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Research Article

Coexistence for an Almost Periodic Predator-Prey Model with Intermittent Predation Driven by Discontinuous Prey Dispersal

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An almost periodic predator-prey model with intermittent predation and prey discontinuous dispersal is studied in this paper, which differs from the classical continuous and impulsive dispersal predator-prey models. The intermittent predation behavior of the predator species only happens in the channels between two patches where the discontinuous migration movement of the prey species occurs. Using analytic approaches and comparison theorems of the impulsive differential equations, sufficient criteria on the boundedness, permanence, and coexistence for this system are established. Finally, numerical simulations demonstrate that, for an intermittent predator-prey model, both the intermittent predation and intrinsic growth rates of the prey and predator species can greatly impact the permanence, extinction, and coexistence of the population.

1. Introduction

In real ecosystems, since the spatial distribution and dynamics of a population are greatly affected by their spatial heterogeneity and population mobility, dispersal becomes one of the dominant themes in mathematical biology. In fact, animal dispersal movements between patches are extremely prevalent in ecological environments; for example, many types of birds and mammals will migrate from cold regions to warm regions in search of a better habitat or a breeding site [1]. Therefore, to take spatial heterogeneity into account, realistic population models should contain the dispersal process. During the past couple of decades, predator-prey models with diffusion in a patchy environment have attracted significant attention from ecologists, biologists, and biomathematicians. Many important works and monographs about the properties of population dynamics in a spatial idiosyncratic environment, for example, permanence, extinction, and global asymptotic stability of positive periodic solutions, have been written (see [2–14]). Teng and Chen [6] considered a nonautonomous predator-prey Lotka-Volterra type dispersal system with periodic coefficients and distributed delays:

$$\begin{aligned}
\dot{x}_1(t) &= x_1(t)\left[a_1(t) - b_1(t)\,x_1(t) - c(t)\int_{-\infty}^{0} k_{12}(s)\,y(t+s)\,ds\right] \\
&\quad + \sum_{j=1}^{n} d_{1j}(t)\left[x_j(t) - x_1(t)\right], \\
\dot{x}_i(t) &= x_i(t)\left[a_i(t) - b_i(t)\,x_i(t)\right] + \sum_{j=1}^{n} d_{ij}(t)\left[x_j(t) - x_i(t)\right], \quad i = 2, 3, \ldots, n, \\
\dot{y}(t) &= y(t)\left[-e(t) + f(t)\int_{-\infty}^{0} k_{21}(s)\,x_1(t+s)\,ds - g(t)\int_{-\infty}^{0} k_{21}(s)\,x_1(t+s)\,ds\right],
\end{aligned} \tag{1}$$

where y is the population density of the predator species confined to the 1st patch. Criteria for the permanence, extinction, and existence of positive periodic solutions for system (1) were established. In this model, the prey dispersal behavior occurs at every point in time and simultaneously

between any patches; that is, it is a continuous bidirectional dispersal.

However in practice, it is often the case that diffusion occurs in the form of regular pulses. For example, when winter comes, birds will migrate between patches to seek a better habitat, whereas they do not diffuse in other seasons, and the dispersion of foliage seeds occurs at a fixed period of time every year. For another example, in the Pacific Northwest, *Larimichthys polyactis* cross over deep water during the winter and migrate to the coast during the spring; then, 3–6 months after spawning, they scatter offshore and return to the depths of the sea during late autumn [15]. All these types of migratory behaviors are appropriately assumed to be in the form of pulses in the modeling process. Thus, impulsive diffusion provides a more natural description. Currently, theories of impulsive differential equations [16] have been introduced into population dynamics. A large number of models have been described by impulsive diffusion (see [14, 17–24]) during the past couple of decades.

Shao [23] considered the following predator-prey models with impulsive prey diffusion between two patches:

$$\begin{aligned}
&\dot{x}_1(t) = x_1(t)\left(r_1 - a_1 x_1(t) - b_1 y(t)\right),\\
&\dot{x}_2(t) = x_2(t)\left(r_2 - a_2 x_2(t)\right),\\
&\dot{y}(t) = y(t)\left(-r_3 + a_3 x_1(t-\tau_1) - b_2 y(t-\tau_2)\right),\\
&\qquad\qquad t \neq n\tau,\\
&\Delta x_1(t) = d_1\left(x_2(t) - x_1(t)\right),\\
&\Delta x_2(t) = d_2\left(x_1(t) - x_2(t)\right),\\
&\Delta y(t) = 0,\\
&\qquad\qquad t = n\tau,
\end{aligned} \tag{2}$$

where the pulse diffusion of the species x occurs in every period T (a positive constant) and d_i is the dispersal rate in the ith patch satisfying $0 < d_i < 1$ for $i = 1, 2$. The system evolves from its initial state without being further affected by diffusion until the next pulse appears. $\Delta x_i(nT) = x_i(nT^+) - x_i(nT)$, where $x_i(nT^+)$ represents the density of the population of the prey species x in the ith patch immediately after the nth diffusion pulse at time $t = nT$; $x_i(nT)$ represents the density of the population of the prey species in the ith patch before the nth diffusion pulse at time $t = nT$, $n = 0, 1, \ldots$. Criteria for the global attractivity and permanence of system (2) were obtained.

Furthermore, migration movements of the population will be influenced by many uncertain factors (such as the landscape and weather). Therefore, the dispersal movement of migratory species will have to be suspended when the environment becomes unavailable. In other words, patches would permit normal movement patterns between patches to occur only during certain time intervals instead of all the time. For instance, in the Canary Islands of Spain, *Anas platyrhynchos* undergo a spring migration from early March to the end of March and a fall migration from late September to the end of October, departing as late as early November, during which they are extensively killed by humans, and other carnivorous animals can prolong the journey [25]. As another example, the wild goose will fly to the south when winter comes; in this process, they will stop to rest in some places and at certain time periods. In other words, their diffusion behavior is neither continuous at all times nor impulsive at a fixed time, but it is intermittent within some time intervals. Therefore, it is more reasonable to model this kind of population dynamics with intermittent dispersals. Zhang et al. [26] considered the following nonautonomous almost periodic single species model with intermittent dispersals and dispersal delays between two patches:

$$\begin{aligned}
&\dot{x}_1(t) = x_1(t)\left[a_1(t) - b_1(t)x_1(t)\right],\\
&\dot{x}_2(t) = x_2(t)\left[a_2(t) - b_2(t)x_2(t)\right],\\
&\qquad\qquad t \in \left[\tau_{2k}, \tau_{2k+1}\right),\\
&x_1(\tau_{2k+1}) = d_1 x_1(\tau_{2k+1}^-),\\
&x_2(\tau_{2k+1}) = d_2 x_2(\tau_{2k+1}^-),\\
&\qquad\qquad t = \tau_{2k+1},\\
&\dot{x}_1(t) = x_1(t)\left[\widetilde{a}_1(t) - \widetilde{b}_1(t)x_1(t)\right]\\
&\qquad + D_{12}(t)\left(x_2(t-\tau_1) - x_1(t)\right),\\
&\dot{x}_2(t) = x_2(t)\left[\widetilde{a}_2(t) - \widetilde{b}_2(t)x_2(t)\right]\\
&\qquad + D_{21}(t)\left(x_1(t-\tau_2) - x_2(t)\right),\\
&\qquad\qquad t \in \left[\tau_{2k+1}, \tau_{2k+2}\right),\\
&x_1(\tau_{2k+2}) = D_1 x_1(\tau_{2k+2}^-),\\
&x_2(\tau_{2k+2}) = D_2 x_2(\tau_{2k+2}^-),\\
&\qquad\qquad t = \tau_{2k+2},
\end{aligned} \tag{3}$$

where all the parameters are almost periodic and the dispersal movement happens only in the time interval $t \in [\tau_{2k+1}, \tau_{2k+2})$ but not in $t \in [\tau_{2k}, \tau_{2k+1})$. Here, $k \in N$. Criteria for the existence, uniqueness, and global attractivity of positive almost periodic solution for system (3) were established.

Moreover, in a real ecological system, there always exist natural enemies during the migratory process between patches. For example, annually, at the end of July, with the arrival of the dry season, millions of wildebeests, zebras, and other herbivorous wildlife form a migratory army, migrating from the Serengeti National Park, Tanzania, Africa, to Kenya's Masai Mara National Nature Reserve to find enough water and food. Along the way, they will be preyed upon by lions, leopards, and so on. Additionally, crocodiles and hippopotamus will wait and ambush the migration species in the Mara River. As the seasons alternate, i.e., when the rainy season comes, the migration movement starts again, and these species will return to the Serengeti National Park, and vice versa [27]. Obviously, the predation behavior for the above situation is intermittent and only happens in the channels where the dispersals of migratory species occur.

Motivated by the above consideration, in this paper, we introduce an almost periodic predator-prey model with intermittent predation and discontinuous prey dispersal between two patches:

$$\begin{aligned}
&\dot{x}_1(t) = x_1(t)\left[a_1(t) - b_1(t)x_1(t)\right],\\
&\dot{x}_2(t) = x_2(t)\left[a_2(t) - b_2(t)x_2(t)\right],\\
&\dot{y}(t) = y(t)\left[a_3(t) - b_3(t)y(t)\right],\\
&\qquad\qquad t \in \left[\tau_{2k}, \tau_{2k+1}\right),\\
&x_1\left(\tau_{2k+1}\right) = d_1 x_1\left(\tau_{2k+1}^-\right),\\
&x_2\left(\tau_{2k+1}\right) = d_2 x_2\left(\tau_{2k+1}^-\right),\\
&y\left(\tau_{2k+1}\right) = y\left(\tau_{2k+1}^-\right),\\
&\qquad\qquad t = \tau_{2k+1},\\
&\dot{x}_1(t) = x_1(t)\left[\widetilde{a}_1(t) - \widetilde{b}_1(t)x_1(t) - c_1(t)y(t)\right]\\
&\quad + D_{12}(t)\left(x_2(t-\tau_1) - x_1(t)\right),\\
&\dot{x}_2(t) = x_2(t)\left[\widetilde{a}_2(t) - \widetilde{b}_2(t)x_2(t) - c_2(t)y(t)\right]\\
&\quad + D_{21}(t)\left(x_1(t-\tau_2) - x_2(t)\right),\\
&\dot{y}(t) = y(t)\\
&\quad \cdot\left[\widetilde{a}_3(t) - \widetilde{b}_3(t)y(t) + \widetilde{c}_1(t)x_1(t) + \widetilde{c}_2(t)x_2(t)\right],\\
&\qquad\qquad t \in \left[\tau_{2k+1}, \tau_{2k+2}\right),\\
&x_1\left(\tau_{2k+2}\right) = D_1 x_1\left(\tau_{2k+2}^-\right),\\
&x_2\left(\tau_{2k+2}\right) = D_2 x_2\left(\tau_{2k+2}^-\right),\\
&y\left(\tau_{2k+2}\right) = y\left(\tau_{2k+2}^-\right),\\
&\qquad\qquad t = \tau_{2k+2},
\end{aligned} \tag{4}$$

where $x_i(t)$ denotes the prey population density in the ith patch ($i = 1, 2$) and $y(t)$ represents the predator population density in channels between two patches. When $t \in [\tau_{2k}, \tau_{2k+1})$ with $k \in N$, the prey species x_i inhabits the ith patch and does not disperse. At the same time, the predator species y inhabits the channels between two patches with other food sources. When $t \to \tau_{2k+1}$, the intrinsic discipline of the species x in each patch changes. The channels between the two patches will open, and the species x disperses bidirectionally from one patch to another; this dispersal movement will continue for the time interval $t \in [\tau_{2k+1}, \tau_{2k+2})$. Meanwhile, the predator species y preys on the species x in the channels. When $t \to \tau_{2k+2}$, the gate of the channels will close, the species x_i will stop dispersing and inhabit patch i. At the same time, the predator species y will also stop preying on species x. Obviously, the predation behavior only happens in the time interval $t \in [\tau_{2k+1}, \tau_{2k+2})$; that is, it is intermittent. Here, $a_i(t), \widetilde{a}_i(t)$ $(i = 1, 2)$ represent the intrinsic growth rates of the species x in the ith patch over the time intervals $[\tau_{2k}, \tau_{2k+1})$ and $[\tau_{2k+1}, \tau_{2k+2})$, respectively; $a_3(t), \widetilde{a}_3(t)$ denote the intrinsic growth rates of the species y over the time intervals $[\tau_{2k}, \tau_{2k+1})$ and $[\tau_{2k+1}, \tau_{2k+2})$, respectively; $b_i(t), \widetilde{b}_i(t)$ $(i = 1, 2)$ represent the intercompetition rates of the species x in the ith patch over the time intervals $[\tau_{2k}, \tau_{2k+1})$ and $[\tau_{2k+1}, \tau_{2k+2})$, respectively; $b_3(t), \widetilde{b}_3(t)$ denote the intercompetition rates of the species y over the time intervals $[\tau_{2k}, \tau_{2k+1})$ and $[\tau_{2k+1}, \tau_{2k+2})$, respectively; d_i represents the survival rates of switching from stage 1 (without dispersal) to stage 2 (dispersal movement); D_i denotes the survival rates of switching from stage 2 to stage 1; $D_{ij}(t)$ is the dispersal rates from the ith patch to the jth patch during the time interval $[\tau_{2k+1}, \tau_{2k+2})$; and τ_i is the time for the population to disperse from patch j to i $(i \neq j,\ i, j = 1, 2)$. During the whole process, the predator species y never disperses.

In allusion to system (4) above, our main purpose in this paper is to establish a series of criteria on the ultimate boundedness, permanence, and coexistence of the two populations for system (4). The methods used in this paper are motivated by the works on the permanence and extinction for periodic predator-prey systems in patchy environments given by Teng and Chen in [6] and the works on the survival analysis for a periodic predator-prey model given by Zhang et al. in [22].

This paper is organized as follows. In Section 2, some definitions, assumptions, and useful lemmas are introduced. In Section 3, we state and prove the main results. Finally, special examples and numerical simulations are illustrated to demonstrate our theoretical results in Section 4.

2. Preliminaries

Let R and R^2 denote the set of real numbers and the 2-dimensional Euclidean linear space, respectively, and τ_k be a time sequence, satisfying $\tau_k < \tau_{k+1}$, with $\tau_k \to \infty$ as $k \to \infty$. Here, $k \in N$. Define $\widehat{\tau} = \max\{\tau_1, \tau_2\}$, $PC([t-\widehat{\tau}, t], R^2) = \{\psi : ([t-\widehat{\tau}, t] \to R^2) \mid \varphi = (\varphi_1, \varphi_2)$ is continuous everywhere except at $t = \tau_k \in [t-\widehat{\tau}, t]$, and $\varphi(\tau_k^+), \varphi(\tau_k^-)$ exist with $\varphi(\tau_k^+) = \varphi(\tau_k)\}$. Define $BPC = \{\varphi \in PC((-\infty, 0], R^2] : \varphi$ is bounded$\}$; the norm of φ is defined by $\|\varphi\| = \sup_{\theta\in[-\widehat{\tau},0]}|\varphi(\theta)|$. Let $BPC_+ = \{\varphi = (\varphi_1, \varphi_2) \in BPC : \varphi_i(\theta) \geq 0$ for all $\theta \in [-\infty, 0]$ and $\varphi_i(0_+) > 0$ for $i = 1, 2\}$.

In this paper, we assume that all solutions of system (4) satisfy the following initial conditions:

$$\begin{aligned}
x_i(s, \phi) &= \phi_i(s),\\
&\quad s \in [-\widehat{\tau}, 0],\\
x_i(0^+, \phi) &= \phi_i(0^+),\\
&\quad s \in [-\widehat{\tau}, 0],
\end{aligned} \tag{5}$$

where $\phi = (\phi_1, \phi_2) \in BPC_+$. It is not hard to prove that the functional of right of system (4) is continuous and satisfies the local Lipschitz condition with respect to ϕ in the space $R\times BPC$. Therefore, by the fundamental theory of the impulsive functional differential equations with finite delays [16, 28, 29], system (4) has a unique solution $(x_1(t, \phi), x_2(t, \phi), y(t))$ satisfying the initial conditions (5). Obviously, the solution

$(x_1(t,\phi), x_2(t,\phi), y(t))$ is positive in its maximal interval of the existence.

Before going into details, we first draw some very useful definitions and lemmas.

Definition 1 (see [30]). The set of sequences $\{\tau_k{}^i \mid \tau_k{}^i = \tau_{k+1} - \tau_k,\ k, i \in N,\ \tau_k \in B\}$ is said to be uniformly almost periodic if, for arbitrary $\varepsilon > 0$, there exists a relatively dense set in R of ε-almost periodic common for all of the sequences; here $B = \{\tau_k \mid \tau_k \in R,\ \tau_k < \tau_{k+1},\ k \in N,\ \lim_{k\to\pm\infty}\tau_k = \pm\infty\}$.

Definition 2 (see [30]). Assume that the following conditions hold:

(1) The set of sequences $\{\tau_k^i\}$ is almost periodic, $k, i \in N$.

(2) For any $\varepsilon > 0$ there exists a real number $\delta > 0$ such that if the points t', t'' belong to the same interval of continuity of $\varphi(t)$ and satisfy the inequality $|t' - t''| < \delta$, then $|\varphi(t') - \varphi(t'')| < \varepsilon$.

(3) For any $\varepsilon > 0$ there exists a relatively dense set C such that if $\sigma \in C$, then $|\varphi(t+\sigma) - \varphi(t)| < \varepsilon$ for all $t \in R$ satisfying the condition $|t - \sigma_k| > \varepsilon$, $k \in N$. The elements of C are called ϵ-almost periods.

We claim that function $\varphi \in PC([t-\widehat{\tau}, t], R^2)$ is almost periodic, and we denote $\varphi \in APC$.

Definition 3 (see [31]). System (4) is said to be permanent if there exist positive constants m and M, such that for any positive solutions $x(t) = (x_1(t), x_2(t), y(t))$ of system (4) with initial value that satisfy condition (5)

$$\begin{aligned} m \le \liminf_{t\to\infty} x_i(t) \le \limsup_{t\to\infty} x_i(t) \le M, \quad i = 1, 2, \\ m \le \liminf_{t\to\infty} y(t) \le \limsup_{t\to\infty} y(t) \le M. \end{aligned} \tag{6}$$

Definition 4 (see [24]). Let $V : R \times R^2 \to R^2$, then V is said to belong to class L if

(i) V is continuous in $(\tau_k, \tau_{k+1}) \times R^2$ for each $x \in R^2$, $k \in N$,

$$\lim_{(t,y)\to(\tau_{k+1}^-, x)} V(t, y) = V(\tau_{k+1}, x). \tag{7}$$

(ii) V is locally Lipschitzian in x.

In this paper, there are some notations and assumptions that shall be used:

(N_1) If $f(t)$, $t \in R$, is an almost periodic function, we define

$$\begin{aligned} A(f) &= \lim_{t\to\infty} \frac{1}{T} \int_t^{t+T} f(s)\, ds, \\ f^u &= \sup_{t\in R} f(t), \\ f^l &= \inf_{t\in R} f(t), \end{aligned} \tag{8}$$

where T is a positive constant.

(N_2) We let $\lambda_i(t) = t - \tau_i$, $\lambda_i{}^{-1}(t)$ be an inverse function of the function $\lambda_i(t)$ $(i = 1, 2)$.

(H_1) Functions $a_i(t), \widetilde{a}_i(t), b_i(t), \widetilde{b}_i(t)$, and $D_{ij}(t)$ are τ-almost periodic and bounded continuous functions for all $t \in R$ and $b_i^l \ge 0$, $\widetilde{b}_i^l \ge 0$, $0 < d_i$, $D_i \le 1$ $(i = 1, 2, 3,\ j = 1, 2)$.

(H_2) $A(a_i(t)) > 0$, $A(\widetilde{a}_i(t)) > 0$, $A(b_i(t)) > 0$, $A(\widetilde{b}_i(t)) > 0$ $(i = 1, 2)$.

(H_3) There exists a constant $\tau > 0$, such that, for any $t \ge 0$,

$$\begin{aligned} \int_t^{t+\tau} \left[\frac{(-1)^k + 1}{2} a_i(s) + \frac{(-1)^{k+1} + 1}{2} \widetilde{a}_i(s) \right] ds \\ + \sum_{t<\tau_{k+1}\le(t+\tau)} \ln h_{ik} > 0 \quad (i = 1, 2,\ k \in N). \end{aligned} \tag{9}$$

(H_4) There exists a constant $\xi_i > 0$ $(i = 1, 2)$, such that, for any $t \ge 0$,

$$\left| \sum_{t<\tau_{k+1}\le(t+s)} \ln h_{ik} \right| \le \xi_i, \quad s \in [0, \tau]. \tag{10}$$

(H_5) $A(a_{ik}(t) - d_{ijk}(t)) > 0$ $(i, j = 1, 2,\ k \in N)$.

(H_6) There exist constants $\theta > 0$, $e_i > 0$ $(i = 1, 2)$, such that $\lim_{t\to\infty}\inf B_i(t) = \lim_{t\to\infty}[e_i(b_{ik}(t)) - d_{ijk}(\lambda_j{}^{-1}(t))/\widetilde{m}] \ge \theta$, where $\widetilde{m} = \max\{1, [m]\}$ and $[m]$ is the integer part of m $(k \in N)$.

(H_7) The set of sequences $\{\tau_k^i\}$, $k \in N$, is uniformly almost periodic, and $\tau_{k+2} = \tau_k + \tau$, $\inf_{k\in N}|\tau_{k+1} - \tau_k| > 0$, where τ belongs to the relatively dense set T $(i = 1, 2, 3)$.

(H_8) There exists a constant $\widetilde{\tau} > 0$, such that, for any $t \ge 0$,

$$\int_t^{t+\widetilde{\tau}} \left[\frac{(-1)^k + 1}{2} a_3(s) + \frac{(-1)^{k+1} + 1}{2} \widetilde{a}_3(s) \right] ds > 0. \tag{11}$$

Now, we give some useful lemmas which will be used in the proofs of the main results.

If there is no predator y in system (4), we have the following predator-free system:

$$\begin{aligned} \dot{x}_i(t) = x_i(t)\left[a_{ik}(t) - b_{ik}(t)\, x_i(t)\right] \\ + d_{ijk}(t)\left[x_j(t - \tau_i) - x_i(t)\right], \\ t \in [\tau_k, \tau_{k+1}), \\ x_i(\tau_{k+1}) = h_{ik} x_i(\tau_{k+1}^-), \quad t = \tau_{k+1}, \end{aligned} \tag{12}$$

where

$$a_{ik}(t) = \frac{(-1)^k + 1}{2} a_i(t) + \frac{(-1)^{k+1} + 1}{2} \widetilde{a}_i(t),$$
$$b_{ik}(t) = \frac{(-1)^k + 1}{2} b_i(t) + \frac{(-1)^{k+1} + 1}{2} \widetilde{b}_i(t),$$
$$d_{ijk}(t) = \frac{(-1)^{k+1} + 1}{2} D_{ij}(t), \tag{13}$$
$$h_{ik} = \frac{(-1)^k + 1}{2} d_i + \frac{(-1)^{k+1} + 1}{2} D_i, \quad i \neq j,\ i, j = 1, 2.$$

For system (12), we have the following result.

Lemma 5 (see [26] Theorem 3.4). *Suppose that assumptions (H_1)–(H_7) hold; then system (12) has a unique globally attractive positive τ-almost periodic solution $x^*(t) = (x_1^*(t), x_2^*(t))$.*

If there is no prey x in system (4), we have the following prey-free system:

$$\dot{y}(t) = y(t)\left[a_{3k}(t) - b_{3k}(t)\, y(t)\right], \quad t \in \left[\tau_k, \tau_{k+1}\right), \tag{14}$$
$$y\left(\tau_{k+1}\right) = y\left(\tau_{k+1}^-\right), \quad t = \tau_{k+1},$$

where

$$a_{3k}(t) = \frac{(-1)^k + 1}{2} a_3(t) + \frac{(-1)^{k+1} + 1}{2} \widetilde{a}_3(t),$$
$$b_{3k}(t) = \frac{(-1)^k + 1}{2} b_3(t) + \frac{(-1)^{k+1} + 1}{2} \widetilde{b}_3(t). \tag{15}$$

For system (14), we have the following result.

Lemma 6 (see [26]). *Suppose assumptions (H_1)-(H_2) and (H_7)-(H_8) hold; then system (14) has a unique globally attractive positive τ-almost periodic solution $y^*(t) \in APC$.*

For $(t, x) \in (\tau_k, \tau_{k+1}) \times R^2$, we define

$$D^+V(t,x) = \limsup_{h \to 0^+} \frac{1}{h}\left[V\left(t + h, x + hf(t,x)\right) - V(t,x)\right], \tag{16}$$

where $f = (f_1, f_2)$ is the right-hand side of system (12). We give the following vector comparison results of the impulsive differential equations.

Lemma 7 (see [1]). *Let $V : R_+ \times R_+^2$ and $V \in L$. Assume that*

$$D^+V(t,x) \leq g(t, V(t,x)) \quad t \in \left[\tau_k, \tau_{k+1}\right),$$
$$V(t, x(t)) \leq \psi_k(V(t,x)), \quad t = \tau_{k+1}, \tag{17}$$

where $g(t,u) : R \times R^2 \to R^2$ is continuous in $(\tau_k, \tau_{k+1}) \times R^2$ and quasi-monotone nondecreasing in u, for $v \in R^2$, $\lim_{(t,u)\to(\tau_{k+1}^-)} g(t,u) = g(\tau_{k+1}^-, v)$ exists, and $\psi_k : R^2 \to R^2$ is nondecreasing. Let $r(t)$ be the maximal solution of the following vector impulsive differential system:

$$\dot{u} = g(t, V(t,x)) \quad t \in \left[\tau_k, \tau_{k+1}\right),$$
$$u(t) = \psi_k\left(u\left(t^-\right)\right),$$
$$u\left(t_0\right) = u\left(t_0^-\right) = u_0 \tag{18}$$
$$t = \tau_{k+1},$$

existing on $[t_0, \infty)$. Then $V(t_0, x_0) \leq u_0$ implies that

$$V(t, x(t)) \leq r(t), \quad t \geq t_0, \tag{19}$$

where $x(t) = x(t, t_0, x_0)$ is any solution of system (12) existing on $[t_0, \infty)$. Here, we state that function $g(t,u)$ is quasi-monotone nondecreasing in u; if $u, v \in R^2$, $u \leq v$, and $x_i = y_i$ for some $1 \leq i \leq 2$, then $g_i(t,x) \leq g_i(t,y)$.

3. Main Results

First, in terms of the ultimate boundedness for system (4), we obtain the following result.

Theorem 8. *Suppose that assumptions (H_1)–(H_9) hold. Then there is a constant $M > 0$ such that*

$$\limsup_{t\to\infty} x_i(t, \phi) \leq M \quad (i = 1, 2),$$
$$\limsup_{t\to\infty} y(t) \leq M \tag{20}$$

for any positive solutions $(x_1(t,\phi), x_2(t,\phi), y(t,\phi))$ of system (4).

Proof. Let $(x_1(t,\phi), x_2(t,\phi), y(t))$ be any positive solution of system (4) satisfying the initial condition (5). From system (4), we have

$$\dot{x}_i(t) \leq x_i(t)\left[a_{ik}(t) - b_{ik}(t)\, x_i(t)\right] + d_{ijk}(t)\left[x_j(t - \tau_i) - x_i(t)\right], \quad t \in \left[\tau_k, \tau_{k+1}\right),$$
$$x_i\left(\tau_{k+1}\right) = h_{ik} x_i\left(\tau_{k+1}^-\right), \quad t = \tau_{k+1}, \tag{21}$$

for all $t \geq 0$, $i, j = 1, 2$ $(i \neq j)$. Then by the Lemmas 5 and 7, we obtain

$$x_i(t) \leq \overline{x}_i(t), \quad \forall t \geq 0,\ i = 1, 2, \tag{22}$$

where $(\overline{x}_1(t), \overline{x}_2(t))$ is the solution of system (12) with the initial condition (5). Under assumptions (H_1)–(H_7), from Lemma 5 we obtain $\overline{x}_i(t) \to x^*(t)$ $(i = 1, 2,\ t \to \infty)$, where $x^*(t) = (x_1^*(t), x_2^*(t))$ is the globally asymptotically stable positive τ-almost periodic solution of system (12). Hence, $\overline{x}_i(t)$ $(i = 1, 2)$ is bounded on R_+. Next, we choose a constant $M_1 = \sup_{t\in R}|x^*(t)|$, where $|x^*(t)| = \sum_{i=1}^2 x_i^*(t)$. Since $\overline{x}_i(t) \to$

$x^*(t)$ $(i = 1, 2,\ t \to \infty)$, there is a pair of $T_1 > 0$ and $\rho > 0$ such that

$$\overline{x}_i(t) \le x_i^*(t) + \rho \triangleq M_1 \quad \forall t \ge T_1,\ i = 1, 2. \tag{23}$$

Hence, by (22) we have

$$x_i(t) \le M_1 \quad \forall t \ge T_1,\ i = 1, 2. \tag{24}$$

Consequently,

$$\limsup_{t\to\infty} x_i(t, \phi) \le M_1, \quad i = 1, 2. \tag{25}$$

Further, we prove that there is a constant $M_2 > 0$ such that

$$\limsup_{t\to\infty} y(t) \le M_2. \tag{26}$$

From systems (4) and (24) we obtain

$$\begin{aligned} \dot{y}(t) &\le y(t) \\ &\cdot \left[a_{3k}(t) + \widetilde{c}_{1k}(t) M_1 + \widetilde{c}_{2k}(t) M_2 - b_{3k}(t) y(t)\right], \\ &\qquad t \in \left[\tau_k, \tau_{k+1}\right), \\ y\left(\tau_{k+1}\right) &= y\left(\tau_{k+1}^-\right), \quad t = \tau_{k+1}, \end{aligned} \tag{27}$$

for all $t \ge T_1$, where

$$\begin{aligned} \widetilde{c}_{1k}(t) &= \frac{(-1)^{k+1} + 1}{2} c_1(t), \\ \widetilde{c}_{2k}(t) &= \frac{(-1)^{k+1} + 1}{2} c_2(t), \\ &\qquad k \in N. \end{aligned} \tag{28}$$

From the comparison theorem of the impulsive differential equations [16, 28, 29], we have $y(t) \le z(t)$ for all $t \ge T_1$, where $z(t)$ is the solution of the following auxiliary equation:

$$\begin{aligned} \dot{z}(t) &= z(t) \\ &\cdot \left[a_{3k}(t) + \widetilde{c}_{1k}(t) M_1 + \widetilde{c}_{2k}(t) M_2 - b_{3k}(t) z(t)\right], \\ &\qquad t \in \left[\tau_k, \tau_{k+1}\right), \\ z\left(\tau_{k+1}\right) &= z\left(\tau_{k+1}^-\right), \quad t = \tau_{k+1}, \end{aligned} \tag{29}$$

with initial value $z(T_1) = y(T_1)$. According to assumptions (H_1)-(H_2) and (H_7)-(H_8), we also have

$$\begin{aligned} \int_t^{t+\tau} \Bigg\{ &\frac{(-1)^k + 1}{2} a_3(s) + \frac{(-1)^{k+1} + 1}{2} \left[\widetilde{a}_3(s)\right. \\ &\left. + \widetilde{c}_{1k}(t) M_1 + \widetilde{c}_{2k}(t) M_1\right] \Bigg\} ds > 0. \end{aligned} \tag{30}$$

Hence from Lemma 6, system (29) has a unique globally attractive positive τ-almost periodic solution $z^*(t)$. For any constant $\lambda > 0$, there is a $T_2 \ge T_1$ such that $z(t) < z^*(t) + \lambda$ for all $t \ge T_2$. Therefore, we have

$$y(t) < z^*(t) + \lambda \le \frac{1}{2} \max_{t\in[0,\tau]} z^*(t) + \lambda \triangleq M_2, \tag{31}$$

for all $t \ge T_2$. Consequently,

$$\limsup_{t\to\infty} y(t) \le M_2. \tag{32}$$

Therefore, (26) holds. Choose a constant $M = \max\{M_1, M_2\}$; then we can see

$$\begin{aligned} \limsup_{t\to\infty} x_i(t, \phi) &\le M \quad (i = 1, 2), \\ \limsup_{t\to\infty} y(t) &\le M; \end{aligned} \tag{33}$$

this completes the proof of Theorem 8. □

Next, on the permanence of system (4), we have the following results.

Theorem 9. *Suppose that assumptions* (H_1)-(H_2) *and* (H_7)-(H_8) *hold. Then the predator species* y *of system (4) is permanent.*

Proof. Let $(x_1(t, \phi), x_2(t, \phi), y(t))$ be any positive solutions of system (4) satisfying the initial condition (5). By system (4), we easily obtain

$$\begin{aligned} \dot{y}(t) &\ge y(t) \left[a_{3k}(t) - b_{3k}(t) y(t)\right], \\ &\qquad t \in \left[\tau_k, \tau_{k+1}\right), \\ y\left(\tau_{k+1}\right) &= y\left(\tau_{k+1}^-\right), \quad t = \tau_{k+1}. \end{aligned} \tag{34}$$

Using comparison theorem of the impulsive differential equations [16, 28, 29] and Lemma 6, we can easily obtain $y(t) \ge \overline{y}(t)$, for all $t \in R$, where $\overline{y}(t)$ is the solution of system (14) with condition (5). Under the assumptions (H_1)-(H_2) and (H_7)-(H_8), by Lemma 6, we can obtain $\overline{y}(t) \to y^*(t)$ as $t \to \infty$, where $y^*(t)$ is the globally asymptotically stable positive τ-almost periodic solution of system (14). Hence, we can easily obtain that, for $\varepsilon_0 = (1/2)\min_{t\in[0,\tau]} y^*(t)$, there exists a $T_1 > 0$ such that

$$y(t) > y^*(t) - \varepsilon_0 \ge \frac{1}{2} \min_{t\in[0,\tau]} y^*(t) \triangleq m_2, \tag{35}$$

for all $t \ge T_1$. Together with Theorem 8, we have that the predator species y of system (4) is permanent. This completes the proof of Theorem 9. □

Remark 10. Set

$$\theta = \int_t^{t+\tau} \left[\frac{(-1)^k + 1}{2} a_3(s) + \frac{(-1)^{k+1} + 1}{2} \widetilde{a}_3(s)\right] ds. \tag{36}$$

In system (4), based on the assumptions and the actual biological meanings of the parameters $a_3(t), b_3(t)$ and $\widetilde{a}_3(t), \widetilde{b}_3(t)$, we can see that the conditions in Theorem 9 are easily satisfied. Additionally, the constant θ represents the minimal total growth rate of the predator species y. If $\theta > 0$, which means that the predator species y has another food resource. As a result, Theorem 9 implies that if $\theta > 0$, then the predator species y will be permanent regardless of whether the prey species x exists or not.

Next, on the permanence of the prey species x of system (4), we have the following result.

Theorem 11. *Suppose that assumptions (H_1)–(H_7) and the following inequality*

$$\int_t^{t+\tau} \frac{(-1)^k+1}{2} a_i(s) + \frac{(-1)^{k+1}}{2}\left(\widetilde{a}_i(s) - c_i(s)\, y^*(s)\right) ds + \sum_{t\le\tau_{k+1}\le t+\tau} \ln h_{ik} > 0 \tag{37}$$

for $(i = 1, 2,\ k \in N)$ hold. Then the prey species x of system (4) is permanent.

Proof. Owing to condition (37), we have that there is a small enough constant $\varepsilon_1 > 0$, such that

$$\int_t^{t+\tau} \frac{(-1)^k+1}{2}\left(a_i(s) - b_i(s)\,\varepsilon_1\right) + \frac{(-1)^{k+1}}{2}\left[\widetilde{a}_i(s) - \widetilde{b}_i(s)\,\varepsilon_1 - c_i(s)\left(y^*(s)+\varepsilon_1\right)\right] ds + \sum_{t<\tau_{k+1}\le(t+\tau)} \ln h_{ik} > 0. \tag{38}$$

Consider the following auxiliary system:

$$\begin{aligned} \dot{n}_i(t) &= n_i(t)\left[a_{ik}(t) - b_{ik}(t)\, n_i(t) - c_{ik}(t)\left(y^*(s)+\varepsilon_1\right)\right] \\ &\quad + d_{ijk}(t)\left[n_j(t-\tau_i) - n_i(t)\right], \quad t \in [\tau_k, \tau_{k+1}), \\ n_i(\tau_{k+1}) &= h_{ik} n_i(\tau_{k+1}^-), \quad t = \tau_{k+1}, \end{aligned} \tag{39}$$

where

$$\begin{aligned} c_{1k}(t) &= \frac{(-1)^{k+1}+1}{2} c_1(t), \\ c_{2k}(t) &= \frac{(-1)^{k+1}+1}{2} c_2(t). \end{aligned} \tag{40}$$

By (38), we have

$$\int_t^{t+\tau} \frac{(-1)^k+1}{2} a_i(s) + \frac{(-1)^{k+1}}{2}\left[\widetilde{a}_i(s) - c_i(s)\left(y^*(s)+\varepsilon_1\right)\right] ds + \sum_{t<\tau_{k+1}\le(t+\tau)} \ln h_{ik} > 0. \tag{41}$$

Then, from (41) and Lemma 5, we know system (39) has a unique globally attractive positive τ-almost periodic solution $n^*_{\varepsilon_1}(t) = (n^*_{1\varepsilon_1}(t), n^*_{2\varepsilon_1}(t))$.

And then, based on assumption H_8 and Lemma 5, for arbitrary constant $\gamma > 0$ the following system

$$\begin{aligned} \dot{u}(t) &= \frac{(-1)^k+1}{2} u(t)\left[a_3(t) - b_3(t)\, u(t)\right] \\ &\quad + \frac{(-1)^{k+1}+1}{2} u(t) \\ &\quad \cdot \left[\widetilde{a}_3(t) + \widetilde{c}_1(t)\,\gamma + \widetilde{c}_2(t)\,\gamma - \widetilde{b}_3(t)\, u(t)\right], \\ &\qquad\qquad t \in [\tau_k, \tau_{k+1}), \\ u(\tau_{k+1}) &= u(\tau_{k+1}^-), \quad t = \tau_{k+1}, \end{aligned} \tag{42}$$

has a unique globally attractive positive τ-almost periodic solution $u^*_\gamma(t)$. For above $\varepsilon_1 > 0$ and $\widehat{M} = \max\{M_2, m_2^{-1}\} > 0$, there is a $t_3 = t(\varepsilon_1, \widehat{M}) > 0$, such that, for any $t_0 \ge 0$ and $u_0 \in [\widehat{M}^{-1}, \widehat{M}]$,

$$\left|u_\gamma(t, t_0, u_0) - u^*_\gamma(t)\right| < \frac{\varepsilon_1}{2} \tag{43}$$

for all $t \ge t_0 + t_3$, where $u_\gamma(t, t_0, u_0)$ is the solution of system (42) with the initial condition $u_\gamma(t_0) = u_0$.

According to the theorem of the continuity of solutions with respect to parameters of the impulsive differential equations [17], there exist $\gamma_0 \in (0, \varepsilon_1)$ and $\gamma_0 < n^*_{\varepsilon_1}(t) - \gamma_0$ such that

$$\left|u^*_{\gamma_0}(t) - y^*(t)\right| < \frac{\varepsilon_1}{2}, \tag{44}$$

for all $t \in [t, t+\tau]$. Also owing to the almost periodic property of $u^*_{\gamma_0}(t)$ and $y^*(t)$, we further have

$$\left|u^*_{\gamma_0}(t) - y^*(t)\right| < \frac{\varepsilon_1}{2}, \tag{45}$$

for all $t \in R$.

Suppose $(x_1(t), x_2(t), y(t))$ is arbitrary positive solution of system (4); from above inequalities (24), (31), and (35), we have

$$\begin{aligned} x(t) &\le M_1, \\ M^{-1} &\le y(t) \le M \end{aligned} \tag{46}$$

for all $t \ge \widehat{T} = \max\{T_1, T_2\}$. If there exists a $t_2 \ge \widehat{T}$, such that $x_i(t) \le \gamma_0$ for all $t \ge t_2$, then from system (4), we can obtain

$$\begin{aligned} \dot{y}(t) &\le \frac{(-1)^k+1}{2} y(t)\left[a_3(t) - b_3(t)\, y(t)\right] \\ &\quad + \frac{(-1)^{k+1}+1}{2} y(t) \\ &\quad \cdot \left[\widetilde{a}_3(t) + \widetilde{c}_1(t)\,\gamma_0 + \widetilde{c}_2(t)\,\gamma_0 - \widetilde{b}_3(t)\, y(t)\right], \\ &\qquad\qquad t \in [\tau_k, \tau_{k+1}), \\ y(\tau_{k+1}) &= y(\tau_{k+1}^-), \quad t = \tau_{k+1}, \end{aligned} \tag{47}$$

for all $t \geq t_2$. By comparison theorem of impulsive differential equations [16, 28, 29], we also have $y(t) \leq u_{\gamma_0}(t)$ for all $t \geq t_2$, where $u_{\gamma_0}(t)$ is the solution of system (42) satisfying $\gamma = \gamma_0$ and with the initial value $u_{\gamma_0}(t_2) = y(t_2)$. For (43), let $t_0 = t_2$ and $u(t_0) = y(t_2)$. From $M^{-1} \leq y(t) \leq M$, we have for $\gamma = \gamma_0$

$$u_{\gamma_0}(t) = u_{\gamma_0}(t, t_2, y(t_2)) < u^*_{\gamma_0}(t) + \frac{1}{2}\varepsilon_1, \tag{48}$$

for all $t \geq t_2 + t_3$. Then $y(t) < u^*_{\gamma_0}(t) + (1/2)\varepsilon_1$ for all $t \geq t_2 + t_3$. Thus, by condition (45), we further get

$$y(t) < y^*(t) + \varepsilon_1. \tag{49}$$

Taking into account system (4) again, from above inequality (49) and system (4), we have

$$\begin{aligned}
&\dot{x}_i(t) \\
&\quad \geq x_i(t)\left[a_{ik}(t) - b_{ik}(t)\, x_i(t) - c_{ik}(t)\left(y^*(t) + \varepsilon_1\right)\right] \\
&\qquad + d_{ijk}(t)\left[x_j(t - \tau_i) - x_i(t)\right], \quad t \in [\tau_k, \tau_{k+1}), \\
&x_i(\tau_{k+1}) = h_{ik} x_i(\tau^-_{k+1}), \quad t = \tau_{k+1},
\end{aligned} \tag{50}$$

for all $t \geq t_2 + t_3$. From Lemma 7, we also have $x_i(t) \geq n_{i\varepsilon_1}(t)$ $(i = 1, 2)$ for all $t \geq t_2 + t_3$, where $n_{\varepsilon_1}(t) = (n_{1\varepsilon_1}(t), n_{2\varepsilon_1}(t))$ is the solution of system (39) satisfying the initial value $n_{i\varepsilon_1}(t_2 + t_3) = x_i(t_2 + t_3)$. Because of the globally attractivity of positive τ-almost periodic solution $n^*_{\varepsilon_1}(t)$ of system (39), we further have that, for previous constant γ_0, there exists a $t_4 \geq t_2 + t_3$ such that

$$n_{\varepsilon_1}(t) \geq n^*_{\varepsilon_1}(t) - \gamma_0 > \gamma_0, \tag{51}$$

for all $t \geq t_4$. Hence, we have $x_i(t) > \gamma_0$ for all $t \geq t_4$, which leads to a contradiction.

Hence, there exists a constant $t_5 \geq t_2$ such that $x_i(t_5) > \gamma_0$. We now prove that

$$x_i(t) \geq \gamma_0 \exp\left\{-\left[\left(M_0 + \varphi_0\right) t_3 + 2\xi_i\right]\right\} \quad \forall t \geq t_5, \tag{52}$$

where

$$\begin{aligned}
\varphi_0 &= \max_{t\in[0,\tau]} \left\{\left|a_{ik}(t) + b_{ik}(t)\,\gamma_0 + c_{ik}(t)\left(y^*(t) + \varepsilon_1\right)\right.\right. \\
&\qquad \left.\left. + d_{ijk}(t)\right|\right\}, \\
M_0 &= \min_{t\in[0,\tau]} \left\{\left|a_{ik}(t) - b_{ik}(t)\, M_1 - c_{ik}(t)\, M - d_{ijk}(t)\right|\right\}.
\end{aligned} \tag{53}$$

If inequality (52) is not true, then from $x_i(t_5) > \gamma_0 > \gamma_0 \exp\{-[(M_0 + \varphi_0)t_3 + 2\xi_i]\}$, there are constants t_6 and t_7 satisfying $t_7 \geq t_6 > t_5$, such that $x_i(t_7) < \gamma_0 \exp\{-[(M_0 + \varphi_0)t_3 + 2\xi_i]\}$, $x_i(t^-_6) \geq \gamma_0$, $x_i(t_6) \leq \gamma_0$. There are two cases for t_7.

Case 1. $t_6 = t_7$.

Case 2. $t_6 < t_7$.

For Case 1, we see that t_6 is an impulsive time. Hence, there is an integer $k > 0$ such that $t_6 = t_k$. We have

$$\begin{aligned}
\gamma_0 \exp\left\{-\left[\left(M_0 + \varphi_0\right) t_3 + 2\xi_i\right]\right\} &> x_i(t_6) = h_{ik} x_i(t^-_6) \\
&\geq \gamma_0 \exp(\ln h_{ik}) \geq \gamma_0 \exp(-\xi_i) \\
&\geq \gamma_0 \exp\left\{-\left[\left(M_0 + \varphi_0\right) t_3 + 2\xi_i\right]\right\},
\end{aligned} \tag{54}$$

which leads to a contradiction.

For Case 2, we have $x_i(t) < \gamma_0$, for all $t \in (t_6, t_7]$. Assume $t_7 - t_6 \leq t_3$, then for any $t \in [t_6, t_7)$ we have

$$\begin{aligned}
&\dot{x}_i(t) \\
&\quad \geq x_i(t)\left[a_{ik}(t) - b_{ik}(t)\, M_1 - c_{ik}(t)\, M\right] \\
&\qquad - d_{ijk}(t)\, x_i(t) \\
&\quad \geq x_i(t)\left[a_{ik}(t) - b_{ik}(t)\, M_1 - c_{ik}(t)\, M - d_{ijk}(t)\right] \\
&\quad \geq x_i(t)\, M_0, \quad t \in [\tau_k, \tau_{k+1}), \\
&x_i(\tau_{k+1}) = h_{ik} x_i(\tau^-_{k+1}), \quad t = \tau_{k+1},
\end{aligned} \tag{55}$$

where

$$\begin{aligned}
M_0 &= \min_{t\in[0,\tau]} \left\{\left|a_{ik}(t) - b_{ik}(t)\, M_1 - c_{ik}(t)\, M - d_{ijk}(t)\right|\right\}.
\end{aligned} \tag{56}$$

Integrating (55) from t_6 to t, for any $t \in [t_6, t_7]$, we obtain

$$\begin{aligned}
x_i(t) &\geq x_i(t_6) \\
&\quad \cdot \exp\left\{\int_{t_6}^{t} \left[a_{ik}(s) - b_{ik}(s)\, M_1 - c_{ik}(s)\, M - d_{ijk}(s)\right] ds \right. \\
&\qquad \left. + \sum_{t_6 < \tau_{k+1} \leq t} \ln h_{ik}\right\} \geq \gamma_0 \exp\left[-\left(M_0 t_3 + \xi_i\right)\right].
\end{aligned} \tag{57}$$

Assume $t_7 - t_6 > t_3$; then when $t \in [t_6, t_6 + t_3]$, according to above discussion we directly have

$$x_i(t) \geq \gamma_0 \exp\left[-\left(M_0 t_3 + \xi_i\right)\right]. \tag{58}$$

Particularly,

$$x_i(t_6 + t_3) \geq \gamma_0 \exp\left[-\left(M_0 t_3 + \xi_i\right)\right]. \tag{59}$$

Since $x_i(t) \leq \gamma_0$ for all $t \in (t_6, t_7]$. Then we have

$$\begin{aligned}
&\dot{y}(t) \leq \frac{(-1)^k + 1}{2}\, y(t)\left[a_3(t) - b_3(t)\, y(t)\right] \\
&\qquad + \frac{(-1)^{k+1} + 1}{2}\, y(t) \\
&\qquad \cdot \left[\widetilde{a}_3(t) + \widetilde{c}_1(t)\,\gamma_0 + \widetilde{c}_2(t)\,\gamma_0 - \widetilde{b}_3(t)\, y(t)\right], \\
&\qquad\qquad\qquad\qquad t \in [\tau_k, \tau_{k+1}), \\
&y(\tau_{k+1}) = y(\tau^-_{k+1}), \quad t = \tau_{k+1},
\end{aligned} \tag{60}$$

for $t \in (t_6, t_7]$. From comparison theorem of impulsive differential equations [16, 28, 29], we also have $y(t) \leqslant u_{\gamma_0}(t)$ for all $t \in (t_6, t_7)$, where $u_{\gamma_0}(t)$ is the solution of system (42) with $\gamma = \gamma_0$ and satisfies $u_{\gamma_0}(t_6) = y(t_6)$. In (43), choose a $t_0 = t_6$ and $u_0 = y(t_6)$, since $M^{-1} \leq y(t_6) \leq M$, we have for $\gamma = \gamma_0$

$$u_{\gamma_0}(t) = u_{\gamma_0}(t, t_6, y(t_6)) \leq u^*_{\gamma_0}(t) + \frac{1}{2}\varepsilon_1 \quad \forall t \geq t_6 + t_3. \tag{61}$$

Hence,

$$y(t) < u^*_{\gamma_0} + \frac{\varepsilon_1}{2} \quad \forall t \geq t_6 + t_3. \tag{62}$$

Thus, from (45) we further have

$$y(t) < y^*(t) + \varepsilon_1 \quad \forall t \geq t_6 + t_3. \tag{63}$$

From (63), we have

$$\begin{aligned} &\dot{x}_i(t) \\ &\quad \geq x_i(t)\left[a_{ik}(t) - b_{ik}(t)\gamma_0 - c_{ik}(t)\left(y^*(t) + \varepsilon_1\right)\right] \\ &\qquad - d_{ijk}(t)\, x_i(t) \quad t \in \left[\tau_k, \tau_{k+1}\right), \\ &x_i\left(\tau_{k+1}\right) = h_{ik} x_i\left(\tau^-_{k+1}\right), \quad t = \tau_{k+1}, \end{aligned} \tag{64}$$

For arbitrary $t \in [t_6 + t_3, t_7)$. We choose an integer $p \geq 0$ such that $t_7 \in [t_6 + t_3 + p\tau, t_6 + t_3 + (p+1)\tau)$; then integrating (64) from $t_6 + t_3$ to t_7, we have

$$\begin{aligned} &\gamma_0 \exp\left\{-\left[\left(M_0 + \varphi_0\right)t_3 + 2\xi_i\right]\right\} > x_i(t_7) = x_i\left(t_6 + t_3\right) \\ &\cdot \exp\left\{\int_{t_6+t_3}^{t_7}\left[a_{ik}(s) - b_{ik}(s)\gamma_0\right.\right. \\ &\left. - c_{ik}(s)\left(y^*(s) + \varepsilon_1\right) - d_{ijk}(s)\right]ds \\ &\left.+ \sum_{t_6+t_3 < \tau_{k+1} \leq t_7} \ln h_{ik}\right\} \geq \gamma_0 \exp\left[-\left(M_0 t_3 + \xi_i\right)\right] \\ &\cdot \exp\left\{\left(\int_{t_6+t_3}^{t_6+t_3+p\omega} + \int_{t_6+t_3+p\omega}^{t_7}\right)\left[a_{ik}(s) - b_{ik}(s)\gamma_0\right.\right. \\ &\left. - c_{ik}(s)\left(y^*(s) + \varepsilon_1\right) - d_{ijk}(s)\right]ds \\ &\left.+ \sum_{t_6+t_3 < \tau_{k+1} \leq t_7} \ln h_{ik}\right\} \geq \gamma_0 \exp\left[-\left(M_0 t_3 + \xi_i\right)\right. \\ &\left. - \left(\varphi_0 t_3 + \xi_i\right)\right] \geq \gamma_0 \exp\left\{-\left[\left(M_0 + \varphi_0\right)t_3 + 2\xi_i\right]\right\}, \end{aligned} \tag{65}$$

which leads to a contradiction. According to the discussion above, we finally have

$$\liminf_{t\to\infty} x_i(t) \geq m_1 \triangleq \gamma_0 \exp\left\{-\left[\left(M_0 + \varphi_0\right)t_3 + 2\xi_i\right]\right\}. \tag{66}$$

Obviously, m_1 is independent of any positive solutions of system (4). This completes the proof of Theorem 11. □

Remark 12. Set

$$\begin{aligned} \lambda = &\int_t^{t+\tau} \frac{(-1)^k + 1}{2} a_i(s) \\ &+ \frac{(-1)^{k+1}}{2}\left(\widetilde{a}_i(s) - c_{ik}(s)\, y^*(t)\right)ds \\ &+ \sum_{t \leq \tau_{k+1} \leq t+\tau} \ln h_{ik}. \end{aligned} \tag{67}$$

Since $y^*(t)$ is the globally asymptotically stable $\tau-$ periodic solution of the prey-free subsystem (14) of system (4), we obtain that the integral $\int_t^{t+\tau}(((-1)^k + 1)/2)a_i(s) + ((-1)^{k+1}/2)(\widetilde{a}_i(s) - c_{ik}(s)y^*(t))ds$ is the net total growth rate of species x within a whole almost period, while $\sum_{t \leq \tau_{k+1} \leq t+\tau} \ln h_{ik}$ is the maximal total survivability of species x within the process in which species x disperses between patches. Therefore, λ is the net total growth rate of species x during a whole period of transference. Theorem 11 tells us that if $\lambda > 0$, then species x in system (4) is permanent.

4. Numerical Simulation and Discussion

In this paper, an almost periodic predator-prey model with intermittent prey dispersals and intermittent predation has been investigated. By using the methods of analysis and comparison theorems of the impulsive differential equations, we obtain sufficient conditions of the boundedness and permanence for this system.

To illustrate our results for system (4), we give some numerical simulations by using the following values of parameters in Table 1.

Due to system (4) being an almost periodic system, we will show the numerical simulation on the following eight intervals in Table 2.

Therefore, we can consider almost periodic system as periodic approximately and the $\tau-$ period is 10.

First, for system (12), when we take each parameter as in Table 1, we have

$$\begin{aligned} &\int_0^{10} a_1(s)\, ds + \sum_{0 < \tau_{k+1} \leq 10} \ln d_1 \approx 4.3918 > 0, \\ &\int_0^{10} \widetilde{a}_1(s)\, ds + \sum_{0 < \tau_{k+1} \leq 10} \ln D_1 \approx 5.4615 > 0, \\ &\int_0^{10} a_2(s)\, ds + \sum_{0 < \tau_{k+1} \leq 10} \ln d_2 \approx 5.0316 > 0, \\ &\int_0^{10} \widetilde{a}_2(s)\, ds + \sum_{0 < \tau_{k+1} \leq 10} \ln D_2 \approx 5.6074 > 0. \end{aligned} \tag{68}$$

We can see that all the assumptions (H_1)–(H_7) of Lemma 5 hold. Therefore, system (12) has a unique positive globally attractive almost periodic solution $x^*(t) = (x_1^*(t), x_2^*(t))$. Numerical simulations of these results can be observed in Figures 1(a) and 1(b). From numerical simulations, we obtain

Table 1: Parameter values used in the simulations of system (4).

Parameter	Value
$a_1(t)$	$0.5 + 0.2\sin(1.2\pi t) + 0.2\sin(1.5\pi t)$
$\widetilde{a}_1(t)$	$0.6 + 0.14\sin(1.2\pi t) + 0.14\sin(1.5\pi t)$
$a_2(t)$	$0.55 + 0.1\sin(1.2\pi t) + 0.1\sin(1.5\pi t)$
$\widetilde{a}_2(t)$	$0.6 + 0.09\sin(1.2\pi t) + 0.09\sin(1.5\pi t)$
$a_3(t)$	$0.5 + 0.3\sin(1.2\pi t) + 0.2\sin(1.5\pi t)$
$\widetilde{a}_3(t)$	$0.1 + 0.12\sin(1.2\pi t) + 0.12\sin(1.5\pi t)$
$b_1(t)$	$0.25 + 0.1\cos(1.2\pi t) + 0.1\cos(1.5\pi t)$
$\widetilde{b}_1(t)$	$0.4 + 0.04\cos(1.2\pi t) + 0.04\cos(1.5\pi t)$
$b_2(t)$	$0.3 + 0.04\cos(1.2\pi t) + 0.04\cos(1.5\pi t)$
$\widetilde{b}_2(t)$	$0.4 + 0.05\cos(1.2\pi t) + 0.05\cos(1.5\pi t)$
$b_3(t)$	$0.2 + 0.1\cos(1.2\pi t) + 0.2\cos(1.5\pi t)$
$\widetilde{b}_3(t)$	$0.5 + 0.10\cos(1.2\pi t) + 0.10\cos(1.5\pi t)$
$c_1(t)$	$0.2 + 0.2\sin(1.2\pi t) + 0.2\sin(1.5\pi t)$
$\widetilde{c}_1(t)$	$0.10 + 0.1\sin(1.2\pi t) + 0.1\sin(1.5\pi t)$
$c_2(t)$	$0.15 + 0.2\sin(1.2\pi t) + 0.2\sin(1.5\pi t)$
$\widetilde{c}_2(t)$	$0.15 + 0.2\sin(1.2\pi t) + 0.2\sin(1.5\pi t)$
$D_{12}(t)$	$0.5 + 0.05\cos(1.2\pi t) + 0.05\cos(1.5\pi t)$
$D_{21}(t)$	$0.5 + 0.05\cos(1.2\pi t) + 0.05\cos(1.5\pi t)$
d_1	0.5
d_2	0.6
D_1	0.55
D_2	0.65
τ_1	1.3
τ_2	1.6

the minimum values of $(x_1(t), x_2(t))$, which are (0.6988, 0.8355).

Then, for system (14), when we also take the above parameters $a_{3k}(t), b_{3k}(t)$, we have

$$\begin{aligned}&\int_0^{10} a_3(s)\,ds + \sum_{0<\tau_{k+1}\le 10} \ln 1 \approx 5.0509 > 0,\\ &\int_0^{10} \widetilde{a}_3(s)\,ds + \sum_{0<\tau_{k+1}\le 10} \ln 1 \approx 1.0509 > 0.\end{aligned} \tag{69}$$

We can easily demonstrate that the assumptions (H_1)-(H_2) and (H_7)-(H_8) of Lemma 6 and Theorem 9 hold. Obviously, system (14) has a unique almost periodic solution $y^*(t)$, and the predator species y of system (4) always has the minimum value of 0.3528 and the maximum value of 1.9017 when taking different initial values; that is, it is permanent. All of these results can be observed in Figure 2(a). However, for system (4), when we keep the other parameters unchanged and only just adjust the value of $\widetilde{a}_3(t) = -0.65 + 0.12\sin(1.2\pi t) + 0.12\sin(1.5\pi t)$, then we have

$$\int_0^{10} \widetilde{a}_3(s)\,ds + \sum_{0<\tau_{k+1}\le 10} \ln 1 \approx -6.4491 < 0. \tag{70}$$

This is contrary to the assumptions (H_1)-(H_2) and (H_7)-(H_8) of Theorem 9, that is, in the whole period, if $\theta < 0$ without considering the prey species x, then the predator species y of system (4) will be extinct (see Figure 2(b)). Nevertheless, if we take the prey species x into account, using the above parameters $\widetilde{c}_1(t), \widetilde{c}_2(t)$ and the minimum values of $(x_1(t), x_2(t))$ with 0.6988, 0.8355 to make

$$\begin{aligned}&\int_0^5 a_3(s)\,ds\\ &\quad + \int_5^{10} \left[\widetilde{a}_3(s) + 0.6988 \times \widetilde{c}_1(s) + 0.8355 \times \widetilde{c}_2(s)\right] ds\\ &\quad + \sum_{0<\tau_{k+1}\le 10} \ln 1 \approx 0.6275 > 0,\end{aligned} \tag{71}$$

then we find an interesting phenomenon where the predator species y of system (4) goes from extinct to permanent (see Figure 3(a)). It turns out that if the predator species y in the time intervals $[\tau_{2k+1}, \tau_{2k+2})$ has no other food resource but can only rely on the prey species x, then the more the predation behavior happens, the more likely the predator species will be permanent.

Moreover, to illustrate the difference between continuous and intermittent predation of the predator species y, we consider the following system [25]:

$$\begin{aligned}&\dot{x}_1(t)\\ &\quad = x_1(t)\left[a_1(t) - b_1(t)x_1(t) - c_1(t)y(t)\right]\\ &\qquad + D_{12}(t)\left[x_2(t) - x_1(t)\right],\\ &\dot{x}_2(t)\\ &\quad = x_2(t)\left[a_2(t) - b_2(t)x_2(t) - c_2(t)y(t)\right]\\ &\qquad + D_{21}(t)\left[x_1(t) - x_2(t)\right],\\ &\dot{y}(t)\\ &\quad = y(t)\left[a_3(t) - b_3(t) + \widetilde{c}_1(t)x_1(t) + \widetilde{c}_2(t)x_2(t)\right].\end{aligned} \tag{72}$$

We keep the other parameters unchanged including the intrinsic growth rates of the species x, the interspecies competition rates of all species, the survival rates of the species x, and the dispersal rates of the species x in system (72). We take another set of parameters in Table 3, and we let $\tau_i = 0$, $d_i = D_i = 1$ $(i = 1, 2)$ for the purpose of contrast.

By simple calculation, we have

$$\begin{aligned}&\int_0^{10} \left[a_3(s) + 0.6988 \times \widetilde{c}_1(s) + 0.8355 \times \widetilde{c}_2(s)\right] ds\\ &\quad + \sum_{0<\tau_{k+1}\le 10} \ln 1 \approx 5.7530 > 0,\\ &\int_0^{10} \left[a_1(s) - 1.9017 \times c_1(s)\right] ds + \sum_{0<\tau_{k+1}\le 10} \ln 1\\ &\quad \approx -0.7009 < 0,\\ &\int_0^{10} \left[a_2(s) - 1.9017 \times c_2(s)\right] ds + \sum_{0<\tau_{k+1}\le 10} \ln 1\\ &\quad \approx -0.2434 < 0.\end{aligned} \tag{73}$$

Table 2: Parameter values used in the simulations of system (4).

Number	Interval	$\tau_{k+1} - \tau_k$	Interval length	Process
1	[0, 5)	$\tau_1 - \tau_0$	5	1
2	[5, 9)	$\tau_2 - \tau_1$	4	2
3	[14, 20)	$\tau_3 - \tau_2$	6	1
4	[20, 25)	$\tau_4 - \tau_3$	5	2
5	[25, 28)	$\tau_5 - \tau_4$	3	1
6	[28, 35)	$\tau_6 - \tau_5$	7	2
7	[40, 46)	$\tau_7 - \tau_6$	6	1
8	[46, 50)	$\tau_8 - \tau_7$	4	2

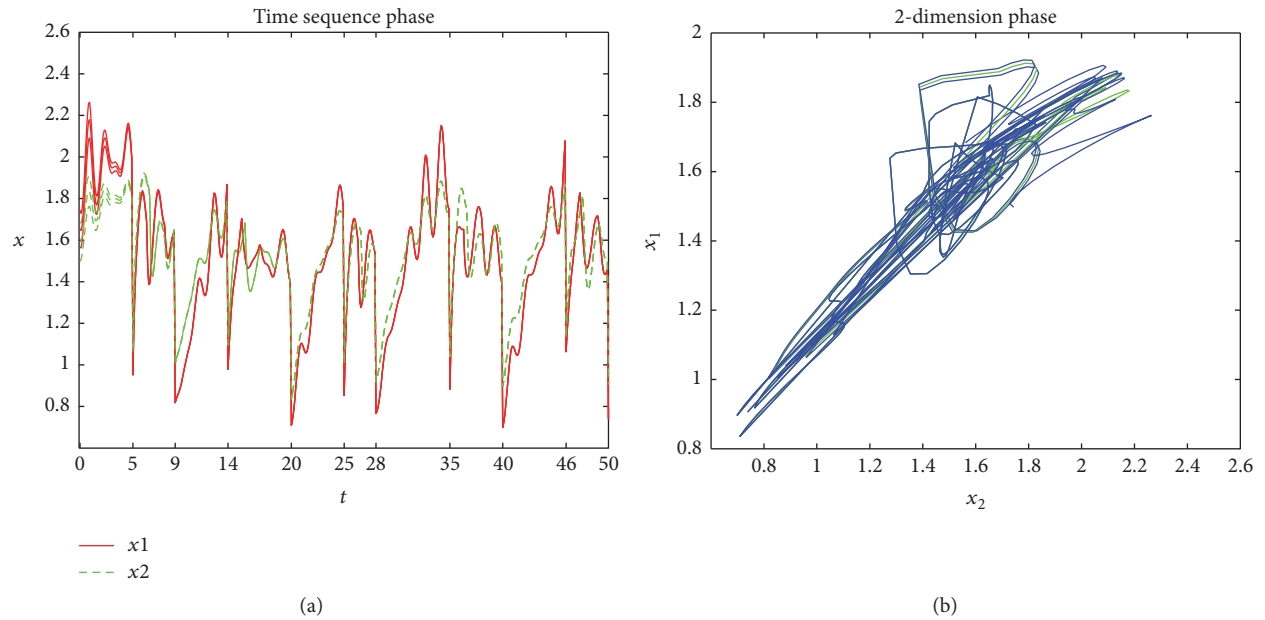

Figure 1: (a) The globally stable positive almost periodic solution of the species x of system (12). (b) The portrait of the globally stable positive almost periodic solution of the species x of system (12). Here, we take the initial values $\phi_i(s) = (\phi_1(s), \phi_2(s)) = (1.5, 1.8), (1.6, 1.7), (1.7, 1.6)$ for all $s \in [-5, 0]$.

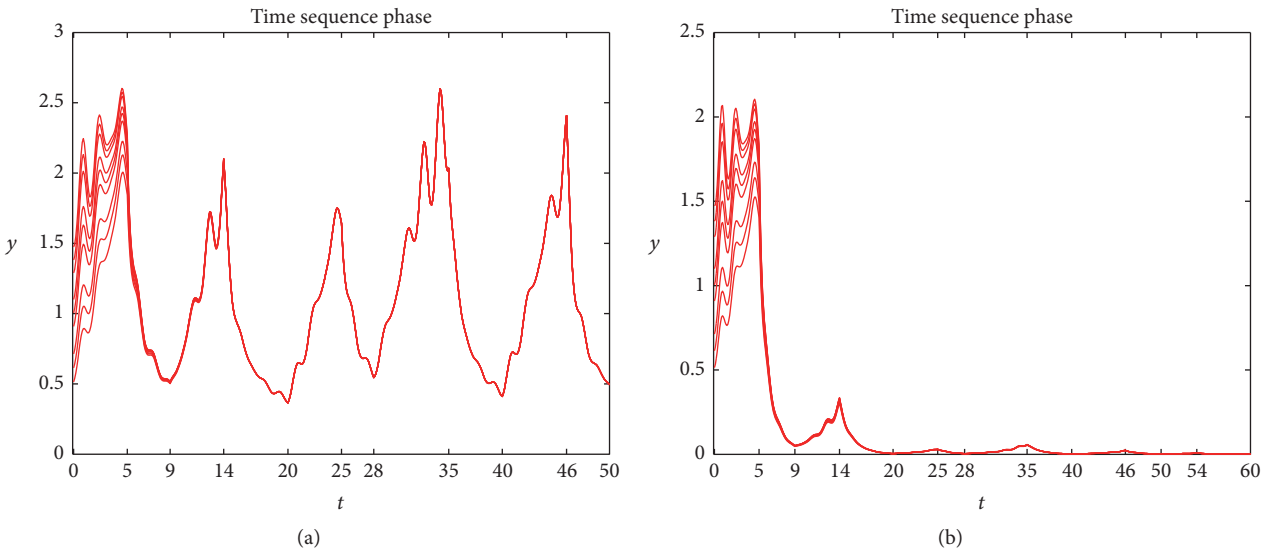

Figure 2: (a) The globally stable positive almost periodic solution of the species y of system (14), and we take the initial condition $\phi_i(s) = (1.2, 0.8, 0.4)$ for all $s \in [-5, 0]$. (b) Extinction of the species y of system (14), and we take the initial values $\phi_i(s) = (0.8, 0.4)$ for all $s \in [-5, 0]$.

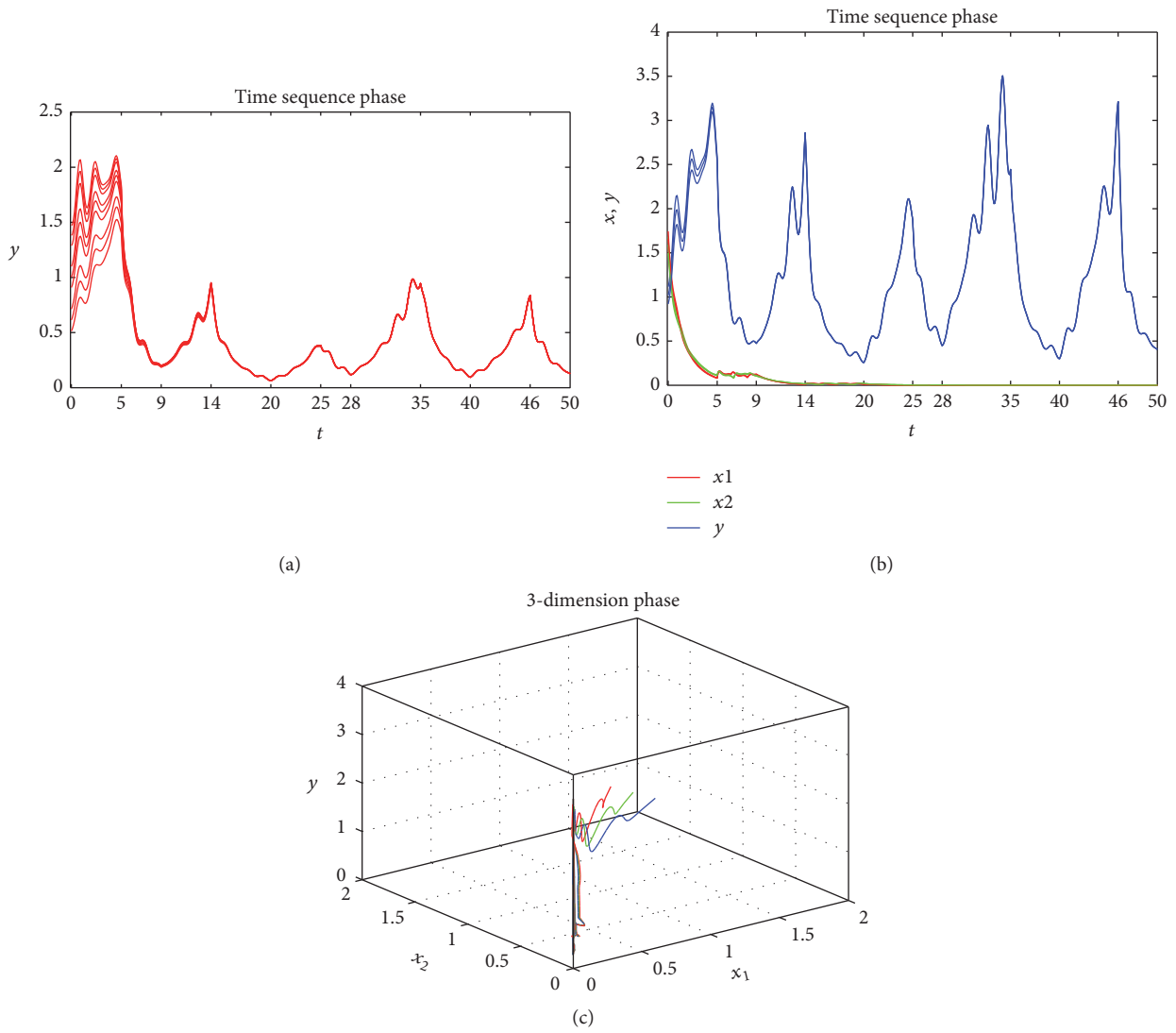

FIGURE 3: (a) The globally stable positive almost periodic solution of the species y of system (14), and the predator species y is permanent just relying on the predation; here, we take the initial condition $\phi_i(s) = (1.2, 0.8, 0.4)$ for all $s \in [-5, 0]$. (b) The time series of almost periodic solution of the species x, y of system (72). (c) The 3-dimension time series of almost periodic solution of the species x, y of system (72).

TABLE 3: Parameter values used in the simulations of system (4).

Parameter	Value
$a_3(t)$	$0.1 + 0.3\sin(1.2\pi t) + 0.2\sin(1.5\pi t)$
$\widetilde{a}_3(t)$	$-0.6 + 0.12\sin(1.2\pi t) + 0.12\sin(1.5\pi t)$
$c_1(t)$	$0.3 + 0.1\sin(1.2\pi t) + 0.1\sin(1.5\pi t)$
$\widetilde{c}_1(t)$	$0.3 + 0.14\sin(1.2\pi t) + 0.1\sin(1.5\pi t)$
$c_2(t)$	$0.3 + 0.1\sin(1.2\pi t) + 0.1\sin(1.5\pi t)$
$\widetilde{c}_2(t)$	$0.3 + 0.14\sin(1.2\pi t) + 0.1\sin(1.5\pi t)$

Then, we find that continuous predation makes the predator permanent and the prey extinct (see Figures 3(b) and 3(c)). However, if we keep all others parameters unchanged and just change system (72) to system (4), that is, changing the model from continuous predation to intermittent predation, we have

$$\int_0^5 a_3(s)\,ds + \int_5^{10} \left[\widetilde{a}_3(s) + 0.6988 \times \widetilde{c}_1(s) + 0.8355 \times \widetilde{c}_2(s)\right] ds$$

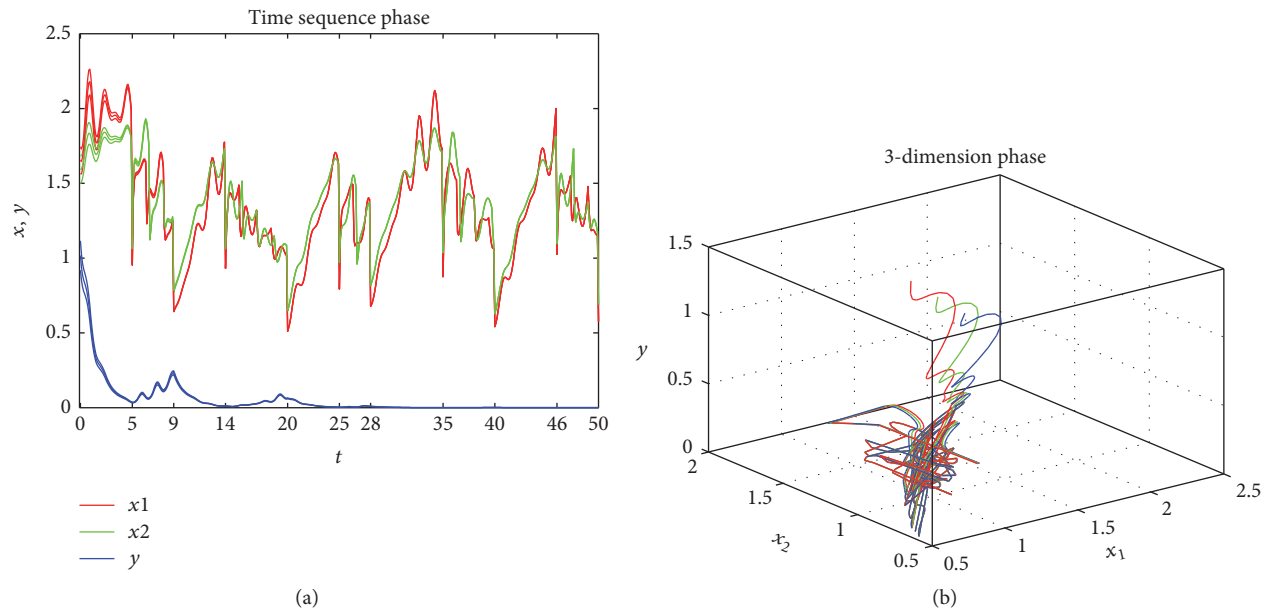

FIGURE 4: (a) The time series of almost periodic solution of the species x of the predator-prey model with continuous predation. (b) The time series of almost periodic solution of the species x of the predator-prey model with intermittent predation. Here, we take the initial condition $\phi_i(s) = (\phi_1(s), \phi_2(s)) = (1.5, 1.8), (1.6, 1.7), (1.7, 1.6)$ for all $s \in [-5, 0]$.

$$
\begin{aligned}
&+ \sum_{0<\tau_{k+1}\le 10} \ln 1 \approx -0.0981 < 0,\\
&\int_0^5 a_1(s)\,ds + \int_5^{10} \left[\widetilde{a}_1(s) - 1.9017 \times c_1(s)\right] ds\\
&+ \sum_{0<\tau_{k+1}\le 10} \ln 1 \approx 2.6762 > 0,\\
&\int_0^5 a_2(s)\,ds + \int_5^{10} \left[\widetilde{a}_1(s) - 1.9017 \times c_2(s)\right] ds\\
&+ \sum_{0<\tau_{k+1}\le 10} \ln 1 \approx 2.8974 > 0.
\end{aligned} \tag{74}
$$

Then, there are some interesting phenomena that the predator goes from permanent to extinct, while the opposite occurs for the prey. All of these results can be observed (Figures 4(a) and 4(b)) by numerical simulations. By the above analysis, we can conclude that if the predator continues preying on prey and the predation intensity is higher than the intrinsic growth of the prey, then the prey will be extinct, while the predator will be permanent. We also obtain that if the predation behavior is intermittent and the predation intensity is small, then the predator will be extinct, while the prey will be permanent. So, for an intermittent predator-prey model, both the intermittent predation and the intrinsic growth rates of the prey and predator species can greatly impact the permanence or extinction of the system.

Finally, for system (4), to test our main results, we use the above parameters in Table 1. We can simply calculate that the assumptions (H_1)–(H_7) in Theorem 11 hold, and for (37), we let $y^*(t) = 1.9017$ then

$$
\begin{aligned}
&\int_0^{10} a_1(s)\,ds + \sum_{0<\tau_{k+1}\le 10} \ln d_1 \approx 4.3918 > 0,\\
&\int_0^{10} \left[\widetilde{a}_1(s) - 1.9017 \times \widetilde{c}_1(s)\right] ds + \sum_{0<\tau_{k+1}\le 10} \ln D_1\\
&\approx 1.4967 > 0,\\
&\int_0^{10} a_2(s)\,ds + \sum_{0<\tau_{k+1}\le 10} \ln d_2 \approx 5.0316 > 0,\\
&\int_0^{10} \left[\widetilde{a}_2(s) - 1.9017 \times \widetilde{c}_1(s)\right] ds + \sum_{0<\tau_{k+1}\le 10} \ln D_2\\
&\approx 2.5934 > 0.
\end{aligned} \tag{75}
$$

From numerical simulations, we obtain that the minimum value of x of system (3) is always 0.8085 when using different initial values; that is, the prey x is permanent (see Figures 5(a) and 5(b)).

Conflicts of Interest

All the authors declare that there are no conflicts of interest regarding the publication of this paper.

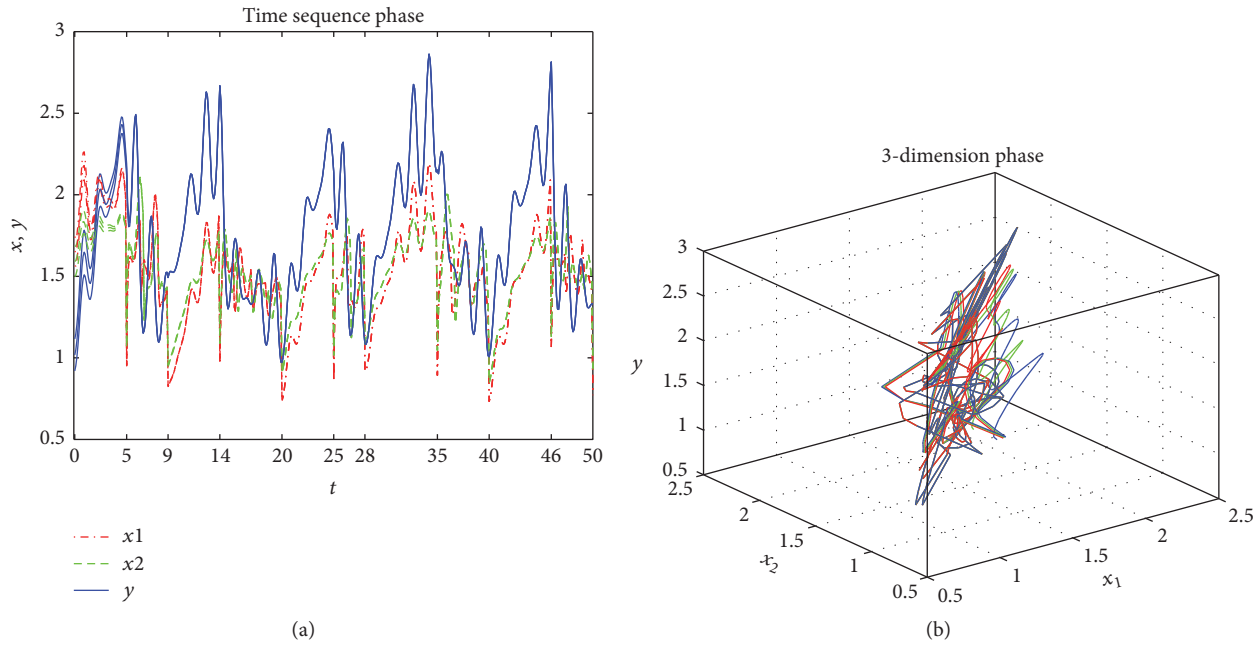

FIGURE 5: (a) The time series of the globally stable positive almost periodic solution of the species x. (b) The portrait of the globally stable positive almost periodic solution of the species x, and we take the initial condition $\phi_i(s) = (\phi_1(s), \phi_2(s)) = (1.5, 1.8), (1.6, 1.7), (1.7, 1.6)$ for all $s \in [-5, 0]$.

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