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Research Article

G/M/N Queuing Model-Based Research on the Parking Spaces for Primary and Secondary School

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In order to ease the traffic congestion near primary and secondary schools during the drop-off and pick-up, traffic behavior was analyzed, and a quantitative allocation method for parking spaces was proposed. The proposed method includes three steps. Firstly, a vehicle queuing model based on G/M/N queue theory was built, and MATLAB was employed to simulate the queuing model to obtain a reasonable parking space for drop-off. Secondly, the reasonable parking space for pick-up is determined based on the cumulative number of vehicles. Finally, an optimization model for the scale of total parking spaces and the short-term parking spaces was proposed to achieve maximum utilization of parking spaces. An empirical analysis and verification were conducted based on actual traffic data of schools. The results show that the proposed model can analyze the overall scale of the parking space for primary and secondary school, as well as the scale of the short-term parking spaces. And the proposed scale of the parking space can well meet the actual parking demand.

1. Introduction

The National Standard for Urban Parking Planning Norms (GBT 51149-2016), which is implemented in China in 2017, requires the vehicle parking spaces for kindergarten, primary, and secondary schools which should not be lower than one parking stall for every 100 students and teachers. Some provinces and cities also have independent standards for parking stalls. However, most of the existing standards just control parking spaces by the number of teachers, which can only satisfy the needs of faculty parking. It is lack of standard to require enough parking spaces for parents to drop-off or pick-up their children. Some cities begin to consider the objective needs of parking spaces for students' parents. Liyang stipulates that parking stalls for drop-off and pick-up should not be less than 30% of the total parking spaces in newly built primary schools. And Nanjing stipulates that temporary parking spaces for primary school should not be less than 4 parking stalls for every 100 students. Despite the rules, there are still many problems in practice. During the drop-off and pick-up time of the primary school, the parking space according to the standard can only meet a small number of vehicles, often resulting in congestion of vehicles. Furthermore, vehicles which need short time to drop-off or pick-up are unwilling to park in prescribed parking area. They tend to park in roadside or spaces near school gate. Irregular parking has a negative impact on the efficiency and safety of traffic around the school. In Europe, the costs attributable to congestion are around 1% of the annual gross domestic product [1]. Furthermore, the costs are also undergone by environmental as loss of biodiversity and pollution [2–5]. Thus, it is essential to use public measures [6] to reduce the negative impact of vehicle congestion during drop-off and pick-up hours around the primary school.

Lots of research studies supporting the scale of parking spaces have been done at home and abroad. By using 12 variables such as the current parking spaces utilization rate and future benchmark utilization rate, Wilson [7] proposed a 12-step toolkit for formulating parking spaces, and this toolkit provides a very useful and rigorously explained standard for the government to arrange the parking spaces. Chen et al. [8] proposed a method for matching shared parking resources based on space-time capacity. Zhang and Li [9] proposed a prediction model for urban parking demand based on a land use model. Cai Yifei et al. [10] proposed a parking space allocation method (PSAM) at the network level, which can allocate the parking demand to a specific parking lot. Mei et al. [11] analyzed and compared the benefits of parking charge and reservation mechanisms and provided operational suggestions for urban parking managers. Khordagui [12] discussed the relationship between parking prices and the decision to drive to work and finally confirmed that parking pricing can indeed be an effective transportation demand management tool. The existing research mainly focuses on parking demand in residential areas, shopping malls, urban CBDs, etc. There are few studies on the scale of drop-off and pick-up parking spaces in primary education buildings. This paper mainly focuses on the characteristics of parking decision during drop-off and pick-up hours around the primary school. Based on the G/M/N queuing theory, a parking space allocation model near the primary education buildings is given, and empirical analysis is made.

2. Data Collection and Traffic Characteristics Analysis

This study conducted a video collection of pick-up and drop-off vehicles around a primary school in Jiangsu Province. Five days' data were obtained from the video collection, of which three days' data will be used to build the parking space allocation model, while the other two days' data will be used to verify the reliability of the proposed model. According to the class time, the statistical time includes two periods, the drop-off period between 7:30 and 8:30, and the pick-up period between 15:00 and 16:20.

Through qualitative observations of the video, it can be found that the parking behavior of primary school has distinctive features from other areas. During the drop-off time, parents only park for a short time to send their children to school. During pick-up time, vehicles must arrive at school in advance to wait for their children. Because of the interval between vehicle's arriving time and the time of class over, there is an accumulation of pick-up vehicles in the parking area.

This study intends to use the G/M/N queuing theory model to calculate the overall scale of parking spaces around primary school. G/M/N means in the queuing system which has n service tables, the time between customers arriving at the system is $G(t)$, and the residence time satisfies the negative exponential distribution. The proposed parking spaces should satisfy both economic and parking demand. Considering some vehicles only need park for a short time, the short-term parking spaces with ease of access will be calculated separately.

3. Modelling on Optimization of the Number of Parking Stalls for Primary School

According to the existing research [13], the degree of risk in the morning peak is greater than that in the evening peak. And the space for parking is limited because of the limited land areas of the city [14]. So, the goal of this study is to satisfy all drop-off parking demand before class begins in the morning to avoid traffic congestion, and the congestion duration after school is within the acceptable length of time.

3.1. Vehicle Queuing Model Based on G/M/N during Drop-Off Time. Considering the vehicle data of the first day as an example, as shown in Figure 1, the residence time satisfies the negative exponential distribution with the parameter μ [15]. Assuming that the number of parking spaces in the system is n and each parking space is independent, then the average service rate of the entire system is $n\mu$. The time between customers arriving at the system $G(t)$ is independent, and with the same distribution, the corresponding distribution density is $g(t)$. The mth car arriving the system is denoted as C_m, and the arriving time and the number of customers already in the system are denoted as t_m and X_m, respectively. X_m is only related to the state of the system at t_m. The mth arrival interval is $T_m = t_m - t_{m-1}$. Y_m is the number of vehicles that will finish picking up or dropping off children within the interval T_m. Assuming that the number of vehicles in the system is X_m when the mth vehicle arrives, the number turns into X_{m+1} after the mth vehicle. Because the residence time satisfies the negative exponential distribution and has no aftereffects, Y_m satisfies Poisson distribution when T_m is given, and its one-step transition matrix can be obtained. The one-step transition matrix p_{ij} should be solved by dividing into three parts as follows [16, 17]:

$$p_{ij} = \begin{cases} \displaystyle\int_0^{\infty} C_{i+1}^{i+1-j}\left(1 - e^{-\mu t}\right)^{i+1-j} e^{-j\mu t} g(t)\mathrm{d}t, & j \le i+1 \le n, \\ \displaystyle\int_0^{\infty} \frac{(n\mu t)^{i+1-j}}{(i+1-j)!} e^{-n\mu t} g(t)\mathrm{d}t, & n \le j \le i+1, \\ \displaystyle\int_0^{\infty}\int_0^{t} g(t) C_n^{j}\left(1 - e^{-\mu(t-\tau)}\right)^{j} e^{-j\mu(t-\tau)} \frac{n\mu(n\mu\tau)^{i-n}}{(i-n)!}\mathrm{d}\tau\mathrm{d}t, & j \le n \le i+1. \end{cases} \tag{1}$$

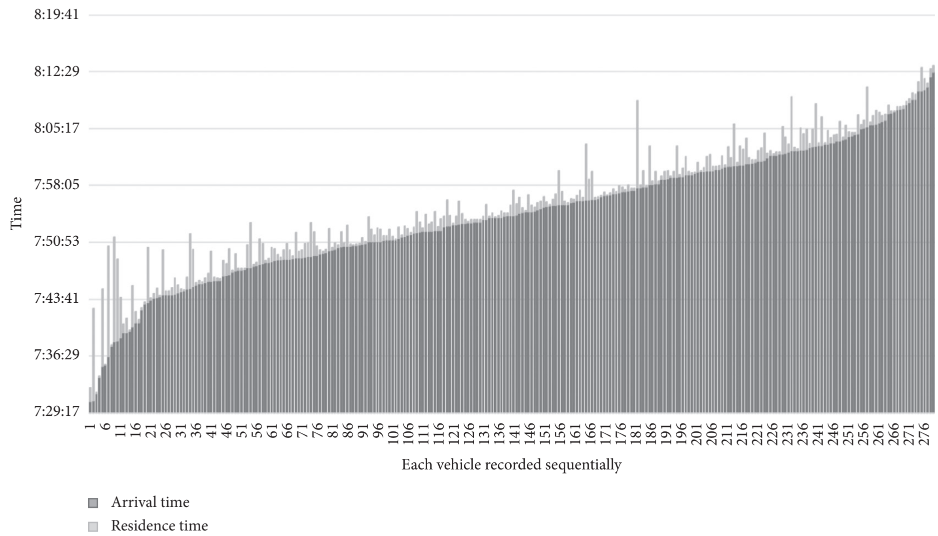

Figure 1: The number of vehicles at a certain time of the first data.

It can be proved that there is a stable distribution of this Markov chain [16], denoted as $\{\overline{p}_i\}$. According to the derivation of the Laplace transform method, the vehicle should enter the queue when it arrives at the system and finds no parking space is available. The probability of waiting for service is $C\alpha/(1-\alpha)$, where α is the undetermined parameter in the process of finding a stationary solution. α can be expressed as $\alpha = G^*(n\mu - n\mu\alpha)$,where $G^*(x)$ is the Laplace transform of $g(t)$. Let $\overline{p}_j = C_1\alpha^j$ and $C = C_1\alpha^{n-1}$, then total number of vehicles in the system and the queue is

$$L_s = C\sum_{j=1}^{n-2} j\gamma_j + C\left(\frac{n-1}{1-\alpha} + \frac{\alpha}{(1-\alpha)^2}\right). \tag{2}$$

The average number of vehicles in the queue is

$$L_q = \frac{C\alpha}{(1-\alpha)^2}. \tag{3}$$

The average waiting time for vehicles entering the system is

$$\widetilde{W}_q = \frac{1}{n\mu(1-\alpha)}. \tag{4}$$

From equations (2)–(4), it can be found that L_q and $\widetilde{W}_q$ are the reduction function of the number of parking space *n*. For this reason, this article intends to seek the minimum number of parking space n to make L_q and $\widetilde{W}_q$ within acceptable range and the number of parking spaces reach the minimum to improve parking space utilization.

To determine the number N_1 of parking spaces during drop-off, the queuing theory model is simulated. In the simulation, the vehicle enters the system and determines whether there is any available parking space in the service system. If there is, the vehicle enters the service system and the residence time starts. The vehicle normally leaves the system once the residence time is over. If there is not, the vehicle enters the queue and the queuing time starts. When any parking spaces are available in the service system, the vehicle will enter the system in order and the residence time starts, and the vehicle normally leaves the system once the residence time is over. The simulation code is shown in Figure 2:

3.2. Method for Deciding Parking Spaces during Pick-Up Time. During pick-up, the vehicle needs to stay for a long time which results in the total number of vehicles in the system to be high. Due to the land limitation near school and the cost limitation of the parking space, if the parking space is allocated according to the peak number of vehicles during the pick-up, it may lead to high allocation cost. To make the number of parking spaces to meet most of the parking needs during the pick-up and to minimize the cost of allocation, t_d is introduced as the parameter of acceptable traffic congestion time. Assuming that vehicles in the system and in the queue during the pick-up submit to the function $F(t)$, and the number of parking spaces to be allocated during the pick-up is N_2, if $F(t)$ is unimodal, there must be two moments t_1 and t_2 that can meet $F(t_1) = F(t_2) = N_2$. Then, t_1 and t_2 satisfy

$$\left|t_1 - t_2\right| = t_d. \tag{5}$$

In the actual situation, t_d can be set according to the environment, and N_2 to be allocated during the pick-up can be calculated. According to the characteristics of traffic behavior during school commuting, the demand for parking spaces during the pick-up is greater than that during drop-off. So, N_1 is included in N_2.

3.3. Method for Deciding Short-Term Parking Spaces. Vehicles just stop for a short time during drop-off, and students' parents are more likely to stop at a convenient place, such as along the road or near the school gate. Therefore, during drop-off, priority can be given to setting up short-term parking spaces along the road or near the school gate to meet short-term parking demand. And other

Algorithm 1 Queuing Simulation Algorithm.

Require: Arrival time of each car At_i; Dwell time of each vehicleDt_i; Number of parking spaces in the system n; Total number of vehicles N;
Ensure: The waiting time of each vehicle Wt_i; The departure time of each car Pt_i; The number of vehicles in the system when each car leavingNt_i;
let $i = 1$
if $i == N$ **then**
    exist
end if
The number of vehicles in the system when ith arriving n_i=the number of (Departure time of all vehicles before the vehicle> ith vehicle arrival time);
if $n_i < n$ **then**
    ith vehicle enters the system;
    $Wt_i = 0$;
    $n_i = n_i + 1$;
    $Pt_i = At_i + Dt_i$;
else
    $Wt_i = Pt_{n_i - n} - At_i$;
    $n_i = n_i + 1$;
    $Pt_i = At_i + Wt_i + Dt_i$;
end if
$i = i + 1$;
goto 2

Figure 2: Code for queuing simulation algorithm.

off-street parking spaces can be set to meet the parking demand that need to stop for a long time.

To pursue configuration optimization and utilization maximization of short-term parking spaces and ordinary parking spaces, it is assumed that short-term parking and long-term parking vehicles can accurately select the corresponding parking spaces according to their own parking duration. That is, vehicles with stay time shorter than a will use the short-term parking spaces and other vehicles only use the normal parking spaces. At the same time, the goal of maximizing the utilization of parking spaces requires that the total time of vehicles stop in ordinary parking spaces is equal to the total time vehicles stop in short-term parking spaces. Vehicles are ranked by different parking durations, and the function of parking durations related to vehicles number is $M(x)$. The total number of vehicles needs to enter the system at drop-off time is n. a is the vehicle whose dwell time divides short-term parking and long-term parking. The total parking time and the relationship between the two types of parking satisfy the following equation:

$$\int_1^a M(x)\mathrm{d}x = \int_a^n M(x)\mathrm{d}x. \quad (6)$$

According to the formula above, variable a can be calculated to help decide which type of parking spaces vehicles should stop. The ratio of ordinary parking spaces to short-term parking spaces during school hours can be calculated as follows:

$$R_{\mathrm{Ord}} = 1 - \frac{a}{n}, \quad (7)$$

$$R_{\mathrm{Short}} = \frac{a}{n}. \quad (8)$$

According to the vehicle queuing model and traffic simulation during the drop-off, the parking spaces N_1 to be allocated during the drop-off can be obtained. And according to the parking space allocation method, the number of parking spaces N_2 to be allocated during the pick-up can be obtained. By calculating the ratio R of short-term parking spaces, the number of ordinary parking spaces Q_{Ord} and the number of short-term parking spaces Q_{Short} can be calculated as follows:

$$Q_{\mathrm{Ord}} = N_2 - \frac{a}{n} * N_1, \quad (9)$$

$$Q_{\mathrm{Short}} = \frac{a}{n} * N_1. \quad (10)$$

4. Case Analysis

4.1. Simulation of Vehicle Queuing during Drop-Off Time. The vehicle arrival data fitting during drop-off of the three-day statistical data was conducted. The vehicle arrival time series is the independent variable, and the arrival time is the dependent variable. It can be obtained by data fitting analysis that the vehicle arrival time function G satisfies the Fourier series expanded in the form of a trigonometric function. The goodness of fit $R^2 = 0.9685$, which shows that the regression curve, fits the observation greatly. The fit is shown in Figure 3.

Fitting result is expanded into Fourier series in the form of a trigonometric function, and the result is as follows:

$$\begin{aligned} f(x) = a_0 &+ a_1 * \cos(x * w) + b_1 * \sin(x * w) \\ &+ a_2 * \cos(2x * w) + b_2 * \sin(2x * w) \\ &+ a_3 * \cos(3x * w) + b_3 * \sin(3x * w) \\ &+ a_4 * \cos(4xw) + b_4 * \sin(4xw), \end{aligned} \tag{11}$$

where $a_0 = -2.874 * 10^{11}$, $a_1 = 0$, $b_1 = 6.94 * 10^{12}$, $a_2 = 6.881 * 10^{11}$, $b_2 = -6.897 * 10^{12}$, $a_3 = -5.227 * 10^{11}$, $b_3 = 2.925 * 10^{12}$, $a_4 = 1.22 * 10^{11}$, $b_4 = -4.805 * 10^{11}$, and $w = 0.0004012$.

The function takes the data of 300 vehicles from 0 to 299 as the arrival time of each vehicle. And the average vehicle parking duration of 907 vehicles in the three days is 1.78 minutes. A negative exponential distribution with $\mu = 1.78$ is used to randomly generate parking duration for 300 vehicles. The number of parking spaces N is given in advance, and vehicles are created in sequence according to the arrival time. According to the availability of parking spaces, queuing, parking, and leaving behavior of the vehicles are simulated. During the simulation, the number of vehicles in the system is continuously recorded. Taking the number of parking spaces from 35 to 39 as an example, the variation of the vehicle number in the system is shown in Figure 4.

It can be found that, as the number of parking spaces increases, the peak value of the vehicle number in the system shows a downward trend. Due to the randomness of the parking duration, situations with different parking spaces from 21 to 40 are repeatedly simulated for 1000 times. The trend of the average peak vehicle number under different parking spaces is shown in Figure 5.

It can be seen from Figure 5 that the peak vehicle number in 1000 times simulation during pick-up is 41.17 when there are 21 parking spaces. And the peak number of the vehicle constantly decreases along with the increasing number of parking places. When the number of parking spaces is 27, the peak number of the vehicle is 33.9, and the descent speed is significantly reduced. According to the given model, the greater the number of parking spaces is, the lower the probability that the number of vehicles in the system exceeding the number of parking spaces. When the number of parking space reaches a certain number, the peak number of vehicles in the system tends to be stable. From Figure 5, it can be found the peak vehicle number in the system keeps 33 or 34 when the number of parking spaces is no less than 30. In order to seek the minimum number of parking space n to make L_q and $\widetilde{W}_q$ within acceptable range and the number of parking spaces reach the minimum to improve parking space utilization, based on these findings, it can be concluded that the number of parking spaces required during pick-up is 33.

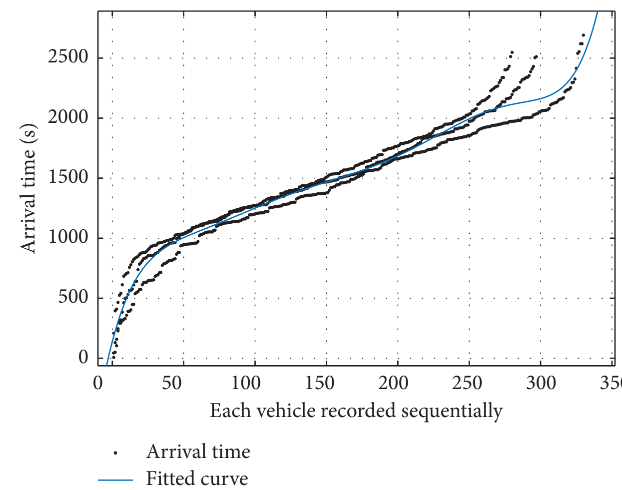

FIGURE 3: Three days' vehicle arrival time fitting graph.

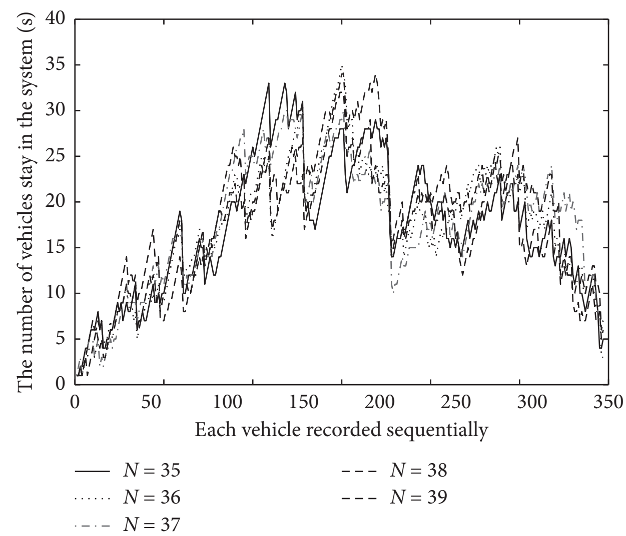

FIGURE 4: The simulation results with the number of parking ranging from 35 to 39.

4.2. Parking Spaces Demanding during Pick-Up Time. For the traffic data at the same period of 3 days, 1 minute was used as the interval to count vehicles entering and leaving the system. The cumulative number of vehicles in the system from 15:00–16:20 during pick-up is shown in Figure 6.

It can be seen from Figure 6 that the peak number of vehicles during the pick-up is 137. The duration that the number of vehicles in the system is more than 90 is short. According to the given model, if the acceptable traffic congestion time t_d is 5 minutes, it can be calculated that $N_2 = 101$. So, the number of parking spaces required during pick-up time is 101.

4.3. Short-Term Parking Spaces Setting. The dwelling time of the vehicles during drop-off is recorded in order from large to small. The quadratic exponential function is used to fit these data by MATLAB. Function $M(t)$ is obtained where the independent variable x is the vehicle number recorded in

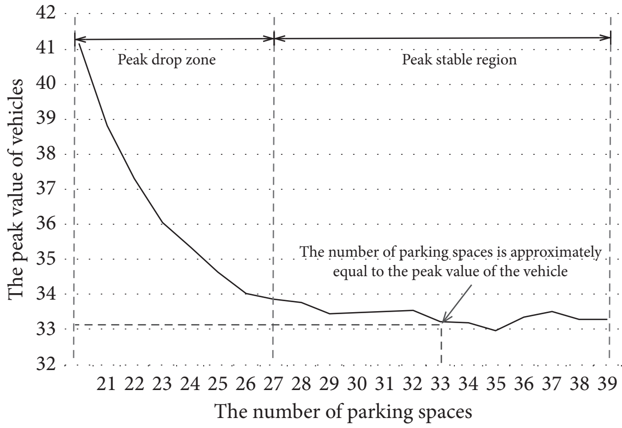

Figure 5: The average peak vehicle number under different parking spaces.

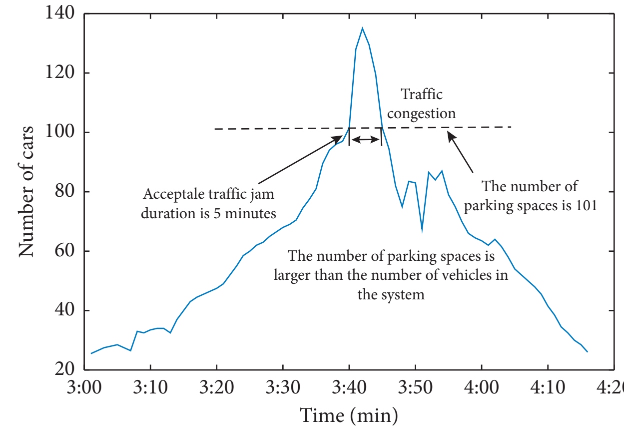

Figure 6: The number of vehicles during the pick-up.

order from large to small, and the dependent variable $M(t)$ is the dwell time of the xth vehicle. The fitting function is shown as follows:

$$M(x) = ae^{bx} + ce^{dx}, \tag{12}$$

where $a = 7.585$, $b = 0.003874$ $c = 1.499 * 10^{-12}$, and $d = 0.0371$. The goodness of fit is $R^2 = 0.995$, close to 1, which means the regression curve fits the data. Then, the following equation should be solved.

$$\int_1^h \left(ae^{bx} + ce^{dx}\right)\mathrm{d}x = \int_h^{907} \left(ae^{bx} + ce^{dx}\right)\mathrm{d}x. \tag{13}$$

By using MATLAB, the variable h for selecting the two types of parking space can be solved. The result shows h is 791, and the corresponding dwell time is 179 seconds. That is, 87 percent of the parking spaces during drop-off should be short-term parking spaces, and the other 13 percent should be ordinary parking spaces.

According to the previous analysis, the total parking spaces demand during the drop-off is 33, and then short-term parking spaces should be 29 and ordinary parking spaces should be 4. Considering the analysis on parking spaces demanding during pick-up time, the total parking spaces around the primary school should be at least 101, of which 72 parking spaces are ordinary type and 29 parking spaces are short-term type.

5. Model Checking

According to research results of the model, it is assumed that a total of 101 parking spaces are allocated, including 29 short-term parking spaces and 72 ordinary parking spaces. The other two days' traffic data were used to verify the given research result. According to the statistic result of the other two days' traffic data, the proportion of vehicles with dwell time less than 179 seconds are 85.23% and 86.38%, respectively. During drop-off, the cumulative number of vehicles with dwell time less than 179 seconds in the system is shown in Figure 7.

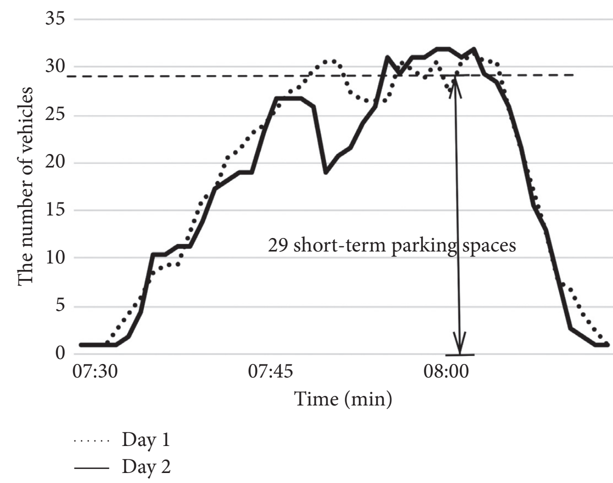

Figure 7: The cumulative number of vehicles with dwell time less than 179 seconds.

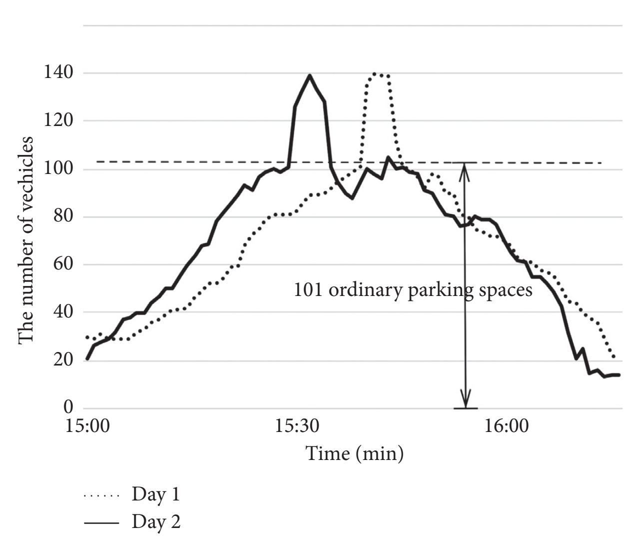

Figure 8: The cumulative number of vehicles during the pick-up.

It can be seen from Figure 7 that the peak vehicle number with dwell time less than 179 is 31 vehicles in the two days. 29 short-term parking spaces are enough to meet the parking needs with short dwell time. Furthermore, the cumulative number of vehicles during pick-up is shown in Figure 8. It is easy to find that the duration time with more than 101 vehicles in the system is less than 5 minutes in the two days. The duration of traffic congestion is acceptable if 101 parking spaces are assigned to the school.

6. Conclusions

The paper studies the allocation method of parking spaces near primary and secondary education buildings. During drop-off, a G/M/N-based vehicle queuing model was proposed, and MATLAB was used to simulate the model to obtain the optimal scale of parking spaces. During pick-up, a given acceptable congestion time was used as a constraint to obtain the optimal scale of parking spaces. Considering the difference of parking duration of different vehicles, the model for allocating short-term parking spaces is constructed to optimize the utilization of parking spaces. To verify the proposed model in this paper, some actual traffic data from one primary school is collected. It is proved that the proposed model can analyze both the overall scale of parking spaces of for primary and secondary education buildings and the short-term parking spaces for parking demand with short dwell time. Furthermore, the analysis results can well meet the actual parking demand.

Along with the stream of this study, several elements of future research can be identified. (1) The actual traffic data are only collected from one school. More different schools should be used to verify the validity of the conclusion in the future. (2) The scale variation of parking spaces may lead to traffic attraction. This possible influence is not yet considered in the proposed model, and deserves to be further studied.

Data Availability

The data used to support the findings of this study can be obtained in "Baidu Web Disk" (https://pan.baidu.com/s/1ipwb6b-0WSazQjcIiz2HTA), and the code is "r3nv."

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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