Hindawi
Discrete Dynamics in Nature and Society
Volume 2019, Article ID 6276250, 9 pages
https://doi.org/10.1155/2019/6276250

Research Article

Estimation of Ask and Bid Prices for Geometric Asian Options

Tao Chen, Kaili Xiang, and Xuemei Luo

School of Economic Mathematics, Southwestern University of Finance and Economics, Chengdu 610074, China

Correspondence should be addressed to Tao Chen; shuxue_ch@sina.com

Received 29 October 2018; Accepted 22 January 2019; Published 13 March 2019

Academic Editor: Emilio Jiménez Macías

Traditional derivative pricing theories usually focus on the risk-neutral price or the equilibrium price. However, in highly competitive financial markets, we observed two prices which are called bid and ask prices; then the unique risk-neutral price fails to hold. In this paper, within the framework of conic finance, we provide a useful approach to evaluate the ask and bid prices of geometric Asian options and obtain the explicit formulas for the ask and bid prices. Finally, numerical examples show that the higher the market liquidity parameter γ, the wider the spread and hence the less the liquidity.

1. Introduction

Asian options give the holder a payoff that depends on the average price of the underlying over some prescribed period. This averaging of the underlying can bring about two significant advantages: one is to reduce the risk of manipulating the underlying asset and the other is that it costs less than standard American options and European options (see Wilmott [1], chap. 25). Asian options are actively traded in both exchanges and over-the-counter markets. In Black-Scholes framework, the study of exotic options has attracted the attention of many scholars.

Although there are many methods, such as the PDE method, Martingale method, Monte Carlo simulation, and the binomial tree, to solve option price, an efficient method has not been found yet. The most important factor is that the real market has many uncertain factors. As we all know, in traditional financial mathematics, the foundations of the option pricing theory are built on the paradigm of frictionless and competitive markets. However, in the real market, the risk elimination is typically unattainable and not available. Furthermore, we observe two prices, one for buying from the market called the ask price and another for selling to the market called the bid price. Hence, in the real market, we can no longer depend on the unique risk-neutral price (or the law of one price or equilibrium price).

There are diversity of theoretical approaches to estimating ask and bid prices. Barles and Soner [2], Cvitanić and Karatzas [3], Constantinides [4], Lo et al. [5], and Jouini and Kallal [6] attempted to study spreads which included transaction costs of trading in liquid markets. Easley and O'Hara [7] and Han and Shino [8] study price formation in securities markets. Copeland and Galai [9] discuss the effects of information on the bid-ask spread. Glosten and Milgrom [10] focus on the effects of heterogeneously informed traders on market makers. In [11–14] the researcher have carried out inventory costs and order processing of liquidity providers. In [15–18], statistical studies are used to model bid-ask spread. However, these models are not effective enough to explain the magnitude of the spreads observed in the markets. A new theory is built up by Cherny and Madan [19, 20], referred to as the conic finance theory. In the conic finance framework, the market acts as a passive counterparty to all transactions, buying at the ask price and selling at the bid price. The spread between bid-ask prices is a measure of illiquidity.

Although there are a number of literatures based on conic finance theory, they focus on credit risk [21, 22], design of portfolio [23, 24], and hedging of financial and insurance risks [25, 26]. To the best of our knowledge, there is no literature research on valuation of ask and bid prices for geometric Asian option. In this paper, within the framework of conic finance, we lead to the explicit formulas for the ask and bid prices of geometric Asian option.

The content of this paper is organized as follows. In Section 2 we introduce the risk-neutral price for the geometric Asian option. Within the framework of conic finance, Section 3 is devoted to estimating the bid-ask prices for geometric Asian option. And we obtain the explicit formulas for the bid-ask prices. In Section 4, we present numerical results for the bid-ask prices. Finally, we finish our paper by concluding remarks in the last section.

2. Geometric Asian Option under the Law of One Price

In this section, we start with a brief description of the geometric Asian option model presented in [27].

Under a probability space $(\Omega, \mathscr{F}, \mathbb{P})$, Kemna and Vorst [27] set up the pricing model in which the underlying asset follows the geometric Brownian motion:

$$dS_t = rS_t dt + \sigma S_t dW_t, \quad 0 \le t \le T, \ S(0) = S_0 \tag{1}$$

where r is the risk-free rate and σ is volatility. And these parameters are usually assumed to be constant. Let T be the maturity date and $[T_0, T]$ be the final time interval over which the average value of the stock is calculated. Let $V(S_t, t)$ be the option price with the underlying price S_t, time to maturity T, and the strike price K. Then, the price of the geometric Asian option under the risk neutral measure $\widetilde{\mathbb{P}}$, at time t, may be represented as follows:

$$V(S_t, t) = e^{-r(T-t)} \widetilde{\mathbb{E}}\left[(G_T - K)^+\right], \tag{2}$$

where G_T is the geometric average of the underlying asset prices during the time to the maturity T.

Kemna and Vorst [27] introduce a process G_T defined by

$$G_T = \exp\left(\frac{1}{T - T_0} \int_{T_0}^{T} (\ln S_u)\, du\right), \tag{3}$$

to represent the geometric average of underlying asset S_t until the time T. And the discrete case equation (3) can be written as follows:

$$G_T = \left(\prod_{i=0}^{n} S_{T_i}\right)^{1/(n+1)}. \tag{4}$$

In both continuous case (3) and discrete case (4), the variable G_T is log-normally distributed so that its expectation and variance values may be calculated explicitly. For the continuous case (3) the log-normal distribution is

$$\begin{aligned} \log G_T \sim N\Bigg(&\frac{1}{2}\left(r - \frac{1}{2}\sigma^2\right)(T - T_0) \\ &+ \log S_{T_0}, \frac{1}{3}\sigma^2 (T - T_0)\Bigg). \end{aligned} \tag{5}$$

Then, the price of geometric Asian call option at time T_0 is given as

$$V\left(S_{T_0}, T_0\right) = S_{T_0} e^{d^*} N(d_1) - K e^{-r(T-T_0)} N(d_2), \tag{6}$$

where

$$\begin{aligned} d^* &= -\frac{1}{2}\left(r + \frac{1}{6}\sigma^2\right)(T - T_0), \\ d_1 &= \frac{\log\left(S_{T_0}/K\right) + (1/2)\left(r + (1/6)\sigma^2\right)(T - T_0)}{\sigma\sqrt{(1/3)(T - T_0)}}, \\ d_2 &= d_1 - \sigma\sqrt{\frac{(T - T_0)}{3}}. \end{aligned} \tag{7}$$

3. Estimation of Bid-Ask Prices Formula

In this section, within the framework of conic finance, we derive the explicit formulas for the bid-ask prices of geometric Asian options. We first present a brief description of conic finance theory. Then we present our main conclusion in the next section.

3.1. Conic Finance Theory. Conic finance is a brand-new quantitative finance theory, which originates from the work by Cherny and Madan [20] and Madan and Cherny [19]. The key to the foundations of the conic finance is an underlying concept of acceptable risks in the economy. Markets are modeled as counterparty accepting at nonnegative stochastic cash flow X that have an acceptability level γ. The theory assumes that price depends on the direction of trade and there are two prices, one for buying from the market called the ask price $a_\gamma(X)$ and one for selling to the market called the bid price $b_\gamma(X)$. The difference between both prices gives rise to the bid-ask spread observed in financial markets.

Let $L^\infty := L^\infty(\Omega, \mathscr{F}, \mathbb{P})$ be the space of all essentially bounded. Madan and Cherny [19] derive these bid and ask prices from the theory of acceptability indices (see [20]), which are functions $\alpha : L^\infty \longrightarrow [0, \infty)$. In particular, they call a net cash flow, or trade, $\widetilde{X} \in L^\infty$ acceptable at an acceptability level γ if and only if $\alpha(\widetilde{X}) \ge \gamma$. Suppose that the market maker sell a cash flow X, for which driven by competition he charges a minimal price of a. Nevertheless, the emerging remaining cash flow $a - X$ ought to be acceptable at level γ. Hence, this price a would be the ask price of X. So the minimal price is given by

$$\begin{aligned} a_\gamma(X) &= \inf\{a : \alpha(a - X) \ge \gamma\} \\ &= \inf\left\{a : \mathbb{E}^{\mathbb{Q}}[a - X] \ge \gamma \text{ for any } \mathbb{Q} \in \mathscr{D}_\gamma\right\} \\ &= \sup_{\mathbb{Q} \in D_\gamma} \mathbb{E}^{\mathbb{Q}}[X], \end{aligned} \tag{8}$$

where a family of sets of probability measures $(D_\gamma)_{\gamma \ge 0}$ is equivalent to the initial probability measure of $\mathbb{P}$.

When the market maker buys X for a price b, it is $X - b$ that must be acceptable at level γ and the maximal price is

$$b_\gamma(X) = \inf_{\mathbb{Q} \in D_\gamma} \mathbb{E}^{\mathbb{Q}}[X]. \tag{9}$$

As proposed by Madan and Cherny [20], parameter family of distortion functions can be used to formulate an operational index of acceptability. The index $\alpha(X)$ is characterized as

$$\alpha(X) = \sup\left\{\gamma \geq 0 : \int_{-\infty}^{\infty} x d\psi^{\gamma}\left(F_X(x)\right)\right\}, \quad (10)$$

or

$$\mathbb{E}^{\mathbb{Q}_\gamma}[X] = \int_{-\infty}^{\infty} x d\psi^{\gamma}\left(F_X(x)\right), \quad (11)$$

where X is an nonnegative stochastic variable, $F_X(x)$ is the distribution function of X, and $(\psi^{\gamma})_{\gamma\geq 0}$ is a pointwise increasing family of concave distortion functions.

Because the distortion function plays a crucial part in the realization of explicit bid and ask prices, Cherny and Madan [20] conclude a series of potential distortion function that can be used. In the following definition, we first give some particular distortion function that has been used extensively in the literature.

Definition 1 (distortion function). A function $\psi : [0,1] \longrightarrow [0,1]$ is a distortion function if and only if it is monotone, $\psi(0) = 0, \psi(1) = 1$.

Definition 2 (Wang transform [28, 29]). Let Φ denote the standard normal cumulative distribution function and let γ be a nonnegative constant. Then a distortion function $\psi^{\gamma} : [0,1] \longrightarrow [0,1]$ defined by

$$\psi^{\gamma}(u) = \Phi\left(\Phi^{-1}(u) + \gamma\right) \quad (12)$$

is called the Wang transform.

Definition 3 (the Maxminvar distortion function [20]). The concave distortion function is given by

$$\psi^{\gamma}(u) = \left(1 - (1-u)^{1+\gamma}\right)^{1/(1+\gamma)}, \quad \forall u \in [0,1],\ \gamma \geq 0. \quad (13)$$

Definition 4 (the Minmaxvar distortion function [20]). The concave distortion function is given by

$$\psi^{\gamma}(u) = 1 - \left(1 - u^{1/(1+\gamma)}\right)^{1+\gamma}, \quad \forall u \in [0,1],\ \gamma \geq 0. \quad (14)$$

From a family of concave distortion functions $(\psi^{\gamma})_{\gamma\geq 0}$ and the properties of the distortion expectation (11), Cherny and Madan [19] lead to the following formulas of bid-ask prices.

$$\begin{gathered} \alpha(a - X) \geq \gamma \Longleftrightarrow \\ \int_{-\infty}^{\infty} x d\psi^{\gamma}\left(F_{a-X}(x)\right) \geq 0 \Longleftrightarrow \\ a + \int_{-\infty}^{\infty} x d\psi^{\gamma}\left(F_{-X}(x)\right) \geq 0, \end{gathered} \quad (15)$$

so that the minimum value of a leads to the ask price:

$$a_{\gamma}(X) = -\int_{-\infty}^{\infty} x d\psi^{\gamma}\left(F_{-X}(x)\right). \quad (16)$$

Analogously, the maximum of b leads to the bid price:

$$\begin{gathered} \alpha(X - b) \geq \gamma \Longleftrightarrow \\ \int_{-\infty}^{\infty} x d\psi^{\gamma}\left(F_{X-b}(x)\right) \geq 0 \Longleftrightarrow \\ -b + \int_{-\infty}^{\infty} x d\psi^{\gamma}\left(F_X(x)\right) \geq 0; \end{gathered} \quad (17)$$

we obtain

$$b_{\gamma}(X) = \int_{-\infty}^{\infty} x d\psi^{\gamma}\left(F_X(x)\right). \quad (18)$$

Under a nonadditive probability using Choquet expectation which introduced by Choquet in [30], the bid-ask prices (16)-(18) may also be presented in the following definition.

Definition 5 (single-period bid-ask prices [19]). Let $(\psi^{\gamma})_{\gamma\geq 0}$ be a pointwise increasing family of concave distortion functions and γ is the market liquidity level. Then, the bid price of a discounted cash flow $X \in L^{\infty}$ is given by

$$\begin{aligned} b_{\gamma}(X) &= \mathbb{E}^{\psi^{\gamma}}[X] = (C)\int X\, d\psi^{\gamma} \circ \mathbb{P} \\ &= -\int_{-\infty}^{0} \psi^{\gamma}\left(\mathbb{P}(X \leq x)\right) dx \\ &\quad + \int_{0}^{\infty} \left(1 - \psi^{\gamma}\left(\mathbb{P}(X \leq x)\right)\right) dx \end{aligned} \quad (19)$$

and its ask price is

$$\begin{aligned} a_{\gamma}(X) &= -\mathbb{E}^{\psi^{\gamma}}[-X] \\ &= \int_{-\infty}^{0} \left(\psi^{\gamma}\left(\mathbb{P}(X > x)\right) - 1\right) dx \\ &\quad + \int_{0}^{\infty} \psi^{\gamma}\left(\mathbb{P}(X > x)\right) dx \end{aligned} \quad (20)$$

In particular, for $\gamma = 0$, the bid-ask prices are equivalent and they reduce to the regular Black-Scholes formula which is undistorted under the risk-neutral probability measure. In addition, we also have the relation as follows:

$$b_{\gamma}(X) \leq \mathbb{E}[X] \leq a_{\gamma}(X). \quad (21)$$

3.2. Bid-Ask Formulas for Geometric Asian Option. In this subsection, we give our main conclusion. For evaluation of the explicit formulas for the bid-ask prices of geometric Asian options, we first use the distortion function based on the Wang Transform from Definition 2. Furthermore, by using Choquet expectation in Definition 5 we can derive the bid-ask price explicit formulas of the geometric Asian call and put options. The following theorem shows our main results.

Theorem 6. *Assume that the distortion function $\psi^{\gamma}(u)$ is the Wang Transform; then the bid-ask prices of the geometric Asian call option at time t is given by*

$$b_{\gamma}(C) = S_t e^{d_b}\Phi(d_{b_1}) - e^{-r(T-t)}K\Phi(d_{b_2}), \tag{22}$$

$$a_{\gamma}(C) = S_t e^{d_a}\Phi(d_{a_1}) - e^{-r(T-t)}K\Phi(d_{a_2}), \tag{23}$$

and the bid-ask prices of the geometric Asian put option at time t are given by

$$b_{\gamma}(P) = e^{-r(T-t)}K\Phi(\tilde{d}_{b_2}) - S_t e^{\tilde{d}_b}\Phi(\tilde{d}_{b_1}), \tag{24}$$

$$a_{\gamma}(P) = e^{-r(T-t)}K\Phi(\tilde{d}_{a_2}) - S_t e^{\tilde{d}_a}\Phi(\tilde{d}_{a_1}), \tag{25}$$

where

$$\begin{aligned}
\tau &= T - t,\\
d_{b_1} &= \frac{\ln(S_t/K) + (1/2)\left(r + (1/6)\sigma^2\right)\tau}{\sigma\sqrt{(1/3)\tau}} - \gamma,\\
d_{b_2} &= d_{b_1} - \sigma\sqrt{\frac{\tau}{3}},\\
d_{a_1} &= \frac{\ln(S_t/K) + (1/2)\left(r + (1/6)\sigma^2\right)\tau}{\sigma\sqrt{(1/3)\tau}} + \gamma,\\
d_{a_2} &= d_{a_1} - \sigma\sqrt{\frac{\tau}{3}},\\
d_b &= -\frac{1}{2}\left(r + \frac{1}{6}\sigma^2\right)\tau - \gamma\sigma\sqrt{\frac{\tau}{3}},\\
d_a &= -\frac{1}{2}\left(r + \frac{1}{6}\sigma^2\right)\tau + \gamma\sigma\sqrt{\frac{\tau}{3}},\\
\tilde{d}_{b_1} &= \frac{\ln(K/S_t) - (1/2)\left(r + (1/6)\sigma^2\right)\tau}{\sigma\sqrt{(1/3)\tau}} - \gamma,\\
\tilde{d}_{b_2} &= \tilde{d}_{b_1} + \sigma\sqrt{\frac{\tau}{3}},\\
\tilde{d}_{a_1} &= \frac{\ln(K/S_t) - (1/2)\left(r + (1/6)\sigma^2\right)\tau}{\sigma\sqrt{1/3\tau}} + \gamma,\\
\tilde{d}_{a_2} &= \tilde{d}_{a_1} + \sigma\sqrt{\frac{\tau}{3}},\\
\tilde{d}_b &= -\frac{1}{2}\left(r + \frac{1}{6}\sigma^2\right)\tau + \gamma\sigma\sqrt{\frac{\tau}{3}},\\
\tilde{d}_a &= -\frac{1}{2}\left(r + \frac{1}{6}\sigma^2\right)\tau - \gamma\sigma\sqrt{\frac{\tau}{3}}.
\end{aligned} \tag{26}$$

In particular, for $\gamma = 0$, the bid-ask prices are equivalent and the bid-ask prices of geometric Asian call option reduce to formula (6).

Proof. Let the payoff of geometric Asian call option be $C_T = (G_T - K)^+$, with G_T being the geometric average of the underlying asset price during the time t to the maturity T. The continuous case of G_T is defined by (3). And (5) shows that random variable G_T has a lognormal distribution; it means that

$$F_{G_T}(x) = \Phi\left(\frac{\log(x/S_t) - (1/2)\left(r - (1/2)\sigma^2\right)(T-t)}{\sigma\sqrt{(1/3)(T-t)}}\right), \tag{27}$$

where Φ is the standard normal cumulative distribution function.

Now, by using Choquet expectation in Definition 5, we can derive the bid price of the geometric Asian call option:

$$\begin{aligned}
b_{\gamma}(C) &= \mathbb{E}^{\psi^{\gamma}}[C_T] = (C)\int C_T\, d\psi^{\gamma} \circ \mathbb{P}\\
&= \int_0^{\infty} x\, d\psi^{\gamma}\left(F_{C_T}(x)\right)\\
&= \int_K^{\infty} (x - K)\, d\psi^{\gamma}\left(F_{G_T}(x)\right)\\
&= \int_K^{\infty} x\, d\psi^{\gamma}\left(F_{G_T}(x)\right) - \int_K^{\infty} K\, d\psi^{\gamma}\left(F_{G_T}(x)\right)\\
&\coloneqq A_1 - B_1.
\end{aligned} \tag{28}$$

If we apply the Wang transform (12) to the distribution function F_{G_T}, we will get the following representation:

$$\begin{aligned}
\psi^{\gamma}\left(F_{G_T}(x)\right) &= \Phi\left(\Phi^{-1}\left(F_{G_T}(x)\right) + \gamma\right)\\
&= \Phi\left(\frac{\log(x/S_t) - (1/2)\left(r - (1/2)\sigma^2\right)(T-t)}{\sigma\sqrt{(1/3)(T-t)}} + \gamma\right).
\end{aligned} \tag{29}$$

And, if $X \sim \text{Lognormal}(\mu, \sigma^2)$ with cumulative distribution function (CDF) $F_X(x)$, then we can obtain

$$\begin{aligned}
\int_K^{\infty} x\, dF_X(x) &= \int_K^{\infty} x\, d\Phi\left(\frac{\ln x - \mu}{\sigma}\right)\\
&= \frac{1}{\sigma\sqrt{2\pi}}\int_{\ln K}^{\infty} e^{y} e^{-(y-\mu)^2/2\sigma^2}\, dy\\
&= \frac{1}{\sigma\sqrt{2\pi}}\int_{\ln K}^{\infty} e^{\mu + 1/2\sigma^2} e^{-(y-(\mu+\sigma^2))^2/2\sigma^2}\, dy\\
&= e^{\mu + 1/2\sigma^2}\Phi\left(\frac{\mu + \sigma^2 - \ln K}{\sigma}\right).
\end{aligned} \tag{30}$$

By using (29) and (30), we can calculate the integral A_1 in (28). It is shown that

$$A_1 = \int_K^{\infty} x d\psi^{\gamma}\left(F_{G_T}(x)\right) = \int_{\ln K}^{\infty} e^y d\Phi\left(\frac{y - \ln S_t - (1/2)\left(r - (1/2)\sigma^2\right)(T-t)}{\sigma\sqrt{(T-t)/3}} + \gamma\right)$$
$$= \frac{1}{\sqrt{2\pi}}\int_{\ln K}^{\infty} \frac{1}{\sigma\sqrt{(T-t)/3}} \cdot \exp\left[-\frac{\left(\left(y - \ln S_t - (1/2)\left(r-(1/2)\sigma^2\right)(T-t)\right)/\sigma\sqrt{(T-t)/3} + \gamma\right)^2}{2} + y\right] dy$$
$$= \exp\left(\ln S_t + \frac{1}{2}\left(r - \frac{1}{2}\sigma^2\right)(T-t) - \gamma\sigma\sqrt{\frac{(T-t)}{3}} + \frac{1}{6}\sigma^2(T-t)\right) \tag{31}$$
$$\cdot \Phi\left(\frac{\ln(S_t/K) + (1/2)\left(r-(1/2)\sigma^2\right)(T-t) + (1/3)\sigma^2(T-t)}{\sigma\sqrt{1/3(T-t)}} - \gamma\right)$$
$$= \exp\left(\ln S_t + \frac{1}{2}\left(r - \frac{1}{6}\sigma^2\right)(T-t) - \gamma\sigma\sqrt{\frac{(T-t)}{3}}\right)\cdot\Phi\left(\frac{\ln(S_t/K) + (1/2)\left(r+(1/6)\sigma^2\right)(T-t)}{\sigma\sqrt{(1/3)(T-t)}} - \gamma\right).$$

And the second integral B_1 in (28) can be calculated as

$$B_1 = \int_K^{\infty} K d\psi^{\gamma}\left(F_{G_T}(x)\right) = \int_K^{\infty} K d\Phi\left(\frac{\ln x - \ln S_t - (1/2)\left(r-(1/2)\sigma^2\right)(T-t)}{\sigma\sqrt{(1/3)(T-t)}} + \gamma\right) = K\left(1 - \Phi\left(\frac{\ln(K/S_t) - (1/2)\left(r-(1/2)\sigma^2\right)(T-t)}{\sigma\sqrt{(1/3)(T-t)}} + \gamma\right)\right)$$
$$= K\Phi\left(\frac{\ln(S_t/K) + (1/2)\left(r-(1/2)\sigma^2\right)(T-t)}{\sigma\sqrt{(1/3)(T-t)}} - \gamma\right). \tag{32}$$

Substituting (31) and (32) into (28) and multiplying by a discount factor $e^{-r(T-t)}$, we can get the following expression for the bid price:

$$b_{\gamma}(C) = S_t \exp\left(-\frac{1}{2}\left(r + \frac{1}{6}\sigma^2\right)(T-t) - \gamma\sigma\sqrt{\frac{(T-t)}{3}}\right)\cdot\Phi\left(\frac{\ln(S_t/K) + (1/2)\left(r+(1/6)\sigma^2\right)(T-t)}{\sigma\sqrt{(1/3)(T-t)}} - \gamma\right) - e^{-r(T-t)}K\Phi\left(\frac{\ln(S_t/K) + (1/2)\left(r-(1/2)\sigma^2\right)(T-t)}{\sigma\sqrt{(1/3)(T-t)}} - \gamma\right). \tag{33}$$

Now, by using Choquet expectation in Definition 5 we can derive the ask price of the geometric Asian call option:

$$a_{\gamma}(C) = -\mathbb{E}^{\psi^{\gamma}}\left[-C_T\right] = -\int_{-\infty}^{0} x d\psi^{\gamma}\left(F_{-C_T}(x)\right)$$
$$= -\int_{-\infty}^{0} x d\psi^{\gamma}\left(1 - F_{G_T}(K-x)\right)$$
$$= -\int_{0}^{\infty} x d\psi^{\gamma}\left(1 - F_{G_T}(K+x)\right) \tag{34}$$
$$= -\int_{K}^{\infty} (x-K) d\psi^{\gamma}\left(1 - F_{G_T}(x)\right)$$
$$= -\int_{K}^{\infty} x d\psi^{\gamma}\left(1 - F_{G_T}(x)\right) + \int_K^{\infty} K d\psi^{\gamma}\left(1 - F_{G_T}(x)\right) =: A_2 + B_2.$$

Similar to the way in which we obtain the bid price. From (27), (30), and Wang transform (12), we get the first integral in (34) as follows:

$$A_2 = -\int_K^{\infty} x d\psi^{\gamma}\left(1 - F_{G_T}(x)\right) = -\int_K^{\infty} x d\psi^{\gamma}\left(\frac{\ln(S_t/x) + (1/2)\left(r-(1/2)\sigma^2\right)(T-t)}{\sigma\sqrt{(T-t)/3}}\right)$$

$$\begin{aligned}
&= -\int_K^{\infty} x d\Phi\left(\frac{\ln(S_t/x) + (1/2)\left(r - (1/2)\sigma^2\right)(T-t)}{\sigma\sqrt{(T-t)/3}} + \gamma\right) \\
&= \int_K^{\infty} x d\Phi\left(\frac{\ln(x/S_t) - (1/2)\left(r - (1/2)\sigma^2\right)(T-t)}{\sigma\sqrt{(T-t)/3}} - \gamma\right) \\
&= \exp\left(\ln S_t + \frac{1}{2}\left(r - \frac{1}{2}\sigma^2\right)(T-t) + \gamma\sigma\sqrt{\frac{(T-t)}{3}} + \frac{1}{6}\sigma^2(T-t)\right) \\
&\quad \cdot \Phi\left(\frac{\ln(S_t/K) + (1/2)\left(r - (1/2)\sigma^2\right)(T-t) + (1/3)\sigma^2(T-t)}{\sigma\sqrt{(1/3)(T-t)}} + \gamma\right),
\end{aligned} \tag{35}$$

and the second integral can be calculated as

$$\begin{aligned}
B_2 &= \int_K^{\infty} K d\psi^{\gamma}\left(1 - F_{G_T}(x)\right) \\
&= \int_K^{\infty} K d\Phi\left(\frac{\ln(S_t/x) + (1/2)\left(r - (1/2)\sigma^2\right)(T-t)}{\sigma\sqrt{(1/3)(T-t)}} + \gamma\right) \\
&= -\int_K^{\infty} K d\Phi\left(\frac{\ln(x/S_t) - (1/2)\left(r - (1/2)\sigma^2\right)(T-t)}{\sigma\sqrt{(1/3)(T-t)}} - \gamma\right) = -K\left(1 - \Phi\left(\frac{\ln(K/S_t) - (1/2)\left(r - (1/2)\sigma^2\right)(T-t)}{\sigma\sqrt{(1/3)(T-t)}} - \gamma\right)\right) \\
&= -K\Phi\left(\frac{\ln(S_t/K) + (1/2)\left(r - (1/2)\sigma^2\right)(T-t)}{\sigma\sqrt{(1/3)(T-t)}} + \gamma\right).
\end{aligned} \tag{36}$$

By combining parts A_2 and B_2 and considering the continuous discount factor $e^{-r(T-t)}$, we have the ask price:

$$\begin{aligned}
a_{\gamma}(C) &= S_t \exp\left(-\frac{1}{2}\left(r + \frac{1}{6}\sigma^2\right)(T-t) + \gamma\sigma\sqrt{\frac{(T-t)}{3}}\right) \\
&\quad \cdot \Phi\left(\frac{\ln(S_t/K) + (1/2)\left(r + (1/6)\sigma^2\right)(T-t)}{\sigma\sqrt{(1/3)(T-t)}} + \gamma\right) \\
&\quad - e^{-r(T-t)} K\Phi\left(\frac{\ln(S_t/K) + (1/2)\left(r - (1/2)\sigma^2\right)(T-t)}{\sigma\sqrt{(1/3)(T-t)}} + \gamma\right)
\end{aligned} \tag{37}$$

In addition, we show the bid-ask prices of the geometric Asian put option. Let $P_T = (K - G_T)^+$; by utilizing Choquet expectation in Definition 5 and transformations (29) and (30), we can derive the bid price of the geometric Asian put option as follows:

$$\begin{aligned}
b_{\gamma}(P) &= \mathbb{E}^{\psi^{\gamma}}[P_T] = \int_0^{\infty} x d\psi^{\gamma}\left(F_{P_T}(x)\right) = \int_0^{\infty} x d\psi^{\gamma}\left(1 - F_{G_T}(K-x)\right) = -\int_0^{K}(K-x)\, d\psi^{\gamma}\left(1 - F_{G_T}(x)\right) \\
&= \int_0^{K}(K-x)\, d\Phi\left(\frac{\ln(x/S_t) - (1/2)\left(r - (1/2)\sigma^2\right)(T-t)}{\sigma\sqrt{(1/3)(T-t)}} - \gamma\right).
\end{aligned} \tag{38}$$

Conducting the same calculation steps that we used during the derivation of the bid price of the call option, we split the integral into two parts as follows:

$$A = \int_0^{K} K d\Phi\left(\frac{\ln(x/S_t) - (1/2)\left(r - (1/2)\sigma^2\right)(T-t)}{\sigma\sqrt{(1/3)(T-t)}}\right.$$

$$- \gamma \Bigg) = K\Phi\left(\frac{\ln(x/S_t) - (1/2)(r - (1/2)\sigma^2)(T-t)}{\sigma\sqrt{(1/3)(T-t)}} - \gamma\right)\Bigg|_0^K = K\Phi\left(\frac{\ln(K/S_t) - (1/2)(r - (1/2)\sigma^2)(T-t)}{\sigma\sqrt{(1/3)(T-t)}} - \gamma\right), \tag{39}$$

and

$$\begin{aligned} B &= \int_0^K x d\Phi\left(\frac{\ln(x/S_t) - (1/2)(r - (1/2)\sigma^2)(T-t)}{\sigma\sqrt{(1/3)(T-t)}} - \gamma\right) \\ &= \left\{\int_0^\infty - \int_K^\infty\right\} x d\Phi\left(\frac{\ln(x/S_t) - (1/2)(r - (1/2)\sigma^2)(T-t)}{\sigma\sqrt{(1/3)(T-t)}} - \gamma\right) \\ &= \exp\left(\ln S_t + \frac{1}{2}\left(r - \frac{1}{2}\sigma^2\right)(T-t) + \gamma\sigma\sqrt{\frac{(T-t)}{3}} + \frac{1}{6}\sigma^2(T-t)\right) \\ &\quad \cdot \left(1 - \Phi\left(\frac{\ln(S_t/K) + (1/2)(r - (1/2)\sigma^2)(T-t) + (1/3)\sigma^2(T-t)}{\sigma\sqrt{(T-t)/3}} + \gamma\right)\right) \\ &= S_t \exp\left(\frac{1}{2}\left(r - \frac{1}{2}\sigma^2\right)(T-t) + \gamma\sigma\sqrt{\frac{(T-t)}{3}} + \frac{1}{6}\sigma^2(T-t)\right) \\ &\quad \cdot \Phi\left(\frac{\ln(K/S_t) - (1/2)(r - (1/2)\sigma^2)(T-t) - (1/3)\sigma^2(T-t)}{\sigma\sqrt{(1/3)(T-t)}} - \gamma\right). \end{aligned} \tag{40}$$

Substituting the results of the two integrals in (38) and multiplying by a discount factor $e^{-r(T-t)}$, we get the bid price of put option as follows:

$$\begin{aligned} b_\gamma(P) &= e^{-r(T-t)} K\Phi\left(\frac{\ln(K/S_t) - (1/2)(r - (1/2)\sigma^2)(T-t)}{\sigma\sqrt{1/3(T-t)}} - \gamma\right) - S_t \exp\left(-\frac{1}{2}\left(r + \frac{1}{6}\sigma^2\right)(T-t) + \gamma\sigma\sqrt{\frac{(T-t)}{3}}\right) \\ &\quad \cdot \Phi\left(\frac{\ln(K/S_t) - (1/2)(r + (1/6)\sigma^2)(T-t)}{\sigma\sqrt{(1/3)(T-t)}} - \gamma\right) \end{aligned} \tag{41}$$

Finally, by the same methods that we used in the calculation process of ask price of the geometric Asian call option, we can get the formula of the ask price of the geometric Asian put option:

$$\begin{aligned} a_\gamma(P) &= -\mathbb{E}^{\psi^\gamma}[-P_T] = -\int_{-\infty}^0 x d\psi^\gamma\left(F_{-P_T}(x)\right) \\ &= -\int_{-\infty}^0 x d\psi^\gamma\left(F_{G_T}(K+x)\right) = -\int_0^\infty x d\psi^\gamma\left(F_{G_T}(K-x)\right) \\ &= \int_0^K (K-x)\, d\psi^\gamma\left(F_{G_T}(x)\right) \\ &= \int_0^K (K-x)\, d\Phi\left(\frac{\ln(x/S_t) - (1/2)(r - (1/2)\sigma^2)(T-t)}{\sigma\sqrt{1/3(T-t)}} + \gamma\right) \\ &= e^{-r(T-t)} K\Phi\left(\frac{\ln(K/S_t) - (1/2)(r - (1/2)\sigma^2)(T-t)}{\sigma\sqrt{1/3(T-t)}} + \gamma\right) \\ &\quad - S_t \exp\left(-\frac{1}{2}\left(r + \frac{1}{6}\sigma^2\right)(T-t) - \gamma\sigma\sqrt{\frac{(T-t)}{3}}\right) \\ &\quad \cdot \Phi\left(\frac{\ln(K/S_t) - (1/2)(r + (1/6)\sigma^2)(T-t)}{\sigma\sqrt{(1/3)(T-t)}} + \gamma\right) \end{aligned} \tag{42}$$

This completes the proof of Theorem 6. □

Table 1: The bid-ask prices for geometric Asian options at different γ with $r = 0.02, \sigma = 0.8, t = 0, T = 3/12$.

Level	S_0	K	Call ask	Call bid	Call spread	Put ask	Put bid	Put spread
0	100	90	10.752	10.752	0	0.048	0.048	0
		100	2.748	2.748	0	1.845	1.845	0
		110	0.178	0.178	0	9.077	9.077	0
0.02	100	90	10.864	10.641	0.223	0.050	0.045	0.005
		100	2.814	2.682	0.131	1.894	1.797	0.097
		110	0.186	0.170	0.016	9.183	8.971	0.212
0.4	100	90	13.031	8.575	4.456	0.129	0.016	0.113
		100	4.250	1.629	2.621	2.985	1.037	1.948
		110	0.420	0.066	0.354	11.224	7.009	4.216
0.9	100	90	15.984	6.068	9.916	0.372	0.003	0.369
		100	6.613	0.737	5.875	4.844	0.434	4.410
		110	1.045	0.016	1.029	13.924	4.669	9.256

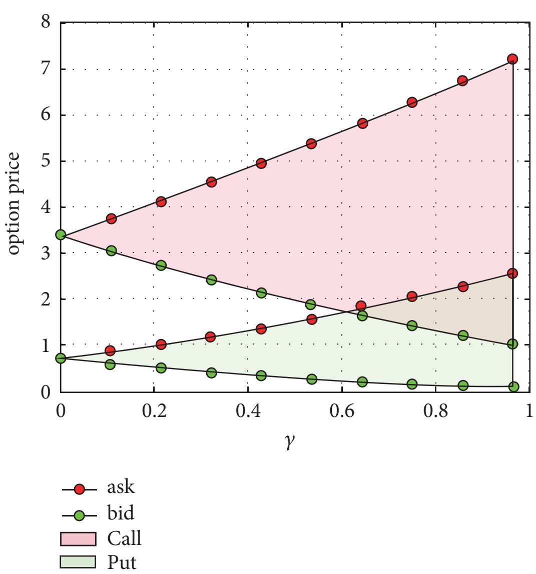

Figure 1: The price information for geometric Asian options at different γ with $S_0 = 78$, $K = 76$, $r = 0.02$, $\sigma = 0.8$, $t = 0$, $T = 3/12$.

4. Numerical Examples

In this section, we present numerical results obtained for the geometric Asian option pricing model proposed in this paper. Assume the risk-free interest rate r being 8% per annum, the stock price volatility σ being 20%, and the geometric Asian option with 3 months to expiry (i.e., $T = 3/12$). We show the bid-ask prices for the geometric Asian option with the different market liquidity parameter γ, which are displayed in Table 1 and Figure 1.

Table 1 provides the numerical results for the bid-ask prices of geometric Asian put and call options. For $\gamma = 0$, the ask and bid prices are equivalent and they reduce to the analytic expression (6) presented by Kemna and Vorst [27]. Figure 1 plots bid-ask spread for the geometric Asian put and call options at different static market liquidity parameter γ. The spread between bid-ask prices is a measure of illiquidity. The nonnegative parameter γ gives an indication of the markets' liquidity: the higher the γ, the wider the spread and hence the less the liquidity.

5. Conclusion

In this paper, within the framework of conic finance, we propose a useful approach to evaluate the ask and bid prices of geometric Asian options and obtain the explicit formulas for the ask and bid prices. Finally, by using the explicit formulas of geometric Asian options, we carry out the impacts of the static market liquidity parameter γ on bid-ask prices.

Data Availability

The numerical simulation data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

This work was supported by the Fundamental Research Funds for the Central Universities (nos. JBK170940 and JBK151126). Thanks to the Guizhou provincial science and Technology Department Project ([2019] 1175) hosted by Congyin Fan for supporting this work.

References

[1] P. Wilmott, *Paul Wilmott on Quantitative Finance*, John Wiley & Sons Ltd, 2nd edition, 2006.

[2] G. Barles and H. M. Soner, "Option pricing with transaction costs and a nonlinear Black-Scholes equation," *Finance and Stochastics*, vol. 2, no. 4, pp. 369–397, 1998.

[3] J. Cvitanić and I. Karatzas, "Hedging and portfolio optimization under transaction costs: a martingale approach," *Mathematical Finance*, vol. 6, no. 2, pp. 133–165, 1996.

[4] G. M. Constantinides, “Capital market equilibrium with transaction costs,” *Journal of Political Economy*, vol. 94, no. 4, pp. 842–862, 1986.

[5] A. Lo, H. Mamaysky, and J. Wang, “Asset prices and trading volume under fixed transactions costs,” *Journal of Political Economy*, vol. 112, no. 5, pp. 1054–1090, 2004.

[6] E. Jouini and H. Kallal, “Martingales and arbitrage in securities markets with transaction costs,” *Journal of Economic Theory*, vol. 66, no. 1, pp. 178–197, 1995.

[7] D. Easley and M. O’Hara, “Price, trade size, and information in securities markets,” *Journal of Financial Economics*, vol. 19, no. 1, pp. 69–90, 1987.

[8] N. Han and T. Shino, “Price, trade size, and information revelation in multi-period securities markets,” *Journal of Financial Markets*, vol. 13, no. 1, pp. 49–76, 2010.

[9] T. E. Copeland and D. Galai, “Information effects on the bid-ask spread,” *The Journal of Finance*, vol. 38, no. 5, pp. 1457–1469, 1983.

[10] L. R. Glosten and P. R. Milgrom, “Bid, ask and transaction prices in a specialist market with heterogeneously informed traders,” *Journal of Financial Economics*, vol. 14, no. 1, pp. 71–100, 1985.

[11] Y. Amihud and H. Mendelson, “Dealership market: Market-making with inventory,” *Journal of Financial Economics*, vol. 8, no. 1, pp. 31–53, 1980.

[12] T. Ho and H. R. Stoll, “Optimal dealer pricing under transactions and return uncertainty,” *Journal of Financial Economics*, vol. 9, no. 1, pp. 47–73, 1981.

[13] T. S. Ho and H. R. Stoll, “The dynamics of dealer markets under competition,” *The Journal of Finance*, vol. 38, no. 4, pp. 1053–1074, 1983.

[14] H. R. Stoll, “The supply of dealer services in securities markets,” *The Journal of Finance*, vol. 33, no. 4, pp. 1133–1151, 1978.

[15] J. Y. Choi, D. Salandro, and K. Shastri, “On the estimation of bid-ask spreads: theory and evidence,” *Journal of Financial and Quantitative Analysis*, vol. 23, no. 2, pp. 219–230, 1988.

[16] T. J. George, G. Kaul, and M. Nimalendran, “Estimation of the bid-ask spread and its components: a new approach,” *Review of Financial Studies*, vol. 4, no. 4, pp. 623–656, 1991.

[17] R. Roll, “A simple implicit measure of the effective bid-ask spread in an efficient market,” *The Journal of Finance*, vol. 39, no. 4, pp. 1127–1139, 1984.

[18] H. R. Stoll, “Inferring the components of the bid-ask spread: theory and empirical tests,” *The Journal of Finance*, vol. 44, no. 1, pp. 115–134, 2012.

[19] D. Madan and A. Cherny, “Markets as a counterparty: An introduction to conic finance,” *International Journal of Theoretical and Applied Finance*, vol. 13, no. 08, pp. 1149–1177, 2010.

[20] A. Cherny and D. Madan, “New measures for performance evaluation,” *Review of Financial Studies*, vol. 22, no. 7, pp. 2571–2606, 2009.

[21] D. B. Madan and W. Schoutens, “Conic CVA and DVA,” *Wilmott Magazine*, pp. 45–50, 2016.

[22] S. Bakel, S. Borovkova, and M. Michielon, “Conic CVA and DVA for option portfolios,” *SSRN*, p. 24, 2017.

[23] D. B. Madan, “Conic portfolio theory,” *International Journal of Theoretical and Applied Finance*, vol. 19, no. 3, pp. 1–42, 2016.

[24] E. Eberlein and D. B. Madan, “Portfolio theory for squared returns correlated across time,” *Probability, Uncertainty and Quantitative Risk*, vol. 1, pp. 1–36, 2016.

[25] D. B. Madan, M. Pistorius, and W. Schoutens, “Dynamic conic hedging for competitiveness,” *Mathematics and Financial Economics*, vol. 10, no. 4, pp. 405–439, 2016.

[26] P. Carr, D. B. Madan, M. Melamed, and W. Schoutens, “Hedging insurance books,” *Insurance: Mathematics and Economics*, vol. 70, pp. 364–372, 2016.

[27] A. G. Z. Kemna and A. C. F. Vorst, “A pricing method for options based on average asset values,” *Journal of Banking & Finance*, vol. 14, no. 1, pp. 113–129, 1990.

[28] S. S. Wang, “A class of distortion operators for pricing financial and insurance risks,” *Journal of Risk and Insurance*, vol. 67, no. 1, pp. 15–36, 2000.

[29] S. S. Wang, “A universal framework for pricing financial and insurance risks,” *ASTIN Bulletin*, vol. 32, no. 2, pp. 213–234, 2002.

[30] G. Choquet, “Theory of capacities,” *Annales Institut Fourier*, vol. 5, pp. 131–295, 1954.


Advances in
Operations Research
Advances in
Decision Sciences
Journal of
Applied Mathematics
The Scientific
World Journal
Journal of
Probability and Statistics
International
Journal of
Mathematics and
Mathematical
Sciences
Hindawi
Submit your manuscripts at
www.hindawi.com
Journal of
Optimization
International Journal of
Engineering
Mathematics
International Journal of
Analysis
Journal of
Complex Analysis
Advances in
Numerical Analysis
Mathematical Problems
in Engineering
International Journal of
Differential Equations
Discrete Dynamics in
Nature and Society
International Journal of
Stochastic Analysis
Journal of
Mathematics
Journal of
Function Spaces
Abstract and
Applied Analysis
Advances in
Mathematical Physics