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Research Article

Output Feedback Stabilization for Stochastic Nonholonomic Systems under Arbitrary Switching

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The output feedback controllers of stochastic nonholonomic systems under arbitrary switching are discussed. We adopt an observer which can simplify the design process. The designed control laws cause the calculation of the gain parameter to be very convenient since the denominator of virtual controllers does not contain the gain parameter. Finally, an example is given to show the effectiveness of controllers.

1. Introduction

In recent years, switched system's control, especially under arbitrary switching, has become an active field [1–3]. The global stabilization of switched systems based on arbitrary switching was given [4–6]. The stabilizing controllers of switched systems with arbitrary switching were given [7–10]. The output feedback controllers of nonlinear systems with arbitrary switching were designed [11, 12].

In the past ten years, the problem of stabilization for stochastic nonholonomic systems (SNSs) received much attention. They mainly can be classified into two types. The first is state-feedback control: stabilization [13, 14], adaptive stabilization [15–18], finite-time stabilization [19], stabilization with time-varying delays [20], and stabilization of mobile robots [21, 22]. The second is the output feedback stabilization [23–25].

Since sometimes part of the system states are unmeasurable, output feedback controllers are needed. Zhang et al. discussed the output feedback stabilizing controllers of SNSs whose virtual control b_i contains gain parameter L [25]. This will lead to a problem where the calculation of L is very difficult, especially for $n \geq 3$, since the inequalities about L were quintic. In addition, to the authors' knowledge, there are some results about state-feedback stabilization of SNSs with Markovian switching [13, 14], with few available results for the output stabilization of SNSs under arbitrary switching. Based on the above analysis, there exists a problem, that is, how to choose a proper observer under arbitrary switching where the virtual control b_i in controllers does not contain gain parameter L, which causes the calculation of L to be easier.

Notations. $\mathbb{R}$ denotes the set of all real numbers. $\mathbb{R}^n$ denotes the real n-dimensional space. For a vector or matrix $X \in \mathbb{R}^{n\times m}$, X^T denotes its transpose, $\|X\|$ denotes the Euclidean norm, $\mathrm{Tr}\{X\}$ is its trace when X is square, and $\mathscr{L}$ is a stochastic differential operator [26].

2. Problem Formulation

The stochastic nonholonomic nonlinear systems are given by

$$dx_0 = u_0 dt + g^T_{[0,\sigma(t)]}(x_0)\, d\omega, \tag{1a}$$

$$\begin{aligned} dx_i &= u_0 x_{i+1} dt + f_{[i,\sigma(t)]}(x_0, \bar{x}_i)\, dt \\ &\quad + g^T_{[i,\sigma(t)]}(x_0, \bar{x}_i)\, d\omega, \quad i = 1, \ldots, n-1, \\ dx_n &= u dt + f_{[n,\sigma(t)]}(x_0, x)\, dt + g^T_{[i,\sigma(t)]}(x_0, x)\, d\omega, \\ y &= [x_0, x_1]^T, \end{aligned} \tag{1b}$$

where u_0 and $u \in \mathbb{R}$ are inputs, $x_0 \in \mathbb{R}$ and $(x_1, \ldots, x_n)^T \in \mathbb{R}^n$ are system states, $\overline{x}_i = (x_1, \ldots, x_i)^T$, $f_i \triangleq f_i(x_0, \overline{x}_i) : \mathbb{R}^{i+1} \to \mathbb{R}$ are smooth functions named as nonlinear drifts with $f_i(0,0) = 0$, and $g_0(x_0) : \mathbb{R} \to \mathbb{R}^m$ and $g_i \triangleq g_i(x_0, \overline{x}_i) : \mathbb{R}^{i+1} \to \mathbb{R}^m$ are smooth functions with $g_0(0) = 0$ and $g_i(0,0) = 0$, $1 \le i \le n$. $\sigma(t) : [0, +\infty) \to M = \{1, 2, \ldots, m\}$ is a piecewise constant switching signal, and $\omega \in \mathbb{R}^m$ is an m-dimensional independent standard Wiener process.

Assumption 1. If there exist smooth function $\nu_{[0,\sigma(t)]}(x_0) : \mathbb{R} \to \mathbb{R}^m$ and positive constant $\overline{m}$, then

$$\begin{aligned} g_{[0,\sigma(t)]}(x_0) &= x_0 \nu_{[0,\sigma(t)]}(x_0), \\ \left\| \nu_{[0,\sigma(t)]}(x_0) \right\| &\le \overline{m}, \end{aligned} \tag{2}$$

where $\nu_{[0,\sigma(t)]}(x_0)$ is a known bounded nonnegative function.

Assumption 2. For $i = 1, \ldots, n$, one has known constants $\overline{\nu}_{[i,\sigma(t)]}$ and $\overline{\gamma}_{[i,\sigma(t)]}$ satisfying

$$\begin{aligned} \left| f_{[i,\sigma(t)]}(x_0, \overline{x}_i) \right| &\le (|x_1| + \cdots + |x_i|)\, \overline{\nu}_{[i,\sigma(t)]}, \\ \left\| g_{[i,\sigma(t)]}(x_0, \overline{x}_i) \right\| &\le (|x_1| + \cdots + |x_i|)\, \overline{\gamma}_{[i,\sigma(t)]}. \end{aligned} \tag{3}$$

Remark 3. There are two main differences between systems (1a) and (1b) and those in [24, 25]. The first is the arbitrary switching mentioned in this paper. The second will be illustrated in Remark 7. In addition, Assumptions 1 and 2 are similar to those in [24, 25], but in fact they hold under arbitrary switching; for example, $\overline{\gamma}_{[i,\sigma(t)]} = \max\{\overline{\gamma}_{[i,1]}, \overline{\gamma}_{[i,2]}, \overline{\gamma}_{[i,3]}, \overline{\gamma}_{[i,4]}\}$ when $\sigma(t) : [0, +\infty) \to M = \{1, 2, 3, 4\}$.

3. Output Feedback Stabilization

The controller design in Sections 3.1 and 3.2 is under $x_0(t_0) \neq 0$. The other one is discussed in Section 3.3.

3.1. Controller u_0 Design. For subsystem (1a), one obtains

$$\mathscr{L}V_0(x_0) \le -2\lambda V_0(x_0). \tag{4}$$

with Lyapunov function $V_0(x_0) = (1/2)x_0^2$ and controller u_0 as follows:

$$\begin{aligned} u_0 &= -\eta_0 x_0, \\ \eta_0 &= \lambda + \frac{1}{2}\overline{m}^2, \end{aligned} \tag{5}$$

where $\lambda > 0$ is a design real number.

Theorem 4. *For system (1a), the closed-loop system with controller (4) is asymptotically stable in probability.*

By (4) and (1a), one has

$$dx_0 = -\eta_0 x_0 dt + g_{[0,\sigma(t)]}^T(x_0)\, d\omega. \tag{6}$$

Remark 5. For any $x_0(t_0) \neq 0$, one has $x_0(t) \neq 0$ in (6) at the time interval $t \in (t_0, +\infty)$ a.s. with a similar proof of Proposition 1 in [25].

3.2. Controller u Design. In order to design controller u, let

$$z_i = \frac{x_i}{u_0^{n-i}}, \quad 1 \le i \le n. \tag{7}$$

Remark 6. For any $x_0(t_0) \neq 0$, from Remark 5, we have that (7) is meaningful a.s.

By (1b) and (7), one has

$$\begin{aligned} dz_i &= z_{i+1} dt + \phi_{[i,\sigma(t)]}(x_0, \overline{z}_i)\, dt + \psi_{[i,\sigma(t)]}^T(x_0, \overline{z}_i)\, d\omega, \\ &\qquad\qquad i = 1, \ldots, n-1, \\ dz_n &= u dt + \phi_{[n,\sigma(t)]}(x_0, x)\, dt + \psi_{[n,\sigma(t)]}^T(x_0, x)\, d\omega, \\ y_1 &= z_1, \end{aligned} \tag{8}$$

where $\overline{z}_i = [z_1, \ldots, z_i]^T$ and

$$\begin{aligned} &\phi_{[i,\sigma(t)]}(x_0, \overline{z}_i) \\ &\quad = \eta_0 (n-i) z_i + \frac{f_{[i,\sigma(t)]}}{u_0^{n-i}} \\ &\qquad + \frac{1}{2}(n-i)(n-i+1) z_i \frac{g_{[0,\sigma(t)]}^T g_{[0,\sigma(t)]}}{x_0^2} \\ &\qquad + \eta_0 (n-i) \frac{g_{[i,\sigma(t)]}^T g_{[0,\sigma(t)]}}{u_0^{n-i+1}}, \\ &\psi_{[i,\sigma(t)]}(x_0, \overline{z}_i) = \frac{g_{[i,\sigma(t)]}}{u_0^{n-i}} - (n-i) z_i \frac{g_{[0,\sigma(t)]}}{x_0}, \\ &\qquad\qquad i = 1, \ldots, n. \end{aligned} \tag{9}$$

We adopt the following observer [27]:

$$\begin{aligned} \dot{\hat{z}}_i &= \hat{z}_{i+1} + L^i a_i (y_1 - \hat{z}_1), \quad i = 1, \ldots, n-1, \\ \dot{\hat{z}}_n &= u + L^n a_n (y_1 - \hat{z}_1), \end{aligned} \tag{10}$$

where $L \ge 1$ is a gain parameter and real numbers $a_i > 0$, $i = 1, \ldots, n$, such that $p(s) = s^n + a_1 s^{n-1} + \cdots + a_n$ is Hurwitz.

Remark 7. The second main difference between this manuscript and [25] is that the observer (10) we choose is the same as that in [27], but different from that in [25]. This observer has two advantages. The first is that it can simplify the process of designing controllers. The second is that b_i in virtual control α_i that we design in the following does not contain the gain parameter L.

Denoting

$$\xi_i = \frac{(z_i - \hat{z}_i)}{L^{i-1}}, \quad i = 1, \ldots, n, \tag{11}$$

one has the error systems

$$d\xi = LA\xi dt + \Phi_{\sigma(t)} dt + \Psi_{\sigma(t)}^T d\omega, \tag{12}$$

where

$$A = \begin{bmatrix} -a_1 & \\ -a_2 & I_{n-1} \\ \vdots & \\ -a_n & 0 \end{bmatrix},$$

$$\Phi_{\sigma(t)} = \begin{bmatrix} \phi_{[1,\sigma(t)]} \\ \frac{\phi_{[2,\sigma(t)]}}{L} \\ \vdots \\ \frac{\phi_{[n-1,\sigma(t)]}}{L^{n-2}} \\ \frac{\phi_{[n,\sigma(t)]}}{L^{n-1}} \end{bmatrix}, \quad (13)$$

$$\Psi^T_{\sigma(t)} = \begin{bmatrix} \psi^T_{[1,\sigma(t)]} \\ \frac{\psi^T_{[2,\sigma(t)]}}{L} \\ \vdots \\ \frac{\psi^T_{[n-1,\sigma(t)]}}{L^{n-2}} \\ \frac{\psi^T_{[n,\sigma(t)]}}{L^{n-1}} \end{bmatrix}.$$

Therefore, for positive definite matrix P, A is a Hurwitz matrix; that is, $A^TP + PA = -I$. Now, one has

$$\begin{aligned} \mathrm{d}\xi &= LA\xi \mathrm{d}t + \Phi_{\sigma(t)}\mathrm{d}t + \Psi^T_{\sigma(t)}\mathrm{d}\omega, \\ \mathrm{d}y_1 &= z_2\mathrm{d}t + \phi_{[1,\sigma(t)]}\mathrm{d}t + \psi^T_{[1,\sigma(t)]}\mathrm{d}\omega, \\ \dot{\hat{z}}_i &= \hat{z}_{i+1} + L^i a_i\left(y_1 - \hat{z}_1\right), \quad i = 1,\dots,n-1, \\ \dot{\hat{z}}_n &= u + L^n a_n\left(y_1 - \hat{z}_1\right). \end{aligned} \quad (14)$$

Proposition 8. *By Assumptions 1 and 2, there exist constants $\tau_1 > 0$ and $\tau_2 > 0$ a.s., such that*

$$\begin{aligned} \left|\phi_{[i,\sigma(t)]}\right| &\le \left(|z_1| + |z_2| + \dots + |z_i|\right)\tau_1, \\ \left|\psi^T_{[i,\sigma(t)]}\right| &\le \left(|z_1| + |z_2| + \dots + |z_i|\right)\tau_2, \end{aligned} \quad (15a)$$

$$\begin{aligned} \left\|\Phi_{\sigma(t)}\left(x_0, z\right)\right\|^2 &\le n\left(|z_1| + \frac{|z_2|}{L} + \dots + \frac{|z_n|}{L^{n-1}}\right)^2\tau_1^2, \\ \left\|\Psi^T_{\sigma(t)}\left(x_0, z\right)\right\|^2 &\le n\left(|z_1| + \frac{|z_2|}{L} + \dots + \frac{|z_n|}{L^{n-1}}\right)^2\tau_2^2, \end{aligned} \quad (15b)$$

where $z = [z_1,\dots,z_n]^T$.

Remark 9. The proof of the above proposition is similar to that of Proposition 2 in [25]. In fact, we only need to let $\overline{\overline{v}}_i = \eta_0 + (|\eta_0\overline{m}|^{i-1} + \dots + 1)\max_{\sigma(t)}\overline{v}_{[i,\sigma(t)]}$.

One can define variables ε_i as follows in order to utilize the backstepping method:

$$\begin{aligned} \varepsilon_1 &= z_1, \\ \varepsilon_i\left(z_1,\hat{z}_2,\dots,\hat{z}_i\right) &= \hat{z}_i - \alpha_{i-1}\left(z_1,\hat{z}_2,\dots,\hat{z}_{i-1}\right), \end{aligned} \quad (16)$$

with virtual smooth controllers α_{i-1} $(i = 2,\dots,n)$.

Step 1. For positive parameter δ, letting $V_0 = \delta\xi^T P\xi$, with a similar method in [24], one has

$$\begin{aligned} \mathscr{L}V_0 \le &-\left[\delta\left(L - 2\|P\|^2\right) - 2nC_e\right]\|\xi\|^2 \\ &+ 2nC_e\left(|z_1|^2 + \frac{|\hat{z}_2|^2}{L^2} + \dots + \frac{|\hat{z}_2|^2}{L^{2n-2}}\right), \end{aligned} \quad (17)$$

where $C_e = n\delta\tau_1^2 + n\delta\lambda_{\max}(P)\tau_2^2$. Defining $V_1 = V_0 + (1/2)\varepsilon_1^2$, we have

$$\begin{aligned} \mathscr{L}V_1 \le &-\left[\delta\left(L - 2\|P\|^2\right) - 2nC_e\right]\|\xi\|^2 \\ &+ 2nC_e\left(|z_1|^2 + \frac{|\hat{z}_2|^2}{L^2} + \dots + \frac{|\hat{z}_n|^2}{L^{2n-2}}\right) \\ &+ \varepsilon_1\left[\varepsilon_2 + \alpha_1 + L\xi_2 + \phi_{[1,\sigma(t)]}\right] \\ &+ \frac{1}{2}\mathrm{Tr}\left[\psi_{[1,\sigma(t)]}\psi^T_{[1,\sigma(t)]}\right]. \end{aligned} \quad (18)$$

The following inequalities hold with Lemma 2.1 in [28]:

$$\begin{aligned} 2nC_e\frac{|\hat{z}_2|^2}{L^2} &\le 4nC_e\frac{\varepsilon_2^2}{L} + 4nC_e\frac{\alpha_1^2}{L^2}, \\ \varepsilon_1\varepsilon_2 &\le \frac{1}{4}L\varepsilon_2^2 + L\varepsilon_1^2, \\ L\varepsilon_1\xi_2 &\le \frac{L}{4}\varepsilon_1^2 + L\xi_2^2, \\ \varepsilon_1\phi_{[1,\sigma(t)]} &\le L\tau_1^2\varepsilon_1^2, \\ \frac{1}{2}\mathrm{Tr}\left[\psi_{[1,\sigma(t)]}\psi^T_{[1,\sigma(t)]}\right] &\le \frac{1}{2}L\tau_2^2\varepsilon_1^2, \\ 2nC_e|z_1|^2 &\le 2nC_eL\varepsilon_1^2, \end{aligned} \quad (19)$$

where, together with (18), $|z_i/L^{i-1}| \le \|\xi_i\| + |\hat{z}_i/L^{i-1}|$. Choosing

$$\begin{aligned} \alpha_1 &= -Lb_1\varepsilon_1, \\ b_1 &= 2n + 1 + 2nC_e + \frac{5}{4} + \tau_1 + \frac{\tau_2^2}{2}, \end{aligned} \quad (20)$$

one has

$$\begin{aligned} \mathscr{L}V_1 \le &-\left[\delta\left(L - 2\|P\|^2\right) - 2nC_e - L\right]\|\xi\|^2 \\ &+ 2nC_e\left[\frac{|\hat{z}_3|^2}{L^4} + \dots + \frac{|\hat{z}_n|^2}{L^{2n-2}}\right] \\ &- \left[(2n+1)L - 4nC_eb_1^2\right]\varepsilon_1^2 + \frac{1}{4L}\varepsilon_2^2 + 4nC_e\frac{\varepsilon_2^2}{L}. \end{aligned} \quad (21)$$

Step i $(2 \le i \le n-1)$. From (16), we have

$$\begin{aligned} \varepsilon_i &= \widehat{z}_i + Lb_{i-1}\widehat{z}_{i-1} + L^2 b_{i-1}b_{i-2}\widehat{z}_{i-2} + \cdots + L^{i-1}b_{i-1}b_{i-2}\cdots b_1 z_1, \\ \mathrm{d}\varepsilon_i &= \left[\widehat{z}_{i+1} + L^i\widetilde{d}_i\xi_1 + L^i d_{i,1}\varepsilon_1 + L^{i-1}d_{i,2}\varepsilon_2 + \cdots + L^2 d_{i,i-1}\varepsilon_{i-1} + Ld_{i,i}\varepsilon_i\right]\mathrm{d}t \\ &\quad + L^{i-1}b_{i-1}b_{i-2}\cdots b_1\left[L\xi_2\mathrm{d}t + \phi_{[1,\sigma(t)]}\mathrm{d}t + \psi^T_{[1,\sigma(t)]}\mathrm{d}\omega\right], \end{aligned} \tag{22}$$

where

$$\begin{aligned} d_{i,1} &= -b_{i-1}\cdots b_2 b_1 b_1, \\ d_{i,j} &= b_{i-1}\cdots b_j b_{j-1} - b_{i-1}\cdots b_j b_j, \quad 2 \le j \le i-1, \\ d_{i,i} &= b_{i-1}, \\ \widetilde{d}_i &= a_i + b_{i-1}b_{i-2}\cdots b_2 a_2 + b_{i-1}\cdots b_3 a_3 + \cdots \\ &\quad + b_{i-1}a_{i-1}. \end{aligned} \tag{23}$$

Now, we have finished the design step $i-1$, and α_{i-1} is chosen as follows:

$$\begin{aligned} \alpha_{i-1} &= -Lb_{i-1}\varepsilon_{i-1}, \\ b_{i-1} &= n + 4nC_e + \frac{5}{4} + \frac{\widetilde{d}^2_{i-1}}{4} + \frac{1}{4}d^2_{i-1,1} + \cdots \\ &\quad + \frac{1}{4}d^2_{i-1,i-1} + \frac{1}{4}\left(b_{i-2}\cdots b_1\right)^2 \\ &\quad + \frac{1}{4}\left(b_{i-2}\cdots b_1\right)^2\tau_1^2 + d_{i-1,i-1}. \end{aligned} \tag{24}$$

Let $V_{i-1} = V_{i-2} + (1/2L^{2i-4})\varepsilon^2_{i-1}$, such that

$$\begin{aligned} \mathscr{L}V_{i-1} &\le -\left[\delta L - 2\delta\|P\|^2 - 2nC_e - (i-1)L\right]\|\xi\|^2 \\ &\quad + 2nC_e\left(\frac{\left|\widehat{z}_{i+1}\right|^2}{L^{2i}} + \cdots + \frac{\left|\widehat{z}_n\right|^2}{L^{2n-2}}\right) \\ &\quad - \left\{[2n - 2(i-1) + 3]L\right. \\ &\quad \left. - \sum_{j=1}^{i-2}\frac{\left(b_j\cdots b_1\right)^2\tau_2^2}{2} - 4nC_e b_1^2\right\}\varepsilon_1^2 - \sum_{j=2}^{i-1}\frac{1}{L^{2j-2}} \\ &\quad \cdot\left[(n+j-i+1)L - 4nC_e b_j^2\right]\varepsilon_j^2 + 4nc_e\frac{\varepsilon_i^2}{L^{2i-3}} \\ &\quad + \frac{\varepsilon_i^2}{4L^{2i-3}}. \end{aligned} \tag{25}$$

Defining $V_i = V_{i-1} + (1/2L^{2i-2})\varepsilon_i^2$, we have

$$\begin{aligned} \mathscr{L}V_i &\le -\left[\delta L - 2\delta\|P\|^2 - 2nC_e - (i-1)L\right]\|\xi\|^2 \\ &\quad + 2nC_e\left(\frac{\left|\widehat{z}_{i+1}\right|^2}{L^{2i}} + \cdots + \frac{\left|\widehat{z}_n\right|^2}{L^{2n-2}}\right) \\ &\quad - \left[[2n - 2(i-1) + 3]L - \sum_{j=1}^{i-2}\frac{\left(b_j\cdots b_1\right)^2\tau_2^2}{2}\right. \\ &\quad \left. - 4nC_e b_1^2\right]\varepsilon_1^2 \\ &\quad - \sum_{j=2}^{i-1}\frac{1}{L^{2j-2}}\left[(n+j-i+1)L - 4nC_e b_j^2\right]\varepsilon_j^2 + 4nC_e \\ &\quad \cdot\frac{\varepsilon_i^2}{L^{2i-3}} + \frac{1}{4L^{2i-3}}\varepsilon_i^2 + \frac{1}{L^{2i-2}}\varepsilon_i\left[\varepsilon_{i+1} + \alpha_i + L^i\widetilde{d}_i\xi_1\right. \\ &\quad + L^i d_{i,1}\varepsilon_1 + L^{i-1}d_{i,2}\varepsilon_2 + \cdots + L^2 d_{i,i-1}\varepsilon_{i-1,p} + Ld_{i,i}\varepsilon_i \\ &\quad \left. + L^{i-1}b_{i-1}b_{i-2}\cdots b_1\left(L\xi_2 + \phi_{[1,\sigma(t)]}\right)\right] \\ &\quad + \frac{1}{2L^{2i-2}}\left(L^{i-1}b_{i-1}b_{i-2}\cdots b_1\right)^2\operatorname{Tr}\left[\psi_{[1,\sigma(t)]}\psi^T_{[1,\sigma(t)]}\right]. \end{aligned} \tag{26}$$

By Lemma 2.1 in [28], one gets

$$\begin{aligned} 2nC_e\frac{\left|\widehat{z}_{i+1}\right|^2}{L^{2i}} &\le 4nC_e\frac{\varepsilon^2_{i+1}}{L^{2i-1}} + 4nC_e\frac{\alpha_i^2}{L^{2i}}, \\ \frac{1}{L^{2i-2}}\varepsilon_i\varepsilon_{i+1} &\le \frac{1}{4L^{2i-1}}\varepsilon^2_{i+1} + \frac{1}{L^{2i-3}}\varepsilon_i^2, \\ \frac{\widetilde{d}_i}{L^{i-2}}\varepsilon_i\xi_1 &\le \frac{\widetilde{d}_i^2}{4L^{2i-3}}\varepsilon_i^2 + L\xi_1^2, \\ \frac{d_{i,1}}{L^{i-2}}\varepsilon_i\varepsilon_1 &\le \frac{d^2_{i,1}}{4L^{2i-3}}\varepsilon_i^2 + L\varepsilon_1^2, \\ &\vdots \\ \frac{d_{i,i-1}}{L^{2i-4}}\varepsilon_i\varepsilon_{i-1} &\le \frac{d^2_{i,i-1}}{4L^{2i-3}}\varepsilon_i^2 + \frac{1}{L^{2i-5}}\varepsilon^2_{i-1}, \\ \frac{1}{L^{i-2}}b_{i-1}b_{i-2}\cdots b_1\varepsilon_i\xi_2 &\le \frac{\left(b_{i-1}b_{i-2}\cdots b_1\right)^2}{4L^{2i-3}}\varepsilon_i^2 + L\xi_2^2, \end{aligned}$$

$$\frac{1}{L^{i-1}}b_{i-1}b_{i-2}\cdots b_1\varepsilon_i\phi_{1p} \le \frac{\left(b_{i-1}b_{i-2}\cdots b_1\tau_1\right)^2}{4L^{2i-3}}\varepsilon_i^2 + L\varepsilon_1^2,$$
$$\frac{1}{2L^{2i-2}}\left(L^{i-1}b_{i-1}b_{i-2}\cdots b_1\right)^2 \operatorname{Tr}\left\{\psi_{[1,\sigma(t)]}\psi_{[1,\sigma(t)]}^T\right\}$$
$$\le \frac{\left(b_{i-1}b_{i-2}\cdots b_1\right)^2}{2}\tau_2^2\varepsilon_1^2. \tag{27}$$

Choosing α_i as

$$\alpha_i = -Lb_i\varepsilon_i,$$
$$b_i = n + 4nC_e + \frac{5}{4} + \frac{\tilde{d}_i^2}{4} + \frac{1}{4}d_{i,1}^2 + \cdots + \frac{1}{4}d_{i,i-1}^2 + \frac{1}{4}\left(b_{i-1}\cdots b_1\right)^2 + \frac{1}{4}\left(b_{i-1}\cdots b_1\right)^2\tau_1^2 + d_{i,i}, \tag{28}$$

one has

$$\begin{aligned}\mathscr{L}V_i \le& -\left[\delta L - 2\delta\|P\|^2 - 2nC_e - iL\right]\|\xi\|^2 \\ &+ 2nC_e\left(\frac{\left|\widehat{z}_{i+2}\right|^2}{L^{2i+2}} + \cdots + \frac{\left|\widehat{z}_n\right|^2}{L^{2n-2}}\right) \\ &- \left\{[2n-2i+3]L - \sum_{j=1}^{i-1}\frac{\left(b_j\cdots b_1\right)^2\tau_2^2}{2} - 4nC_eb_1^2\right\} \\ &\cdot \varepsilon_1^2 - \sum_{j=2}^{i}\frac{1}{L^{2j-2}}\left[(n+j-i)L - 4nC_eb_j^2\right]\varepsilon_j^2 + 4nC_1 \\ &\cdot \frac{\varepsilon_{i+1}^2}{L^{2i-1}} + \frac{1}{4L^{2i-1}}\varepsilon_{i+1}^2.\end{aligned} \tag{29}$$

Step n. Letting $V_n = V_{n-1} + (1/2L^{2n-2})\varepsilon_n^2$ and choosing

$$u = -Lb_n\varepsilon_n,$$
$$b_n = n + 4nC_e + \frac{5}{4} + \frac{\tilde{d}_n^2}{4} + \frac{d_{n,1}^2}{4} + \cdots + \frac{d_{n,n-1}^2}{4} + \frac{\left(b_{n-1}\cdots b_1\right)^2}{4} + \frac{\left(b_{n-1}\cdots b_1\right)^2\tau_1^2}{4} + d_{n,n}, \tag{30}$$

one gets

$$\begin{aligned}\mathscr{L}V_n &\le -h_0\|\xi\|^2 - h_1\varepsilon_1^2 - \sum_{j=2}^{n-1}h_j\frac{1}{L^{2j-2}}\varepsilon_j^2 - h_n\frac{1}{L^{2n-3}}\varepsilon_n^2 \\ &\le -\tilde{h}V_n,\end{aligned} \tag{31}$$

where $\tilde{h} = \min\{h_0/\delta, 2h_1, \ldots, 2h_n\}$ and

$$\begin{aligned}h_0 &= \delta L - 2\delta\|P\|^2 - 2nC_e - nL \ge 0, \\ h_1 &= 3L - \sum_{j=1}^{n-1}\frac{\left(b_j\cdots b_1\right)^2\tau_2^2}{2} - 4nC_eb_1^2 \ge 0, \\ h_j &= jL - 4nC_eb_j^2 \ge 0, \quad j = 2,\ldots,n-1, \\ h_n &= n \ge 0, \quad \delta > n.\end{aligned} \tag{32}$$

Remark 10. From (20), one has that b_1 do not contain L under arbitrary switching. From (28), one has that all b_i do not contain L under the designed controllers.

Remark 11. For example, with $n = 3$, by (41) in [25] and (32), we have

$$\begin{aligned}h_0 &= \delta\left(L - 2\|P\|^2 - \|B\|^2\right) - 6C_e - 3L \ge 0, \\ h_1 &= 6L - 4L - 2\frac{\eta_0^2}{L} - \frac{\left(b_2b_1\right)^2\tau_2^2}{2} - 12C_eb_1^2 \ge 0, \\ h_2 &= 2L - \frac{\eta_0^2}{L} - 12C_eb_2^2 \ge 0, \\ h_3 &= 3 \ge 0, \quad \delta > 3,\end{aligned} \tag{33}$$

$$\begin{aligned}h_0 &= \delta\left(L - 2\|P\|^2\right) - 6C_e - 3L \ge 0, \\ h_1 &= 7L - 4L - \sum_{j=1}^{2}\frac{\left(b_j\cdots b_1\right)^2\tau_2^2}{2} - 12C_eb_1^2 \ge 0, \\ h_2 &= 3L - 12C_eb_2^2 \ge 0, \\ h_3 &= 3 \ge 0, \quad \delta > 3,\end{aligned} \tag{34}$$

respectively. It is easy to see that h_1 and h_2 in (33) are all quintic functions about L, but they are linear functions about L in (34). So, the calculation of δ and L will be more simple with the method in this manuscript.

Choose $V = V_0 + V_n$, which together with (5) and (31) result in

$$\mathscr{L}V \le -hV, \tag{35}$$

where $h = \min\{\lambda, \tilde{h}\}$.

Theorem 12. *For system (1b), the closed-loop system with controller (30) is asymptotically stable in probability.*

3.3. Switching Control. In the above two subsections, we give the controllers u_0 and u with $x_0(t_0) \ne 0$ as (4) and (30). Now, we turn to the case of $x_0(t_0) = 0$. If $x_0(t_0) = 0$, we firstly choose constant control $u_0 = u_0^* \ne 0$. Secondly, there will exist a time $t_s^* > 0$ such that $x_0(t_s^*) \ne 0$. After that, controllers u_0 and u as (4) and (30) can be applied.

Theorem 13. *If we apply the above switching procedure, systems (1a) and (1b) will be asymptotically stabilized in probability.*

4. A Simulation Example

Consider systems (1a) and (1b) with $\sigma : [0,+\infty) \to \{1,2\}$ and $g_{[0,1]} = c_1x_0\cos(x_0)$, $g_{[0,2]} = c_1x_0\sin(x_0)$, $f_{[1,1]} = c_1x_1\cos^2(x_1)$, $f_{[1,2]} = c_1x_1\cos(x_1)$, $g_{[1,1]} = c_1x_1\sin^2(x_1)$, $g_{[1,2]} = c_1x_1\sin(x_1)$, $f_{[2,1]} = f_{[2,2]} = 0$, $g_{[2,1]} = c_1x_2\sin^2(x_2)$, $g_{[2,2]} = c_1x_2\sin(x_2)$, and $c_1 = 0.001$.

From (8), we have $\phi_{[1,1]} = \eta_0z_1 + c_1z_1\cos^2(x_1) + c_1^2z_1\cos^2(x_0) - c_1^2z_1\sin^2(x_1)\cos(x_0)$, $\phi_{[2,1]} = 0$, $\psi_{[1,1]} =$

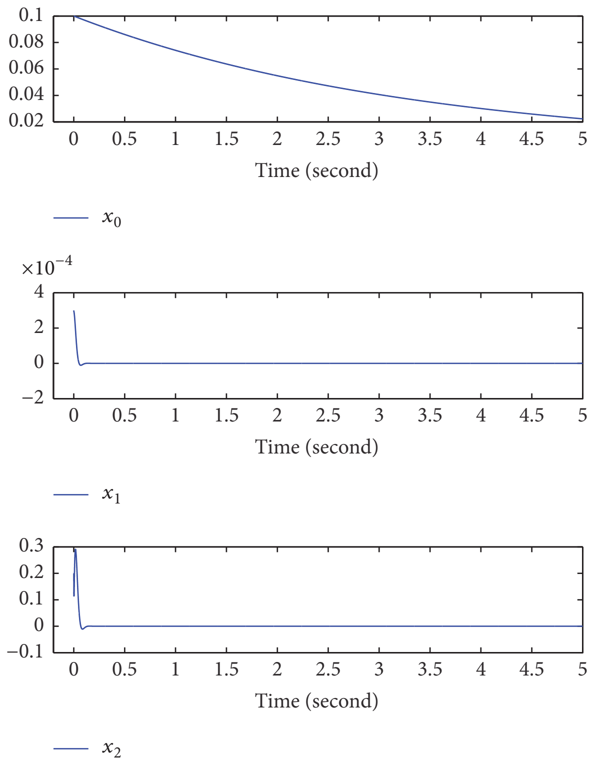

FIGURE 1: The responses of states x_0, x_1, and x_2.

$c_1 z_1 \sin^2(x_1) - c_1 z_1 \cos(x_0)$, $\psi_{[2,1]} = c_1 z_2 \sin^2(z_2)$, $\phi_{[1,2]} = \eta_0 z_1 + c_1 z_1 \cos^2(x_1) + c_1^2 z_1 \sin^2(x_0) - c_1^2 z_1 \sin(x_1)\sin(x_0)$, $\phi_{[2,2]} = 0$, $\psi_{[1,2]} = c_1 z_1 \sin(x_1) - c_1 z_1 \sin(x_0)$, and $\psi_{[2,2]} = c_1 z_2 \sin(z_2)$.

Letting $\eta_0 = 0.3$, by Proposition 8, we have $\tau_1 = 0.3023$ and $\tau_2 = 0.001$. Choosing $a_1 = 0.75$ and $a_2 = 1.25$, then $\lambda_{\max}(P) = 2$ and $\|P\| = 2$.

From (20) and (30), one gets

$$\begin{aligned} b_1 &= 5 + 4C_e + 1 + \frac{1}{4} + \tau_1 + \frac{1}{2}\tau_2^2, \\ b_2 &= 2 + 8C_e + 1 + \frac{1}{4} + \frac{(a_2 + b_1 a_1)^2}{4} + \frac{b_1^2}{4} + b_1 + \frac{b_1^2}{4} \\ &\quad + \frac{b_1^2 \tau_1^2}{4}, \\ u &= -L b_2 \hat{z}_2 - L^2 b_2 b_1 z_1, \end{aligned} \tag{36}$$

where $C_e = 0.4131\delta$. From (32) and (35), we have

$$\begin{aligned} h_0 &= (\delta - 2) L - 2\delta \|P\|^2 - 4C_e \geq 0, \\ h_1 &= 2L - \frac{b_1^2 \tau_2^2}{2} - 8 C_e b_1^2 \geq 0, \\ h_2 &= 2 \geq 0, \quad \delta > 2. \end{aligned} \tag{37}$$

Solving the above inequalities, one has $\delta = 2.26$ and $L = 76$, which means $h_0 = 0.0277$, $h_1 = 5.5526$, and $\mathscr{L}V \leq 0$.

If we choose initial values $x_0(0) = 0.1$, $x_1(0) = 0.0003$, $x_2(0) = 0.2$, $\hat{z}_1(0) = -0.001$, and $\hat{z}_2(0) = 7.5$, responses of systems are as in Figures 1, 2, and 3.

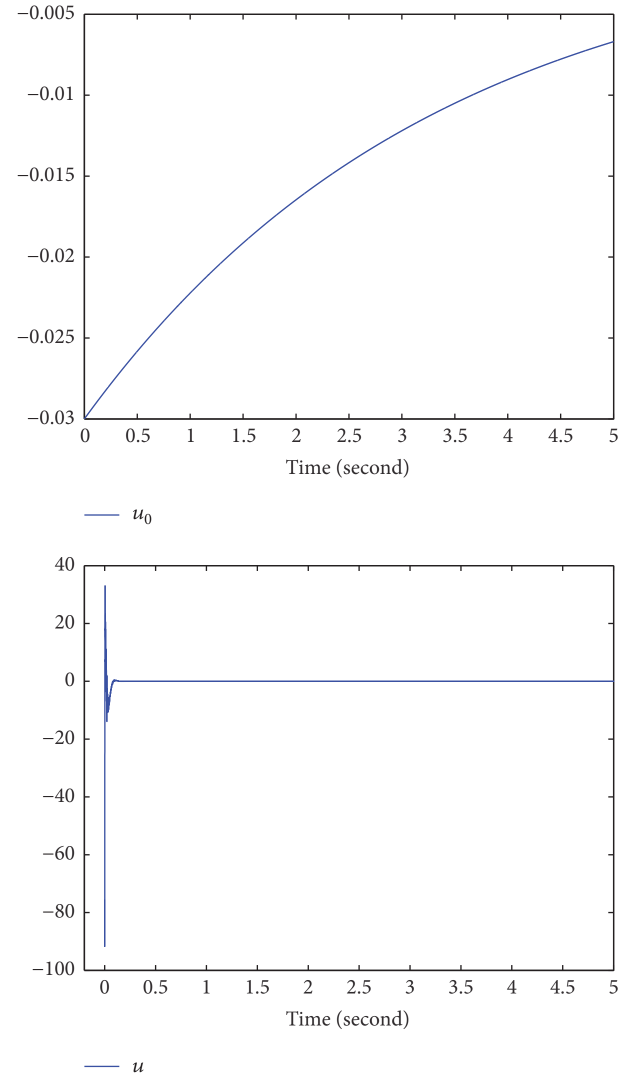

FIGURE 2: The responses of controllers u_0 and u.

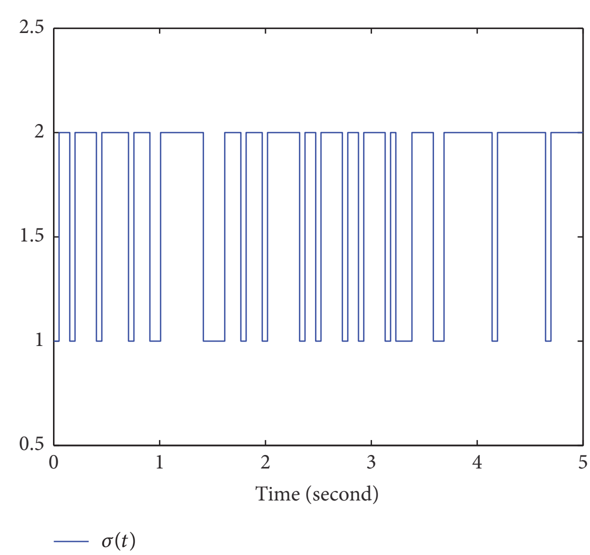

FIGURE 3: The responses of switching signals $\sigma(t)$.

Remark 14. From the above example, the observer we adopted is the same as that in [27], but it can simplify the calculation of δ and L compared with the observer in [25].

5. Conclusions

The output feedback stabilization for SNSs under arbitrary switching is discussed. We proposed an observer which is different from that in [25]. b_i of the designed output feedback stabilizing control laws do not contain the gain parameter. We will give some new results, for example, how to design an adaptive controller with the method LMI based on results in [29, 30].

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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