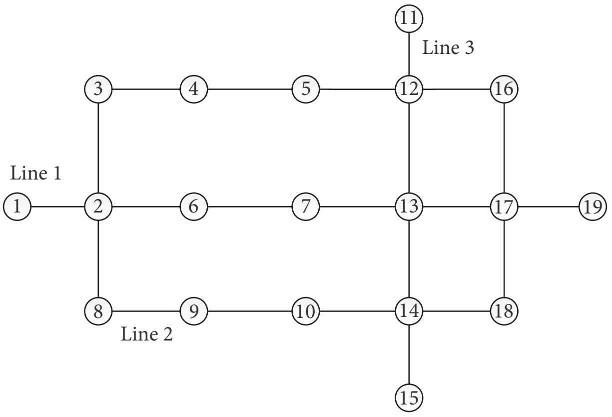


Figure 3: A practical tested rail network.

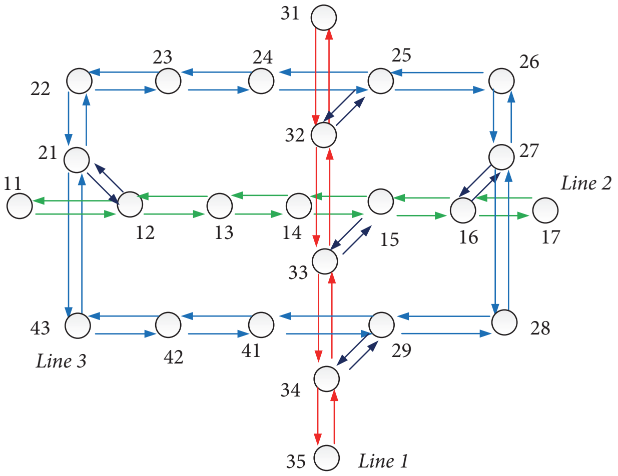


Figure 4: A grid network for illustrating solution process.

where $\varepsilon_m^s(t)$ and $\varepsilon_l^s(t)$ denote the minimum instantaneous impedance from nodes l to s and nodes m to s, respectively.

Step 6. According to the conservation principle, the inflow to be assigned at each node is $\int_t^{t+c_a(t)} q_a^s(w)dw$. Use an all-or-nothing algorithm to calculate the initial feasible assigned inflow (denoted as $\overline{p_a^{s^n}}$) for each link in the network. Then, calculate the optimal convergent step length as follows:

$$Z = \sum_a \int_0^{p_a^n(t)+\alpha\cdot[p_a^n(t)-p_a^n(t)]} c_a(w)\,dw + \sum_a \sum_s \left\{p_a^{s^n}(t) + \alpha\left[\overline{p_a^{s^n}} - p_a^{s^n}\right]\right\}\left[\varepsilon_m^s(t) - \varepsilon_l^s(t)\right]. \tag{16}$$

Step 7. Update the assigned inflow for each link as follows:

$$p_a^{s^{n+1}}(t) = p_a^{s^n}(t) + \alpha^n\left[\overline{p_a^{s^n}}(t) - p_a^{s^n}(t)\right]. \tag{17}$$

Step 8. Convergence checking: if $\sqrt{\sum_a (\overline{c}_a^n(t) - \overline{c}_a^{n-1}(t))^2} / \sum_a \overline{c}_a^n(t) \leq \varepsilon$, then stop the algorithm; otherwise, set $n = n + 1$, and go to Step 5.

The above Frank-Wolfe solution can be directly used to solve the proposed passenger flow assignment model. Note that ε is a predetermined precision parameter.

4. Case Study

4.1. Experiments in a Practical Network. In this section, a numerical example is presented to illustrate the performance of the proposed model and algorithm in a tested network as shown in Figure 3.

According to Section 2, we divided each transfer station into two individual platform nodes locating on different lines. Then, the grid network is extracted from the practical tested rail network as shown in Figure 4. This network consists of 24 nodes and 44 direct links. Note that direct links between these two nodes represent transfer activity. According to the above section, we consider a penalty for each transfer activity to indicate extra cost.

The travel times of each link are randomly generated as shown in Table 1. The value of parameters ω, ρ, α, and c are set to 2, 1.5, 1.5, and 200, respectively. Assume that train disturbance occurs at direct link (12, 14) at time interval 7, which causes a 15-minute train delay. To obtain the solution for the given case, the proposed model and Frank-Wolfe algorithm are implemented in a personal computer with 4.00 GB Memory and an Intel Core i5 processor.

All the networks were empty initially. For descriptive convenience, nodes 11 and 31 are selected as the origin and destination station, respectively. The study period is set to 50 minutes, which is from 7:00 to 7:50. The calculation results are shown in Table 2.

4.2. Results Analysis. As shown in the computational results, two feasible paths are selected to assign passenger flow between nodes 11 (origin) and 31 (destination). One path is 11→12→21→22→23→24→25→32→31 (path one), and the other is 11→12→13→14→15→33→32→31 (path two). Figure 5 lists these two feasible paths between the given OD pair. The transfer activity stations are represented by solid circles. Obviously, the impedance value of path 1 is greater compared to path 2.

For intervals 1–3, all passengers select path 2. This indicates that no congestion and train operation disturbance occurred during the beginning intervals. When congestion exists in path 1, it is very easy to see that part of the passengers select path 2. Moreover, in DUO, passengers make decisions at each reached node dynamically. This implies that passengers may change their original path using real-time information at any nodes.

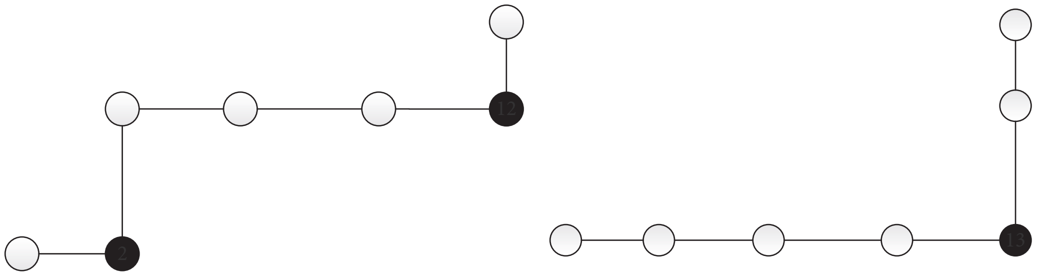
According to the assumption mentioned above, train disturbance occurs at direct link (12, 14) at the beginning of interval 7. That is, the impedance of path 2 is suddenly increased. The selected result above shows that 110 passengers are traveling through directed link (13, 12) at time 7. This indicates that some of the passengers who initially choose path 1, which is influenced by train operation disturbance, suddenly choose path 2 instead of path 1.

Table 1: The travel time of each link without overload delays (min).

Directed link	Travel time	Directed link	Travel time	Directed link	Travel time
11-12	5	12-13	5	13-14	5
12-11	5	13-12	5	14-13	5
14-15	5	15-16	6	16-17	4
15-14	5	16-15	6	17-16	4
43-21	5	21-22	5	22-23	6
21-43	5	22-21	5	23-22	6
23-24	4	24-25	4	25-26	6
24-23	4	25-24	4	26-25	6
26-27	3	27-28	5	28-29	5
27-26	3	28-27	5	29-28	5
29-41	4	41-42	5	42-43	4
41-29	4	42-41	5	43-42	4
31-32	5	32-33	6	33-34	5
32-31	5	33-32	6	34-33	5
34-35	5	21-12	5	32-25	4
35-34	5	12-21	5	25-32	4
33-15	3	34-29	3	16-27	3
15-33	3	29-34	3	27-16	3

Table 2: The assigned passengers for directed links during each interval.

Link	Time interval index									
	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$	$t = 6$	$t = 7$	$t = 8$	$t = 9$	$t = 10$
11-12	300	300	300	300	300	300	300	300	300	300
12-13	0	300	300	450	183	110	0	205	300	300
12-21	0	0	0	150	267	190	490	205	0	0
13-14	0	0	300	0	300	183	33	33	238	355
14-15	0	0	0	300	0	300	150	0	0	183
15-33	0	0	0	0	300	0	300	150	150	0
33-32	0	0	0	0	0	300	0	300	0	150
32-31	0	0	0	0	0	0	300	0	300	0
13-12	0	0	0	0	0	0	110	0	0	0
21-22	0	0	0	0	150	267	0	490	205	205
23-24	0	0	0	0	0	77	150	267	0	490
22-23	0	0	0	0	0	150	267	0	490	0
24-25	0	0	0	0	0	0	0	150	417	267
25-32	0	0	0	0	0	0	0	0	0	150



Path 1

Path 2

Figure 5: Valid paths.

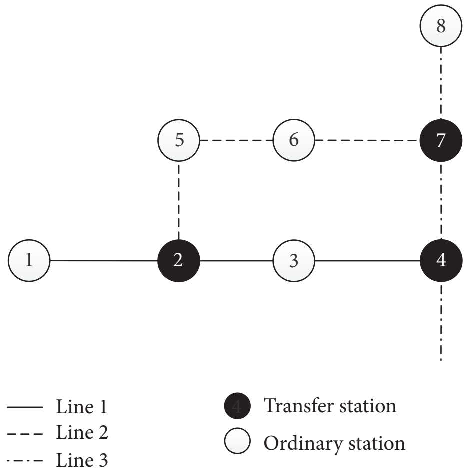


Figure 6: Tested network for comparative analysis of traditional and DUO approaches.

Table 3: Valid paths for given OD pair (Node1-Node 8).

Path	Nodes	Transfer nodes
1	1-2-3-4-7-8	4
2	1-2-5-6-7-8	2, 7

4.3. Comparative Analysis of Traditional and DUO Approaches. To test the effectiveness and performance of the proposed model, we compared it with the commonly used approach. The model and algorithm proposed in this paper are referred to as M1. The model proposed by Zhou and Xu [22] is referred to as M2. The test network is shown in Figure 6. Nodes 1 and 8 are selected as the origin and destination station, respectively. The travel time of each link without overload delays is set to 5 minutes.

In the following, M1 and M2 will be tested in four types of practical scenarios:

(1) Oversaturated and disturbance of train operation conditions occurring at direct link (3, 4).

(2) Undersaturated and disturbance of train operation conditions occurring at direct link (3, 4).

(3) Oversaturated and without disturbance.

(4) Undersaturated and without disturbance.

Two valid paths are obtained by the Dial algorithm, as shown in Table 3. The calculated results of M1 and M2 under different scenarios are shown in Table 4. The selection results of these two models are consistent in scenarios 3 and 4. That is, M1 is similar to M2 when the trains are operating strictly according to the predetermined timetable.

Varying degrees of separation exist in scenarios 1 and 2. According to the assumption, train operation disturbance occurs at direct link (3, 4). The impedance of path 2 is suddenly increased. Passengers who originally select path 1 are confronted with the disturbance of train delay. Therefore, a certain part of the passengers will change their original path.

Table 4: Calculated results of M1 and M2 (%).

Scenario number	M1		M2	
	Path 1	Path 2	Path 1	Path 2
1	22	78	82	18
2	33	67	88	12
3	88	12	87	13
4	93	7	95	5

The numerical result shows that M2 does not consider the influence of train operation disturbance when passengers make decisions. M1 provides the dynamically calculated process, which can overcome the shortcomings of M2 to some extent. Therefore, M1 is better than M2 for describing passenger route choice behavior when train operation disturbance occurs.

5. Conclusions

This paper proposed an enhanced passenger flow assignment model for metro networks. Compared with the traditional static model, the DUO-based model is more suitable for describing dynamic passenger distribution during rush hour. In addition, passenger route choice behavior in URT is different from car driver's route choice behavior on the road. Therefore, the method of determining the impedance in the metro network was improved in this study.

A more favorable characteristic for the DUO-based model was found: train operation disturbance. Indeed, this paper found that passengers experiencing oversaturated condition are more sensitive to disturbances than those experiencing undersaturated conditions. Furthermore, the DUO-based model proposed in this paper is robust and may be applied to oversaturated conditions for URT networks.

In terms of future research, it is necessary to calibrate relevant parameters under different travel purposes and mentalities. Moreover, randomness of passenger travel time should also be considered.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

This work is jointly supported by Young Teachers Training Funding of Universities in Shanghai (ZZGCD15114), Scientific Research Foundation for Doctors in Shanghai University of Engineering Science (2016-11), National Key Project of Research and Development Plan of China (Grant no. 2016YFC0802505), National Natural Science Foundation of China (Grant no. 71601110), and Foundation of Shanghai Committee of Science and Technology (Grant no. 15590501400).

References

[1] J. G. Wardrop, “Some theoretical aspects of road traffic research,” in *Proceedings of the Institute of Civil Engineering, Part II*, pp. 325–378, 1952.

[2] W. Vickrey, “Congestion theory and transport investment,” *American Economic Review*, vol. 59, pp. 251–261, 1969.

[3] J. Yang and G. Jiang, “Development of an enhanced route choice model based on cumulative prospect theory,” *Transportation Research Part C: Emerging Technologies*, vol. 47, part 2, pp. 168–178, 2014.

[4] F. Zhou, J. Shi, and R. Xu, “Estimation method of path-selecting proportion for urban rail transit based on AFC data,” *Mathematical Problems in Engineering*, vol. 2015, Article ID 350397, 9 pages, 2015.

[5] B. Ran and D. E. Boyce, *Modeling Dynamic Transportation Network: An Intelligent Transportation System Oriented Approach*, Springer, Heidelberg, Germany, 1996.

[6] T. L. Friesz, K. Han, P. A. Neto, A. Meimand, and T. Yao, “Dynamic user equilibrium based on a hydrodynamic model,” *Transportation Research Part B: Methodological*, vol. 47, pp. 102–126, 2013.

[7] M. Blumberg-Nitzani and H. Bar-Gera, “The effect of signalised intersections on dynamic traffic assignment solution stability,” *Transportmetrica A: Transport Science*, vol. 10, no. 7, pp. 622–646, 2014.

[8] D. K. Merchant and G. L. Nemhauser, “A model and an algorithm for the dynamic traffic assignment,” *Transportation Science*, vol. 12, no. 3, pp. 183–199, 1978.

[9] A. K. Ziliaskopoulos, “Linear programming model for the single destination system optimum dynamic traffic assignment problem,” *Transportation Science*, vol. 34, no. 1, pp. 37–49, 2000.

[10] W. Shen and H. M. Zhang, “System optimal dynamic traffic assignment: Properties and solution procedures in the case of a many-to-one network,” *Transportation Research Part B: Methodological*, vol. 65, pp. 1–17, 2014.

[11] S. C. Dafermos and F. T. Sparrow, “The traffic assignment problem for a general network,” *Journal of Research of the National Bureau of Standards*, vol. 73B, pp. 91–118, 1969.

[12] T. L. Friesz, D. Bernstein, T. E. Smith, R. L. Tobin, and B.-W. Wie, “A variational inequality formulation of the dynamic network user equilibrium problem,” *Operations Research*, vol. 41, no. 1, pp. 179–191, 1993.

[13] T. L. Friesz and R. Mookherjee, “Solving the dynamic network user equilibrium problem with state-dependent time shifts,” *Transportation Research Part B: Methodological*, vol. 40, no. 3, pp. 207–229, 2006.

[14] H. Dong, B. Ning, B. Cai, and Z. Hou, “Automatic train control system development and simulation for high-speed railways,” *IEEE Circuits and Systems Magazine*, vol. 10, no. 2, pp. 6–18, 2010.

[15] H. Dong, S. Gao, B. Ning, and L. Li, “Extended fuzzy logic controller for high speed train,” *Neural Computing and Applications*, vol. 22, no. 2, pp. 321–328, 2013.

[16] G. Wen, Z. Duan, W. Yu, and G. Chen, “Consensus of multi-agent systems with nonlinear dynamics and sampled-data information: a delayed-input approach,” *International Journal of Robust and Nonlinear Control*, vol. 23, no. 6, pp. 602–619, 2013.

[17] R. Balcombe, *The Demand for Public Transport: A Practical Guide*, TRL Limited, 2004.

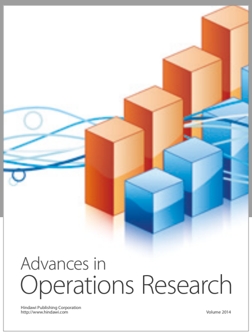
[18] D. Hasselstrom, *Public Transportation Planning-A Mathematical Programming Approach [Ph.D. thesis]*, Department of Business Administration, University of Gothenburg, Gothenburg, Sweden, 1981.

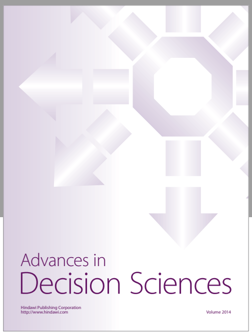
[19] M. Carey, “Nonconvexity of the dynamic traffic assignment problem,” *Transportation Research. Part B. Methodological. An International Journal*, vol. 26, no. 2, pp. 127–133, 1992.

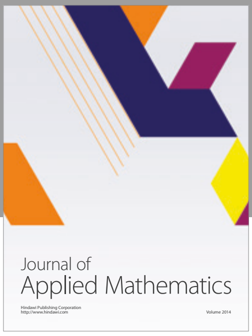
[20] H. K. Chen, *Dynamic Travel Choice Model: A Variational Inequality Approach*, Springer, 1999.

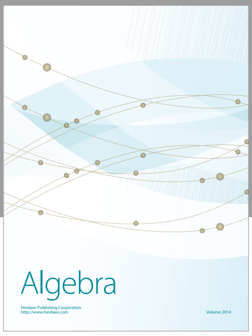
[21] M. Frank and P. Wolfe, “An algorithm for quadratic programming,” *Naval Research Logistics Quarterly*, vol. 3, pp. 95–110, 1956.

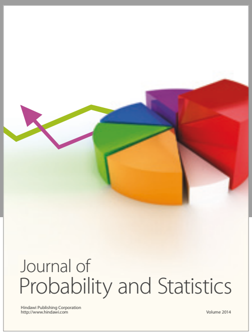
[22] F. Zhou and R.-H. Xu, “Model of passenger flow assignmentfor Urban rail transit based on entryand exit time constraints,” *Transportation Research Record*, no. 2284, pp. 57–61, 2012.


Advances in
Operations Research

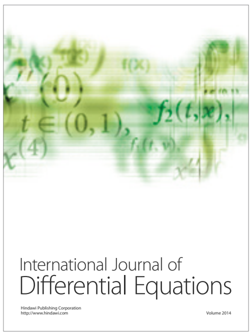

Advances in
Decision Sciences

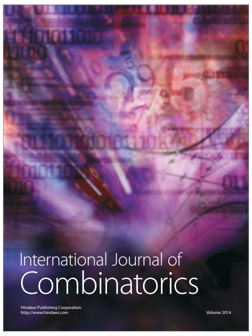

Journal of
Applied Mathematics

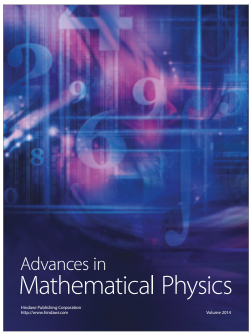

Algebra

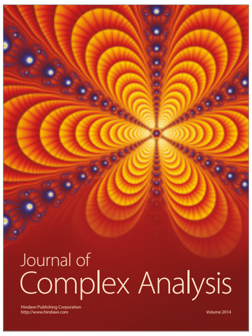

Journal of
Probability and Statistics

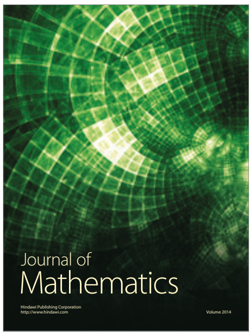

The Scientific
World Journal

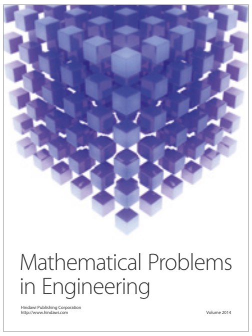

International Journal of
Differential Equations

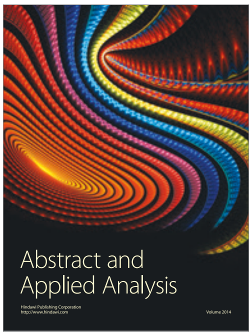

International Journal of
Combinatorics

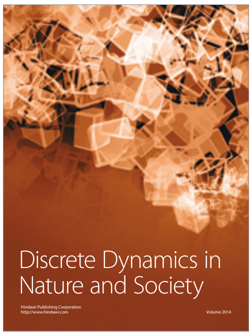

Advances in
Mathematical Physics

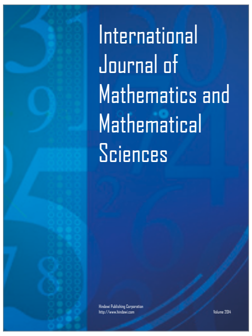

Journal of
Complex Analysis


Journal of
Mathematics


Mathematical Problems
in Engineering

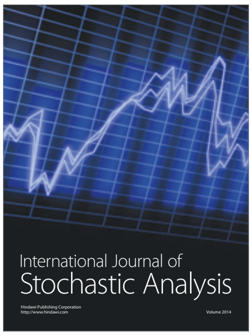

Abstract and
Applied Analysis


Discrete Dynamics in
Nature and Society


International
Journal of
Mathematics and
Mathematical
Sciences


Journal of
Discrete Mathematics


Journal of
Function Spaces


International Journal of
Stochastic Analysis


Journal of
Optimization