Hindawi
Discrete Dynamics in Nature and Society
Volume 2019, Article ID 9181093, 10 pages
https://doi.org/10.1155/2019/9181093

Research Article

Distributed Adaptive Optimization for Generalized Linear Multiagent Systems

Shuxin Liu,[1,2] Haijun Jiang ,[3] Liwei Zhang,[1] and Xuehui Mei[3]

[1] *Institute of Operations Research and Control Theory, School of Mathematical Sciences, Dalian University of Technology, Dalian 116024, China*
[2] *College of Mathematics and Physics, Xinjiang Agricultural University, Urumqi 830052, China*
[3] *College of Mathematics and System Sciences, Xinjiang University, Urumqi 830046, China*

Correspondence should be addressed to Haijun Jiang; jianghaijunxju@163.com

Received 29 May 2019; Revised 3 July 2019; Accepted 8 July 2019; Published 1 August 2019

Academic Editor: Ewa Pawluszewicz

In this paper, the edge-based and node-based adaptive algorithms are established, respectively, to solve the distribution convex optimization problem. The algorithms are based on multiagent systems with general linear dynamics; each agent uses only local information and cooperatively reaches the minimizer. Compared with existing results, a damping term in the adaptive law is introduced for the adaptive algorithms, which makes the algorithms more robust. Under some sufficient conditions, all agents asymptotically converge to the consensus value which minimizes the cost function. An example is provided for the effectiveness of the proposed algorithms.

1. Introduction

Recently, the consensus problems of multiagent systems have been extensively investigated on account of its widespread application, such as cooperative reconnaissance and unmanned aerial vehicles formation. Many meaningful results about consensus algorithms [1–7] have been established. Specially, the consensus problems can solve the distributed optimization, when the consensus value minimizes the global cost function, which is a sum of local objective functions each of which is known by one agent.

Meanwhile, with the development of various network systems, various distributed optimization problems have emerged. Extensive efforts to design the efficient algorithm have been put into the distributed optimization problems. Because of the advantages of decentralized and distributed structure, the distributed algorithms based on multiagent systems are more efficient than the centralized algorithm. A large number of consensus-based optimization algorithms have been presented over the past two decades. For instance, in [8, 9], the subgradient algorithms upon multiagent systems have been presented. In [10], the algorithms with fixed step size have been proposed. In [11], edge-based fixed-time consensus algorithms have been established. More results can be found in [12–15].

In the above works, the proposed optimization algorithms depend on the selection of constant control gains. In fact, the lower bound of the gains is determined by the smallest nonzero eigenvalue of Laplacian matrix for the communication graph, i.e., algebraic connectivity which is global information. These consensus algorithms are not fully distributed. To face this challenge, some well-performing adaptive algorithms have been proposed, where the adaptive gain updating laws rely on local information of the agents. For example, in [16, 17], the adaptive algorithms of the finite time convergence have been established upon integrator multiagent systems. In [18], the adaptive algorithms have been proposed upon general linear multiagent systems from the edge-based and node-based design.

Note that more agent networks in practical applications are described by generalized linear systems. Moreover, from the results in [19], the control gains in [17, 18] monotonically increase very quickly when the value of the initial states is large. Overlarge control gains will magnify the control input and will lead to unstable algorithms. Based on the above considerations, from the edge-based and node-based standpoint two distributed adaptive algorithms for solving the distributed convex optimization problems are proposed upon general linear multiagent systems in this paper. Compared with the existing algorithms, the proposed algorithms based on generalized linear dynamical systems are more practical and its theoretical analysis is more difficult than that based on integrator systems. The algorithm does not require global information, which makes it fully distributed and easier to implement. The proposed algorithms introduce a damping term in the update law, which makes the algorithm robust under input disturbance and the high-gain update law.

The article is organized as follows. In next section, some preliminaries and lemmas are introduced. In Section 3, the edge-based and node-based consensus protocols are designed for solving the distributed convex optimization; the asymptotic convergence is guaranteed for the adaptive algorithms. In Section 4, the effectiveness of the performance for the algorithm to solve the distributed convex optimization problems is presented by a typical example. In Section 5, a brief conclusion is given.

2. Notations and Preliminaries

In this paper, $\mathbb{R}^l$ is the set of all l-dimensional real vectors. $\mathbb{R}^{m\times q}$ is the set of $m \times q$ real matrices. $\mathbf{1} = [1,1,\dots,1]^T$ means the vector that all the elements are one. I_m is the m-dimensional identity matrix. For a real matrix B, $B > 0$ ($B \geq 0$) means B is positive definite (semidefinite). $(\cdot)^T$ represents the transpose. $\|\cdot\|_1$, $\|\cdot\|$, and $\|\cdot\|_\infty$ denote the 1-norm, 2-norm and infinite norm, respectively. Given $y \in \mathbb{R}^l$, $\nabla h(y) = (\partial h(y)/\partial y_1, \partial h(y)/\partial y_2, \dots, \partial h(y)/\partial y_l)^T$ represents the gradient of the function $h(y)$. The function $g(t) : \mathbb{R} \longrightarrow \mathbb{R}^l$ is $g(t) \in \mathbb{L}_2$, if $\int_0^\infty g(t)^T g(t)dt < 0$ and $g(t) \in \mathbb{L}_\infty$ if for any element $g_i(t)$ for $g(t)$, $\sup_t |g_i(t)| < \infty, i = 1,\dots,l$.

An undirected graph G consists of a node set $V = \{1,2,\dots,m\}$ and an edge set $\mathscr{E} \subseteq V \times V$, the edge $(i,j) \in \mathscr{E}$ if and only if $(j,i) \in \mathscr{E}$, and the self-edge (i,i) is not allowed. The incidence matrix $D = [d_{ij}]_{m\times|\mathscr{E}|}$ corresponds to a graph G with an arbitrary orientation, namely, whose edges have a terminal node and initial node. For jth edge, if the node i is initial, $d_{ij} = 1$, if the node i is terminal, $d_{ij} = -1$; otherwise, $d_{ij} = 0$. The graph Laplacian of G is defined as $L = DD^T$. A path in the graph G from i to j is a sequence of distinct nodes starting with i and ending with j. The graph G is connected, if there is a path between any two nodes.

Assumption 1. The undirected graph G is connected.

Lemma 2 (see [20]). *Under Assumption 1, L is positive semidefinite,* 0 *is a simple eigenvalue for L, and* 1 *is the associated eigenvector; its smallest nonzero eigenvalue* $\lambda_2 = \min_{y\neq 0, \mathbf{1}^T y=0} y^T Lx / y^T y$.

Lemma 3 (see [21]). *Let* $h(y) : \mathbb{R}^l \longrightarrow \mathbb{R}$ *be a continuously differentiable convex function; then* $h(y)$ *is minimized if and only if* $\lim_{u\longrightarrow y} \nabla h(u) = 0$.

Lemma 4 (see [22]). *If* $g(t), \dot{g}(t) \in \mathbb{L}_\infty$, *and* $g(t) \in \mathbb{L}_p$, $p \in [1, +\infty)$, *then* $\lim_{t\longrightarrow\infty} g(t) = 0$.

3. Problem Formulation

Consider the following multiagent system; each agent satisfies the dynamics:

$$\dot{y}_i(t) = By_i(t) + Cu_i(t), \quad i \in \mathscr{T} = \{1,2,\dots,m\}, \tag{1}$$

where $u_i(t) \in \mathbb{R}^q$ is the protocol, $y_i(t)$ is the state for ith agent, and $B \in \mathbb{R}^{l\times l}$ and $C \in \mathbb{R}^{l\times q}$ are the constant matrices.

Our aim is to design $u_i(t)$ for (1) by using only local information, such that all agents cooperatively arrive the consensus value y^* which minimizes the convex problem

$$\begin{aligned} &\min \quad \sum_{i=1}^{m} h_i(y), \\ &\text{subject to} \quad y \in \mathbb{R}^l, \end{aligned} \tag{2}$$

where $h_i(y) : \mathbb{R}^l \longrightarrow \mathbb{R}$ is a local cost function, which is known only to agent i.

We give the following hypothesis.

Assumption 5. The local cost function $h_i(y)$ is convex and differentiable; its gradient satisfies

$$\nabla h_i(y) = \widetilde{B}y + \widetilde{C}\psi_i(y), \tag{3}$$

where B, C are defined in (1), $\widetilde{B} = KB, \widetilde{C} = KC$, $K \in \mathbb{R}^{l\times l}$ is some negative define matrix, $\psi_i(y)$ is continuous, and there exists $\eta > 0$ such that $\|\psi_i(y) - \psi_j(y)\|_\infty \leq \eta$, $i, j \in \mathscr{T}$, for all $y \in \mathbb{R}^l$.

The equivalent characterization for (2) is that the agents achieve consensus value which minimizes the function $\sum_{i=1}^{m} h_i(y_i)$, namely,

$$\begin{aligned} &\min \quad \sum_{i=1}^{m} h_i(y_i), \\ &\text{subject to} \quad \lim_{t\longrightarrow\infty} \left\| y_i(t) - y_j(t) \right\| = 0, \quad i, j \in \mathscr{T}. \end{aligned} \tag{4}$$

Assumption 6. Each set $\Omega_i = \{y_i : \nabla h_i(y_i) = 0\}, i \in \mathscr{T}$ is nonempty; that is to say, there exists $y_i^* \in \mathbb{R}^l$ such that $h_i(y_i^*)$ is minimum.

4. Edge-Based Adaptive Designs

To solve problem (3), the edge-based adaptive algorithm for (1) is presented as follows:

$$\begin{aligned} u_i(t) = &-\sum_{j\in N_i} \xi_{ij} \left(P\left(y_i(t) - y_j(t) \right) \right) \\ &- \sum_{j\in N_i} \varsigma_{ij} \operatorname{sign}\left[P\left(y_i(t) - y_j(t) \right) \right] + \psi_i(y_i), \end{aligned} \tag{5}$$

$$\dot{\xi}_{ij} = \left\| P\left(y_i(t) - y_j(t)\right) \right\|^2 - \sigma_{ij}\left(\xi_{ij} - \tilde{\xi}_{ij}\right), \quad (6)$$

$$\dot{\varsigma}_{ij} = \left\| P\left(y_i(t) - y_j(t)\right) \right\|_1 - \sigma_{ij}\left(\varsigma_{ij} - \tilde{\varsigma}_{ij}\right), \quad (7)$$

$$\begin{aligned} \dot{\tilde{\xi}}_{ij}(t) &= \rho_{ij}\left(\xi_{ij} - \tilde{\xi}_{ij}\right), \\ \dot{\tilde{\varsigma}}_{ij} &= \rho_{ij}\left(\varsigma_{ij} - \tilde{\varsigma}_{ij}\right), \end{aligned} \quad (8)$$

where ξ_{ij}, ς_{ij} are adaptive coupling strengths, Q is a feedback matrix, the initial states satisfy $\xi_{ij}(0) \geq \tilde{\xi}_{ij}(0)$, and $\varsigma_{ij}(0) \geq \tilde{\varsigma}_{ij}(0)$.

From (5), system (1) can be written as follows:

$$\begin{aligned} y_i(t) &= By_i(t) - C\sum_{j\in N_i}\xi_{ij}\left(P\left(y_i(t) - y_j(t)\right)\right) \\ &\quad - C\sum_{j\in N_i}\varsigma_{ij}\operatorname{sign}\left[P\left(y_i(t) - y_j(t)\right)\right] \\ &\quad + C\psi_i\left(y_i(t)\right), \end{aligned} \quad (9)$$

Denote $y(t) = (y_1(t)^T, y_2(t)^T, \ldots, y_m(t)^T)^T$ and $\Psi(t) = (\psi_1^T(y_1(t)), \psi_2^T(y_2(t)), \cdots, \psi_m^T(y_m(t)))^T$. Thus, (9) can be represented by the compact form

$$\begin{aligned} \dot{y}(t) &= \left(I_m \otimes B\right)y(t) - \left(D\mathscr{A}D^T \otimes CP\right)y(t) \\ &\quad - (D\mathscr{B} \otimes C)\operatorname{sign}\left[\left(D^T \otimes P\right)y(t)\right] \\ &\quad + (M \otimes C)\Psi\left(y(t)\right), \end{aligned} \quad (10)$$

where D is the incidence matrix, $\mathscr{A} = \operatorname{diag}\{\xi_{ij}\}$, and $\mathscr{B} = \operatorname{diag}\{\varsigma_{ij}\}$. The solution of the system (10) is understood by the sense of Filippov [23].

Note that $MD = DM = D$; multiplying both sides of (10) by $M \otimes I_m$, we get the error system

$$\begin{aligned} \dot{\epsilon}(t) &= \left(I_m \otimes B\right)\epsilon(t) - \left(D\mathscr{A}D^T \otimes CP\right)\epsilon(t) \\ &\quad - (D\mathscr{B} \otimes C)\operatorname{sign}\left[\left(D^T \otimes P\right)\epsilon(t)\right] \\ &\quad + (M \otimes C)\Psi\left(y(t)\right), \end{aligned} \quad (11)$$

where $M = I_m - (1/m)\mathbf{1}\mathbf{1}^T$, $\epsilon(t) = (\epsilon_1(t)^T, \epsilon_2(t)^T, \ldots, \epsilon_m(t)^T)^T = (M \otimes I_m)y(t)$. Obviously, $y_1(t) = y_2(t)\cdots = y_m(t)$ if and only if $\lim_{t\longrightarrow\infty}\epsilon(t) = 0$.

Theorem 7. *Under Assumptions 1–6 and Lemma 3, problem (4) can be solved by algorithm (5), if there exists the feedback matrix $P = C^TQ$, such that*

$$QB + B^TQ - QCC^TQ < 0, \quad (12)$$

where Q is the positive-definite matrix. Meanwhile, as $t \longrightarrow \infty$, we have $\xi_{ij}(t)$ and $\tilde{\xi}_{ij}(t)$ ($\varsigma_{ij}(t)$ and $\tilde{\varsigma}_{ij}(t)$) will converge to the same positive number.

Proof. Choose the Lyapunov function candidate

$$\begin{aligned} V(t) &= \epsilon(t)^T\left(I_m \otimes Q\right)\epsilon(t) + \frac{1}{2}\sum_{i=1}^{m}\sum_{j\in N_i}\left(\xi_{ij} - a\right)^2 \\ &\quad + \sum_{i=1}^{m}\sum_{j\in N_i}\frac{\sigma_{ij}}{2\rho_{ij}}\left(\tilde{\xi}_{ij} - a\right)^2 + \frac{1}{2}\sum_{i=1}^{m}\sum_{j\in N_i}\left(\varsigma_{ij} - b\right)^2 \\ &\quad + \sum_{i=1}^{m}\sum_{j\in N_i}\frac{\sigma_{ij}}{2\rho_{ij}}\left(\tilde{\varsigma}_{ij} - b\right)^2, \end{aligned} \quad (13)$$

where a and b are the positive constants; Q is the positive-definite matrix.

Take $P = C^TQ$; we have

$$\begin{aligned} \dot{V}(t) &= \epsilon(t)^T \\ &\quad \cdot\left[I_m \otimes \left(QB + B^TQ\right) - 2D\mathscr{A}D^T \otimes QCC^TQ\right]\epsilon(t) \\ &\quad - 2\epsilon(t)^T(D\mathscr{B} \otimes QC)\operatorname{sign}\left[\left(D^T \otimes C^TQ\right)\epsilon(t)\right] \\ &\quad + 2\epsilon(t)^T(M \otimes QC)\Psi\left(y(t)\right) + \sum_{i=1}^{m}\sum_{j\in N_i}\left(\xi_{ij} - a\right)\dot{\xi}_{ij} \\ &\quad + \sum_{i=1}^{m}\sum_{j\in N_i}\frac{\sigma_{ij}}{\rho_{ij}}\left(\tilde{\xi}_{ij} - a\right)\dot{\tilde{\xi}}_{ij} + \sum_{i=1}^{m}\sum_{j\in N_i}\left(\varsigma_{ij} - b\right)\dot{\varsigma}_{ij} \\ &\quad + \sum_{i=1}^{m}\sum_{j\in N_i}\frac{\sigma_{ij}}{\rho_{ij}}\left(\tilde{\varsigma}_{ij} - b\right)\dot{\tilde{\varsigma}}_{ij}. \end{aligned} \quad (14)$$

From the third term in (14) and $\|(M \otimes I_q)\Psi(y(t)\|_1 \leq \eta$, we get

$$\begin{aligned} &\epsilon(t)^T(M \otimes QC)\Psi\left(y(t)\right) \\ &\quad \leq \left\|\left(M \otimes C^TQ\right)\epsilon(t)\right\|_1\left\|\left(M \otimes I_q\right)\Psi(y(t)\right\|_\infty \\ &\quad \leq \frac{\eta}{m}\sum_{i=1}^{m}\sum_{j=1}^{m}\left\|C^TQ\left(\epsilon(t)_i - \epsilon(t)_j\right)\right\|_1 \\ &\quad \leq \frac{\eta}{m}\sum_{i=1}^{m}\max_i\left\{\sum_{j=1}^{m}\left\|C^TQ\left(\epsilon(t)_i - \epsilon(t)_j\right)\right\|_1\right\} \\ &\quad = \eta\max_i\left\{\sum_{j=1}^{m}\left\|C^TQ\left(\epsilon(t)_i - \epsilon(t)_j\right)\right\|_1\right\} \\ &\quad \leq \frac{\eta}{2}(m-1)\sum_{i=1}^{m}\sum_{j\in N_i}\beta_{ij}\left\|C^TQ\left(\epsilon_i(t) - \epsilon_j(t)\right)\right\|_1. \end{aligned} \quad (15)$$

According to the fourth term in (14), we obtain

$$\sum_{i=1}^{m}\sum_{j\in N_i}\left(\xi_{ij} - a\right)\dot{\xi}_{ij} + \sum_{i=1}^{m}\sum_{j\in N_i}\frac{\sigma_{ij}}{\rho_{ij}}\left(\tilde{\xi}_{ij} - a\right)\dot{\tilde{\xi}}_{ij}$$

$$
\begin{aligned}
&= \sum_{i=1}^{m}\sum_{j\in N_i}\left(\xi_{ij}-a\right)\left\|P\left(y_i(t)-y_j(t)\right)\right\|^2 \\
&\quad - \sum_{i=1}^{m}\sum_{j\in N_i}\sigma_{ij}\left(\xi_{ij}-a\right)\left(\xi_{ij}-\tilde{\xi}_{ij}\right) \\
&\quad + \sum_{i=1}^{m}\sum_{j\in N_i}\sigma_{ij}\left(\tilde{\xi}_{ij}-a\right)\left(\xi_{ij}-\tilde{\xi}_{ij}\right) \\
&= y(t)^T\left(2D\mathscr{A}D^T\otimes QCC^TQ-2aL\otimes QCC^TQ\right)y(t) \\
&\quad - \sum_{i=1}^{m}\sum_{j\in N_i}\sigma_{ij}\left(\xi_{ij}-\tilde{\xi}_{ij}\right)^2 \\
&= \epsilon(t)^T\left(2D\mathscr{A}D^T\otimes QCC^TQ-2aL\otimes QCC^TQ\right)\epsilon(t) \\
&\quad - \sum_{i=1}^{m}\sum_{j\in N_i}\sigma_{ij}\left(\xi_{ij}-\tilde{\xi}_{ij}\right)^2 .
\end{aligned} \tag{16}
$$

Note that

$$
2y(t)^T(D\mathscr{B}\otimes QC)\operatorname{sign}\left[\left(D^T\otimes C^TQ\right)y(t)\right] = \sum_{i=1}^{m}\sum_{j\in N_i}\varsigma_{ij}\left\|C^TQ\left(y_i(t)-y_j(t)\right)\right\|_1 , \tag{17}
$$

$$
2\epsilon(t)^T(D\otimes QC)\operatorname{sign}\left[\left(D^T\otimes C^TQ\right)\epsilon(t)\right] = \sum_{i=1}^{m}\sum_{j\in N_i}\varsigma_{ij}\left\|C^TQ\left(\epsilon_i(t)-\epsilon_j(t)\right)\right\|_1 , \tag{18}
$$

by the fourth term in (14), we obtain

$$
\begin{aligned}
&\sum_{i=1}^{m}\sum_{j\in N_i}\left(\varsigma_{ij}-b\right)\dot{\varsigma}_{ij}+\sum_{i=1}^{m}\sum_{j\in N_i}\frac{\sigma_{ij}}{\rho_{ij}}\left(\tilde{\varsigma}_{ij}-b\right)\dot{\tilde{\varsigma}}_{ij} \\
&\quad = \sum_{i=1}^{m}\sum_{j\in N_i}\left(\varsigma_{ij}-b\right)\left\|P\left(y_i(t)-y_j(t)\right)\right\|_1 \\
&\quad - \sum_{i=1}^{m}\sum_{j\in N_i}\sigma_{ij}\left(\varsigma_{ij}-b\right)\left(\varsigma_{ij}-\tilde{\varsigma}_{ij}\right) \\
&\quad - \sum_{i=1}^{m}\sum_{j\in N_i}\sigma_{ij}\left(\tilde{\varsigma}_{ij}-b\right)\left(\varsigma_{ij}-\tilde{\varsigma}_{ij}\right) \\
&\quad = \sum_{i=1}^{m}\sum_{j\in N_i}\left(\varsigma_{ij}-b\right)\left\|P\left(y_i(t)-y_j(t)\right)\right\|_1 \\
&\quad - \sum_{i=1}^{m}\sum_{j\in N_i}\sigma_{ij}\left(\varsigma_{ij}-\tilde{\varsigma}_{ij}\right)^2 \\
&\quad = y(t)^T(D\mathscr{B}\otimes QC)\operatorname{sign}\left[\left(D^T\otimes C^TQ\right)y(t)\right] \\
&\quad + y(t)^T(bD\otimes QC)\operatorname{sign}\left[\left(D^T\otimes C^TQ\right)y(t)\right] \\
&\quad - \sum_{i=1}^{m}\sum_{j\in N_i}\sigma_{ij}\left(\varsigma_{ij}-\tilde{\varsigma}_{ij}\right)^2 \\
&\quad = \epsilon(t)^T(D\mathscr{B}\otimes QC)\operatorname{sign}\left[\left(D^T\otimes C^TQ\right)\epsilon(t)\right] \\
&\quad - b\sum_{i=1}^{m}\sum_{j\in N_i}\left\|C^TQ\left(\epsilon_i(t)-\epsilon_j(t)\right)\right\|_1 \\
&\quad - \sum_{i=1}^{m}\sum_{j\in N_i}\sigma_{ij}\left(\varsigma_{ij}-\tilde{\varsigma}_{ij}\right)^2 .
\end{aligned} \tag{19}
$$

From (15), (16), and (19), we get

$$
\begin{aligned}
&\dot{V}(t) \\
&\quad \le \epsilon(t)^T\left[I_m\otimes\left(QB+B^TQ\right)-2aL\otimes QCC^TQ\right]\epsilon(t) \\
&\quad + \left(\eta(m-1)-b\right)\sum_{i=1}^{m}\sum_{j\in N_i}\left\|C^TQ\left(\epsilon_i(t)-\epsilon_j(t)\right)\right\|_1 \\
&\quad - \sum_{i=1}^{m}\sum_{j\in N_i}\sigma_{ij}\left(\xi_{ij}-\tilde{\xi}_{ij}\right)^2-\sum_{i=1}^{m}\sum_{j\in N_i}\sigma_{ij}\left(\varsigma_{ij}-\tilde{\varsigma}_{ij}\right)^2 ,
\end{aligned} \tag{20}
$$

and choose $a > 1/2\lambda_2$, $b \ge \eta(m-1)$, due to (12),

$$
\begin{aligned}
&\dot{V}(t) \\
&\quad \le \epsilon(t)^T\left[I_m\otimes\left(QB+B^TQ\right)-2a\lambda_2\otimes QCC^TQ\right]\epsilon(t) \\
&\quad \le \epsilon(t)^T\left(I_m\otimes\left(QB+B^TQ-QCC^TQ\right)\right)\epsilon(t)\le 0,
\end{aligned} \tag{21}
$$

i.e., $V(t)$ is nonincreasing. Moreover, from (13), we have $\epsilon(t), \dot{\epsilon}(t), \xi_{ij}, \tilde{\xi}_{ij}, \varsigma_{ij}, \tilde{\varsigma}_{ij} \in \mathbb{L}_\infty$. Meanwhile, integrating both sides of (21), one has $\epsilon(t) \in \mathbb{L}_2$. Thus, $\epsilon(t) \in \mathbb{L}_2 \cap \mathbb{L}_\infty$. From Lemma 4, $\lim_{t\to\infty}\epsilon(t) = 0$; that is, $\lim_{t\to\infty}(y_i(t) - (1/m)\sum_{i=1}^{m} y_i(t)) = 0$.

Let consensus value $y^*(t) = (1/m)\sum_{i=1}^{m} y_i(t)$; then

$$
\begin{aligned}
\frac{dy^*(t)}{dt} &= \frac{1}{m}\sum_{i=1}^{m}\frac{dy_i(t)}{dt} = \frac{1}{m}\sum_{i=1}^{m}\Bigg\{By_i(t) \\
&\quad - C\sum_{j\in N_i}\xi_{ij}\left(P\left(y_i(t)-y_j(t)\right)\right) \\
&\quad - C\sum_{j\in N_i}\varsigma_{ij}\operatorname{sign}\left[P\left(y_i(t)-y_j(t)\right)\right]+C\psi_i\left(y_i(t)\right)\Bigg\} \\
&= \frac{1}{m}\sum_{i=1}^{m}\left\{By_i(t)+C\psi_i\left(y_i(t)\right)\right\},
\end{aligned} \tag{22}
$$

According to Assumption 5, we have

$$\begin{aligned}\frac{dy^*(t)}{dt} &= \frac{1}{m}K^{-1}\sum_{i=1}^{m}\left\{\widetilde{B}y^*(t)+\widetilde{C}\psi_i\left(y^*(t)\right)\right\}\\ &= \frac{1}{m}K^{-1}\sum_{i=1}^{m}\nabla h_i\left(y^*(t)\right),\end{aligned} \quad (23)$$

thus,

$$\begin{aligned}\frac{d\sum_{i=1}^{m}h_i\left(y^*(t)\right)}{dt} &= \frac{1}{m}\sum_{i=1}^{m}\left(\nabla h_i\left(y^*(t)\right)\right)^T\frac{dy^*(t)}{dt}\\ &= \frac{1}{m}\sum_{i=1}^{m}\left(\nabla h_i\left(y^*(t)\right)\right)^T K^{-1}\frac{1}{m}\sum_{i=1}^{m}\nabla h_i\left(y^*(t)\right)\le 0.\end{aligned} \quad (24)$$

From Assumption 1, $\sum_{i=1}^{m}(h_i(y^*(t)))$ is bounded below; that is, $\lim_{t\longrightarrow\infty}(d\sum_{i=1}^{m}h_i(y^*(t))/dt) = 0$. Thus, we obtain $\lim_{t\longrightarrow\infty}(1/m)\sum_{i=1}^{m}(\nabla h_i(y^*(t))) = 0$. From Lemma 2, when $t\longrightarrow\infty$, $y^*(t)$ minimizes the cost function $\sum_{i=1}^{m}h_i(y(t))$.

Let $\widehat{\varsigma}_{ij}=\varsigma_{ij}-\widetilde{\varsigma}_{ij}$, then

$$\begin{aligned}\dot{\widehat{\varsigma}}_{ij} &= \dot{\varsigma}_{ij}-\dot{\widetilde{\varsigma}}_{ij}\\ &= \left\|P\left(y_i(t)-y_j(t)\right)\right\|_1-\sigma_{ij}\left(\varsigma_{ij}-\widetilde{\varsigma}_{ij}\right)\\ &\quad -\rho_{ij}\left(\varsigma_{ij}-\widetilde{\varsigma}_{ij}\right)\\ &= \left\|P\left(y_i(t)-y_j(t)\right)\right\|_1-\left(\sigma_{ij}+\rho_{ij}\right)\widehat{\varsigma}_{ij}\end{aligned} \quad (25)$$

To solve (25), one has

$$\begin{aligned}\widehat{\varsigma}_{ij}(t) &= e^{-(\sigma_{ij}+\rho_{ij})t}\widehat{\varsigma}_{ij}(0)\\ &\quad +\int_0^t e^{-(\sigma_{ij}+\rho_{ij})(t-s)}\left\|P\left(y_i(s)-y_j(s)\right)\right\|_1 ds\\ &= e^{-(\sigma_{ij}+\rho_{ij})t}\left(\widehat{\varsigma}_{ij}(0)\right.\\ &\quad \left.+\int_0^t e^{(\sigma_{ij}+\rho_{ij})s}\left\|P\left(y_i(s)-y_j(s)\right)\right\|_1 ds\right)\end{aligned} \quad (26)$$

By $y_i\in\mathbb{L}_2$, we have $\widehat{\varsigma}_{ij}(t)\in\mathbb{L}_\infty$. From $\varsigma_{ij}(0)\ge\widetilde{\varsigma}_{ij}(0)$, we obtain $\widehat{\varsigma}_{ij}(0)\ge 0$. By (26), $\widehat{\varsigma}_{ij}(t)\ge 0$. Now, let us prove $\lim_{t\longrightarrow\infty}\widehat{\varsigma}_{ij}(t)=0$. Consider the following two cases.

(1) $\int_0^t e^{(\sigma_{ij}+\rho_{ij})s}\|P(y_i(s)-y_j(s))\|_1 ds$ is bounded. Then,

$$\begin{aligned}\lim_{t\longrightarrow\infty}\widehat{\varsigma}_{ij}(t) &= \lim_{t\longrightarrow\infty}e^{-(\sigma_{ij}+\rho_{ij})t}\\ &\quad\cdot\left(\widehat{\varsigma}_{ij}(0)+\int_0^t e^{(\sigma_{ij}+\rho_{ij})s}\left\|P\left(y_i(s)-y_j(s)\right)\right\|_1 ds\right)\\ &= 0.\end{aligned} \quad (27)$$

(2) $\lim_{t\longrightarrow\infty}\int_0^t e^{(\sigma_{ij}+\rho_{ij})s}\|P(y_i(s)-y_j(s))\|_1 ds=\infty$. Then,

$$\begin{aligned}&\lim_{t\longrightarrow\infty}\widehat{\varsigma}_{ij}(t)\\ &= \lim_{t\longrightarrow\infty}e^{-(\sigma_{ij}+\rho_{ij})t}\left(\widehat{\varsigma}_{ij}(0)+\int_0^t e^{(\sigma_{ij}+\rho_{ij})s}\left\|P\left(y_i(s)\right.\right.\right.\\ &\quad\left.\left.\left.-y_j(s)\right)\right\|_1 ds\right)\\ &= \lim_{t\longrightarrow\infty}\frac{\widehat{\varsigma}_{ij}(0)+\int_0^t e^{(\sigma_{ij}+\rho_{ij})s}\left\|P\left(y_i(s)-y_j(s)\right)\right\|_1 ds}{e^{(\sigma_{ij}+\rho_{ij})t}}\\ &= \lim_{t\longrightarrow\infty}\frac{\left\|P\left(y_i(t)-y_j(t)\right)\right\|_1}{\sigma_{ij}+\rho_{ij}}=0.\end{aligned} \quad (28)$$

Based on $\widetilde{\varsigma}_{ij}(t)\in\mathbb{L}_\infty$ and $\widetilde{\varsigma}_{ij}(t)$ is monotonically increasing, when $t\longrightarrow\infty$, we acquire that $\widetilde{\varsigma}_{ij}(t)$ will converge to some positive constant. From $\lim_{t\longrightarrow\infty}\widehat{\varsigma}_{ij}(t)=0$, we have $\varsigma_{ij}(t)$ and $\widetilde{\varsigma}_{ij}(t)$ will converge to the same positive number. For $\xi_{ij}(t)$ and $\widetilde{\xi}_{ij}(t)$, we have the same results. □

Remark 8. Here, damping terms $\widetilde{\xi}_{ij}(t),\widetilde{\varsigma}_{ij}(t)$ are designed for the gain adaption law of the adaptive algorithm (5). $\widetilde{\varsigma}_{ij}(t)$ increases monotonically and converges to a finite positive constant. Choosing $\widehat{\varsigma}_{ij}(0)=0$, by (26), $\lim_{t\longrightarrow\infty}\widehat{\varsigma}_{ij}(t)=0$, and $\widehat{\varsigma}_{ij}(t)\ge 0$, we have that $\widehat{\varsigma}_{ij}(t)$ will increase from zero and then decrease to zero. In other words, $\varsigma_{ij}(t)$ will increase at first and then decrease for some time upon the select of σ_{ij} and ρ_{ij}. For $\xi_{ij}(t)$, we have similar results. Compared with the adaptive law in [14, 15], the advantage of the adaptive law introducing the damping term is that the adaptive control gains and the amplitude of the control inputs are smaller; this makes our algorithm more robust.

5. Node-Based Adaptive Designs

To solve problem (3), the node-based protocol for (1) is proposed as follows:

$$u_i(t)=-\zeta_i\,\mathrm{sign}\left[P\sum_{j\in N_i}\left(y_i(t)-y_j(t)\right)\right]+\psi_i\left(y_i\right), \quad (29)$$

$$\dot{\zeta}_i=\left\|P\sum_{j\in N_i}\left(y_i(t)-y_j(t)\right)\right\|_1-\sigma_i\left(\zeta_i-\widetilde{\zeta}_i\right), \quad (30)$$

$$\dot{\widetilde{\zeta}}_i=\rho_i\left(\zeta_i-\widetilde{\zeta}_i\right), \quad (31)$$

where ζ_i are adaptive coupling strengths, Q is a feedback matrix, and the initial states satisfy $\zeta_i(0)\ge\widetilde{\zeta}_i(0)$.

From (1) and (29), the closed-loop system can be represented as follows:

$$y_i(t) = By_i(t) - \zeta_i C \operatorname{sign}\left[P\sum_{j\in N_i}\left(y_i(t) - y_j(t)\right)\right] + C\psi_i\left(y_i(t)\right), \quad (32)$$

where the first and third terms take the role to minimize the function $h_i(y_i(t))$, and the second term makes the agents achieve consensus.

Denote $y(t) = (y_1(t)^T, y_2(t)^T, \ldots, y_m(t)^T)^T$, $\zeta = \operatorname{diag}\{\xi_{ij}\}$, and $\Psi(t) = (\psi_1^T(y_1(t)), \psi_2^T(y_2(t)), \ldots, \psi_m^T(y_m(t)))^T$; by (32) we obtain

$$\begin{aligned}\dot{y}(t) &= \left(I_m \otimes B\right) y(t) - (\zeta \otimes C) \operatorname{sign}\left[(L \otimes P)\, y(t)\right] \\ &\quad + \left(I_m \otimes C\right) \Psi\left(y(t)\right).\end{aligned} \quad (33)$$

Note that $ML = DL = D$; multiply both sides of (33) by $M \otimes I_m$; the following error system is obtained:

$$\begin{aligned}\dot{\epsilon}(t) &= \left(I_m \otimes B\right) \epsilon(t) - (\zeta \otimes C) \operatorname{sign}\left[(L \otimes P)\, \epsilon(t)\right] \\ &\quad + \left(I_m \otimes C\right) \Psi\left(y(t)\right),\end{aligned} \quad (34)$$

where $M = I_m - (1/m)\mathbf{1}\mathbf{1}^T$, $\epsilon(t) = (\epsilon_1(t)^T, \epsilon_2(t)^T, \ldots, \epsilon_m(t)^T)^T = (M \otimes I_m) y(t)$.

Theorem 9. *Under Assumptions 1–6 and Lemma 3, problem (29) can be solved by the nosed-based algorithm (5) if $P = C^TQ$, where $Q > 0$ satisfies*

$$QB + B^TQ < 0, \quad (35)$$

Meanwhile, as $t \longrightarrow \infty$, we have $\zeta_i(t)$ and $\tilde{\zeta}_i(t)$ will converge to the same positive number.

Proof. Choose the Lyapunov function candidate

$$\begin{aligned}V(t) &= \epsilon(t)^T (L \otimes Q)\, \epsilon(t) + \frac{1}{2}\sum_{i=1}^{m}\left(\zeta_i - a\right)^2 \\ &\quad + \sum_{i=1}^{m}\frac{\sigma_i}{2\rho_i}\left(\tilde{\zeta}_i - a\right)^2,\end{aligned} \quad (36)$$

where $Q > 0$; a is a positive constant.

Calculate the derivative of $V(t)$ along system (34) and choose $P = C^TQ$; we have

$$\begin{aligned}\dot{V}(t) &= \epsilon(t)^T \left[L \otimes \left(QB + B^TQ\right)\right] \epsilon(t) \\ &\quad + 2\epsilon(t)^T (L \otimes QC)\, \Psi\left(y(t)\right) \\ &\quad - 2\epsilon(t)^T (L \otimes QC) \operatorname{sign}\left[\left(L \otimes C^TQ\right) \epsilon(t)\right] \\ &\quad + \sum_{i=1}^{m}\left(\zeta_i - a\right)\dot{\zeta}_i + \sum_{i=1}^{m}\frac{\sigma_i}{\rho_i}\left(\tilde{\zeta}_i - a\right)\dot{\tilde{\zeta}}_i.\end{aligned} \quad (37)$$

From the third term in (37) and $\|(M \otimes I_q)\Psi(y(t)\|_1 \leq \eta$, we have

$$\begin{aligned}&\epsilon(t)^T (M \otimes QC)\, \Psi\left(y(t)\right) \\ &\quad \leq \left\|\left(M \otimes C^TQ\right) \epsilon(t)\right\|_1 \left\|\left(M \otimes I_q\right) \Psi(y(t)\right\|_1 \\ &\quad \leq \eta\sum_{i=1}^{m}\zeta_i\left\|\sum_{j\in N_i}C^TQ\left(\epsilon_i(t) - \epsilon_j(t)\right)\right\|_1.\end{aligned} \quad (38)$$

By the results

$$\begin{aligned}&y(t)^T (L \otimes QC)\, \zeta \operatorname{sign}\left[\left(L \otimes C^TQ\right) y(t)\right] \\ &\quad = \sum_{i=1}^{m}\zeta_i\left\|C^TQ\sum_{j\in N_i}\left(y_i(t) - y_j(t)\right)\right\|_1, \\ &\epsilon(t)^T (aL \otimes QC) \operatorname{sign}\left[\left(L \otimes C^TQ\right) \epsilon(t)\right] \\ &\quad = a\sum_{i=1}^{m}\left\|C^TQ\sum_{j\in N_i}\left(\epsilon_i(t) - \epsilon_j(t)\right)\right\|_1,\end{aligned} \quad (39)$$

we obtain

$$\begin{aligned}&\sum_{i=1}^{m}\left(\zeta_i - a\right)\dot{\zeta}_i + \sum_{i=1}^{m}\frac{\sigma_i}{\rho_i}\left(\tilde{\zeta}_i - a\right)\dot{\tilde{\zeta}}_i \\ &\quad = \sum_{i=1}^{m}\left(\zeta_i - a\right)\left\|P\sum_{j\in N_i}\left(y_i(t) - y_j(t)\right)\right\|_1 \\ &\quad - \sum_{i=1}^{m}\sigma_i\left(\zeta_i - a\right)\left(\zeta_i - \tilde{\zeta}_i\right) + \sum_{i=1}^{m}\sigma_i\left(\tilde{\zeta}_i - a\right)\left(\zeta_i - \tilde{\zeta}_i\right) \\ &\quad = \sum_{i=1}^{m}\left(\zeta_i - a\right)\left\|P\sum_{j\in N_i}\left(y_i(t) - y_j(t)\right)\right\|_1 \\ &\quad - \sum_{i=1}^{m}\sigma_i\left(\zeta_i - \tilde{\zeta}_i\right)^2 \\ &\quad = y(t)^T (L \otimes QC)\, \zeta \operatorname{sign}\left[\left(L \otimes C^TQ\right) y(t)\right] \\ &\quad + y(t)^T (aL \otimes QC) \operatorname{sign}\left[\left(L \otimes C^TQ\right) y(t)\right] \\ &\quad - \sum_{i=1}^{m}\sigma_i\left(\zeta_i - \tilde{\zeta}_i\right)^2 \\ &\quad = \epsilon(t)^T (L \otimes QC)\, \zeta \operatorname{sign}\left[\left(L \otimes C^TQ\right) \epsilon(t)\right] \\ &\quad - a\sum_{i=1}^{m}\left\|C^TQ\sum_{j\in N_i}\left(\epsilon_i(t) - \epsilon_j(t)\right)\right\|_1, \\ &- \sum_{i=1}^{m}\sigma_i\left(\zeta_i - \tilde{\zeta}_i\right)^2.\end{aligned} \quad (40)$$

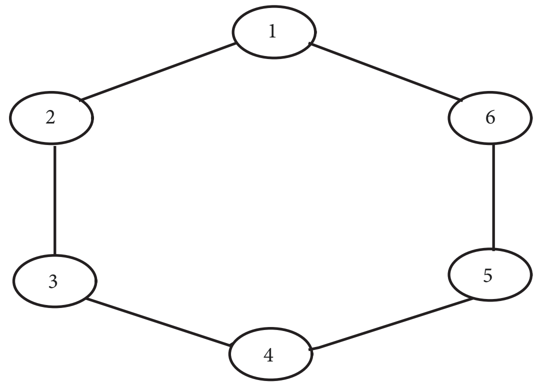

Figure 1: The communication topology for the distributed optimization.

From (37), (38), and (40), we have

$$\begin{aligned}\dot{V}(t) \leq\ & \epsilon(t)^T \left[I_m \otimes \left(QB + B^T Q\right)\right] \epsilon(t) \\ & + (\eta - a) \sum_{i=1}^{m} \sum_{j \in N_i} \left\| C^T Q \left(\epsilon_i(t) - \epsilon_j(t)\right)\right\|_1 \\ & - \sum_{i=1}^{m} \sigma_i \left(\zeta_i - \tilde{\zeta}_i\right)^2,\end{aligned} \tag{41}$$

Choose $a > \eta$, we get

$$\dot{V}(t) \leq \epsilon(t)^T \left[I_m \otimes \left(QB + B^T Q\right)\right] \epsilon(t) \tag{42}$$

Thus, similar to the proof of Theorem 7, the nosed-based algorithm (29) can solve the problem (29). □

Remark 10. It is easy to see that the node-based algorithm is very different from the edge-based algorithm. The advantage is that the nosed-based algorithm has a less computation and more concise form than the edge-based algorithm. The disadvantage is that the nosed-based algorithm converges more slowly than the edge-based algorithm.

Remark 11. Based on the edge and node standpoints, two adaptive algorithms have been established for solving the distributed convex optimization problems; the algorithms are fully distributed without depending on any global information; that is, the adaptive algorithms can get the optimal solution via local information. Moreover, the connectivity of the networks takes a key role in the rate of convergence of the algorithms during the calculation process.

6. Simulations

Consider the following distributed optimization:

$$\begin{aligned} \min \quad & \sum_{i=1}^{6} h_i(y_i) \\ \text{subject to} \quad & y_i = y_j \in \mathbb{R}^2, \quad i, j = 1, \ldots, 6, \end{aligned} \tag{43}$$

where

$$h_i(y_i) = \begin{pmatrix} y_{i1} \\ y_{i2} \\ \psi_i \end{pmatrix}^T \begin{pmatrix} 1 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 3 \end{pmatrix} \begin{pmatrix} y_{i1} \\ y_{i2} \\ \psi_i \end{pmatrix}, \tag{44}$$

$\psi_i = i/2$, $i = 1, 2, \ldots, 6$. Figure 1 is the communication topology. Choose

$$K = \begin{pmatrix} -1 & 0 \\ 0 & -\frac{1}{2} \end{pmatrix}, \text{ thus, } B = \begin{pmatrix} -1 & 2 \\ 1 & -3 \end{pmatrix}, \ C = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{45}$$

Simulation results employing edge-based algorithm (5) and node-based algorithm (29) are presented in Figures 2–4 and Figures 5–7, respectively. In Figures 2 and 5, it can be observed that the states of consensus are achieved. In Figures 3 and 6, the adaptive control gains first increase, then decrease, and finally converge to some constants, which is consistent with our theoretical analysis in Remark 8. In Figures 4 and 7, the evolution of $\sum_{i=1}^{6} h_i(y_i)$ is presented, which reaches the optimal value 31.5.

7. Conclusion

In this paper, two distributed adaptive algorithms to solve the distributed convex optimization problems are designed based on general linear multiagent systems from the edge-based and node-based standpoint. Compared with the existing algorithms, through introducing a damping term in the update law, the algorithms are robust in the face of input disturbances. In this paper, the multiagent networks are undirected graph. Next, we will focus on the case that the multiagent networks are directed graph.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

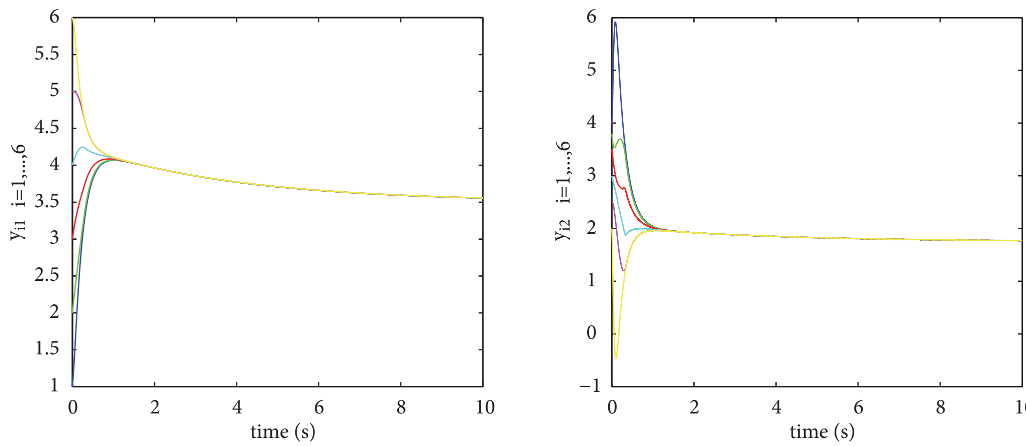

Figure 2: The state variables $y(t)$ under the edge-based algorithm.

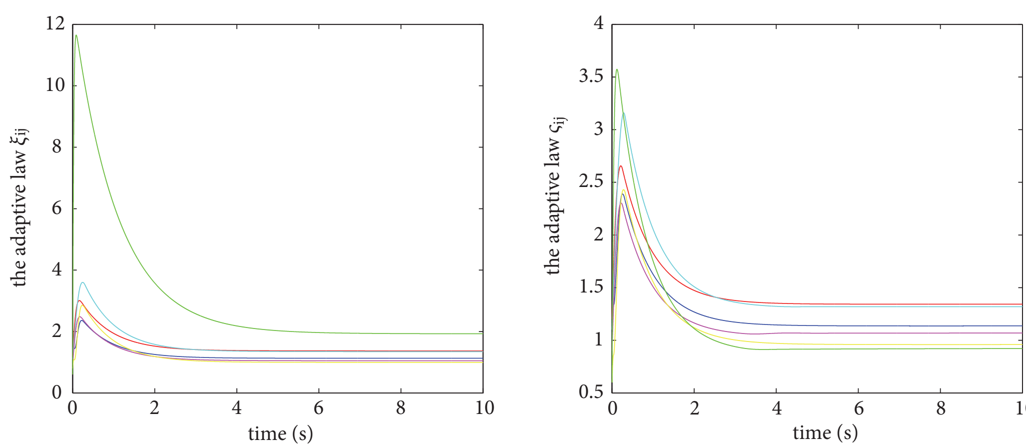

Figure 3: The adaptive law under the edge-based algorithm.

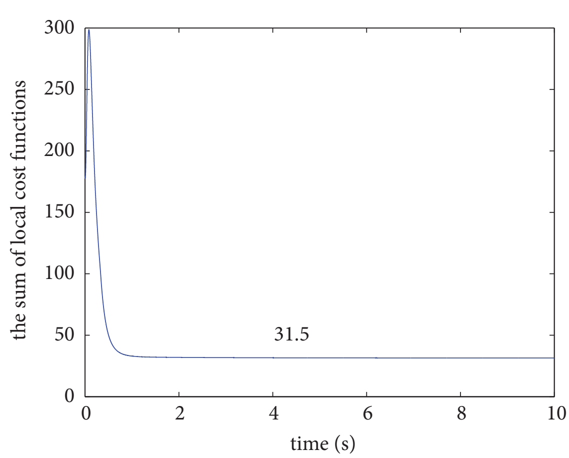

Figure 4: The global cost function under the edge-based algorithm.

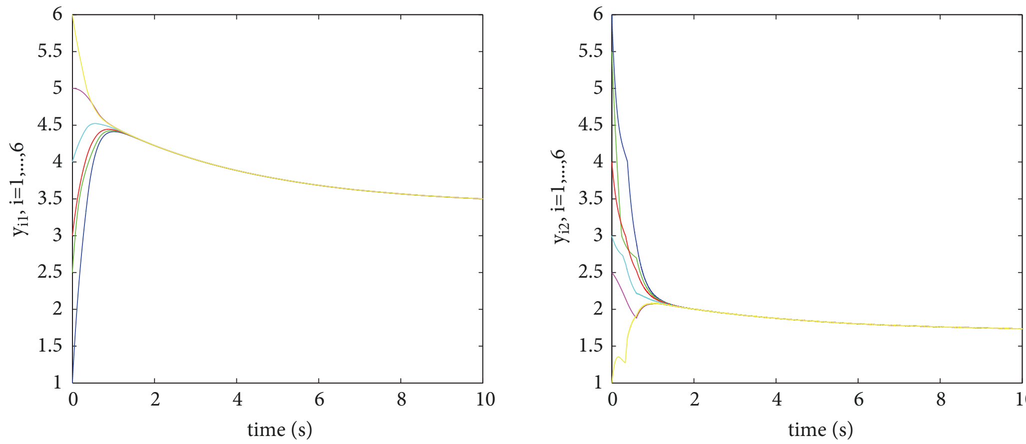

Figure 5: The state variables $y(t)$ under the node-based algorithm.

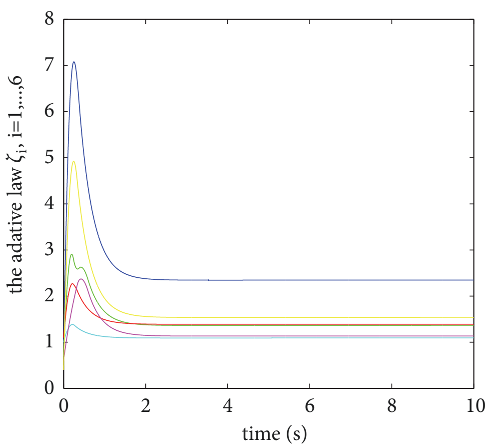

Figure 6: The adaptive law under the node-based algorithm.

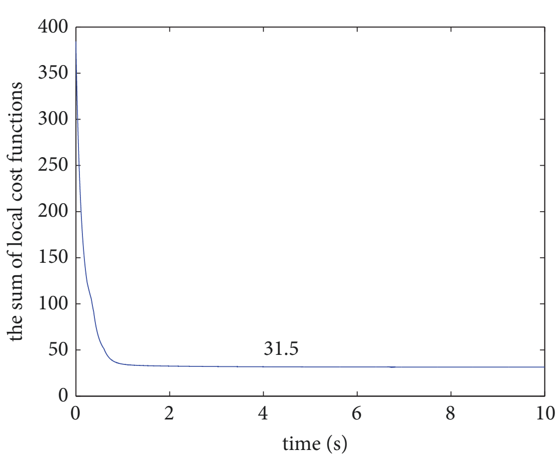

Figure 7: The global cost function under the node-based algorithm.

Acknowledgments

This work was supported by the Scientific Research Program of the Higher Education Institution of Xinjiang (Grant no. XJEDU2017T001), the Natural Science Foundation of Xinjiang Uygur Autonomous Region (Grant no. 2018D01C039), and the National People's Republic of China (Grants nos. U1703262 and 61563048).

References

[1] R. Olfati-Saber, J. A. Fax, and R. M. Murray, "Consensus and cooperation in networked multi-agent systems," *Proceedings of the IEEE*, vol. 95, no. 1, pp. 215–233, 2007.

[2] H. Hong, W. Yu, J. Fu, and X. Yu, "Finite-time connectivity-preserving consensus for second-order nonlinear multiagent systems," *IEEE Transactions on Control of Network Systems*, vol. 6, no. 1, pp. 236–248, 2019.

[3] Z. Yu, H. Jiang, C. Hu, and J. Yu, "Necessary and sufficient conditions for consensus of fractional-order multiagent systems via sampled-data control," *IEEE Transactions on Cybernetics*, vol. 47, no. 8, pp. 1892–1901, 2017.

[4] Z. Yu, D. Huang, H. Jiang, C. Hu, and W. Yu, "Distributed consensus for multiagent systems via directed spanning tree based adaptive control," *SIAM Journal on Control and Optimization*, vol. 56, no. 3, pp. 2189–2217, 2018.

[5] J. Mei, W. Ren, and J. Chen, "Distributed consensus of second-order multi-agent systems with heterogeneous unknown inertias and control gains under a directed graph," *IEEE Transactions on Automatic Control*, vol. 61, no. 8, pp. 2019–2034, 2016.

[6] B. Ning, Q. Han, Z. Zuo, J. Jin, and J. Zheng, "Collective behaviors of mobile robots beyond the nearest neighbor rules with switching topology," *IEEE Transactions on Cybernetics*, vol. 48, no. 5, pp. 1577–1590, 2018.

[7] B. Ning and Q. Han, "Prescribed finite-time consensus tracking for multiagent systems with nonholonomic chained-form dynamics," *IEEE Transactions on Automatic Control*, vol. 64, no. 4, pp. 1686–1693, 2019.

[8] A. Nedic and A. Ozdaglar, "Distributed subgradient methods for multi-agent optimization," *IEEE Transactions on Automatic Control*, vol. 54, no. 1, pp. 48–61, 2009.

[9] Y. A. Nedic, A. Ozdaglar, and P. A. Parrilo, "Constrained consensus and optimization in multi-agent networks," *IEEE Transactions on Automatic Control*, vol. 55, no. 4, pp. 922–938, 2010.

[10] Q. Liu, S. Yang, and Y. Hong, "Constrained consensus algorithms with fixed step size for distributed convex optimization over multiagent networks," *IEEE Transactions on Automatic Control*, vol. 62, no. 8, pp. 4259–4265, 2017.

[11] B. Ning, Q. Han, and Z. Zuo, "Distributed optimization for multiagent systems: an edge-based fixed-time consensus approach," *IEEE Transactions on Cybernetics*, vol. 49, no. 1, pp. 122–132, 2019.

[12] B. Gharesifard and J. Cortes, "Distributed continuous-time convex optimization on weight-balanced digraphs," *IEEE Transactions on Automatic Control*, vol. 59, no. 3, pp. 781–786, 2014.

[13] G. Droge, H. Kawashima, and M. B. Egerstedt, "Continuous-time proportional-integral distributed optimisation for networked systems," *Journal of Control and Decision*, vol. 1, no. 3, pp. 191–213, 2014.

[14] X. Shi, J. Cao, G. Wen, and M. Perc, "Finite-time consensus of opinion dynamics and its applications to distributed optimization over digraph," *IEEE Transactions on Cybernetics*, vol. 49, no. 10, pp. 3767–3779, 2019.

[15] K. Li, Q. Liu, S. Yang, J. Cao, and G. Lu, "Distributed robust adaptive optimization for nonlinear multiagent systems," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*.

[16] Z. Li, Z. Ding, J. Sun, and Z. Li, "Distributed adaptive convex optimization on directed graphs via continuous-time algorithms," *IEEE Transactions on Automatic Control*, vol. 63, no. 5, pp. 1434–1441, 2018.

[17] P. Lin, W. Ren, and J. A. Farrel, "Distributed continuous-time optimization: nonuniform gradient gains, finite-time convergence, and convex constraint set," *IEEE Transactions on Automatic Control*, vol. 62, no. 5, pp. 2239–2253, 2016.

[18] Y. Zhao, Y. Liu, G. Wen, and G. Chen, "Distributed optimization for linear multiagent systems: edge- and node-based adaptive designs," *IEEE Transactions on Automatic Control*, vol. 62, no. 7, pp. 3602–3609, 2017.

[19] P. Ioannou and P. Kokotovic, "Instability analysis and improvement of robustness of adaptive control," *Automatica*, vol. 20, no. 5, pp. 583–594, 1984.

[20] C. Godsil and G. Royle, *Algebraic Graph Theory*, Springer, New York, NY, USA, 2001.

[21] S. Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge University Press, Cambridge, UK, 2004.

[22] S. Sastry, *Nonlinear Systems: Analysis, Stability, and Control*, Springer, New York, NY, USA, 1999.

[23] J.-P. Aubin and A. Cellina, *Differential Inclusions: Set-Valued Maps and Viability Theory*, Springer, Berlin, Germany, 1984.