Hindawi
Discrete Dynamics in Nature and Society
Volume 2019, Article ID 5481926, 7 pages
https://doi.org/10.1155/2019/5481926

Research Article

Modeling and Analysis on Teacher-Student Relationship

Li Xu [1,2] **and Qi Yang** [3]

[1]*School of Science, Tianjin University of Commerce, Tianjin 300134, China*
[2]*School of Mathematics, Tianjin University, Tianjin 300072, China*
[3]*School of Public Management, Tianjin University of Commerce, Tianjin 300134, China*

Correspondence should be addressed to Qi Yang; yangqi@tjcu.edu.cn

Received 30 August 2019; Revised 27 October 2019; Accepted 29 October 2019; Published 29 December 2019

Academic Editor: Rafael J. Villanueva

Although the teacher-student relationship has been addressed in some studies, the cooperation or reciprocal relations between teachers and students have not been explored sufficiently. In this paper, a difference equation model is applied to express the relationship, stability analysis at the positive steady state of the discrete model is done to verify that the performance output is not empty, and hypothesis testing is conducted to show the validity of the model by means of sample data from a college. Then some reasonable suggestions are proposed to improve the performance output of teachers and students.

1. Introduction

The relationship between teachers and students is the most basic relationship in the process of education and also a special kind of interpersonal and social relations, which is key to students' academic, social, and emotional development, and may altogether affect the educational environment [1]. Good teacher-student relationship, characterized by high levels of closeness, plays an active role in the development of both teachers and students, which contributes to students' social-emotional, behavioral, and academic adjustment [2, 3]. These studies provide insight into how teacher-student relationships influence students' outcomes. The teacher-student relationships can be regarded as predictors of students' academic outcomes [4], which also indicates the central role of relationship between students and teachers, especially with the teachers' role shifting from "a sage on the stage" to "a guide on the side." [5].

It is clear that teacher-student relationship lies at the heart of teaching and learning, but it is not easy to clarify and measure the relationship better from the perspective of teachers or students or both [6]. The interaction between teachers and students is a complex system which will involve age, sex, family socioeconomic status, feelings, beliefs, and other factors. At different stage from fundamental education through to college, the teacher-student relationship will have different characteristics, the ways how to build a good relationship between teachers and students will be different. For students studying at university, they are more mature and more independent, so that they do not want to be just a student. They may wish a teacher-student relationship as one between fisherman and fishing net, in which the student is a fishing net, the teacher is a fisherman who should cherish his/her students and be able to release the net and allow students to work on their own rather than controlling the students strictly, and the fisherman cannot fish without the net [7].

In fact, students' characteristics including academic achievement contribute to not only the formation of teachers' preference and acceptance for students, but teachers' academic outcomes. This is especially true in colleges and universities. Although good academic performance is very important to students, scientific literacy, or research ability is more important. These students will join some teachers' group to make some research with the teachers, which will also improve the teachers' academic performance. The development of teacher-student relationships will also be influenced by personal and demographic characteristics of both relationship partners by means of developmental systems theory [8]. It will be true that cooperation or reciprocal relation is one of the most common forms of teacher-student interaction, which emphasizes the cooperation between teachers and students.

The cooperation or reciprocal relations between students and teachers can be explored with the application of qualitative research methods. Many theories and perspectives can be

applied to study the teacher-student relationship, which mainly include attachment theory [9, 10], interpersonal theory [11, 12], power dynamics [7], and phenomenological perspective [13]. Combined with these theories or perspectives, qualitative study or quantitative study or both of them is used to explore the nature of teacher student interactions (for example, see [4, 14–17]). In [14], phenomenographic methods can reveal that teachers experience pedagogic engagements with students. Quantitative study with three competing models shows the hypothesized directionality of influence in relations between teacher acceptance, student perceived teacher support, and academic achievement in [4].

In almost all of the quantitative study, sample data must be collected firstly by means of questionnaires and observation. Descriptive statistics can help us find out some correlation of the data, then further statistical analysis is necessary. Modeling is an effective tool to analyze data with statistical analysis. Structural equation, hierarchical linear model, logistic and negative binomial regression models, growth mixture model can be found in [12, 18–20]. A longitudinal, cross-lagged path analysis was conducted to determine the patterns of influence among teacher preference, peer rejection, and student aggression in [21], but it failed to enable conclusions about reciprocal relations. In [4], structural equation modeling was used to analyze the relations between teacher acceptance, student-perceived teacher support, and academic achievement, which can support the reciprocal relation.

The teacher-student relationship can be defined as a dynamic system, in accordance with ecological systems theory in human development [22]. It is a whole structure, which is composed of interrelated two parts: teachers and students. The development of the teacher-student society resembles that of the biological population system, and biological models can be used to examine the micro-behavior of groups and individuals. The Lotka-Volterr model is widely used for studying interspecific competition or cooperation and explaining the outcomes of the competitive or cooperation interactions among populations (e.g. [23, 24]). To the best of our knowledge, the Lotka-Volterr model has not been used for the teacher-student relationship. In this paper, we will apply a discrete model in biology, the discrete Lotka-Volterra model for investigating the teacher-student relationship, with an aim to provide practical suggestion for more performance output. And this study is focused on answering the following questions:

(1) Why can the discrete model express the relationship between teachers and students?
(2) Why do stability analysis needs to be done?
(3) What valuable conclusions can be drawn from our study?

So the paper is organized as follows. In Section 2, sample data can be obtained according to the quantitative criteria of performance output on teachers and students in a college. A two species discrete-time model is used to express the teacher-student relationship, and local stability conditions at the positive fixed point can be deduced in Section 3. Least square method can be used to estimate the parameters, hypothesis testing, and predictive function are finished to guarantee the validity of the model, and reasonable suggestions are proposed in Section 4. The final section is the discussion and conclusion.

2. Data

The present study was based on data from 15 teachers working in statistics department, Tianjin University of Commerce, China, and 25 students enrolled in 2015 and studying in the same department in the college. Teachers matched to the students are almost certainly the ones who taught the class. The data come from their performance output in seven terms from 2015 to 2019, which rang over four-year study cycle for college students (the data of the eighth semester have not been collected yet). The principles for quantifying the performance output of teachers and students are listed as follows.

2.1. Performance Output on Teachers. Performance output on teacher mainly includes course teaching and other work including teaching achievement award, the one from guiding student competition award, university student innovation, and entrepreneurship contest. Considering that scientific research can promote teaching, research projects and papers can also be considered. According to calculation by the school, the currency value of 16 teaching hours is almost equivalent to a scientific research score. Then how course teaching is converted to scores is based on the following regulations of the college.

$$\text{teaching score} = \frac{\text{teaching hours}}{16}. \tag{1}$$

Other output can be quantified into specific scores according to the quantitative criteria of the teaching and scientific research which is as in Table 1.

The total performance output score for every teacher can be obtained as follows.

$$\text{performance output score} = \text{teaching score} + \text{other work score}. \tag{2}$$

The performance output on every teacher can be found in Table 2.

2.2. Performance Output on Students. For students, the main output includes test scores and some additional scores. The additional scores come from the College English Test Band Four and Six, the discipline competition, a professional qualification and scientific research. These output can also be quantified into specific scores according to the quantitative criteria of the college. The list of specific options for additional scores is as in Table 3.

The total performance output score for every student can be obtained as follows.

$$\text{performance output score} = \text{test score} + \text{the additional score}. \tag{3}$$

Table 1: The quantitative criteria of performance output on teachers with the exception of course teaching.

Output	Level	Score
Teaching achievement award	The first-class national award	480
	The second-class national award	320
	The first-class provincial award	65
	The second-class provincial award	35
Guiding students' contest	The first-class national award	9
	The second-class national award	8
	The first-class provincial award	7
	The second-class provincial award	6
Guiding students' innovation contest	National award	4
	Provincial award	3
Papers	Q1 (Scientific Journal Ranking)	20
	Q2 (Scientific Journal Ranking)	15
	Q3 (Scientific Journal Ranking)	10
	Q4 (Scientific Journal Ranking)	8
Research project	National major or key scientific research projects	100
	Sub-task in national major or key scientific research projects	50
	General national scientific research projects	25
	Major scientific research projects at the provincial or ministerial level	20
	Key scientific research projects at the provincial or ministerial level	15
	General provincial or ministerial scientific research projects	10

Table 2: Performance output on each teacher.

Teacher number	Term 1	Term 2	Term 3	Term 4	Term 5	Term 6	Term 7
1	8.36	16.73	9.51	19.03	11.20	22.40	20.88
2	19.65	41.30	27.81	61.62	43.32	76.64	24.33
3	29.12	50.25	34.21	58.43	22.23	51.46	30.91
4	20.22	38.44	23.24	43.47	21.53	46.06	28.73
5	16.81	28.61	23.76	39.51	24.47	42.95	30.46
6	14.33	24.66	22.72	39.45	19.62	39.24	11.22
7	36.64	64.27	29.62	48.24	23.45	43.91	21.31
8	20.80	36.59	15.22	30.45	17.22	34.43	21.83
9	13.46	26.92	14.04	28.08	16.09	32.19	23.33
10	14.15	28.29	21.53	35.06	15.62	32.24	17.50
11	1.98	21.96	14.75	27.49	13.96	28.92	16.25
12	13.55	27.10	18.53	34.07	18.89	37.78	24.78
13	10.45	20.91	22.66	40.33	31.24	52.47	17.24
14	16.42	32.83	17.39	34.77	20.59	44.18	26.75
15	10.43	19.87	10.41	19.82	9.51	19.03	15.44

Table 3: The quantitative criteria of additional performance output on students.

Output	Level	Score
Research paper	Indexed by SCI or EI	1
	others	0.5
The discipline competition	The first-class national award	3
	The second-class national award	2
	The first-class provincial award	1
	The second-class provincial award	0.5
A professional qualification	Data analyst qualification	0.25
	The securities qualification	0.25
	Teaching qualification	0.25
English test band four and six	CET4	0.5
	CET6	0.5

The performance output on every student can be found in Table 4.

3. Model and Stability Analysis

In this section, we will make a discrete model express the relationship between teachers and students. Stability analysis at the positive steady state of the discrete model is done so that we wish to verify that the performance output is not empty and keep at an equilibrium level over time.

Cooperation in ecology can be divided into interspecific (mutualism) and intraspecific cooperation. Mutualism involves cooperation between different species or groups. In these kinds of interspecific interactions, both groups can benefit from each other. The more there is of one group, the better for the other one [24, 25]. Teacher-student interaction has such cooperative characteristics. Continuous or discrete Lotka-Volterr cooperation models could qualitatively explain the main features associated with the underlying kinetics of cooperation.

The discrete models governed by difference equations are more appropriate than the continuous ones when the populations have nonoverlapping generations, and the discrete ones can also provide efficient computations for numerical simulations [26, 27]. In fact, the performance output of teachers and students is not time continuous, and the data of performance output are collected at the end of every semester or the beginning of the next semester, which is discrete. Then we consider the system of difference equations as follows.

$$\begin{aligned} \Delta x(k) &= x(k+1) - x(k) = x(k)\big(r_1 - a_{11}x(k) + a_{12}y(k)\big), \\ \Delta y(k) &= y(k+1) - y(k) = y(k)\big(r_2 + a_{21}x(k) - a_{22}y(k)\big), \\ & k = 1, 2, \ldots, m, \end{aligned} \tag{4}$$

or

$$\begin{aligned} x(k+1) &= x(k)\big(r_1 + 1 - a_{11}x(k) + a_{12}y(k)\big), \\ y(k+1) &= y(k)\big(r_2 + 1 + a_{21}x(k) - a_{22}y(k)\big), \end{aligned} \quad k = 1, 2, \ldots, m, \tag{5}$$

where $\Delta x(k)$ is incremental output in adjacent semesters for teachers, $x(k)$ is the mean of performance output for all the

Table 4: Performance output on each student.

Student number	Term 1	Term 2	Term 3	Term 4	Term 5	Term 6	Term 7
1	74.09	66.44	66.70	51.95	68.63	69.70	69.45
2	82.91	73.78	81.87	67.95	85.42	81.20	89.64
3	81.18	75.29	79.43	71.95	83.21	74.20	74.00
4	79.41	69.30	80.87	69.10	75.95	66.40	83.09
5	70.14	63.07	68.22	63.80	70.53	69.80	66.09
6	89.59	80.93	86.87	82.60	89.37	83.30	94.09
7	89.68	81.59	87.09	83.40	90.47	79.00	92.82
8	81.86	68.52	78.26	67.90	84.00	77.80	82.73
9	73.09	64.30	77.17	65.05	78.63	71.70	86.55
10	79.14	72.22	81.48	68.70	88.36	88.50	83.45
11	81.50	73.56	81.26	76.45	81.00	73.50	80.91
12	73.55	69.15	73.57	72.90	78.47	68.90	85.36
13	79.09	67.37	81.70	69.85	82.58	82.80	86.91
14	74.36	63.48	70.74	69.70	80.21	66.60	78.73
15	76.36	74.63	81.35	76.65	79.42	71.20	76.09
16	72.59	63.56	68.48	67.90	67.59	67.40	73.18
17	85.82	80.56	85.57	86.25	89.59	88.70	92.82
18	84.32	80.96	89.91	87.95	84.16	86.10	89.00
19	80.27	75.41	88.43	83.55	89.53	81.70	83.18
20	82.09	70.78	83.65	72.95	84.32	83.70	91.18
21	86.95	82.85	89.09	90.95	85.74	89.40	82.73
22	80.64	76.22	84.87	80.25	87.79	85.00	88.73
23	71.91	67.41	70.48	67.00	71.00	68.90	69.64
24	65.86	59.96	62.96	50.30	33.63	46.40	58.09
25	73.64	59.44	77.43	63.85	71.21	74.10	82.27

teachers at the kth term, $\Delta y(k)$ is incremental output in adjacent semesters for students, $y(k)$ is the mean of performance output for all the students at the kth term. The $r_1, r_2, a_{11}, a_{12}, a_{12}$, and a_{12} are assumed to be positive constants, in which $r_i (i = 1, 2)$ is its effort level, rate, a_{ii} is the level of individual capacity constraints $(i = 1, 2)$, a_{12} and a_{12} are the inter-specific cooperation rates.

According to the definition of fixed points, the fixed points of map (4) are solved by direct calculation yields four fixed points $E_0(0,0), E_1(0, r_2/a_{22}), E_2(r_1/a_{11}, 0)$ and $E_3(x^*, y^*) = ((r_1a_{22} + r_2a_{12})/(a_{11}a_{22} - a_{12}a_{21}), (r_1a_{21} + r_2a_{11})/(a_{11}a_{22} - a_{12}a_{21}))$, E_3 is the only positive one if $a_{11}a_{22} - a_{12}a_{21} > 0$ hold. As we have mentioned before, good teacher-student relationship aims to better performance output for the both groups, zeros output is not a desirable one. Then we only care about the positive fixed point in this paper, and investigate stability and changes on performance output over time.

There are some previous studies which think about stability and changes in teacher-student relationships over time. For instance, the authors found that the degree of closeness in teacher-student relationships decreased from kindergarten to sixth grade, whereas teacher-student conflict increased over time form the viewpoint of statistical analysis in [28]. In this paper, local asymptotic stability of the positive fixed point will be obtained by means of Proposition 1 in [29]. We can obtain the conditions to make the system (4) stable at the positive fixed point E_3. Namely, the system (4) at the fixed point E_3 is stable when the conditions

Table 5: Results of parameter estimation.

Variable	Coefficient	Std. error	t-Statistic	Prob.
r_1	0.910960	0.110035	17.36684	0.01808
a_{11}	0.056204	0.005016	11.20494	0.00152
a_{12}	0.061591	0.013208	4.663159	0.01861
r_2	0.994115	0.397325	5.018851	0.01520
a_{21}	0.004564	0.001811	2.520155	0.08620
a_{22}	0.014540	0.004769	3.048857	0.05550

$$1 - a_{11}x^* - a_{22}y^* < (1 - a_{11}x^*)(1 - a_{22}y^*) < 1 + a_{12}a_{21}x^*y^* \tag{6}$$

hold.

4. Parameter Estimation and Hypothesis Test

The purpose of the section is to determine the parameters which best fit the simulations to the measurements, and to conduct hypothesis testing on certain explanatory variables effect.

We denote (4)

$$\begin{aligned} U(k+1) &= \begin{pmatrix} x(k+1) \\ y(k+1) \end{pmatrix} = \begin{pmatrix} x(k)(r_1 + 1 - a_{11}x(k) + a_{12}y(k)) \\ y(k)(r_2 + 1 + a_{12}x(k) - a_{12}y(k)) \end{pmatrix} \\ &= \begin{pmatrix} x(k)(R_1 - a_{11}x(k) + a_{12}y(k)) \\ y(k)(R_2 + a_{12}x(k) - a_{12}y(k)) \end{pmatrix} \\ &= \begin{pmatrix} x(x) & -x^2(k) & x(k)y(k) & 0 & 0 & 0 \\ 0 & 0 & 0 & y(k) & x(k)y(k) & -y^2(k) \end{pmatrix} \\ &\quad \cdot \begin{pmatrix} r_1 + 1 \\ a_{11} \\ a_{12} \\ r_2 + 1 \\ a_{21} \\ a_{22} \end{pmatrix} = H(U(k))C, \end{aligned} \tag{7}$$

where

$$\begin{aligned} H(U(k)) &= \begin{pmatrix} x(x) & -x^2(k) & x(k)y(k) & 0 & 0 & 0 \\ 0 & 0 & 0 & y(k) & x(k)y(k) & -y^2(k) \end{pmatrix}, \\ C &= (r_1 + 1, a_{11}, a_{12,} r_2 + 1, a_{21}, a_{22})^T. \end{aligned} \tag{8}$$

We apply the observed data provided in Section 2 into (4), a linear system with variable C can be obtained. Least square method can be used to deal with the parameter estimation. By means of software R, estimation results are shown in Table 5. Then $E_3(x^*, y^*) = ((r_1a_{22} + r_2a_{12})/(a_{11}a_{22} - a_{12}a_{21}), (r_1a_{21} + r_2a_{11})/(a_{11}a_{22} - a_{12}a_{21})) = (35.26, 79.44)$, we can easily verify the fixed point $(35.26, 79.44)$ which can hold stability condition (6), namely, the mean performance output of the teachers and students will tend to $(35.26, 79.44)$ under the system environment over time.

Under the significance level at 0.05, r_1, a_{11}, and a_{12} are the remarkable factors affecting the performance output of all the teachers, which show that teachers' own efforts are more important, teachers' overall performance output is constrained by the environment, and the overall performance of students also plays a greater role in promoting teachers' performance. According to the meaning of parameters r_1, a_{11}, and a_{12}, three suggestions for teachers are given:

Table 6: F-test.

	F-Statistic	Critical values	Prob.
Equation (3)	162.1	9.277	0.00878
Equation (3)	19.18	9.277	0.01954

(1) The teacher should continuously improve the level of personal teaching and scientific research.

(2) The teacher should have a good competition and cooperation environment to decrease the negative impact of the environment.

(3) The teacher should incorporate more students into his research team and improve the cooperation with students.

Under the significance level at 0.05, r_2, a_{21}, and a_{22} are the remarkable factors affecting the performance output of all the students. The fact r_2 is significance shows that self-effort is more important. The fact a_{21} and a_{22} are the remarkable shows that learning environment and teacher level also affect students' performance. Then according to the meaning of parameters r_2, a_{21}, and a_{22}, three suggestions for students can also be given:

(1) The student must study hard.

(2) The student should create a harmonious learning environment with good competition and cooperation to reduce the negative restraint of the environment.

(3) The student should actively join teachers' scientific research to improve their scientific literacy.

To verify the whole effectiveness of the estimation, F test is carried out in the following part. Given certain significant level $\alpha = 0.05$, the test result in (4) can be shown in Table 6.

It is proven that the whole regression equation is significant.

Model (4) has predictive function. In order to assess the predictive performance of the model, we need to form both a prediction and a metric that quantifies the difference between the prediction and the observed data. When the system parameters are fixed, output of different terms $\overline{x}(k)$ and $\overline{y}(k)$ ($k > 1$) can be calculated with the initial observed data $x(1)$ and $y(1)$, and a statistical indices (relative error (RE)) is used to evaluate the accuracy of the prediction model.

$$\begin{aligned} \mathrm{RE}_{\mathrm{teacher}}(k) &= \frac{|\overline{x}(k)-x(k)|}{x(k)} \times 100\%, \\ \mathrm{RE}_{\mathrm{student}}(k) &= \frac{|\overline{y}(k)-y(k)|}{y(k)} \times 100\%, \end{aligned} \quad k > 1. \tag{9}$$

where $x(k)$ and $y(k)$ are teachers' and students' performance observed data, respectively. $\overline{x}(k)$ and $\overline{y}(k)$ are the prediction from model (4). And mean absolute error (MAE) [30, 31] can also be obtained.

$$\begin{aligned} \mathrm{MAE}_{\mathrm{teacher}} &= \frac{1}{n-1}\sum_{k=2}^{n-1}|\overline{x}(k) - x(k)| \times 100\%, \\ \mathrm{MAE}_{\mathrm{student}} &= \frac{1}{n-1}\sum_{k=2}^{n-1}|\overline{y}(k) - y(k)| \times 100\%. \end{aligned} \tag{10}$$

Table 7: Comparisons between predicted and true values on teachers' output.

k	$x(k)$	$\overline{x}(k)$	$\mathrm{RE}_{\mathrm{teacher}}(k)$
1	17.02	—	—
2	31.92	33.62043	5.327
3	20.36	19.72913	3.099
4	37.32	37.33658	0.044
5	20.53	20.47468	0.269
6	40.26	37.98324	5.655
7	22.06	23.29558	5.600
8	—	40.71044	—

Table 8: Comparisons between predicted and true values on students' output.

k	$y(k)$	$\overline{y}(k)$	$\mathrm{RE}_{\mathrm{student}}(k)$
1	78.63	—	—
2	70.18	73.00906	4.031
3	79.36	78.55912	1.009
4	73.87	74.04986	0.243
5	79.65	80.54765	1.127
6	76.98	76.01625	1.252
7	81.95	81.47751	0.577
8	—	74.25011	—

Comparisons between predicted and true values on teachers' and students' output are shown in Tables 7 and 8, and $\mathrm{MAE}_{\mathrm{teacher}} = 3.333$, $\mathrm{MAE}_{\mathrm{student}} = 1.373$.

5. Conclusion

We give several concluding remarks in this section.

(1) The main aim of this study was to explore the cooperation or reciprocity mechanism between teacher group and student group. A two species discrete-time model is used to express the teacher-tudent relationship from the viewpoint of ecology. As shown in Tables 5 and 6, the model can exhibit an acceptable fit, the factors affecting performance output are reasonable and significant. It should be noted that there will be two issues to be discussed.

(a) In addition to the internal and external effects mentioned in Model (3), performance output is also affected by the underlying uncertain disturbances, which could be summarized as environmental noises [32]. In population ecology, environmental noises prove to have notable impacts on the ecological system [33]. In this context, random perturbations associated with changes of resource availability, policy environment, and teachers' and students' behavior may also play a formidable role on performance output. It will be more reasonable to establish a stochastic model.

(b) In this paper, we pay attention on the dynamics between teacher group and student group. In fact, we can also discuss the relationship between teacher group and student individual, teacher individual and student group, and teacher individual and student individual. As an example, the quantitative analysis model of the relationship between teacher group and student individual can be shown as follows

$$\begin{aligned} y_i(k+1) &= y_i(k)\left(r_i - \sum_{j=1,j\neq i}^{n} n_{ij} y_j(k) + l_i x(k)\right), \\ x(k+1) &= x(k)\left(r + \sum_{i=1}^{n} m_i y_j(k) - a x(k)\right), \end{aligned} \quad k = 1, 2, \ldots, m, \tag{11}$$

where $x(k)$ is the performance output of the teacher group at the kth term, $y_i(k)$ is the performance output of the ith student individual at the kth term. The r_i, r, n_{ij}, l_i, m_i, and a are positive constants and n is positive integer which denote the number of students. It should be noted that the least square method will be inappropriate to estimate model (10), since there are more variables than sample data. Partial least square regression or other methods can be considered.

(2) Besides of parameter estimation and hypothesis testing, stability analysis at the positive steady state of the discrete model (4) aims to obtain the conditions to ensure the performance output of teachers and students stabilize in a desired state. In other words, how to control individual effort and degree of cooperation can make teachers and students achieve more performance output.

(3) This work may contribute a small piece in the understanding of the relation between teacher-student relationships and provide some empirical support for the reciprocal nature of this relationship. The reciprocal dynamics is indicated to represent some important potential factors that can contribute to the stability of the system between teachers and students. Some feasible suggestions for teachers and students are also put forward for better adjustment which can result into more performance.

Data Availability

All data are fully available without restriction.

Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this paper.

Acknowledgments

The authors would like to thank the reviewers for their constructive comments and suggestions which improved the quality of the paper. This work was supported by Tianjin Municipal Education Commission Scientific Research Project of China (Grant No. 161082) and Applied Study Program (Grant Nos. 171006901B, 60204, and WH18012).

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