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Research Article

Asymptotic Solutions of Time-Space Fractional Coupled Systems by Residual Power Series Method

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This paper focuses on the asymptotic solutions to time-space fractional coupled systems, where the fractional derivative and integral are described in the sense of Caputo derivative and Riemann-Liouville integral. We introduce the Residual Power Series (for short RPS) method to construct the desired asymptotic solutions. Furthermore, we apply this method to some time-space fractional coupled systems. The simplicity and efficiency of RPS method are shown by the application.

1. Introduction

Fractional derivative was mentioned in a letter from L'Hopital to Leibniz in 1695. In the letter, L'Hopital proposed a question "What is the result of $d^n y/dx^n$ if $n = 1/2$?" The answer of Leibniz was "$d^{1/2}x$ will be equal to $x\sqrt{dx : x}$. This is an apparent paradox, from which, one day useful consequences will be drawn" [1, 2]. Furthermore, the generalization of this framework indicates that it is more appropriate to talk of integration and differentiation of, such as fractional order, real number order and even complex number order just as the development of number system. However, there is a basic question: "What is fractional integral and derivative?" Or "How to define the fractional integral and derivative?" More and more mathematicians focused on this problem, such as J. L. Lagrange, P. S. Laplace, and Joseph B. J. Fourier. Some different definitions of fractional integrals and derivatives have been defined according to different needs, like Riemann-Liouville integral, Caputo derivative, Weyl derivative, and so on [2, 3]. But there is no uniform definition of fractional integral and derivative, and the frequently used definition is Riemann-Liouville integral and Caputo derivative.

Fractional differential equations, which involve fractional order derivatives, are applied in many engineering and scientific disciplines as the mathematical modelling of systems and processes in the fields of physics, chemistry, aerodynamics, electrodynamics of complex medium, polymer rheology, and so on. Fractional differential equations also serve as an excellent tool for the description of hereditary properties of various materials and processes. An essential topic is to construct the solutions to fractional differential equations. And there are some effective methods to obtain different kinds of solutions, like Sumudu transform and variational iteration method [4], fractional Taylor vector approximate method [5], iterative method [6–8], Residual Power Series (RPS) method [9–13], and so on [14–16]. On the other hand, the study of coupled systems which involve fractional differential equations is also important because fractional coupled systems occur in many fields [17–21]. In this paper, we generalize the RPS method to time-space fractional coupled systems and obtain the asymptotic series solutions.

The organization of this paper is as follows: In Section 2, some concepts and lemmas on fractional calculus are presented. In Section 3, we introduce the algorithm of RPS method for time-space fractional coupled system. In Section 4, asymptotic solutions of some examples are solved via RPS method. In Section 5, some concluding remarks are presented.

2. Preliminaries

In this section, some concepts and main lemmas we need in this paper are presented [2, 3, 22–24]. And more details about fractional calculus can be found in [2, 23, 24].

Definition 1. A real function $f(x)$ is said to be in the space $C_\mu, \mu \in \mathbb{R}$ if there exists a real number $\rho > \mu$ such that $f(x) = x^\rho f_1(x)$, where $f_1(x) \in C[0,\infty)$. And it is said to be in the space C_μ^n if $f^{(n)}(x) \in C_\mu,\ n \in \mathbb{N}$.

Definition 2. The Riemann-Liouville fractional integral operator of order $\alpha \geqslant 0$ of a function $f \in C_\mu,\ \mu \geqslant -1$ is defined as

$$I_t^\alpha f(t) := \begin{cases} \dfrac{1}{\Gamma(\alpha)} \displaystyle\int_0^t (t-\tau)^{\alpha-1} f(\tau)\,\mathrm{d}\tau, & \alpha > 0,\ t > \tau \geqslant 0, \\ f(t), & \alpha = 0, \end{cases} \quad (1)$$

where the symbol I_t^α represents the αth Riemann-Liouville fractional integral of f.

Definition 3. The Caputo fractional derivative of order $\alpha > 0$ of $f \in C_{-1}^n,\ n \in \mathbb{N}$ is defined as

$$D_t^\alpha f(t) := \begin{cases} I_t^{n-\alpha} f^{(n)}(t), & n-1 < \alpha < n,\ t > 0, \\ \dfrac{\mathrm{d}^n f(t)}{\mathrm{d}t^n}, & \alpha = n, \end{cases} \quad (2)$$

where the symbol $D_t^\alpha f(t)$ represents the αth Caputo fractional derivative of f.

Definition 4. The power series

$$\sum_{n=0}^{\infty} c_n (t-t_0)^{n\alpha} = c_0 + c_1 (t-t_0) + c_2 (t-t_0)^2 + \cdots \quad (3)$$

is called a fractional power series about $t = t_0$, where t is a variable and c_n $(n = 0, 1, 2 \ldots)$ are the coefficients of the series, $\alpha \in \mathbb{R}^+$.

Remark 5. For convenience, we shall treat $t_0 = 0$. In fact, the transformation $\mathcal{T} : t' = t - t_0$ reduces the fractional power series about $t = t_0$ to the fractional power series about $t = 0$ and meanwhile the transformation $\mathcal{T}$ is reversible.

Definition 6. A function $f(t)$ is analytical at $t = 0$ if $f(t)$ can be written as a form of fractional power series.

Lemma 7. *Suppose that $f(t)$ is an analytic function at $t = 0$; then $f(t)$ can be written as follows:*

$$f(t) = \sum_{n=0}^{\infty} c_n t^{n\alpha}, \quad 0 < \alpha \le 1,\ |t| < R. \quad (4)$$

Furthermore, if $f(t) \in C(-R,R)$ and $D_t^{n\alpha} f(t) \in C(-R,R)$ for $n = 0, 1, 2 \ldots$ then the coefficients c_n will take the form

$$c_n = \frac{D_t^{n\alpha} f(t)\big|_{t=0}}{\Gamma(n\alpha+1)}, \quad (5)$$

where $D_t^{n\alpha} = \underbrace{D_t^\alpha \cdot D_t^\alpha \cdots D_t^\alpha}_{n\text{-times}},\ n = 0, 1, 2 \ldots.$

Proof. First of all, notice that if we put $t = 0$ into (5), it yields

$$c_0 = f(0) = \frac{D_t^{0\alpha} f(t)\big|_{t=0}}{\Gamma(0\alpha+1)}. \quad (6)$$

Applying the operator D_t^α one time on (4),

$$c_1 = \frac{D_t^{\alpha} f(t)\big|_{t=0}}{\Gamma(\alpha+1)}. \quad (7)$$

Again, by applying the operator D_t^α two times on (4),

$$c_2 = \frac{D_t^{2\alpha} f(t)\big|_{t=0}}{\Gamma(2\alpha+1)}. \quad (8)$$

Analogously

$$c_n = \frac{D_t^{n\alpha} f(t)\big|_{t=0}}{\Gamma(n\alpha+1)}, \quad n = 0, 1, 2, \ldots. \quad (9)$$

This completes the proof. □

3. Algorithm of RPS Method for Coupled Systems

In this section, we consider the following system:

$$\begin{aligned} D_t^{p\alpha} u + F(x,t) &= 0, \\ D_t^{q\alpha} v + G(x,t) &= 0, \\ D_t^{r\alpha} w + H(x,t) &= 0 \end{aligned} \quad (10)$$

with initial values

$$\begin{aligned} D_t^{i\alpha} u(x,t)\big|_{t=0} &= a_i(x), \quad i = 0, 1, 2, \ldots, p-1, \\ D_t^{j\alpha} v(x,t)\big|_{t=0} &= b_j(x), \quad j = 0, 1, 2, \ldots, q-1, \\ D_t^{k\alpha} w(x,t)\big|_{t=0} &= c_k(x), \quad k = 0, 1, 2, \ldots, r-1 \end{aligned} \quad (11)$$

as a generalized illustration for the main idea of RPS method, where the symbol $D_t^{(\cdot)\alpha}$ represents the $(\cdot)$th fractional derivative in the Caputo sense, $\max\{(p-1)/p, (q-1)/q, (r-1)/r\} < \alpha \le 1$ $(p, q, r \in \mathbb{N})$, the functions u, v, w, F, G, H are analytic at $t = 0$, and the initial functions a_i, b_j, and c_k are infinitely many times differentiable for all $i = 0, 1, 2, \ldots, p$; $j = 0, 1, 2, \ldots, q$ and $k = 0, 1, 2, \ldots, r$.

Since u, v, w, F, G, H are analytic at $t = 0$, then they can be expanded in the form of fractional power series at $t = 0$ as follows:

$$\begin{aligned} u(x,t) &= \sum_{i=0}^{\infty} \frac{u_i(x)}{\Gamma(i\alpha+1)} t^{i\alpha}, \\ F(x,t) &= \sum_{i=0}^{\infty} \frac{f_i(x)}{\Gamma(i\alpha+1)} t^{i\alpha}; \\ v(x,t) &= \sum_{j=0}^{\infty} \frac{v_j(x)}{\Gamma(j\alpha+1)} t^{j\alpha}, \\ G(x,t) &= \sum_{j=0}^{\infty} \frac{g_j(x)}{\Gamma(j\alpha+1)} t^{j\alpha}; \\ w(x,t) &= \sum_{k=0}^{\infty} \frac{w_k(x)}{\Gamma(k\alpha+1)} t^{k\alpha}, \\ H(x,t) &= \sum_{k=0}^{\infty} \frac{h_k(x)}{\Gamma(k\alpha+1)} t^{k\alpha}, \end{aligned} \tag{12}$$

where $x \in \mathbb{R}$, $t \in (-R, R)$, R is the minimum convergence radius of functions $u(x,t), v(x,t), w(x,t), F(x,t), G(x,t), H(x,t)$, and

$$\begin{aligned} a_i(x) &= D_t^{i\alpha} u(x,t)\Big|_{t=0}, \\ f_i(x) &= D_t^{i\alpha} F(x,t)\Big|_{t=0}, \\ &\qquad i = 0, 1, 2, \ldots; \\ b_j(x) &= D_t^{j\alpha} u(x,t)\Big|_{t=0}, \\ g_j(x) &= D_t^{j\alpha} G(x,t)\Big|_{t=0}, \\ &\qquad j = 0, 1, 2, \ldots; \\ c_k(x) &= D_t^{k\alpha} u(x,t)\Big|_{t=0}, \\ h_k(x) &= D_t^{k\alpha} H(x,t)\Big|_{t=0}, \\ &\qquad k = 0, 1, 2, \ldots. \end{aligned} \tag{13}$$

According to the initial conditions,

$$\begin{aligned} u_i(x) &= a_i(x), \quad i = 0, 1, 2, \ldots, p-1; \\ v_j(x) &= b_j(x), \quad j = 0, 1, 2, \ldots, q-1; \\ w_k(x) &= c_k(x), \quad k = 0, 1, 2, \ldots, r-1. \end{aligned} \tag{14}$$

Thus the initial approximation of the solution u, v, w is as follows:

$$\begin{aligned} u^{\text{init}}(x,t) &= \sum_{i=0}^{p-1} \frac{u_i(x)}{\Gamma(i\alpha+1)} t^{i\alpha}, \\ v^{\text{init}}(x,t) &= \sum_{j=0}^{q-1} \frac{v_j(x)}{\Gamma(j\alpha+1)} t^{j\alpha}, \\ w^{\text{init}}(x,t) &= \sum_{k=0}^{r-1} \frac{w_k(x)}{\Gamma(k\alpha+1)} t^{k\alpha}. \end{aligned} \tag{15}$$

Then we calculate the coefficients $u_i(x)$, $v_j(x)$, and $w_k(x)$ for $i = p, p+1, \ldots$; $j = q, q+1, \ldots$; $k = r, r+1, \ldots$. Firstly, some symbols are given as follows:

$$\begin{aligned} \text{Res}_u(x,t) &= D_t^{p\alpha} u + F(x,t), \\ \text{Res}_{u,l}(x,t) &= D_t^{p\alpha} u_l + F(x,t); \\ \text{Res}_v(x,t) &= D_t^{q\alpha} u + G(x,t), \\ \text{Res}_{v,m}(x,t) &= D_t^{q\alpha} v_m + F(x,t); \\ \text{Res}_w(x,t) &= D_t^{r\alpha} u + H(x,t), \\ \text{Res}_{w,n}(x,t) &= D_t^{r\alpha} u_n + F(x,t), \end{aligned} \tag{16}$$

where

$$\begin{aligned} u_l(x,t) &= \sum_{i=0}^{l} \frac{u_i(x)}{\Gamma(i\alpha+1)} t^{i\alpha} \\ &= u^{\text{init}}(x,t) + \sum_{i=p}^{l} \frac{u_i(x)}{\Gamma(i\alpha+1)} t^{i\alpha}, \\ &\qquad l = p, p+1, \ldots; \\ v_m(x,t) &= \sum_{j=0}^{m} \frac{v_j(x)}{\Gamma(j\alpha+1)} t^{j\alpha} \\ &= v^{\text{init}}(x,t) + \sum_{i=q}^{m} \frac{v_i(x)}{\Gamma(i\alpha+1)} t^{i\alpha}, \\ &\qquad m = q, q+1, \ldots; \\ w_n(x,t) &= \sum_{k=0}^{n} \frac{w_k(x)}{\Gamma(k\alpha+1)} t^{k\alpha} \\ &= w^{\text{init}}(x,t) + \sum_{i=r}^{n} \frac{w_i(x)}{\Gamma(i\alpha+1)} t^{i\alpha}, \\ &\qquad n = r, r+1, \ldots. \end{aligned} \tag{17}$$

Hence there are two facts:

$(\mathbf{F}_1)$

$$\lim_{l\to\infty} u_l(x,t) = u(x,t),$$
$$\lim_{m\to\infty} v_m(x,t) = u(x,t), \quad (18)$$
$$\lim_{n\to\infty} w_n(x,t) = w(x,t).$$

$(\mathbf{F}_2)$

$$\lim_{l\to\infty} \mathrm{Res}_{u,l}(x,t) = \mathrm{Res}_{u,\infty}(x,t) = \mathrm{Res}_u(x,t) \equiv 0;$$
$$\lim_{m\to\infty} \mathrm{Res}_{v,m}(x,t) = \mathrm{Res}_{v,\infty}(x,t) = \mathrm{Res}_v(x,t) \equiv 0; \quad (19)$$
$$\lim_{n\to\infty} \mathrm{Res}_{w,n}(x,t) = \mathrm{Res}_{w,\infty}(x,t) = \mathrm{Res}_w(x,t) \equiv 0.$$

Furthermore

$$\begin{aligned}
0 &= D_t^{(i-p)\alpha}\mathrm{Res}_{u,\infty}(x,t)\Big|_{t=0} = u_i(x) + D_t^{(i-p)\alpha}F(x,t), \quad i = p, p+1,\ldots \\
&\Longrightarrow u_i(x) = f_{i-p}(x) \triangleq a_i(x), \quad i = p, p+1,\ldots; \\
0 &= D_t^{(j-q)\alpha}\mathrm{Res}_{v,\infty}(x,t)\Big|_{t=0} = v_j(x) + D_t^{(j-q)\alpha}G(x,t), \quad j = q, q+1,\ldots \\
&\Longrightarrow v_j(x) = g_{j-q}(x) \triangleq b_j(x), \quad j = q, q+1,\ldots; \\
0 &= D_t^{(k-r)\alpha}\mathrm{Res}_{w,\infty}(x,t)\Big|_{t=0} = w_k(x) + D_t^{(k-q)\alpha}H(x,t), \quad k = r, r+1,\ldots \\
&\Longrightarrow w_k(x) = h_{k-p}(x) \triangleq c_k(x), \quad k = r, p+1,\ldots.
\end{aligned} \quad (20)$$

Thus the solutions of coupled system (10) are

$$u(x,t) = \sum_{i=0}^{\infty} \frac{a_i(x)}{\Gamma(i\alpha+1)} t^{n\alpha},$$
$$v(x,t) = \sum_{j=0}^{\infty} \frac{b_j(x)}{\Gamma(j\alpha+1)} t^{n\alpha}, \quad (21)$$
$$w(x,t) = \sum_{k=0}^{\infty} \frac{c_k(x)}{\Gamma(k\alpha+1)} t^{n\alpha}.$$

Remark 8. If

$$F(x,t) = f(x,t,u,v,w),$$
$$G(x,t) = g(x,t,u,v,w), \quad (22)$$
$$H(x,t) = h(x,t,u,v,w)$$

and even F, G, H include the term of space fractional derivative and the term of time-fractional derivative whose order is less than the order of the system, then RPS method is also effective in calculating the asymptotic solutions for coupled system (10). In fact, F, G, H could be expanded in the form of fractional power series about time variable t at the initial time t_0, and facts $(\mathbf{F}_1)$ and $(\mathbf{F}_2)$ are reasonable as well; thus the coefficients appearing in the asymptotic solutions could be obtained successfully.

4. Application of RPS Method to Time-Space Fractional Coupled Systems

4.1. The Time-Space Fractional Coupled KdV System. KdV equation plays an important role in nonlinear evolution equation for its wide application in physics and engineering. Coupled KdV system was introduced by Hirota and Satsuma [25] to describe the iterations of water waves and they claimed that the system exits a soliton solution. In [26], Fan and Zhang got several kinds of solutions by an improved homogeneous method. In [20], Bhrawy et al. reduced the time-fractional coupled KdV equations into a problem consisting of a system of algebraic equations that greatly simplifies the problem via the shifted Legendre polynomials. The time-fractional coupled KdV equation is a generalization of the classical coupled KdV equation and in this subsection we generalize time-fractional coupled KdV system to time-space fractional coupled system (23) and obtain the asymptotic solution using RPS method.

Consider the time-space fractional coupled KdV system:

$$\begin{gathered}
D_t^{\alpha}u - aD_x^{3\beta}u - 6auD_x^{\gamma}u - 2bvD_x^{\delta}v = 0, \\
D_t^{\alpha}v + D_x^{3\lambda}v + 3uD_x^{\tau}v = 0
\end{gathered} \quad (23)$$

with initial values

$$\begin{gathered}
u(x,0) = a_0(x), \\
v(x,0) = b_0(x),
\end{gathered} \quad (24)$$

where $0 < \alpha, \gamma, \delta, \tau \le 1$, $2/3 < \beta, \lambda \le 1$, $u = u(x,t)$, $v = v(x,t)$, $(x,t) \in \mathbb{R} \times \mathbb{R}$.

If $u(x,t)$ and $v(x,t)$ are analytic at $t = 0$, then they can be expanded in the form of fractional power series

$$u(x,t)=\sum_{i=0}^{\infty}\frac{u_i(x)}{\Gamma(i\alpha+1)}t^{i\alpha},$$
$$v(x,t)=\sum_{j=0}^{\infty}\frac{v_j(x)}{\Gamma(j\alpha+1)}t^{j\alpha},$$
$$F(x,t)=\sum_{n=0}^{\infty}\frac{t^{n\alpha}}{\Gamma(n\alpha+1)}\left\{-aD_x^{3\beta}u_n(x)-\sum_{s=0}^{n}\frac{\Gamma(n\alpha+1)}{\Gamma(s\alpha+1)\Gamma((n-s)\alpha+1)}\left(6au_s(x)D_x^{\gamma}u_{n-s}(x)+2bv_s(x)D_x^{\delta}v_{n-s}(x)\right)\right\},$$
$$G(x,t)=\sum_{n=0}^{\infty}\frac{t^{n\alpha}}{\Gamma(n\alpha+1)}\left\{D_x^{3\lambda}v_n+\sum_{s=0}^{n}\frac{\Gamma(n\alpha+1)}{\Gamma(s\alpha+1)\Gamma((n-s)\alpha+1)}\cdot 3u_s(x)D_x^{\tau}v_{i-1-s}(x)\right\}. \quad (25)$$

Under the initial conditions,

$$u_0(x)=a_0(x),\quad v_0(x)=b_0(x); \quad (26)$$

that is, the initial approximate solutions are

$$u^{\text{initial}}(x,t)=a_0(x),\quad v^{\text{initial}}(x,t)=b_0(x). \quad (27)$$

Set

$$\begin{aligned}\text{Res}_u(x,t)&=D_t^{\alpha}u-aD_x^{3\beta}u-6auD_x^{\gamma}u-2bvD_x^{\delta}v,\\ \text{Res}_v(x,t)&=D_t^{\alpha}v+D_x^{3\lambda}v+3uD_x^{\tau}v,\\ \text{Res}_{u,l}(x,t)&=D_t^{\alpha}u_l-aD_x^{3\beta}u-6au_lD_x^{\gamma}u_l\\ &\quad-2bv_mD_x^{\delta}v_m,\\ \text{Res}_{v,m}(x,t)&=D_t^{\alpha}v_m+D_x^{3\lambda}v_m+3u_lD_x^{\tau}v_m,\end{aligned} \quad (28)$$

where

$$u_l(x,t)=a_0(x)+\sum_{i=1}^{l}\frac{u_i(x)}{\Gamma(i\alpha)+1}t^{i\alpha},\quad v_m(x,t)=b_0(x)+\sum_{j=1}^{m}\frac{v_j(x)}{\Gamma(j\alpha)+1}t^{j\alpha}. \quad (29)$$

Using RPS method

$$\begin{aligned}u_i(x)&=aD_x^{3\beta}u_{i-1}(x)+\sum_{s=0}^{i-1}\frac{\Gamma((i-1)\alpha+1)}{\Gamma(s\alpha+1)\Gamma((i-1-s)\alpha+1)}\\ &\quad\cdot\left\{6au_s(x)D_x^{\gamma}u_{i-1-s}(x)+2bv_s(x)D_x^{\delta}v_{i-1-s}(x)\right\}\\ &\triangleq a_i(x),\quad i=1,2,\ldots,\end{aligned}$$

$$\begin{aligned}v_j(x)&=-D_x^{3\lambda}v_{j-1}(x)\\ &\quad-\sum_{s=0}^{j-1}\frac{\Gamma((j-1)\alpha+1)}{\Gamma(s\alpha+1)\Gamma((j-1-s)\alpha+1)}\\ &\quad\cdot 3u_s(x)D_x^{\tau}v_{j-1-s}(x)\triangleq b_j(x),\quad j=1,2,\ldots.\end{aligned} \quad (30)$$

Thus the fractional power series solutions of coupled system (23) are

$$u(x,t)=\sum_{i=0}^{\infty}\frac{a_i(x)}{\Gamma(i\alpha+1)}t^{i\alpha},\quad v(x,t)=\sum_{j=0}^{\infty}\frac{b_j(x)}{\Gamma(j\alpha+1)}t^{j\alpha}. \quad (31)$$

4.2. The Time-Space Fractional Coupled KdV System of Generalized Hirota-Satsuma Type. In this subsection, we consider the time-space fractional coupled KdV system of generalized Hirota-Satsuma type

$$\begin{aligned}D_t^{\alpha}u-\frac{1}{2}D_x^{3\beta}u+3uD_x^{\gamma}u-3D_x^{\delta}(vw)&=0,\\ D_t^{\alpha}v+D_x^{3\lambda}v-3uD_x^{\tau}v&=0,\\ D_t^{\alpha}w+D_x^{3\theta}w-3uD_x^{\sigma}w&=0,\\ u(x,0)&=a_0(x),\\ v(x,0)&=b_0(x),\\ w(x,0)&=c_0(x),\end{aligned} \quad (32)$$

where $0<\alpha,\gamma,\delta,\lambda,\tau\le 1$, $2/3<\beta,\sigma,\theta\le 1$, $u=u(x,t)$, $v=v(x,t)$, $w=w(x,t)$, $(x,t)\in\mathbb{R}\times\mathbb{R}$. The equation describes an interaction of two long waves with different dispersion relations.

If u, v, w are analytic at $t = 0$, then u, v, w can be written as the form of fractional power series

$$\begin{aligned} u(x,t) &= \sum_{i=0}^{\infty} \frac{u_i(x)}{\Gamma(i\alpha+1)} t^{i\alpha}, \\ v(x,t) &= \sum_{j=0}^{\infty} \frac{v_j(x)}{\Gamma(j\alpha+1)} t^{j\alpha}, \\ w(x,t) &= \sum_{k=0}^{\infty} \frac{w_k(x)}{\Gamma(k\alpha+1)} t^{k\alpha}, \\ F(x,t) &= -\frac{1}{2} D_x^{3\beta} u + 3u D_x^{\gamma} u - 3D_x^{\delta}(vw) \\ &= \sum_{n=0}^{\infty} \frac{t^{n\alpha}}{\Gamma(n\alpha+1)} \cdot \left\{ -\frac{1}{2} D_x^{3\beta} u_n(x) \right. \\ &\quad + 3\sum_{s=0}^{n} \frac{\Gamma(n\alpha+1)}{\Gamma(s\alpha+1)\Gamma((n-s)\alpha+1)} \left(u_s(x) \right. \\ &\quad \left. \left. \cdot D_x^{\gamma} u_{n-s}(x) - v_n(x) w_{n-s}(x) \right) \right\}, \end{aligned} \tag{33}$$

$$\begin{aligned} G(x,t) &= D_x^{3\lambda} v - 3u D_x^{\tau} v = \sum_{n=0}^{\infty} \frac{t^{n\alpha}}{\Gamma(n\alpha+1)} \\ &\quad \cdot \left\{ D_x^{3\lambda} v_n(x) - 3\sum_{s=0}^{n} \frac{\Gamma(n\alpha+1)}{\Gamma(s\alpha+1)\Gamma((n-s)\alpha+1)} \right. \\ &\quad \left. \cdot u_s(x) D_x^{\tau} v_{n-s}(x) \right\}, \\ H(x,t) &= D_x^{3\theta} w - 3u D_x^{\sigma} w = \sum_{n=0}^{\infty} \frac{t^{n\alpha}}{\Gamma(n\alpha+1)} \\ &\quad \cdot \left\{ D_x^{3\theta} w_n(x) - 3\sum_{s=0}^{n} \frac{\Gamma(n\alpha+1)}{\Gamma(s\alpha+1)\Gamma((n-s)\alpha+1)} \right. \\ &\quad \left. \cdot u_s(x) D_x^{\sigma} w_{n-s}(x) \right\}. \end{aligned}$$

With the initial values

$$\begin{aligned} u_0(x) &= a_0(x), \\ v_0(x) &= b_0(x), \\ w_0(x) &= c_0(x), \end{aligned} \tag{34}$$

the initial approximate solutions are

$$\begin{aligned} u^{\text{initial}}(x,t) &= a_0(x), \\ v^{\text{initial}}(x,t) &= b_0(x), \\ w^{\text{initial}}(x,t) &= c_0(x). \end{aligned} \tag{35}$$

Set

$$\begin{aligned} \text{Res}_u(x,t) &= D_t^{\alpha} u - \frac{1}{2} D_x^{3\beta} u + 3u D_x^{\gamma} u - 3D_x^{\delta}(vw) \\ &= 0, \\ \text{Res}_v(x,t) &= D_t^{\alpha} v + D_x^{3\lambda} v - 3u D_x^{\tau} v = 0, \\ \text{Res}_w(x,t) &= D_t^{\alpha} w + D_x^{3\theta} w - 3u D_x^{\sigma} w = 0, \\ \text{Res}_{u,l}(x,t) &= D_t^{\alpha} u_l - \frac{1}{2} D_x^{3\beta} u_l + 3u_l D_x^{\gamma} u_l \\ &\quad - 3D_x^{\delta}(v_m w_n), \\ \text{Res}_{v,m}(x,t) &= D_t^{\alpha} v_m + D_x^{3\lambda} v - 3u_l D_x^{\tau} v_m, \\ \text{Res}_{w,n}(x,t) &= D_t^{\alpha} w_n + D_x^{3\theta} w_n - 3u_l D_x^{\sigma} w_n, \end{aligned} \tag{36}$$

where

$$\begin{aligned} u_l(x,t) &= a_0(x) + \sum_{i=1}^{l} \frac{u_i(x)}{\Gamma(i\alpha+1)} t^{\alpha}, \\ v_m(x,t) &= b_0(x) + \sum_{j=1}^{m} \frac{v_i(x)}{\Gamma(j\alpha+1)}, \\ w_n(x,t) &= c_0(x) + \sum_{k=1}^{n} \frac{w_k(x)}{\Gamma(k\alpha+1)} t^{k\alpha}, \end{aligned} \tag{37}$$

with the results of RPS method:

$$\begin{aligned} u_i(x) &= \frac{1}{2} D_x^{3\beta} u_{i-1}(x) \\ &\quad - 3\sum_{s=0}^{i-1} \frac{\Gamma((i-1)\alpha+1)}{\Gamma(s\alpha+1)\Gamma((i-1-s)\alpha+1)} \\ &\quad \cdot \left\{ u_s(x) D_x^{\gamma} u_{i-1-s}(x) - D_x^{\delta}(v_s(x) w_{i-1-s}(x)) \right\} \\ &\triangleq a_i(x), \quad i = 1, 2, \ldots, \\ v_j(x) &= -D_x^{3\sigma} v_{j-1}(x) \\ &\quad + 3\sum_{s=0}^{j-1} \frac{\Gamma((j-1)\alpha+1)}{\Gamma(j\alpha+1)\Gamma((j-1-s)\alpha+1)} u_s(x) \\ &\quad \cdot D_x^{\lambda} v_{j-1-s}(x) \triangleq b_j(x), \quad j = 1, 2, \ldots, \\ w_k(x) &= -D_x^{3\theta} w_{k-1}(x) \\ &\quad + 3\sum_{s=0}^{k-1} \frac{\Gamma((k-1)\alpha+1)}{\Gamma(j\alpha+1)\Gamma((k-1-s)\alpha+1)} u_s(x) \\ &\quad \cdot D_x^{\tau} w_{k-1-j}(x) \triangleq c_k(x), \quad k = 1, 2, \ldots. \end{aligned} \tag{38}$$

So the fractional power series solutions of coupled system (32) are

$$u(x,t) = \sum_{i=0}^{\infty} \frac{a_i(x)}{\Gamma(i\alpha+1)} t^{i\alpha},$$
$$v(x,t) = \sum_{j=0}^{\infty} \frac{b_j(x)}{\Gamma(j\alpha+1)} t^{j\alpha}, \quad (39)$$
$$w(x,t) = \sum_{k=0}^{\infty} \frac{c_k(x)}{\Gamma(k\alpha+1)} t^{k\alpha}.$$

4.3. The Time-Space Fractional Coupled Whitham-Broer-Kaup (WBK) System. Whitham [27], Broer [28], and Kaup [29] obtained nonlinear WBK system. In [30], Rashidi et al. obtained the approximate traveling wave solutions of the coupled WBK system in shallow water using homotopy analysis method. In [31], Kadem and Baleanu applied the homotopy perturbation method to find an analytical approximate solution for the coupled WBK system. In this subsection, we consider the time-space fractional coupled WBK system and construct the approximate solution by RPS method.

Consider the time-space fractional coupled WBK system:

$$D_t^{\alpha} u + u D_x^{\beta} u + D_x^{\gamma} v + a D_x^{2\delta} u = 0,$$
$$D_t^{\alpha} v + D_x^{\lambda}(uv) - a D_x^{2\tau} v + b D_x^{3\theta} u = 0, \quad (40)$$
$$u(x,0) = a_0(x),$$
$$v(x,0) = b_0(x),$$

where $0 < \alpha, \sigma, \tau, \lambda \le 1$, $1/2 < \delta, \eta \le 1$, $2/3 < \theta \le 1$, $(x,t) \in \mathbb{R} \times \mathbb{R}$, $a, b \in \mathbb{R}$ represent different dispersive power, $u = u(x,t)$ is the field of horizontal velocity, and $v = v(x,t)$ is the height deviating equilibrium position of liquid. And this is a very good model to describe dispersive wave.

If u, v are analytic at $t = 0$, then u, v can be expanded in the form of fractional power series

$$u(x,t) = \sum_{i=0}^{\infty} \frac{u_i(x)}{\Gamma(i\alpha+1)} t^{i\alpha},$$
$$v(x,t) = \sum_{j=0}^{\infty} \frac{v_j(x)}{\Gamma(j\alpha+1)} t^{j\alpha},$$
$$F(x,t) = \sum_{n=0}^{\infty} \frac{t^{n\alpha}}{\Gamma(n\alpha+1)} \cdot \left\{ \sum_{s=0}^{n} \frac{\Gamma(n\alpha+1)}{\Gamma(s\alpha+1)\Gamma((n-s)\alpha+1)} u_s(x) D_x^{\beta} u_{n-s}(x) + D_x^{\gamma} v_n(x) + a D_x^{2\delta} u_n(x) \right\},$$

$$G(x,t) = \sum_{n=0}^{\infty} \frac{t^{n\alpha}}{\Gamma(n\alpha+1)} \left\{ \sum_{s=0}^{n} \frac{\Gamma(n\alpha+1)}{\Gamma(s\alpha+1)\Gamma((n-s)\alpha+1)} \cdot D_x^{\lambda}(u_s(x) v_{n-s}(x)) + a D_x^{2\tau} v_n(x) + b D_x^{3\theta} u_n(x) \right\}. \quad (41)$$

Under the initial conditions

$$u_0(x) = a_0(x),$$
$$v_0(x) = b_0(x), \quad (42)$$

and the initial approximate solutions are

$$u^{\text{initial}}(x,t) = a_0(x),$$
$$v^{\text{initial}}(x,t) = b_0(x,t). \quad (43)$$

Set

$$\text{Res}_u(x,t) = D_t^{\alpha} u + u D_x^{\beta} u + D_x^{\gamma} v + a D_x^{2\delta} u,$$
$$\text{Res}_v(x,t) = D_t^{\alpha} v + D_x^{\lambda}(uv) - a D_x^{2\tau} v + b D_x^{3\theta} u,$$
$$\text{Res}_{u,l}(x,t) = D_t^{\alpha} u_l + u_l D_x^{\beta} u_l + D_x^{\gamma} v_m + a D_x^{2\delta} u_l, \quad (44)$$
$$\text{Res}_{v,m}(x,t) = D_t^{\alpha} v_m + D_x^{\lambda}(u_l v_m) - a D_x^{2\tau} v_m + b D_x^{3\theta} u_l,$$

where

$$u_l(x,t) = a_0(x) + \sum_{i=1}^{l} \frac{u_i(x)}{\Gamma(i\alpha+1)} t^{\alpha},$$
$$v_m(x,t) = b_0(x) + \sum_{j=1} \frac{v_j(x)}{\Gamma(j\alpha+1)} t^{j\alpha}. \quad (45)$$

With RPS method

$$u_i(x) = -\sum_{s=0}^{i-1} \frac{\Gamma((i-1)\alpha+1)}{\Gamma(s\alpha+1)\Gamma((i-1-s)\alpha+1)} u_s(x) \cdot D_x^{\beta} u_{i-1-s}(x) + D_x^{\gamma} v_{i-1}(x) + a D_x^{2\delta} u_{i-1}(x) \triangleq a_i(x), \quad i = 1, 2, \ldots,$$
$$v_j(x) = -\sum_{s=0}^{j-1} \frac{\Gamma((j-1)\alpha+1)}{\Gamma(s\alpha+1)\Gamma((j-1-s)\alpha+1)} \cdot D_x^{\lambda}(u_s(x) v_{j-1-s}(x)) + a D_x^{2\tau} v_{j-1}(x) - b D_x^{3\theta} u_{j-1}(x) \triangleq b_j(x), \quad j = 1, 2, \ldots. \quad (46)$$

So the fractional power series solution of coupled system (40) is

$$u(x,t)=\sum_{i=0}^{\infty}\frac{a_i(x)}{\Gamma(i\alpha+1)}t^{i\alpha},\qquad v(x,t)=\sum_{j=0}^{\infty}\frac{b_j(x)}{\Gamma(j\alpha+1)}t^{j\alpha}. \tag{47}$$

Remark 9. When $\alpha=\sigma=\tau=\delta=\lambda=\eta=\theta=1$, $\beta\neq 0$, $\gamma=0$, (40) reduces to the classical long-wave equations that describe the shallow water wave with diffusion.

Remark 10. When $\alpha=\sigma=\tau=\delta=\lambda=\eta=\theta=1$, $\beta=0$, $\gamma=1$, (40) reduces to the variant Boussinesq equation.

4.4. The Time-Space Fractional Coupled Shallow Water System. Shallow water systems are widely used in predicting hydrodynamics of surface flows such as water flows in rivers, channels, flood plains, and coastal regions. It is well known that the shallow water systems can accurately predict the hydraulic parameters under conditions of slow erosion and low sediment concentration of the time-space fractional coupled shallow water system [32]. In this subsection, consider the time-space fractional coupled shallow water system

$$D_t^{\alpha}u+uD_x^{\beta}u+D_x^{\gamma}v+aD_x^{2\delta}u=0,\qquad D_t^{\alpha}v+vD_x^{\lambda}u+uD_x^{\tau}v-aD_x^{2\theta}v+bD_x^{3\sigma}u=0, \tag{48}$$

with initial values

$$u(x,0)=a_0(x),\qquad v(x,0)=b_0(x), \tag{49}$$

where $0<\alpha,\beta,\gamma,\lambda,\tau\le 1$, $1/2<\delta,\theta\le 1$, $2/3<\sigma\le 1$, $u=u(x,t)$, $v=v(x,t)$, $(x,t)\in\mathbb{R}\times\mathbb{R}$.

If u,v are analytic at $t=0$, then u,v can be written as the form of fractional power series:

$$u(x,t)=\sum_{i=0}^{\infty}\frac{u_i(x)}{\Gamma(i\alpha+1)}t^{i\alpha},$$

$$v(x,t)=\sum_{j=0}^{\infty}\frac{v_j(x)}{\Gamma(j\alpha+1)}t^{j\alpha},$$

$$F(x,t)=\sum_{n=0}^{\infty}\frac{t^{n\alpha}}{\Gamma(n\alpha+1)}\cdot\left\{\sum_{s=0}^{n}\frac{\Gamma(n\alpha+1)}{\Gamma(s\alpha+1)\Gamma((n-s)\alpha+1)}u_s(x)D_x^{\beta}u_{n-s}(x)+D_x^{\gamma}v_n(x)+aD_x^{2\delta}u_n(x)\right\},$$

$$G(x,t)=\sum_{n=0}^{\infty}\frac{t^{n\alpha}}{\Gamma(n\alpha+1)}\cdot\left\{\sum_{s=0}^{n}\frac{\Gamma(n\alpha+1)}{\Gamma(s\alpha+1)\Gamma((n-s)\alpha+1)}\cdot\left(v_s(x)D_x^{\lambda}u_{n-s}(x)+u_s(x)D_x^{\tau}v_{n-s}(x)\right)-aD_x^{2\theta}v_n(x)+bD_x^{3\sigma}u_n(x)\right\}. \tag{50}$$

With the initial conditions

$$u_0(x)=a_0(x),\qquad v_0(x)=b_0(x), \tag{51}$$

and the initial approximate solutions are

$$u^{\text{initial}}(x,t)=a_0(x),\qquad v^{\text{initial}}(x,t)=b_0(x). \tag{52}$$

Set

$$\begin{aligned}
\mathrm{Res}_u(x,t)&=D_t^{\alpha}u+uD_x^{\beta}u+D_x^{\gamma}v+aD_x^{2\delta}u,\\
\mathrm{Res}_v(x,t)&\,D_t^{\alpha}v+vD_x^{\lambda}u+uD_x^{\tau}v-aD_x^{2\theta}v+bD_x^{3\sigma}u,\\
\mathrm{Res}_{u,l}(x,t)&=D_t^{\alpha}u_l+u_lD_x^{\beta}u_l+D_x^{\gamma}v_m+aD_x^{2\delta}u_l,\\
\mathrm{Res}_{v,m}(x,t)&=D_t^{\alpha}v_m+v_mD_x^{\lambda}u_l+u_lD_x^{\tau}v_m-aD_x^{2\theta}v_m+bD_x^{3\sigma}u_l,
\end{aligned} \tag{53}$$

where

$$u_l(x,t)=a_0(x)+\sum_{i=1}^{l}\frac{u_i(x)}{\Gamma(i\alpha+1)}t^{i\alpha},\qquad v_m(x,t)=b_0(x)+\sum_{j=1}^{m}\frac{v_j(x)}{\Gamma(j\alpha+1)}t^{j\alpha}. \tag{54}$$

Using the RPS method

$$\begin{aligned}
u_i(x)&=-\sum_{s=0}^{i-1}\frac{\Gamma((i-1)\alpha+1)}{\Gamma(s\alpha+1)\Gamma((i-1-s)\alpha+1)}u_s(x)\cdot D_x^{\beta}u_{i-1-s}(x)-D_x^{\gamma}v_{i-1}(x)+aD_x^{2\delta}u_{i-1}(x)\\
&\triangleq a_i(x),\quad i=1,2,\ldots,\\
v_j(x)&=-\sum_{s=0}^{i-1}\frac{\Gamma((i-1)\alpha+1)}{\Gamma(s\alpha+1)\Gamma((i-1-s)\alpha+1)}\left(v_s(x)\cdot D_x^{\lambda}u_{j-1-s}+u_s(x)D_x^{\tau}v_{j-1-s}(x)\right)+aD_x^{2\theta}v_{j-1}(x)\\
&\quad-bD_x^{3\sigma}u_{i-1}(x)\triangleq b_j(x),\quad j=1,2,\ldots.
\end{aligned} \tag{55}$$

So the solutions of coupled system (48) are

$$u(x,t)=\sum_{n=0}^{\infty}\frac{a_n(x)}{\Gamma(n\alpha+1)}t^{n\alpha},$$
$$v(x,t)=\sum_{n=0}^{\infty}\frac{b_n(x)}{\Gamma(n\alpha+1)}t^{n\alpha}. \quad (56)$$

5. Concluding Remarks

This paper introduced a new analytical iterative technique to construct asymptotic solutions to time-space fractional coupled systems, which is based on the general Residual Power Series method. Furthermore, we apply this method to some specific time-space fractional coupled systems to obtain asymptotic solutions with respect to initial values, which shows that this method is efficient and does not require linearization or perturbation.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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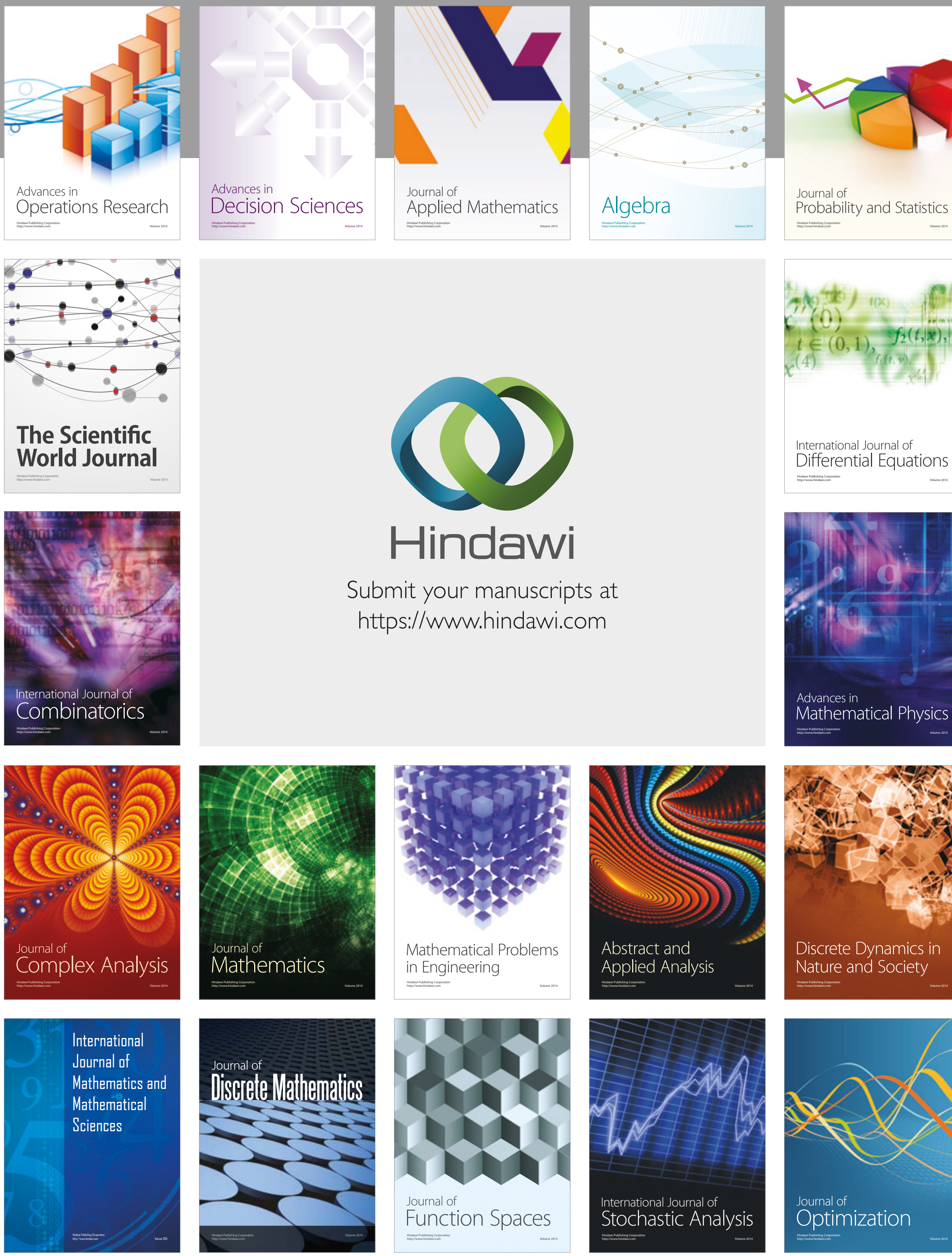
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