Hindawi
Discrete Dynamics in Nature and Society
Volume 2019, Article ID 3234295, 9 pages
https://doi.org/10.1155/2019/3234295

*Research Article*

# State Estimation for Switched Time-Varying Systems with Delay and Nonlinear Disturbance: An Integral Inequality Method

**Fanwei Meng and Yuangong Sun**

*School of Mathematical Sciences, Qufu Normal University, Qufu 273165, Shandong, China*

Correspondence should be addressed to Yuangong Sun; sunyuangong@163.com

Received 18 December 2018; Accepted 6 March 2019; Published 26 March 2019

Academic Editor: Allan C. Peterson



This paper focuses on the problem of state estimation for certain switched time-varying systems with time-varying delay and nonlinear disturbance. By using an integral inequality technique and a method used in positive systems, we have established several explicit criteria for state estimation of the system, which reduce to stability criteria for some particular cases. The involved nonlinear disturbance of the system takes more general form including both the internal disturbance and the external disturbance. Three numerical examples are also given to verify the validity of the obtained theoretical results.

## 1. Introduction

Switched system is a particular hybrid system containing a number of subsystems and a switching signal. Each subsystem is usually described by a definite differential equation or difference equation. For switched systems, the issue of stability plays a key role in system analysis. As a result, stability of switched systems has received considerable attention during the past several decades owing to its extensive applications in automotive engine control system [1], chemical process control system [2], multiagent systems [3, 4], and so on.

Some basic problems in stability and design of switching systems were put forward by Liberzon and Morse [5]. Later, there are several very important monographs devoted to the stability analysis and design of switched systems; e.g., see Liberzon [6] and Sun and Ge [7]. There are also many interesting results for stability of switched systems in [8–15]. In most of the existing references, the Lyapunov-Krasovskii functional method was most commonly used for switched time-invariant systems. It seems to us that little attention has been paid to the stability of switched time-varying systems. Recently, by using a positive system method, exponential stability of switched time-varying systems with delay and nonlinear disturbance was investigated in [16, 17].

Integral inequality plays an important role in qualitative analysis of delay systems [18–22, 22–25]. For example, stabilization of switched systems with impulsive effects and disturbances was studied in [18] by using the Gronwall integral inequality. By introducing a generalized Gronwall-Bellman inequality, the authors established stability criteria under arbitrary switching for switched systems with general nonlinear disturbances in [21]. Later, the main results in [21] were extended to switched delay systems with nonlinear disturbances in [25]. The same method was also applied to study a class of switched delay systems in [23], where global exponential stability criteria for the system were established.

Note that time delay has attracted much attention in the theory analysis of switched systems due to its detrimental effects on system performance such as oscillation [24, 26–30] and stability [31–36]. Inspired by the work in [18, 25], we will use a delay integral inequality technique and a method developed in positive systems [37, 38] to study the problem of state estimation for a class of switched time-varying systems with time-varying delay and nonlinear disturbance. The main contributions of this paper are as follows: (1) unlike most existing results in the literature, all the subsystems considered in this paper are time-varying; (2) explicit global (local) state estimation criteria will be established for the cases when the nonlinear disturbance satisfies linear and nonlinear growth conditions, respectively; (3) the nonlinear disturbance of the system has more general form which contains both the internal state disturbance and the external input disturbance, and hence it contains some cases in the literature.

The rest of this paper is organized as follows. In Section 2, we introduce some notations and preliminaries that are essential for deriving the main results of this paper. Section 3 then focuses on establishing explicit state estimation criteria for the system. Simulations are given to illustrate the main results in Section 4. Finally, conclusions are drawn in Section 5.

## 2. Problem Statements and Preliminaries

In the sequel, denote by $\mathbb{R}^{n\times n}$ the set of $n\times n$-dimensional real matrices, and denote by $\mathbb{R}^n$ the $n$-dimensional Euclidean space with the vector norm $\|x\| = \sum_{i=1}^{n}|x_i|$, where $x=(x_i)\in\mathbb{R}^n$ for $i\in\langle n\rangle=\{1,2,\dots,n\}$. Set $|x| = (|x_i|)$ and $|x|^p = (|x_i|^p)$. For two vectors $x=(x_i)\in\mathbb{R}^n$ and $y=(y_i)\in\mathbb{R}^n$, we write $x \succ (\prec,\preceq) y$ if $x_i > (<,\le) y_i$ for $i\in\langle n\rangle$. For a matrix $A=[a_{ij}]\in\mathbb{R}^{n\times n}$, we write $A\succeq 0$ if $a_{ij}\ge 0$ for $i,j\in\langle n\rangle$.

Now, we consider the following switched time-varying system of the form

$$\begin{aligned}\dot{x}(t) &= A_{\sigma(t)}(t)\,x(t) + B_{\sigma(t)}(t)\,x(t-\tau(t)) \\ &\quad + F_{\sigma(t)}(t, x(t), x(t-\tau(t))), \quad t\ge 0,\end{aligned} \tag{1}$$

where $x(t)\in\mathbb{R}^n$ is the state, $\sigma(t):[0,\infty)\longrightarrow\langle m\rangle=\{1,2,\dots,m\}$ is the switching signal which is a piecewise constant function, $A_k(t)=[a_{ij}^{(k)}(t)]\in\mathbb{R}^{n\times n}$ and $B_k(t)=[b_{ij}^{(k)}(t)]\in\mathbb{R}^{n\times n}$ are continuous matrix functions for $k\in\langle m\rangle$ and $i,j\in\langle n\rangle$, $\tau(t):[0,\infty)\longrightarrow[0,\infty)$ is the time-varying delay satisfying $0\le\tau(t)\le h$, $h>0$ is a constant, $F_k(t,x,y):[0,\infty)\times\mathbb{R}^n\times\mathbb{R}^n\longrightarrow\mathbb{R}^n$ is a continuous vector function such that system (1) has a unique solution for each initial condition $x(t)=\phi(t)$, $t\in[-h,0]$, and $\phi(t):[-h,0]\longrightarrow\mathbb{R}^n$ is a continuous vector function.

We need the following assumptions for establishing the main results of this paper.

$(H_1)$ There exist continuous functions $\lambda(t)\ge 0$ and $\eta(t)\ge 0$ for $t\ge 0$, constant matrices $\overline{A}_k=[\overline{a}_{ij}^{(k)}]$ and $\overline{B}_k=[\overline{b}_{ij}^{(k)}]$, $k\in\langle m\rangle$, such that $|a_{ij}^{(k)}(t)|\le\overline{a}_{ij}^{(k)}\lambda(t)$ if $i\ne j$, $a_{ij}^{(k)}(t)\le\overline{a}_{ij}^{(k)}\lambda(t)$ if $i=j$, and $|b_{ij}^{(k)}(t)|\le\overline{b}_{ij}^{(k)}\eta(t)$ for $i,j\in\langle n\rangle$.

$(H_2)$ There exist continuous functions $\widetilde{\lambda}(t)\ge 0$ and $\widetilde{\eta}(t)\ge 0$ for $t\ge 0$, constant matrices $M_k=[m_{ij}^{(k)}]\succeq 0$ and $N_k=[n_{ij}^{(k)}]\succeq 0$, $k\in\langle m\rangle$, an $n$-dimensional vector function $\alpha(t)=(\alpha_i(t))\succeq 0$, and a constant $p>0$ such that

$$\begin{aligned}&\left|F_k(t,x(t),x(t-\tau(t)))\right| \\ &\quad\preceq \widetilde{\lambda}(t)\,M_k|x(t)|^p + \widetilde{\eta}(t)\,N_k|x(t-\tau(t))|^p + \alpha(t),\end{aligned} \tag{2}$$

where the first two parts in the right-hand side of above inequality are interpreted as the internal state disturbances, and the third part $\alpha(t)$ is defined as the external input disturbance.

The following two lemmas play a crucial role in the state estimation of system (1).

**Lemma 1** (see [39]). *Assume that $c\ge 0$ is a constant, $u(t)$ and $f(t)$ are nonnegative continuous functions defined on $[0,\infty)$, and $u(t)$ satisfies the following integral inequality*

$$u(t)\le c+\int_0^t f(s)\,u(s)\,ds, \quad t\ge 0. \tag{3}$$

*Then we have*

$$u(t)\le c e^{\int_0^t f(s)ds}, \quad t\ge 0. \tag{4}$$

**Lemma 2** (see [21]). *Assume that $u(t)$, $f(t)$, and $g(t)$ are nonnegative continuous functions defined on $[0,\infty)$, and $u(t)$ satisfies the following integral inequality*

$$u(t)\le k+\int_0^t\left(f(s)\,u(s)+g(s)\,u^p(s)\right)ds, \quad t\ge 0, \tag{5}$$

*where $k\ge 0$ and $p>0$ $(p\ne 1)$ are constants. If*

$$k^{1-p}-(p-1)\int_0^t g(s)\,e^{\int_0^s (p-1)f(\tau)d\tau}ds>0, \quad t\ge 0 \tag{6}$$

*then we have*

$$\begin{aligned}u(t)&\le e^{\int_0^t f(s)ds}\Big[k^{1-p} \\ &\quad -(p-1)\int_0^t g(s)\,e^{\int_0^s (p-1)f(v)dv}ds\Big]^{1/(1-p)}, \quad t\ge 0.\end{aligned} \tag{7}$$

## 3. Main Results

We first study the case of $p=1$ in Assumption $(H_2)$.

**Theorem 3.** *Assume that $(H_1)$ and $(H_2)$ with $p=1$ hold. If there exists an $n$-dimensional vector $\xi=(\xi_i)\succ 0$ such that*

$$\xi^T\overline{A}_k\prec 0, \quad k\in\langle m\rangle, \tag{8}$$

*and*

$$\int_0^\infty e^{\int_0^t[\gamma_1\lambda(s)-\gamma_3\widetilde{\lambda}(s)]ds}\xi^T\alpha(t)\,dt<\infty, \tag{9}$$

*then all solutions of system (1) satisfy*

$$\begin{aligned}\|x(t)\|&\le\frac{\widetilde{k}_1}{\min_{i\in\langle n\rangle}\xi_i} \\ &\quad\cdot e^{\int_0^t\{[\gamma_2\eta(s)+\gamma_4\widetilde{\eta}(s)]e^{\int_{s-h}^s[\gamma_1\lambda(v)-\gamma_3\widetilde{\lambda}(v)]dv}-\gamma_1\lambda(s)+\gamma_3\widetilde{\lambda}(s)\}ds}, \\ &\quad t\ge 0.\end{aligned} \tag{10}$$

*where*

$$\begin{aligned}\widetilde{k}_1 &= \|\phi\|_h \max_{i\in\langle n\rangle} \xi_i + \int_0^\infty e^{\int_0^t [\gamma_1\lambda(s)-\gamma_3\widetilde{\lambda}(s)]ds} \xi^T \alpha(s)\,ds,\\ \gamma_1 &= \min_{k\in\langle m\rangle, i\in\langle n\rangle} \frac{\eta_i^{(k)}}{\xi_i},\\ \gamma_2 &= \max_{k\in\langle m\rangle, i\in\langle n\rangle} \frac{\theta_i^{(k)}}{\xi_i},\\ \gamma_3 &= \max_{k\in\langle m\rangle, i\in\langle n\rangle} \frac{\widetilde{\eta}_i^{(k)}}{\xi_i},\\ \gamma_4 &= \max_{k\in\langle m\rangle, i\in\langle n\rangle} \frac{\widetilde{\theta}_i^{(k)}}{\xi_i},\end{aligned} \tag{11}$$

$\|\phi\|_h = \max_{-h\le t\le 0}\|\phi(t)\|$, $-\eta_i^{(k)}$, $\theta_i^{(k)}$, $\widetilde{\eta}_i^{(k)}$, $\widetilde{\theta}_i^{(k)}$ *are the ith entry of vectors* $\xi^T\overline{A}_k$, $\xi^T\overline{B}_k$, $\xi^T M_k$, *and* $\xi^T N_k$, *respectively.*

*Proof.* Let

$$V(x(t)) = \xi^T |x(t)|. \tag{12}$$

Without loss of generality, assume that $\sigma(t) = k$. Denote by $D_+V(x(t))$ the right derivative of $V(x(t))$ along the trajectory of system (1). We get from assumptions $(H_1)$ and $(H_2)$ that

$$\begin{aligned}D_+ |x_i(t)| &\le \dot{x}_i(t)\,\mathrm{sign}\, x_i(t)\\ &\le \sum_{j=1}^n \left[\lambda(t)\overline{a}_{ij}^{(k)} |x_j(t)| + \eta(t)\overline{b}_{ij}^{(k)} |x_j(t-\tau(t))|\right]\\ &\quad + \sum_{j=1}^n \left[\widetilde{\lambda}(t) m_{ij}^{(k)} |x_j(t)| + \widetilde{\eta}(t) n_{ij}^{(k)} |x_j(t-\tau(t))|\right]\\ &\quad + \alpha_i(t), \quad i\in\langle n\rangle, \ t\ge 0.\end{aligned} \tag{13}$$

Therefore,

$$\begin{aligned}D_+V(x(t)) &\le \xi^T\left[\lambda(t)\overline{A}_k + \widetilde{\lambda}(t) M_k\right]|x(t)|\\ &\quad + \xi^T\left[\eta(t)\overline{B}_k + \widetilde{\eta}(t) N_k\right]|x(t-\tau(t))|\\ &\quad + \xi^T\alpha(t), \quad t\ge 0.\end{aligned} \tag{14}$$

According to definitions of $\gamma_i$ for $i = 1, 2, 3, 4$, we derive from the above inequality that

$$\begin{aligned}D_+V(x(t)) &\le \left[-\gamma_1\lambda(t) + \gamma_3\widetilde{\lambda}(t)\right] V(x(t))\\ &\quad + \left[\gamma_2\eta(t) + \gamma_4\widetilde{\eta}(t)\right] V(x(t-\tau(t)))\\ &\quad + \xi^T\alpha(t), \quad t\ge 0.\end{aligned} \tag{15}$$

Multiply the above inequality by $e^{\int_0^t [\gamma_1\lambda(s)-\gamma_3\widetilde{\lambda}(s)]ds}$ and let

$$u(t) = e^{\int_0^t [\gamma_1\lambda(s)-\gamma_3\widetilde{\lambda}(s)]ds} V(x(t)). \tag{16}$$

Then we have

$$\begin{aligned}&D_+u(t)\\ &\quad\le \left[\gamma_2\eta(t) + \gamma_4\widetilde{\eta}(t)\right] e^{\int_{t-\tau(t)}^t [\gamma_1\lambda(s)-\gamma_3\widetilde{\lambda}(s)]ds} u(t-\tau(t))\\ &\qquad + e^{\int_0^t [\gamma_1\lambda(s)-\gamma_3\widetilde{\lambda}(s)]ds} \xi^T\alpha(t), \quad t\ge 0.\end{aligned} \tag{17}$$

Integrating it from 0 to $t$, we obtain

$$\begin{aligned}u(t) &\le u(0) + \int_0^\infty e^{\int_0^t [\gamma_1\lambda(s)-\gamma_3\widetilde{\lambda}(s)]ds} \xi^T\alpha(t)\,dt\\ &\quad + \int_0^t \left[\gamma_2\eta(s) + \gamma_4\widetilde{\eta}(s)\right]\\ &\quad \cdot e^{\int_{s-h}^s [\gamma_1\lambda(v)-\gamma_3\widetilde{\lambda}(v)]dv} u(s-\tau(s))\,ds, \quad t\ge 0.\end{aligned} \tag{18}$$

Set

$$\begin{aligned}\omega(t) &= \widetilde{k}_1 + \int_0^t \left[\gamma_2\eta(s) + \gamma_4\widetilde{\eta}(s)\right]\\ &\quad \cdot e^{\int_{s-h}^s [\gamma_1\lambda(v)-\gamma_3\widetilde{\lambda}(v)]dv} u(s-\tau(s))\,ds, \quad t\ge 0,\end{aligned} \tag{19}$$

$$\omega(t) = \widetilde{k}_1, \quad -h\le t\le 0, \tag{20}$$

where $\widetilde{k}_1$ is defined as in Theorem 3. Since $\omega(t)$ is a monotone nondecreasing function on $[-h, \infty)$, we get

$$u(t-\tau(t)) \le \omega(t-\tau(t)) \le \omega(t), \quad t\ge 0. \tag{21}$$

Consequently,

$$\begin{aligned}u(t) &\le \omega(t)\\ &\le \widetilde{k}_1\\ &\quad + \int_0^t \left[\gamma_2\eta(s) + \gamma_4\widetilde{\eta}(s)\right] e^{\int_{s-h}^s [\gamma_1\lambda(v)-\gamma_3\widetilde{\lambda}(v)]dv} \omega(s)\,ds,\\ &\qquad t\ge 0.\end{aligned} \tag{22}$$

Combining this and Lemma 1, it implies that

$$\begin{aligned}u(t) \le \omega(t) \le \widetilde{k}_1 e^{\int_0^t [\gamma_2\eta(s)+\gamma_4\widetilde{\eta}(s)] e^{\int_{s-h}^s [\gamma_1\lambda(v)-\gamma_3\widetilde{\lambda}(v)]dv} ds},\\ t\ge 0.\end{aligned} \tag{23}$$

By using the definition of $u(t)$, we have

$$\begin{aligned}&V(x(t))\\ &\quad\le \widetilde{k}_1 e^{\int_0^t \{[\gamma_2\eta(s)+\gamma_4\widetilde{\eta}(s)] e^{\int_{s-h}^s [\gamma_1\lambda(v)-\gamma_3\widetilde{\lambda}(v)]dv} - \gamma_1\lambda(s)+\gamma_3\widetilde{\lambda}(s)\} ds},\\ &\qquad t\ge 0.\end{aligned} \tag{24}$$

Therefore, (10) holds. This completes the proof of Theorem 3. □

*Remark 4.* For the particular case when $\lambda(t) \equiv \tilde{\lambda}(t) \equiv \eta(t) \equiv \tilde{\eta}(t) \equiv 1$, we get from Theorem 3 that all solutions of system (1) satisfy

$$\|x(t)\| \leq \frac{\tilde{k}_1}{\min_{i\in\langle n\rangle}\xi_i} e^{[(\gamma_2+\gamma_4)e^{(\gamma_1-\gamma_3)h}-\gamma_1+\gamma_3]t}, \quad t \geq 0. \tag{25}$$

It implies that system (1) is globally exponentially stable if $(\gamma_2+\gamma_4)e^{(\gamma_1-\gamma_3)h} - \gamma_1 + \gamma_3 < 0$.

Next, we consider the case of $p > 1$ in Assumption $(H_2)$.

**Theorem 5.** *Assume that $(H_1)$ and $(H_2)$ with $p > 1$ hold. If there exists an n-dimensional vector $\xi = (\xi_i) \succ 0$ such that (8) holds,*

$$\int_0^\infty e^{\gamma_1 \int_0^t \lambda(s)ds} \xi^T \alpha(t)\, dt < \infty, \tag{26}$$

*and*

$$\tilde{k}_2^{1-p} - (p-1)\int_0^t g(s)\, e^{\int_0^s (p-1)f(v)dv} ds > 0, \quad t \geq 0, \tag{27}$$

*then the corresponding solutions of system (1) satisfy*

$$\|x(t)\| \leq \frac{e^{\int_0^t [f(s)-\gamma_1\lambda(s)]ds}}{\min_{i\in\langle n\rangle}\xi_i}\left[\tilde{k}_2^{1-p} - (p-1)\int_0^t g(s)\, e^{\int_0^s (p-1)f(v)dv} ds\right]^{1/(1-p)}, \quad t \geq 0, \tag{28}$$

*where*

$$f(s) = \gamma_2 \eta(s)\, e^{\gamma_1 \int_{s-h}^s \lambda(v)dv},$$

$$g(s) = e^{\gamma_1(1-p)\int_0^s \lambda(v)dv}\left[\gamma_5 \tilde{\lambda}(s) + \gamma_6 \tilde{\eta}(s)\, e^{\gamma_1 p \int_{s-h}^s \lambda(v)dv}\right],$$

$$\tilde{k}_2 = \|\phi\|_h \max_{i\in\langle n\rangle} \xi_i + \int_0^\infty e^{\gamma_1 \int_0^t \lambda(s)ds} \xi^T \alpha(t)\, dt, \tag{29}$$

$$\gamma_5 = \max_{k\in\langle m\rangle, i\in\langle n\rangle} \frac{\tilde{\eta}_i^{(k)}}{\xi_i^p},$$

$$\gamma_6 = \max_{k\in\langle m\rangle, i\in\langle n\rangle} \frac{\tilde{\theta}_i^{(k)}}{\xi_i^p},$$

*$\gamma_1$, $\gamma_2$, $\tilde{\eta}_i^{(k)}$, and $\tilde{\theta}_i^{(k)}$ are defined as in Theorem 3.*

*Proof.* Let $V(x(t)) = \xi^T|x(t)|$. Following the same discussion in Theorem 3, we have

$$\begin{aligned} D_+V(x(t)) \leq{}& \lambda(t)\, \xi^T \overline{A}_k |x(t)| \\ &+ \eta(t)\, \xi^T \overline{B}_k |x(t-\tau(t))| \\ &+ \tilde{\lambda}(t)\, \xi^T M_k |x(t)|^p \\ &+ \tilde{\eta}(t)\, \xi^T N_k |x(t-\tau(t))|^p + \xi^T\alpha(t), \\ & t \geq 0. \end{aligned} \tag{30}$$

By using the definitions of $\gamma_i$ for $i = 5, 6, 7, 8$ and the basic inequality $(a+b)^p \geq a^p + b^p$ for $a, b \geq 0$ and $p > 1$, we can derive from the above inequality that

$$\begin{aligned} D_+V(x(t)) \leq{}& -\gamma_1 \lambda(t)\, V(x(t)) \\ &+ \gamma_2 \eta(t)\, V(x(t-\tau(t))) \\ &+ \gamma_5 \tilde{\lambda}(t)\, V^p(x(t)) \\ &+ \gamma_6 \tilde{\eta}(t)\, V^p(x(t-\tau(t))) + \xi^T\alpha(t), \\ & t \geq 0. \end{aligned} \tag{31}$$

By multiplying the above inequality by $e^{\gamma_1 \int_0^t \lambda(s)ds}$ and letting $u(t) = e^{\gamma_1 \int_0^t \lambda(s)ds} V(x(t))$, we obtain

$$\begin{aligned} D_+u(x(t)) &\leq e^{\gamma_1 \int_0^t \lambda(s)ds} \xi^T \alpha(t) + \gamma_2 \eta(t)\, e^{\gamma_1 \int_{t-\tau(t)}^t \lambda(s)ds} u(t-\tau(t)) \\ &+ \gamma_5 \tilde{\lambda}(t)\, e^{\gamma_1(1-p)\int_0^t \lambda(s)ds} u^p(t) \\ &+ \gamma_6 \tilde{\eta}(t)\, e^{\gamma_1(1-p)\int_0^t \lambda(s)ds} e^{\gamma_1 p \int_{t-\tau(t)}^t \lambda(s)ds} u^p(t-\tau(t)), \\ & t \geq 0. \end{aligned} \tag{32}$$

Integrating it from 0 to $t$, we get

$$\begin{aligned} u(t) \leq{}& \xi^T|x(0)| + \int_0^\infty e^{\gamma_1 \int_0^t \lambda(s)ds} \xi^T \alpha(t)\, dt \\ &+ \int_0^t \gamma_2 \eta(s)\, e^{\gamma_1 \int_{s-h}^s \lambda(v)dv} u(s-\tau(s))\, ds \\ &+ \int_0^t \gamma_5 \tilde{\lambda}(s)\, e^{\gamma_1(1-p)\int_0^s \lambda(v)dv} u^p(s)\, ds + \int_0^t \gamma_6 \tilde{\eta}(s) \\ &\cdot e^{\gamma_1(1-p)\int_0^s \lambda(v)dv} e^{\gamma_1 p \int_{s-h}^s \lambda(v)dv} u^p(s-\tau(s))\, ds, \\ & t \geq 0. \end{aligned} \tag{33}$$

Set

$$\begin{aligned} \omega(t) ={}& \tilde{k}_2 + \int_0^t \gamma_2 \eta(s)\, e^{\gamma_1 \int_{s-h}^s \lambda(v)dv} u(s-\tau(s))\, ds \\ &+ \int_0^t \gamma_5 \tilde{\lambda}(s)\, e^{\gamma_1(1-p)\int_0^s \lambda(v)dv} u^p(s)\, ds + \int_0^t \gamma_6 \tilde{\eta}(s) \\ &\cdot e^{\gamma_1(1-p)\int_0^s \lambda(v)dv} e^{\gamma_1 p \int_{s-h}^s \lambda(v)dv} u^p(s-\tau(s))\, ds, \\ & t \geq 0, \end{aligned} \tag{34}$$

$$\omega(t) = \tilde{k}_2, \quad -h \leq t \leq 0, \tag{35}$$

where $\tilde{k}_2$ is defined as in Theorem 5. It can be seen that $\omega(t)$ is a monotone nondecreasing function on $[-h, \infty)$ and $u(t - \tau(t)) \leq \omega(t-\tau(t)) \leq \omega(t)$ for $t \geq 0$. Therefore,

$$\omega(t) \leq \tilde{k}_2 + \int_0^t \gamma_2 \eta(s)\, e^{\gamma_1 \int_{s-h}^s \lambda(v)dv} \omega(s)\, ds$$

$$+\int_0^t e^{\gamma_1(1-p)\int_0^s \lambda(v)dv}\left[\gamma_5\tilde{\lambda}(s) + \gamma_6\tilde{\eta}(s)e^{\gamma_1 p\int_{s-h}^s \lambda(v)dv}\right]\omega^p(s)\,ds, \quad t \geq 0. \tag{36}$$

That is,

$$\omega(t) \leq \tilde{k}_2 + \int_0^t f(s)\,\omega(s)\,ds + \int_0^t g(s)\,\omega^p(s)\,ds, \quad t \geq 0, \tag{37}$$

where $f(s)$ and $g(s)$ are defined as in Theorem 5. Note that (27) holds. We conclude from Lemma 2 that

$$u(t) \leq w(t) \leq e^{\int_0^t f(s)ds}\left[\tilde{k}_2^{1-p} - (p-1)\int_0^t g(s)\,e^{\int_0^s (p-1)f(v)dv}ds\right]^{1/(1-p)}, \quad t \geq 0. \tag{38}$$

It implies that (28) holds true. This completes the proof of Theorem 5. □

*Remark 6.* Note that Theorem 5 gives a local state estimation result of system (1). That is, the state estimation (28) only holds for the solutions of system (1) satisfying (27). Let $\lambda(t) \equiv \tilde{\lambda}(t) \equiv \eta(t) \equiv \tilde{\eta}(t) \equiv 1$. If $\gamma_1 - \gamma_2 e^{\gamma_1 h} > 0$ and $\tilde{k}_2 < ((\gamma_5+\gamma_6 e^{\gamma_1 ph})/(\gamma_1-\gamma_2 e^{\gamma_1 h}))^{1/(1-p)}$, we have that condition (27) holds and the corresponding solutions of system (1) satisfy

$$\|x(t)\| \leq \frac{\tilde{k}_2^{1/(1-p)}}{\min_{i\in\langle n\rangle}\xi_i}e^{-(\gamma_1-\gamma_2 e^{\gamma_1 h})t}, \quad t \geq 0. \tag{39}$$

That is, system (1) is locally exponentially stable.

Finally, we introduce the following assumption on the nonlinear disturbance

$(H_3)$ There exist continuous functions $\tilde{\lambda}(t) \geq 0$ and $\tilde{\eta}(t) \geq 0$ for $t \geq 0$, an $n$-dimensional vector function $\alpha(t) \geq 0$, and constants $\tilde{\gamma}_5 \geq 0$, $\tilde{\gamma}_6 \geq 0$, $p > 0$ such that

$$\xi^T\left|F_k(t, x(t), x(t-\tau(t)))\right| \preceq \tilde{\gamma}_5\tilde{\lambda}(t)\left(\xi^T|x(t)|\right)^p + \tilde{\gamma}_6\tilde{\eta}(t)\left(\xi^T|x(t-\tau(t))|\right)^p + \alpha(t), \tag{40}$$

where $\xi \succ 0$ is an $n$-dimensional vector.

Similar to the proof of Theorem 5, the following result is immediate. Hence, we omit its proof.

**Theorem 7.** *Assume that $(H_1)$ holds and $p > 0$ $(p \neq 1)$. If there exists an $n$-dimensional vector $\xi = (\xi_i) \succ 0$ such that (8), (26), $(H_3)$, and (27) hold, where $\gamma_5$ and $\gamma_6$ are replaced by $\tilde{\gamma}_5$ and $\tilde{\gamma}_6$, respectively, then the corresponding solutions of system (1) satisfy (28).*

*Remark 8.* Note that condition (27) is always valid for $0 < p < 1$. Therefore, Theorem 7 gives a global state estimation result for system (1). For the particular case when $\lambda(t) \equiv \tilde{\lambda}(t) \equiv \eta(t) \equiv \tilde{\eta}(t) \equiv 1$ and $0 < p < 1$, a straightforward computation yields that all solutions of system (1) satisfy

$$\|x(t)\| \leq \frac{\left[\tilde{k}_2^{1-p}e^{-(1-p)(\gamma_1-\gamma_2 e^{\gamma_1 h})t} + \left(\gamma_5+\gamma_6 e^{\gamma_1 ph}\right)/\left(\gamma_1-\gamma_2 e^{\gamma_1 h}\right)\right]^{1/(1-p)}}{\min_{i\in\langle n\rangle}\xi_i}, \quad t \geq 0, \tag{41}$$

where $\tilde{k}_2$, $\gamma_1$, $\gamma_2$, $\gamma_5$, and $\gamma_6$ are defined as in Theorems 3 and 7. Consequently, all solutions of system (1) are bounded if $\gamma_1 - \gamma_2 e^{\gamma_1 h} > 0$.

## 4. Numerical Examples

In this section, three examples are given to illustrate the main results.

*Example 1.* Consider system (1) with $m = 2$, $\tau(t) = 0.2 + 0.1\sin t$,

$$\begin{aligned}
A_1(t) &= \begin{bmatrix} 0.5\cos t - 1.5 & 0.2\sin t \\ 0.2\cos t & 0.5\sin t - 1.5 \end{bmatrix}, \\
B_1(t) &= \begin{bmatrix} \dfrac{0.2t}{1+t} & 0.1\sin t \\ 0.1\cos t & 0.2\sin t \end{bmatrix}, \\
A_2(t) &= \begin{bmatrix} 0.5\sin t - 2 & 0.7\sin t \\ 0.7\cos t & 0.5\cos t - 2 \end{bmatrix}, \\
B_2(t) &= \begin{bmatrix} 0.1\sin t & 0.1\cos t \\ \dfrac{0.1t}{1+t} & \dfrac{0.1t}{1+t} \end{bmatrix}, \\
F_1(t, x(t), x(t-\tau(t))) &= \begin{bmatrix} 0.1\sin x_1(t) + 0.1\sin x_2(t-\tau(t)) + e^{-t} \\ 0.2\sin x_2(t) + 0.1\sin x_1(t-\tau(t)) + e^{-t} \end{bmatrix}, \\
F_2(t, x(t), x(t-\tau(t))) &= \begin{bmatrix} 0.2\sin x_2(t) + 0.1\sin x_1(t-\tau(t)) + e^{-t} \\ 0.2\sin x_1(t) + 0.2\sin x_2(t-\tau(t)) + e^{-t} \end{bmatrix}.
\end{aligned} \tag{42}$$

A straightforward computation yields

$$\overline{A}_1 = \begin{bmatrix} -1 & 0.2 \\ 0.2 & -1 \end{bmatrix},$$

$$\overline{B}_1 = \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.2 \end{bmatrix},$$

$$\overline{A}_2 = \begin{bmatrix} -1.5 & 0.7 \\ 0.7 & -1.5 \end{bmatrix},$$
$$\overline{B}_2 = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.1 \end{bmatrix},$$
$$M_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix},$$
$$N_1 = \begin{bmatrix} 0 & 0.1 \\ 0.1 & 0 \end{bmatrix},$$
$$M_2 = \begin{bmatrix} 0 & 0.2 \\ 0.2 & 0 \end{bmatrix},$$
$$N_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}. \quad (43)$$

Choosing $\xi = (1,1)^T$, then $\xi^T \overline{A}_i \prec 0$ for $i = 1,2$, $\gamma_1 = 0.8$, $\gamma_2 = 0.3$, $\gamma_3 = \gamma_4 = 0.2$, $h = 0.3$. Note that $(\gamma_2+\gamma_4)e^{(\gamma_1-\gamma_3)h}-\gamma_1+\gamma_3 < 0$, and $\int_0^{\infty} e^{\gamma_1 t} e^{-t} dt < \infty$. By Theorem 3 and Remark 4, system (1) is exponentially stable. Figure 1 shows the state trajectories of the system, where $\lambda = 1$ and the switching signal is defined as in Figure 2.

*Example 2.* Consider system (1) with $m = 2$, $\tau(t) = 0.1 + 0.05 \sin t$,

$$A_1(t) = \begin{bmatrix} -3+\sin t & -\cos t \\ \sin t & -3+\cos t \end{bmatrix},$$
$$B_1(t) = \begin{bmatrix} -0.3\cos t & 0.2\sin t \\ -0.1\sin t & -0.3\cos t \end{bmatrix},$$
$$A_2(t) = \begin{bmatrix} -4+\cos t & 2\sin t \\ -2\cos t & -4+\sin t \end{bmatrix},$$
$$B_2(t) = \begin{bmatrix} -0.4\sin t & -0.1\cos t \\ 0.2\cos t & -0.3\sin t \end{bmatrix}, \quad (44)$$
$$F_1(t, x(t), x(t-\tau(t))) = \begin{bmatrix} 0.1x_1^2(t) + 0.05x_2^2(t-\tau(t)) + 0.5e^{-2t} \\ 0.1x_2^2(t) + 0.05x_1^2(t-\tau(t)) + 0.5e^{-2t} \end{bmatrix},$$
$$F_2(t, x(t), x(t-\tau(t))) = \begin{bmatrix} 0.1x_2^2(t) + 0.0x_1^2(t-\tau(t)) + 0.5e^{-2t} \\ 0.1x_1^2(t) + 0.05x_2^2(t-\tau(t)) + 0.5e^{-2t} \end{bmatrix}.$$

A straightforward computation yields that

$$\overline{A}_1 = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix},$$
$$\overline{B}_1 = \begin{bmatrix} 0.3 & 0.2 \\ 0.1 & 0.3 \end{bmatrix},$$
$$\overline{A}_2 = \begin{bmatrix} -3 & 2 \\ 2 & -3 \end{bmatrix},$$
$$\overline{B}_2 = \begin{bmatrix} 0.4 & 0.2 \\ 0.1 & 0.3 \end{bmatrix},$$
$$M_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad (45)$$
$$N_1 = \begin{bmatrix} 0 & 0.05 \\ 0.05 & 0 \end{bmatrix},$$
$$M_2 = \begin{bmatrix} 0 & 0.1 \\ 0.1 & 0 \end{bmatrix},$$
$$N_2 = \begin{bmatrix} 0.05 & 0 \\ 0 & 0.05 \end{bmatrix}.$$

Choosing $\xi = (1,1)^T$, it can be seen that $\xi^T \overline{A}_i \prec 0$ for $i = 1,2$, $\gamma_1 = 1$, $\gamma_2 = 0.5$, $\gamma_5 = 0.1$, $\gamma_6 = 0.05$, $h = 0.15$. Note that $\gamma_1 > \gamma_2 e^{\gamma_1 h}$ and $\int_0^{\infty} e^{\gamma_1 t} e^{-2t} dt < \infty$. By Theorem 5 and Remark 6, system (1) is exponentially stable if $\|\phi\|_h < ((\gamma_5 + \gamma_6 e^{2\gamma_1 h})/(\gamma_1 - \gamma_2 e^{\gamma_1 h}))^{1/(1-2)} - 1 \cong 0.5$. Figure 3 shows the state trajectories of the system, where the switching signal is defined as in Figure 2.

*Example 3.* Consider system (1) with $m = 2$, $\tau(t) = 0.5 + 0.5 \sin t$,

$$A_1(t) = \begin{bmatrix} 0.5\sin t - 2 & \sin t \\ \cos t & 0.5\cos t - 2 \end{bmatrix},$$
$$B_1(t) = \begin{bmatrix} -0.2\sin t & 0.1\cos t \\ 0.1\cos t & -0.2\sin t \end{bmatrix},$$
$$A_2(t) = \begin{bmatrix} \cos t - 2 & \dfrac{0.5t}{1+t} \\ 0.5\sin t & \sin t - 2 \end{bmatrix},$$
$$B_2(t) = \begin{bmatrix} 0.1\sin t & 0.1\cos t \\ 0.2\sin t & 0.1\cos t \end{bmatrix},$$
$$F_1(t, x(t), x(t-\tau(t)))$$

$$= \begin{bmatrix} 0.2x_1^{1/3}(t) + 0.1x_2^{1/3}(t) + 0.1x_2^{1/3}(t-\tau(t)) + e^{-t} \\ 0.1x_1^{1/3}(t) + 0.1x_2^{1/3}(t) + 0.2x_1^{1/3}(t-\tau(t)) + e^{-t} \end{bmatrix},$$

$$F_2(t, x(t), x(t-\tau(t)))$$

$$= \begin{bmatrix} 0.3x_1^{1/3}(t) + 0.2x_2^{1/3}(t) + 0.2x_2^{1/3}(t-\tau(t)) + e^{-t} \\ 0.2x_1^{1/3}(t) + 0.3x_2^{1/3}(t) + 0.1x_1^{1/3}(t-\tau(t)) + e^{-t} \end{bmatrix}. \tag{46}$$

Based on a direct computation, we get

$$\overline{A}_1 = \begin{bmatrix} -1.5 & 1 \\ 1 & -1.5 \end{bmatrix},$$
$$\overline{B}_1 = \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.2 \end{bmatrix},$$
$$\overline{A}_2 = \begin{bmatrix} -1 & 0.5 \\ 0.5 & -1 \end{bmatrix},$$
$$\overline{B}_2 = \begin{bmatrix} 0.1 & 0.1 \\ 0.2 & 0.1 \end{bmatrix}, \tag{47}$$
$$M_1 = \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.1 \end{bmatrix},$$
$$N_1 = \begin{bmatrix} 0 & 0.1 \\ 0.2 & 0 \end{bmatrix},$$
$$M_2 = \begin{bmatrix} 0.3 & 0.2 \\ 0.2 & 0.3 \end{bmatrix},$$
$$N_2 = \begin{bmatrix} 0 & 0.2 \\ 0.1 & 0 \end{bmatrix}.$$

If we choose $\xi = (1,1)^T$, we have that $\xi^T \overline{A_i} \prec 0$ for $i = 1, 2$, $\gamma_1 = 0.5$, $\gamma_2 = 0.3$, $h = 1$. Note that $\gamma_1 > \gamma_2 e^{\gamma_1 h}$, $\int_0^\infty e^{\gamma_1 t} e^{-t} dt < \infty$, and $(H_3)$ holds for appropriate constants $\tilde{\gamma}_5$ and $\tilde{\gamma}_6$. By Theorem 7 and Remark 8, we conclude that each solution of system (1) is bounded. Figure 4 shows the state trajectories of the system, where the switching signal is defined as in Figure 2.

## 5. Conclusions

The problem of state estimation for switched time-varying systems with time-varying delay and nonlinear disturbance has been discussed in this paper. When the nonlinear disturbance satisfies linear and nonlinear growth conditions, explicit global (local) state estimation criteria have been established. For some particular cases, exponential stability and boundness of the system are taken into consideration. The method used in this paper is mainly based on the integral inequality technique. Finally, three numerical examples demonstrate the effectiveness of our main results.

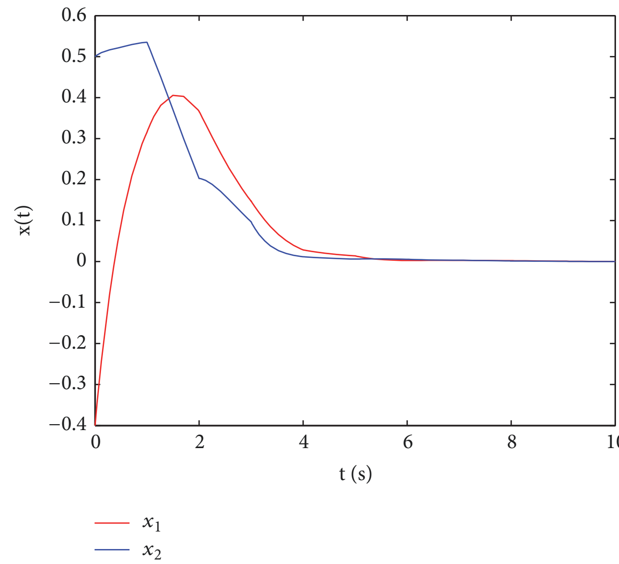


FIGURE 1: The state trajectories of system (1) with $\tau(t) = 0.2 + 0.1 \sin t$.

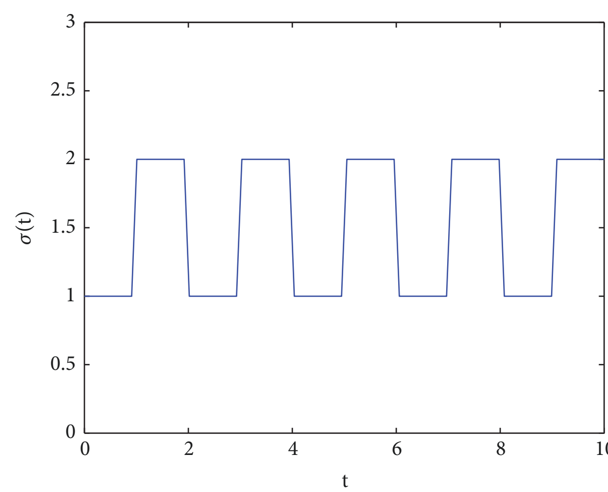


FIGURE 2: The given switching signal.

## Data Availability

We claim that all the data supporting the conclusions of the study has been contained in the paper. The readers can directly access it.

## Conflicts of Interest

We declare that there are no conflicts of interest regarding this paper.

## Acknowledgments

This work is supported by the National Natural Science Foundation of China under grants nos. 61873110, 61473133, and 11671227.

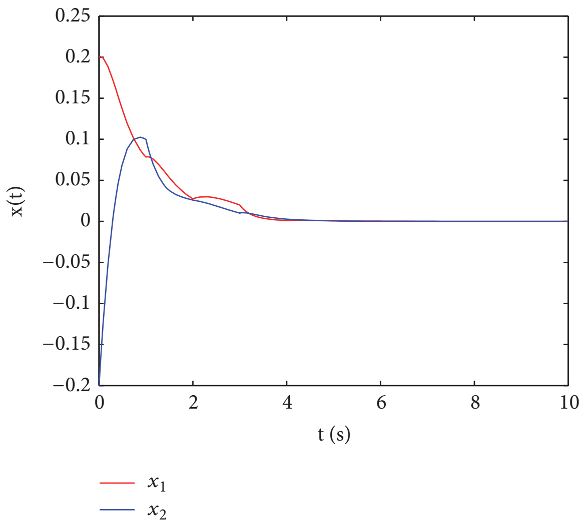


FIGURE 3: The state trajectories of system (1) with $\tau(t) = 0.1 + 0.05 \sin t$.

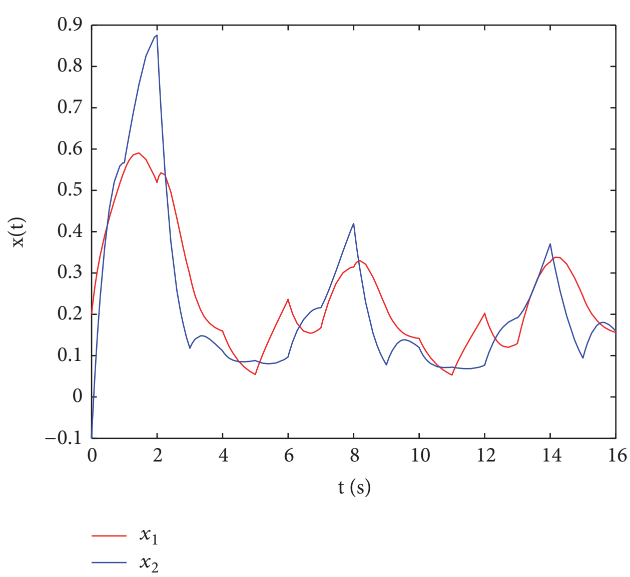


FIGURE 4: The state trajectories of system (1) with $\tau(t) = 0.5 + 0.5 \sin t$.

## References


[1] F. A. Wyzalek, "Hybrid electric vehicles," *IEEE Aerospace and Electronic Systems Magazine*, vol. 16, pp. 15–25, 2001.

[2] B. Lennartson, M. Tittus, B. Egardt, and S. Pettersson, "Hybrid systems in process control," *IEEE Control Systems Magazine*, vol. 16, no. 5, pp. 45–56, 1996.

[3] J. Qu, Z. Ji, C. Lin, and H. Yu, "Fast consensus seeking on networks with antagonistic interactions," *Complexity*, vol. 2018, pp. 1–15, 2018.

[4] Z. Ji and H. Yu, "A new perspective to graphical characterizition of multi-agent controllability," *IEEE Trans. Cybern*, vol. 47, pp. 1471–1483, 2017.

[5] D. Liberzon and A. S. Morse, "Basic problems in stability and design of switched systems," *IEEE Control Systems Magazine*, vol. 19, no. 5, pp. 59–70, 1999.

[6] D. Liberzon, *Switching in Cystems and Control*, Birkhäuser, Boston, Mass, USA, 2003.

[7] Z. Sun and S. S. Ge, *Stability Theory of Switched Dynamical Systems*, Springer, London, UK, 2011.

[8] Z. Wu and Y. Sun, "On easily verifiable conditions for the existence of common linear copositive Lyapunov functions," *Institute of Electrical and Electronics Engineers Transactions on Automatic Control*, vol. 58, no. 7, pp. 1862–1865, 2013.

[9] X. Zhao, X. Liu, S. Yin, and H. Li, "Improved results on stability of continuous-time switched positive linear systems," *Automatica*, vol. 50, no. 2, pp. 614–621, 2014.

[10] M. Xiang and Z. Xiang, "Exponential stability of discrete-time switched linear positive systems with time-delay," *Applied Mathematics and Computation*, vol. 230, pp. 193–199, 2014.

[11] L. Zhang, S. Zhuang, P. Shi, and Y. Zhu, "Uniform tube based stabilization of switched linear systems with mode-dependent persistent dwell-time," *Institute of Electrical and Electronics Engineers Transactions on Automatic Control*, vol. 60, no. 11, pp. 2994–2999, 2015.

[12] Y. Sun, "Stability analysis of positive switched systems via joint linear copositive Lyapunov functions," *Nonlinear Analysis: Hybrid Systems*, vol. 19, pp. 146–152, 2016.

[13] Y. Sun, Y. Tian, and X.-J. Xie, "Stabilization of positive switched linear systems and its application in consensus of multiagent systems," *Institute of Electrical and Electronics Engineers Transactions on Automatic Control*, vol. 62, no. 12, pp. 6608–6613, 2017.

[14] Y. Sun, Z. Wu, and F. Meng, "Common weak linear copositive lyapunov functions for positive switched linear systems," *Complexity*, vol. 2018, Article ID 1365960, 7 pages, 2018.

[15] X. Yang, Y. Tian, and X. Li, "Finite-time boundedness and stabilization of uncertain switched delayed neural networks of neutral type," *Neurocomputing*, vol. 314, pp. 468–478, 2018.

[16] Y. Li, Y. Sun, and F. Meng, "New criteria for exponential stability of switched time-varying systems with delays and nonlinear disturbances," *Nonlinear Analysis: Hybrid Systems*, vol. 26, pp. 284–291, 2017.

[17] Y. Li, Y. Sun, F. Meng, and Y. Tian, "Exponential stabilization of switched time-varying systems with delays and disturbances," *Applied Mathematics and Computation*, vol. 324, pp. 131–140, 2018.

[18] Y. Sun, "Stabilization of switched systems with nonlinear impulse effects and disturbances," *IEEE Transactions on Automatic Control*, vol. 56, no. 11, pp. 2739–2743, 2011.

[19] L. Li, F. Meng, and P. Ju, "Some new integral inequalities and their applications in studying the stability of nonlinear integro-differential equations with time delay," *Journal of Mathematical Analysis and Applications*, vol. 377, no. 2, pp. 853–862, 2011.

[20] Q. Feng and F. Meng, "Some generalized Ostrowski-Gruss type integral inequalities," *Computers Mathematics with Applications*, vol. 63, no. 3, pp. 652–659, 2012.

[21] J.-G. Dong, "Stability analysis of switched systems with general nonlinear disturbances," *Mathematical and Computer Modelling*, vol. 58, no. 7-8, pp. 1563–1567, 2013.

[22] Y. Tian, M. Fan, and F. Meng, "A generalization of retarded integral inequalities in two independent variables and their applications," *Applied Mathematics and Computation*, vol. 221, pp. 239–248, 2013.

[23] Y. Zhang, M. Wang, H. Xu, and K. L. Teo, “Global stabilization of switched control systems with time delay,” *Nonlinear Analysis: Hybrid Systems*, vol. 14, pp. 86–98, 2014.

[24] J. Gu and F. Meng, “Some new nonlinear Volterra-Fredholm type dynamic integral inequalities on time scales,” *Applied Mathematics and Computation*, vol. 245, pp. 235–242, 2014.

[25] Y. Tian, Y. Cai, and Y. Sun, “Asymptotic behavior of switched delay systems with nonlinear disturbances,” *Applied Mathematics and Computation*, vol. 268, pp. 522–533, 2015.

[26] F. Meng and Y. Huang, “Interval oscillation criteria for a forced second-order nonlinear differential equations with damping,” *Applied Mathematics and Computation*, vol. 218, no. 5, pp. 1857–1861, 2011.

[27] H. Liu, F. Meng, and P. Liu, “Oscillation and asymptotic analysis on a new generalized Emden-Fowler equation,” *Applied Mathematics and Computation*, vol. 219, no. 5, pp. 2739–2748, 2012.

[28] J. Shao, Z. Zheng, and F. Meng, “Oscillation criteria for fractional differential equations with mixed nonlinearities,” *Advances in Difference Equations*, p. 323, 2013.

[29] J. Shao, F. Meng, and Z. Zheng, “Oscillation theorems for linear matrix Hamiltonian systems,” *Applied Mathematics and Computation*, vol. 253, pp. 402–409, 2015.

[30] H. Liu and F. Meng, “Interval oscillation criteria for second-order nonlinear forced differential equations involving variable exponent,” *Advances in Difference Equations*, Paper No. 291, 14 pages, 2016.

[31] J. Qi and Y. Sun, “Global exponential stability of certain switched systems with time-varying delays,” *Applied Mathematics Letters*, vol. 26, no. 7, pp. 760–765, 2013.

[32] G. Zong, H. Ren, and L. Hou, “Finite-time stability of interconnected impulsive switched systems,” *IET Control Theory & Applications*, vol. 10, no. 6, pp. 648–654, 2016.

[33] Y. Yin, G. Zong, and X. Zhao, “Improved stability criteria for switched positive linear systems with average dwell time switching,” *Journal of The Franklin Institute*, vol. 354, no. 8, pp. 3472–3484, 2017.

[34] G. Zong and H. Zhao, “Input-to-state stability of switched nonlinear delay systems based on a novel Lyapunov-Krasovskii functional method,” *Journal of Systems Science & Complexity*, vol. 31, no. 4, pp. 875–888, 2018.

[35] D. Yang, X. Li, J. Shen, and Z. Zhou, “State-dependent switching control of delayed switched systems with stable and unstable modes,” *Mathematical Methods in the Applied Sciences*, vol. 41, no. 16, pp. 6968–6983, 2018.

[36] H. Gao, J. Xia, and G. Zhuang, “Robust Finite-time Extended Dissipative Control for a Class of Uncertain Switched Delay Systems,” *International Journal of Control, Automation, and Systems*, vol. 16, no. 3, pp. 1459–1468, 2018.

[37] Yuangong Sun and Fanwei Meng, “Reachable Set Estimation for a Class of Nonlinear Time-Varying Systems,” *Complexity*, vol. 2017, Article ID 5876371, 6 pages, 2017.

[38] N. Zhang, Y. Sun, and F. Meng, “State bounding for switched homogeneous positive nonlinear systems with exogenous input,” *Nonlinear Analysis: Hybrid Systems*, vol. 29, pp. 363–372, 2018.

[39] R. Bellman, “The stability of solutions of linear differential equations,” *Duke Mathematical Journal*, vol. 10, pp. 643–647, 1943.